CHAPTER 1
INTRODUCTION

1.1. BACKGROUND

Logistics may be defined as "the provision of goods and services from a supply point to a demand point"[201]. A complete logistics system covers the entire process of moving raw materials and input requirements from suppliers to plants, the conversion of the inputs into products at certain plants, the movement of the products to various warehouses or depots, and the eventual delivery of these products to the final customers. The distribution activities of a firm comprise all movements and storage of goods "downstream" from the plants. The last step in these movements (from distribution centers to customers), which may be called local transportation or delivery, is the most costly link of the distribution chain[135]. For this step to be carried out effectively, the firm must approach the planning and execution of its transportation activities in a rational manner in full view of the economies involved.

Effective distribution management presents a variety of decision-making problems at all three levels of strategic, tactical, and operational planning. Decisions relating to the location of facilities (plants, warehouses, or depots) may be viewed as strategic, while the problems of fleet size and mix determination could be termed tactical. Finally, on the operational level, various decisions concerning the routing and scheduling of vehicles and the staffing of such vehicles with crews require ongoing attention on a day-to-day basis. Clearly, the distinction between strategic, tactical, and operational planning should not be interpreted too rigidly, especially in view of the close interaction between the decisions involved. For example, the location of plants, warehouses, and fleet depots has a major impact on the distribution activities of a firm. Generally, the locations of all facilities are required as input data for planning the local transportation activities. Conversely, such siting decisions rely upon distribution or transportation costs between various geographic locations. The resulting combined production/inventory/distribution problem is shared by many firms in the private sector. The work by Geoffrion and Graves[268] is a good instance of a modeling effort that integrates the various aspects of the combined problem just described. For private firms, a detailed study of the distribution problem involves an explicit consideration of vehicle routing and fleet size and mix issues. These issues, in turn, are analyzed in conjunction with a plan for routing and scheduling the vehicles assigned to the distribution function. Thus, whereas highly aggregate location models traditionally employ rough estimates of the point-to-point transportation costs, location decisions for a firm with an in-house fleet must incorporate the level of detail associated with the economies of customer deliveries or collections. At this level the need for effective and flexible planning tools for routing and scheduling activities becomes quite evident.

In addition to the location of depots, effective planning of deliveries generally requires inputs concerning a variety of other "exogenous" decisions which include:

—districting the size of the area served out of each depot
—fleet size and mix available at each depot
—allocation of delivery activities between the in-house fleet and an outside common carrier
—customer service levels: frequency of deliveries to each customer.

Given the decisions listed above, the firm may then route and schedule its vehicles to perform the assigned functions at minimal cost. This step requires an optimum-seeking algorithm to identify the best configuration of routes and schedules which brings us to the main focus of this report. The main goal of the preceding discussion is to emphasize the impact of various "higher level" decisions on the final design of a routing and scheduling system. Furthermore, it should be remarked that recent advances in routing and scheduling procedures now hold the promise of integrating the planning of outbound transportation with some of the higher-level decisions mentioned before. For example, Christofides[135] describes a successful integration of delivery decisions with issues relating to customer service and fleet size determination. Fisher *et al.*[221] discuss another successful implemention where the outbound transportation and inventory management functions are integrated.

The issues raised in the preceding discussion are in no way limited to the private sector. For example, the location/distribution problem arises in various public services. In solid waste collection, communities must decide on the number and locations of disposal facilities and determine how the refuse ought to be transported to such locations. Similarly, in mass transit systems, one must determine the locations of garages to house buses so as to allow for a cost-effective servicing of existing bus lines by the fleet of vehicles. Similar issues arise in the location of emergency units, e.g. fire or police stations in a city (see Larson and Odoni[421] and Beltrami[62]).

The importance of distribution problems is evident from the magnitude of the associated distribution costs. Surveys by Kearny[379] show that physical distribution costs account for about 16% of the sales value of an item. Of this, about a fourth is due to downstream distribution of the final product from distribution centers to customers. The same study estimates the annual distribution costs at approximately $400 billion in the United States and some £15 billion in the United Kingdom. The following examples should impart some flavor of the distribution and transportation-related costs in both private and public sectors.

