An Assignment Routing Problem

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ABSTRACT

This paper examines a routing design problem in which the objective is to assign customer demand points to days of the week in order to solve the resulting node routing problems over the entire week most effectively. The emphasis is on obtaining approximate solutions for this type of combinatorial problem. Several heuristics are developed and tested on a large scale refuse collection problem. Computational results, as well as a discussion of expected benefits, are presented.

1. INTRODUCTION

This paper addresses a routing design problem in which the objective is to assign customer demand points to days of the week in such a way that the resulting node routing problems yield a near-optimal solution. It is assumed that the demand points offer some flexibility in their assignment to days of the week; each point may require service anywhere from one to seven days per week. It is further assumed that the resulting node routing problem on each day of the week is a single depot vehicle dispatch problem whose objective is to minimize the distance or time required to service customer demand points, and to minimize the number of vehicles required.

The routing design problem arises in many delivery or collection type problems. Refuse collection, mail collection and delivery, and scheduled retail and wholesale delivery problems are examples of assignment routing problems. The literature is replete with work done on vehicle dispatch and node routing problems [2,3,5,6,8,14,15]. In some cases, applications have been reported revealing dramatic improvements in route efficiency with the application of scientific routing techniques.

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Krolak [9] has reported an analysis of a school bus routing problem which resulted in a 50% reduction in distance traveled and a decrease in buses from 13 to 9. Russell [12] rerouted a fleet of 4 industrial refuse collection vehicles to achieve a 33% reduction in distance traveled per week. Many other successful routing applications exist, however, most treat the assignment of demand points to days of the week as being fixed.

In this paper we develop practical heuristic procedures for assigning points to days of the week in order to minimize weekly travel distance (time) and to balance vehicle requirements. Computational experience on a large scale problem is reported as well as an assessment of potential benefits. The necessity of using heuristic approaches is easily seen given the size of the application and the inherent intractability of node routing problems. The vehicle dispatch problem is NP-complete as it is a generalization of the traveling salesman problem. Thus, the assignment routing problem is NP-complete.

Little work has been done on this assignment routing problem. Beltrami and Bodin [1] addressed the assignment of hoist compactor trucks in New York City. Their analysis, however, was limited to demand points specifically requiring service either 3 or 6 times per week. Recently Foster and Ryan [4] have developed a "relaxed integer" programming approach which has obtained good results for routing problems of limited size. They do not report computational experience on problems involving more than 100 demand points, however, and in order to space multiple deliveries throughout the week, their model requires an extra constraint for each disallowed day combination. It is doubtful that their approach could be applied to really large scale assignment routing problems.

2. PROBLEM DESCRIPTION

It is possible to formulate the assignment routing problem as a mixed integer programming model even though the model is computationally intractable for moderately sized problems. (The mixed integer formulation is available from the authors.) However, a verbal description of the constraints and input parameters should help clarify the exact nature of the problem.

Given n points to be assigned to a given number of days of the week (usually 6 or 7), we assume that point i requires service S_i $(1 \leq S_i \leq 7)$ times per week, and must be assigned to S_i distinct days of the week. Thus, S_i is a predetermined input parameter to the problem, and it is not a decision variable.

Other input parameters include the following:

V = number of vehicles available daily
C_k = load capacity of vehicle k
D_k = maximum distance (time) allowed for vehicle k on any route
Q_{id} = demand at node i on day d
U_d = maximum allowable load on day d
d_{ij} = distance (time) from node i to node j
P_j = set of permissible day assignments for node j.

The objective in the assignment routing problem is to assign the points to days of the week in order to facilitate the solution of the resulting node routing problem on each day. Thus, we wish to assign points in order to minimize the total distance (time) traveled per week subject to the following types of constraints:

1. The total demand of the points assigned to day i does not exceed U_i
2. No more than V vehicles are required each day
3. The demand load for each vehicle does not exceed C_k for any route
4. The total distance traveled for any vehicle does not exceed D_k for any route
5. Each point is serviced by only one vehicle on each day of the week
6. The assignment of points requiring service more than once per week satisfies certain day assignment spacings. For example, it may be undesirable to assign a point requiring service three times per week to Monday-Tuesday-Wednesday, whereas Monday-Wednesday-Friday may be acceptable.

