Step 3. Insert node k^* in subtour between nodes i^* and j^*.

Step 4. Repeat Steps 2 and 3 until a Hamiltonian cycle is obtained.

Worst case Behavior. Unknown. However, Yang[696] has demonstrated the negative result that there does not exist a constant that bounds the ratio of "greatest angle" tour length to optimal tour length.

Number of computations. Same as for cheapest insertion. *Comments*: Norback and Love[514] seem to have first suggested this approach as well as a related procedure known as the eccentric ellipse method.

(c8) *Difference × ratio insertion* (*Or* [518])

Procedure

Same as for greatest angle insertion except that Step 2 is replaced by the following step.

Step 2′. Choose the node k^* not in the subtour and the arc (i^*, j^*) in the subtour such that the product $\{c_{i^*k^*} + c_{k^*j^*} - c_{i^*j^*}\} \times \{(c_{i^*k^*} + c_{k^*j^*})/c_{i^*j^*}\}$ is smallest possible.

Worst case behavior. Unknown

Number of computations. Same as for cheapest insertion.

(d) *Minimal spanning tree approach* (Kim [387])

Procedure.

Step 1. Find a minimal spanning tree T of G.

Step 2. Double the edges in the minimal spanning tree (MST) to obtain an Euler cycle.

Step 3. Remove polygons over the nodes with degree greater than 2 and transform the Euler cycle into a Hamiltonian cycle.

Worst case behavior.

$$\frac{\text{length of MST approach tour}}{\text{length of optimal tour}} \leq 2.$$

Number of computations. This approach requires on the order of n^2 computations.

(e) *Christofides' heuristic* Christofides[132] recently proposed the following interesting technique for solving TSP's.

Procedure

Step 1. Find a minimal spanning tree T of G

Step 2. Identify all the odd degree nodes in T. Solve a minimum cost perfect matching on the odd degree nodes using the original cost matrix. Add the branches from the matching solution to the branches already in T, obtaining an Euler cycle. In this subgraph, every node is of even degree although some nodes may have degree greater than 2.

Step 3. Remove polygons over the nodes with degree greater than 2 and transform the Euler cycle into a Hamiltonian cycle.

Worst case behavior.

$$\frac{\text{length of Christofides' tour}}{\text{length of optimal tour}} \leq 1.5.$$

Cornuejols and Nemhauser[151] have improved this bound slightly (although not asymptotically) in obtaining a tight bound for every $n \geq 3$.

Number of computations. Since the most time-consuming component of this procedure is the minimum matching segment which requires $O(n^3)$ operations, this heuristic is $O(n^3)$. In most cases, the number of odd nodes will be considerably less than n.

(f) *Nearest merger* (Rosenkrantz *et al.* [568])

The nearest merger method when applied to a TSP on n nodes constructs a sequence $S_1, \ldots, S_n$ such that each S_i is a set of $n - i + 1$ disjoint subtours covering all the nodes.