—Fisher and Jaikumar[220] estimate deliveries by trucks during 1975 to account for 69 billion miles of travel and some $5.7 billion worth of fuel.

—In 1979, the transportation costs for a major U.S. pharmaceutical company totalled over $1.2 million a month.

—The annual distribution costs of one of the twenty largest distributors of propane gas in the United States exceeded $6 million in 1979.

—The monthly operating cost for the aircraft and crews of a major freight airline was over $2.5 million in 1979. Large passenger-carrying airlines in the U.S. have comparable operating costs.

—In the mid-1970's, the annual budget of New York City's Department of Sanitation was about $200 million of which roughly 80% covered the wages and benefits of some 11,000 sanitation workers. The major activity of this department was refuse collection.

—In 1974, New York State budgeted between 150 and 200 million dollars annually for school bus transportation. This budget was supplemented by funds for school transportation from local school districts. In 1974, the annual cost of leasing a school bus for 6 hours/day was roughly $11,000. This figure has increased to about $20,000 for 1981.

As these examples indicate, the costs associated with operating vehicles and crews for delivery purposes form an important component of total distribution costs. Consequently small percentage savings in these expenses could result in substantial total savings over a number of years. The significance of detecting these potential savings has become increasingly apparent due to escalating fuel costs, higher capital costs for the replacement of vehicles, growing salaries for crews, etc. These factors have caused a larger percentage of the total operating costs of an organization to be devoted to routing and scheduling activities. The use of analytic routing and scheduling models and techniques can be instrumental in realizing the savings alluded to before. When coupled with an effective management information system, the routing and scheduling methodology can assume a crucial role in the operational planning of distribution activities. Furthermore, these methods can be used as tactical planning tools, for instance, in the evaluation of how future demand patterns would impact the delivery system and the proposed fleet composition. This allows one to plan for potential capital savings by restructuring the availability of various resources.

Although cost minimization is the primary objective of most routing and scheduling problems, other objectives may assume primary importance especially in the context of service operations in the public sector. Safety and convenience are two other objectives that certain problems may focus on. For example, in school bus routing and scheduling, the objective is to minimize the total number of student-minutes on the bus since this measure is perceived to be highly correlated with safety. Consequently, school bus routes and schedules must be designed to ensure that no student stays on the bus for an excessive length of time. Given this objective, it may be desirable to service the stops with the largest number of students near the end of a route (close to the school), even if the resulting routes turn out to be relatively long and hence costly in terms of crew and vehicle costs. Another example where safety is a major consideration concerns the routing and scheduling of street sweepers and vehicles for household refuse collection (see Section 4.4, Bodin and Kursh[88] and McBride[474]). Here, one attempts to minimize the number of u-turns or left-hand turns since such turns are dangerous to make on major arteries. This objective is usually achieved at the expense of increased "deadheading" (defined to be nonproductive travel time) resulting in additional costs. As a result, a set of routes and schedules that seem to be attractive when the safety criterion is invoked, may well deviate substantially from the minimal cost configuration of routes in the absence of safety considerations. This deviation from the minimum cost may be viewed as a premium paid for safety. In dial-a-ride services for the elderly or the handicapped, once again the primary objective is to provide convenient service to all users. Consequently an appropriate objective is to minimize the total inconvenience of all users. Measures of inconvenience must then be identified in a quantifiable form to allow the problem to be viewed as an optimization problem. A discussion of such measures may be found in Section 4.9.

In certain routing and scheduling problems, the choice of an appropriate objective function or measure of performance constitutes an important modeling question in its own right. For instance in school bus transportation for the State of New York, the aid formula according to which the state subsidizes school districts complicates the choice of an objective. The state subsidizes up to 90% of most transportation costs while the school district is responsible for developing the routes and schedules of school buses (see Bodin and Berman[82]). Consequently the school district may easily depart from an overall minimal cost solution in order to minimize its own costs possibly at the expense of the state.

To take an example from the private sector, the design of delivery routes and schedules for a commercial firm's distribution activities must take account of inventory costs and customer service levels as well as purely distribution-related costs (crews and vehicles). Generally, these are conflicting objectives as a decrease in distribution costs usually implies an increase in inventory costs and a lower customer service level (see Christofides[135]). An interesting example of using a composite objective function that incorporates both inventory and routing costs may be found in Fisher *et al.*[221].