Foster and Ryan [4] propose a different type of mixed integer model to solve a related problem. Their model limits the set of possible tours to those in a petal type configuration. This restriction does not guarantee optimality but facilitates solution efficiency. Nevertheless, the Foster and Ryan approach is limited to problems of moderate size. In this paper, we resort to heuristic methods in order to effectively solve large scale problems having more than 700 points.

3. ASSIGNMENT ROUTING APPLIED TO REFUSE COLLECTION

We test our heuristic procedures on an industrial refuse collection problem located in a large southwestern city. The problem consists of sequencing a fleet of 4 hoist compactor trucks through 490 accounts which must be serviced from 1 to 6 times per week. A pure node routing analysis of this problem has been reported previously [12]. This previous analysis assumed that each account's assignment to days of the week was fixed--as designated by management. Thus, removing this assumption will allow us to determine the advantages of an assignment routing analysis over that of a pure routing analysis.

For accounts requiring service more than once a week, we create multiple copies of that point and force these multiple copies to be assigned to different days of the week; Beltrami and Bodin [1] used this artifice in their assignment routing analysis. Creating multiple copies expands the problem size from 490 to 776 points. Table I presents the composition of the points comprising the data set.

Table 1

Composition of 776 Point Problem

No. Scheduled Pick-ups Per Week	No. Accounts	No. Points Resulting
6	6	36
5	17	85
4	3	12
3	36	108
2	107	214
1	321	321
	490	776

In large problems, where many points require service 5 or 6 times per week, the creation of multiple copies can lead to exceptionally large problems. Fortunately, the assignment of points requiring service 5 or 6 times per week is "combinatorially simpler" than the assignment of points requiring service 3 times per week in that there are fewer combinations of assignment. Nevertheless, for large expanded problems involving more than 500 points it may be necessary to effectively reduce the problem size by clustering groups of points together.

In order to facilitate computational testing, we clustered the original problem having 490 accounts and 776 expanded points into a subproblem consisting of 126 accounts and 192 total points. Points are grouped according to service frequency and are clustered only if their frequency of required service is identical. The criterion for clustering is based on proximity. The procedure is an iterative one in which points in groups that are within .1 miles are clustered together. The point nearest the centroid of the cluster assumes the combined demand load of the points clustered. The procedure is repeated for distances of .2, .3 miles and so on until the problem is reduced to an acceptable size. The assignment routing solution to the clustered problem can then be expanded to the original problem. The application of an effective traveling salesman algorithm can be used to refine the daily routes of the original problem.

The composition of the 192 point subproblem is presented in Table II. Complete details for other interested researchers are shown in Table III; the details of the 776 point problem are available from the authors.

Our analysis of the 776 point refuse collection day assignment problem is limited to a 6 day work week, Monday through Saturday. Furthermore, the spacing constraints (6) are observed only in the case of 2 and 3 day a week points. Even though a M W F or T TH S day assignment would seem most desirable for a 3 day account, management reported some M W TH or T TH F day assignments. Thus our spacing constraints accepted permutations in which no 2 or 3 day assignments could be totally consecutive. That is, at least one day must elapse (without service) between the first and last day of collection.

Table II

Composition of 192 Point Problem

No. Scheduled Pick-ups Per Week	No. Accounts	No. Points Resulting
6	1	6
5	3	15
4	1	4
3	12	36
2	22	44
1	87	87
	126	192

Table III

Details of the 192 Point Problem (126 Accounts)