1.2. CLASSIFICATION OF ROUTING AND SCHEDULING PROBLEMS

The basic output of all routing and scheduling systems is essentially the same: For each vehicle or driver, a route and a schedule is provided. Generally, the route specifies the sequence of locations to be visited and the schedule identifies the times at which the activities at these locations are to be carried out. An example of this output is displayed in Fig. 1.1 for a dial-a-ride subscriber service in Baltimore, Maryland (described more fully in Section 4.9). The information in Fig. 1.1 completely specifies the nature and order of the tasks that have to be performed.

One objective of this paper is to provide a classification of various routing and scheduling problems. These problems are first divided into the three groups: (1) routing, (2) scheduling, and (3) routing and scheduling, and later subdivided according to a more detailed classification scheme.

Since various routing and scheduling systems share the same type of output, one must distinguish between these problems based on other problem characteristics and the assumptions that surround a given problem. Take, for example, the problem of delivering goods to various locations by a fleet of vehicles based at a depot. If there are no *a priori* restrictions on delivery times and if all goods can be delivered within a short period of time (say, three hours), then one may ignore temporal considerations to obtain a pure vehicle routing problem (an example for this scenario may be newspaper delivery). If, however, the times of visits to various locations are of primary importance, as in the case where customers only accept deliveries within a given span of time, then temporal characteristics may no longer be ignored and in fact the time restrictions guide the routing and scheduling activities. The resulting problem must then be attacked by methods that are different from those used for the earlier example. In general, the

Location	Activity	Customer Number	Time	Number on Vehicle
3405 Powhatan Avenue	Pick-up	25	12:00	1
2901 Strickland Street	Deliver	25	12:15	0
861 Park Ave. -Waxter Center	Pick-up	26	13:45	1
5604 Woodmont Avenue	Deliver	26	14:10	0
1111 E. Coldspring Lane	Pick-up	5	14:35	1
1701 Bloomingdale Road	Deliver	5	14:58	0
Levindale Avenue	Pick-up	29	15:15	1
6514 Eberle Drive	Deliver	29	15:27	0
1111 E. Coldspring Lane	Pick-up	12	16:00	1
1818 N. Collington Avenue	Deliver	12	16:12	0
1111 E. Coldspring Lane	Pick-up	51	16:24	1
1111 E. Coldspring Lane	Pick-up	48	16:24	2
1111 E. Coldspring Lane	Pick-up	47	16:24	3
1111 E. Coldspring Lane	Pick-up	50	16:24	4
1111 E. Coldspring Lane	Pick-up	49	16:24	5
1600 Mount Royal Avenue	Deliver	47	16:56	4
301 McMechen Street	Deliver	48	17:00	3
1032 W. Franklin Street	Deliver	50	17:12	2
2013 Madison Avenue	Deliver	51	17:23	1
708 North Gilmore	Deliver	49	17:34	0
1600 Charles-Penn Station	Pick-up	63	17:47	1
500 Calvert Street	Pick-up	34	18:01	2
500 Calvert Street	Pick-up	33	18:01	3
1616 Melby Court	Deliver	63	18:46	2
3006 Pinewood Avenue	Deliver	33	18:56	1
6239 Northwood Drive	Deliver	34	19:10	0

Fig. 1.1. Sample route and schedule for vehicle

characteristics and restrictions associated with various service activities lead to different categories of problems that require different modeling assumptions. The following discussion focuses on certain frequently-encountered problem characteristics.

A routing and scheduling system deals with a collection of entities requiring service. A *precedence relationship* between two such entities states that one of these is to be serviced before the other. For example, in dial-a-ride systems, a passenger must be picked up at one location and delivered to his destination location. The pick-up activity must precede the delivery activity thus establishing a precedence relation between these two activities. An entity requiring service is said to have a *definitive time for service* if the starting and ending times of the service are specified in advance. In a mass transit system, for instance, a timetable provides definitive times for the start and termination of each trip in the timetable, requiring the vehicle to cover the entire trip without interruption. Somewhat less restrictive time constraints may be imposed by *time windows.* A *two-sided window* $[s, t]$ restricts the service time of an entity to fall into a specified interval of time from s to t. For example, a warehouse may only accept deliveries between 9 : 00 AM and 4 : 00 PM. The time window associated with such deliveries is then 9 :00 AM to 4 : 00 PM. Clearly, if the two endpoints of a time window coincide, a definitive time for service will result. Thus an activity that must be carried out at 9 : 00 AM may receive a time window with $s = t = 9:00$ AM. A *one-sided* time window is of the form $[-\infty, t]$ or $[s, \infty]$. The first window requires that the service be provided *before* time t and the second restricts the service to occur *after* time s. Some dial-a-ride systems (see Section 4.9) have one-sided time windows.

Table 1.1. Characteristics of routing and scheduling problems.

CHARACTERISTICS	POSSIBLE OPTIONS
1. Size of Available Fleet	one vehicle multiple vehicles
2. Type of Available Fleet	homogeneous (only one vehicle type) heterogeneous (multiple vehicle types) special vehicle types (compartmentalized, etc.)
3. Housing of Vehicles	single depot (domicile) multiple depots
4. Nature of Demands	deterministic (known) demands stochastic demand requirements partial satisfaction of demand allowed
5. Location of Demands	at nodes (not necessarily all) on arcs (" " ") mixed
6. Underlying Network	undirected directed mixed euclidean
7. Vehicle Capacity Restrictions	imposed (all the same) imposed (different vehicle capacities) not imposed (unlimited capacity)
8. Maximum Route Times	imposed (same for all routes) imposed (different for different routes). not imposed
9. Operations	pickups only drop-offs (deliveries) only mixed (pick ups and deliveries) split deliveries (allowed or disallowed)
10. Costs	variable or routing costs fixed operating or vehicle acquisition costs common carrier costs (for unserviced demands)
11. Objectives	minimize total routing costs minimize sum of fixed and variable costs minimize number of vehicles required maximize utility function based on service or convenience. maximize utility function based on customer priorities

If the entities to be serviced have no temporal restrictions and there are no precedence relations among these entities, then we have a *routing* problem. Routing problems form the subject of Chapter 2. If each entity has a definitive service time, then a *scheduling* problem results. Scheduling problems are discussed in Chapter 3. Otherwise, one is dealing with a *combined routing and scheduling* problem. Generally, routing and scheduling problems involve both precedence relations and time windows. This class of problems is studied in Chapter 4.

In addition to the division of problems into the three major classes given above, one may further characterize routing and scheduling problems through a more detailed list of their characteristics. Table 1.1 presents some broad characteristics in which various routing and scheduling problems may differ and is based on a similar enumeration by Bodin and Golden[85].

The entries in Table 1.1 may be used to provide a quick description of a routing or scheduling problem to be studied. Taking different combinations of options within various characteristics on the l.h.s. of Table 1.1 results in a large number of possible problem settings. Two examples, couched in the terminology of the table, should illustrate this classification approach. Consider a problem with a single domicile, a single vehicle of unlimited capacity, deterministic (i.e. known) demands that must all be serviced at nodes of an undirected network, no restriction on route time, and the objective of minimizing routing costs alone in the form of total distance traveled. This problem is the celebrated traveling salesman problem discussed extensively in Chapter 2. To take another example, the Dilworth problem (see Ford and Fulkerson[227]) may be characterized as a multi-vehicle scheduling problem with a homogeneous fleet, deterministic demands at all nodes of a directed network, unrestricted vehicle capacity and route time, fixed vehicle acquisition costs, and the objective is to minimize the number of vehicles required to service all demands. This problem is utilized in the UCOST procedure for scheduling vehicles in mass transit systems (see Section 3.3 and Bodin, Rosenfield, and Kydes[92]). By varying the choices for the costs and the objective, that is, options within categories 10 and 11, other versions of this scheduling problem for mass transit systems may be obtained. For example, minimizing routing costs resulted in the RUCUS formulation of mass transit vehicle scheduling[69] whereas the minimization of the sum of fixed and variable costs yields the formulation of Bodin and Dial[83].

1.3. COMPLEXITY AND COMPUTATIONAL BURDEN OF ROUTING AND SCHEDULING PROBLEMS

An important consideration in the formulation and solution of routing and scheduling problems is the computational burden associated with various solution techniques for these problems. The computational burden of solving a given problem clearly increases as the size of the problem becomes larger. The nature of this growth in computation time as a function of problem size is an issue of both theoretical and practical interest. If this growth is too rapid, the computational burden soon becomes prohibitive even for moderate problem sizes thereby limiting the applicability of a solution technique in a realistic environment where the problems encountered are typically large scale.

Most routing and scheduling problems of interest may be formulated as network problems. A measure of the problem size is then available in the number of nodes (and possibly arcs) of the resulting network. Table 1.2 lists a number of standard network problems that are frequently encountered in solving routing and scheduling problems. To impart some feel for the problem sizes that are currently manageable, the table provides the network size that can be solved within a few minutes on a computer comparable to the UNIVAC 1108. Here, an upper limit of 5000 nodes is chosen since larger problems may run into storage difficulties. Thus, when 5000 appears in the table, it should not necessarily be taken to mean that larger problem sizes are impossible to handle. Although the actual numbers in Table 1.2 (first presented in Golden *et al.*[298]) are subject to frequent changes as advances in designing computational procedures are made, they still serve to indicate the "relative" burden of solving the problems listed.

While Table 1.2 represents a practical way of comparing the computational burden of various network problems, this issue may also be approached by means of a theoretical schema that involves the notion of "polynomially-bounded" algorithms. A polynomially-bounded algorithm for a problem is a procedure whose computational burden increases only polynomi-

Table 1.2. Selected network problems and algorithms

Problem Name	Heuristic Algorithm: Size Handled Easily	Heuristic Algorithm: References	Exact Algorithm: Size Handled Easily	Exact Algorithm: References
Shortest Path from s to t	NN		5000	Golden & Ball [297]
Shortest Path from s to all other nodes	NN		5000	Denardo & Fox [173], Golden [288], Pape [538], Gilsinn & Witzgall [277], Dial et al. [186]
Shortest Paths Between All Nodes	NN		500	Kelton & Law [382]
K Shortest Paths	NN		500 (K<5)	Shier [597],[598]
Minimal Spanning Tree	NN		5000	Kershenbaum & Van Slyke [385]
Capacitated Minimal Spanning Tree!	1000	Kershenbaum [384]	40	Chandy & Lo [117]
Transportation Problem	NN		5000	Mulvey [498], Bradley et al. [100], Glover et al. [280]
Max Flow	NN		5000	Cheung [122], Glover, et al. [280]
Min Cost Flow	NN		5000	Bradley et al. [100], Barr et al. [41]
Matching	NN		500	Cunningham & Marsh [157], Derigs [174], Derigs & Kazakidis [177]
Traveling Salesman Problem!	1000	Webb [667], Golden & Bodin [299], Golden et al. [302]	100	Miliotis [480],[481], Held & Karp [337], [338], Padberg & Hong [528], Balas & Christofides [29]
Vehicle Routing Problem !	750	Golden et al. [306]	30	Christofides et al. [143],[144]

! indicates problem is NP-hard
NN indicates heuristic or approximate algorithms are not necessary

ally with problem size in the worst case. The class of all problems for which polynomially-bounded algorithms are known to exist is denoted by P. Problems in the class P can generally be solved to optimality quite efficiently. In Table 1.2, all problems for which the symbol ! does *not* appear belong to the class P. The practical implication of obtaining a polynomially-bounded algorithm for a problem may be illustrated through a small example.

Consider two network problems, labeled A and B, both defined on a network of size N (say, the number of nodes). Assume that Problem A belongs to P and that an algorithm requiring $1000N^2$ computations in the worst case is available for solving an instance of A with size N. For Problem B, however, suppose that the best known algorithm is not polynomial and requires 2^N computations in the worst case. For $N = 15$, Problem A will be significantly more time-consuming to solve than Problem B. However, as the problem size N increases, this situation is rapidly reversed. For $N = 30$, Problem B would require over one billion computations as opposed to 900,000 computations for A (this latter number corresponds to a couple of seconds of Univac 1108 computer time). One may conclude that solving large-scale problems with an algorithm of exponential computational burden is impractical. Consequently, it is useful to obtain some theoretical insight into the class of problems for which no polynomially-bounded algorithm may be expected to exist.

In contrast to the class P, there is a large class of network and combinatorial problems for which no polynomially-bounded algorithm has yet been found. Problems in this class are called "NP-hard". Loosely speaking, this class is characterized by the property that if a polynomially-bounded algorithm exists for any particular problem in this class, then all other problems of the class are also solvable in polynomial time. As such, the class of "NP-hard" problems may be viewed as forming a hard core of problems that polynomial algorithms have not been able to penetrate so far. This suggests (but does not prove) that the effort required to solve "NP-hard" problems increases exponentially with problem size in the worst case. Although there are some problems that have not yet been classified as either NP-hard or members of the class P, most problems reviewed in this paper fall into one of these two classes. In fact, all problems with the symbol ! in Table 1.2 are NP-hard. Research into the computational complexity of various combinatorial problems has been an area of intense activity in the recent years. The interested reader may consult the works of Garey and Johnson[242], Karp[374], Lenstra and Rinnooy Kan[434], Lewis and Papadimitriou [443], Papadimitriou and Steiglitz[536], and Tarjan[642] for precise definitions, further details, and extensive classifications of NP-hard problems. In the area of routing and scheduling, Lenstra and Rinnooy Kan[436] provide a concise overview of complexity results known to date. As expected, most routing and scheduling problems of interest are NP-hard. The practical utility of the available complexity results therefore lies in identifying components or subproblems of NP-hard routing and scheduling problems that belong to the class P and thus may be used effectively in attacking the larger problem. To take one example, Section 3.5.3 describes an approach for the crew/vehicle scheduling problem (itself NP-hard) that utilizes a matching subproblem. It may be cautioned that, even among problems that belong to the class P, apparently minor changes in problem characteristics may result in radical changes in the computational complexity of the resulting problems. For example, although the Chinese Postman Problem (CPP) (defined in Section 2.9) on directed undirected networks belongs to the class P, the mixed-CPP (when a mixture of directed and undirected arcs is allowed) is NP-hard. (For a recent survey of the mixed CPP, see Minieka[487]). The Dilworth problem mentioned in the previous section is also in P as long as all vehicles are housed at the same depot. The multiple depot case, however, is NP-hard[436].

Given that most routing and scheduling problems are NP-hard, known approaches for solving these problems optimally suffer from an exponential growth in computational burden with problem size. When faced with an NP-hard problem, one frequently resorts to heuristic or approximate procedures to obtain near-optimal solutions in lieu of seeking optimal solutions. A heuristic algorithm is a procedure that uses the problem structure in a mathematical (and usually intuitive) way to provide feasible or near-optimal solutions. A heuristic is considered effective if the solutions it provides are consistently close to the optimal solution. In many cases it is possible to obtain bounds on the deviation of the heuristic solution from the optimal one in the worst case. Examples of such bounds for traveling salesman heuristics may be found in

Section 2.3. The design of effective heuristics with known average of worst-case error bounds is an active area of current research (see the survey papers by Fisher[215], Graham *et al.*[314], Klee[390], Sahni[576], and Silver *et al.*[599]). The two papers by Lenstra and Rinnooy Kan[435] and Ball and Magazine[39] discuss approximation algorithms and issues in the design of effective heuristics in the specific context of routing and scheduling problems.

1.4. SET PARTITIONING AND SET COVERING PROBLEMS

Many routing and scheduling problems can be formulated as instances of a special class of zero-one integer programs known as set partitioning or set covering problems. Basically, a set covering problem involves a given 0–1 matrix with costs attached to all columns. The objective is to choose a minimum-cost collection of columns such that the number of 1's appearing in each row of the selected columns is at least one. If this number is required to be exactly one, the set partitioning problem results. In this case the rows are partitioned into a number of subsets each "covered" by exactly one selected column. Set covering and set partitioning problems have been studied extensively over the last two decades (see Balas and Padberg[31], Garfinkel and Nemhauser[246], [247]).

The idea of using set partitioning or covering in routing and scheduling problems dates back to at least 1961[610]. In these applications the rows represent the entities that require service whereas the columns of the integer program correspond to ways in which service may be provided to a subset of the demand entities. For example, in vehicle routing from a central depot for delivery to a set of customers, the rows correspond to the customers and the columns stand for feasible round trips based at the depot through a given cluster of customers. The column cost would then equal the total cost associated with such a trip. By the same token, in air crew scheduling, the rows represent the flight legs to be flown and the columns enumerate possible round trips that a crew might fly (see Marsten and Shepardson[471]).

While set covering and partitioning provides a valid conceptual framework for the formulation of many routing and scheduling problems, its practical utility may be limited if the

Table 1.3. Representative applications of set covering and partitioning to routing and scheduling problems.

I. Air Crew Scheduling
Arabeyre et al. [11], Baker et al. [23], Marsten et al. [470], Marsten and Shepardson [471], Rubin [449], Spitzer [610].
II. Airline Fleet Planning
Levin [440], [441]
III. Mass Transit Crew Scheduling
Koljonen and Tamminen [396], Mitra and Welsh [490], Parker and Smith [539], Ryan and Foster [574], Ward et al. [665]
IV. Vehicle Routing and Scheduling
Balinski and Quandt [32], Cullen, Jarvis, and Ratliff [156], Dantzig and Ramser [167], Fisher, Jaikumar, and Bell [221], Foster and Ryan [228], Pierce [545].

resulting integer program is too large. In the last decade, it has been possible to solve set partitioning problems of some 150 rows and a few thousand columns optimally (see, for example, Marsten[469]). However since all columns corresponding to feasible options must be enumerated, in most cases a full-scale representation of a routing and scheduling problem as an integer program leads to problems of enormous size. Moreover, even the enumeration of all feasible columns is often a difficult and time consuming task. As a result, researchers have employed heuristics or clustering approaches for decomposing set partitioning problems (Marsten and Shepardson[471]), or have embedded these problems within an interactive computing environment (Cullen, Jarvis, and Ratliff[156]). Clearly, heuristics can also be used to provide approximate solutions to set partitioning or set covering problems—an approach that is frequently resorted to in practice (see Baker *et al.*[23]). Further discussion of the application of set partitioning and covering problems may be found in Section 3.5. A representative (but not exhaustive) list of applications in the area of routing and scheduling is presented in Table 1.3. The reader may consult this table as a guide to further references in this research.

1.5. ORGANIZATION OF THIS REPORT

We wish to conclude this chapter with a brief review of the plan of this paper. Chapters 2 and 3 focus on routing and scheduling problems respectively. The major problems of interest and the associated solution techniques are discussed. Chapter 4 describes combined routing and scheduling problems where routing and scheduling features are present simultaneously. This is an area of great potential for future research and the approaches discussed in Chapter 4 must be viewed as first steps along a relatively unexplored path. Chapter 5 deals with implementation issues for both routing and scheduling systems. The discussion in this chapter reviews the difficulties associated with data collection and explores database design issues and the options provided by manual, automated, and interactive man–machine systems. Finally, Chapter 6 presents conclusions and directions for future research. The report concludes with a comprehensive list of references on all aspects of routing and scheduling problems.

A report of this scope cannot avoid making certain disclaimers. The authors must emphasize that the main focus of this report is vehicle and crew routing and scheduling. The main routing and scheduling decisions draw upon and interact with a variety of "higher-level" and peripheral planning issues. These issues can not be fully treated if the size of this report is to be kept within reasonable limits. For instance, a variety of related problems involving location, districting, and aggregation or clustering questions are not discussed in any detail within this report.

The authors realize that even in the specific area of routing and scheduling problems, the report may have suffered from certain inadvertent omissions. In particular, some degree of provincialism is unavoidable as the authors tend to dwell upon the research best-known to them in a research area of vast scope and much recent activity. The authors wish to apologize for all potential shortcomings in advance and hope that the readers consider whatever omissions they might detect as being due to mere oversight.