Account No.	Location X	Location Y	Management Assignment	Daily Loads
1	-20.656	- 6.313	MTWThFS	54,54,54,54,54,10
2	17.781	.075	MTWThF	10,10,10,10,10
3	- 9.156	- 9.250	MTWThF	8,5,5,5,41
4	-10.469	- 6.875	MTWThF	140,15,135,15,105
5	-20.656	- 6.031	MTThS	26,26,26,26
6	- 8.156	- 4.156	MThF	20,20,20
7	-19.031	- 5.063	MWF	9,9,9
8	-14.219	-12.313	MWF	8,8,8
9	-16.000	-17.218	MWF	24, 24, 24
10	2.781	-15.688	MWF	12,12,12
11	2.563	-11.063	MWF	31,31,31
12	9.125	-13.781	MWF	10,10,10
13	9.125	-24.218	MWF	9,9,9
14	3.531	- 5.188	MWF	11,11,11
15	- 4.438	- 4.594	MWF	18,18,18
16	- 1.500	1.344	MThF	41,41,41
17	7.094	2.531	TThS	6,6,6
18	- 7.906	-11.563	MTh	35,35
19	- 6.688	- 4.719	MTh	18,18
20	3.250	-11.375	MTh	22,22
21	5.031	-12.093	MW	25,25
22	11.844	-17.406	MW	12,12
23	2.813	- 9.344	MW	17,17
24	2.375	- 5.531	MW	23,23
25	- 3.281	- 4.250	MTh	11,11
26	- 7.125	- 3.188	MTh	26,26
27	- 5.750	7.375	MTh	11,11
28	-19.500	- 7.125	MF	23,23
29	- 8.875	- 8.594	TF	21,21
30	-13.563	-13.625	TTh	11,11
31	- 5.813	-13.813	TF	15,15
32	3.781	-12.406	TF	28,28
33	- 5.688	- 4.156	TF	26,26
34	.688	1.250	TF	17,17
35	- 4.469	- .719	TF	22,22
36	-11.906	- 4.500	TTh	10,10
37	- 3.250	- 1.906	TF	33,33
38	- 2,281	2.156	TF	7,7
39	- 1.688	- 4.594	WS	21,21
40	7.563	-12.219	M	19
41	2.406	- 5.313	M	12
42	3.281	- 5.500	M	31
43	9.875	- 3.063	M	10
44	15.250	- 3.063	M	5
45	8.656	- .906	M	16
46	5.906	- 3.063	M	23
47	- 7.344	- 4.500	M	11
48	- 6.844	1.625	M	11
49	- 3.594	9.594	M	5
50	- 5.750	5.594	M	9
51	- 2.063	4.438	M	13
52	-23.156	- 5.781	T	16
53	-26.125	- 5.063	T	19
54	-14.405	- 6.625	T	8
55	-12.719	- 7.125	T	5
56	-11.281	- 8.813	T	5
57	- 5.813	-13.813	T	6
58	- 6.000	-15.750	T	11
59	- 4.219	-13.625	T	9
60	-11.125	-13.813	T	10
61	2.313	-11.250	T	10
62	2.813	-10.188	T	11
63	3.750	-10.813	T	18
64	4.156	-12.313	T	11
65	5.281	-13.188	T	11
66	5.063	-12.313	T	23
67	- .531	- 7.219	T	5
68	- .750	- 4.906	T	4
69	- 5.688	- 3.969	T	18
70	1.563	4.844	T	4
71	2.844	.750	T	8
72	7.875	- 4.938	S	12
73	- 5.344	- 2.594	T	16
74	- 5,750	- 1.563	T	10
75	-11.625	- 4.500	W	10
76	-18.688	- 7.000	W	8
77	-10.094	- 7.094	W	18
78	-24.063	-19.468	W	2
79	-15.656	-17.500	W	26
80	4.313	-11.906	W	33
81	6.344	-11.500	W	20
82	20.156	-19.250	W	3
83	11.438	- 9.344	W	9
84	1.469	- 4.969	W	7
85	13.500	1.750	Th	3
86	2.844	1.938	W	17
87	1.250	- 3.063	W	18
88	- .281	- 3.063	W	9
89	- 1.688	- 4.281	W	9
90	- 6.656	- 5.281	W	14
91	- 3.594	- 2.688	W	26
92	- 8.188	- 3.594	W	25
93	- 2.156	.188	W	4
94	- 2.375	- 4.969	Th	13
95	- 2.813	- 6.000	Th	12
96	- 4.875	- .625	Th	6
97	4.656	- 4.969	Th	8
98	4.750	- 6.00	Th	5
99	18.500	.094	Th	14
100	7.125	5.781	Th	29
101	2.813	- 2.250	Th	10
102	2.813	.250	Th	12
103	1.750	- 3.125	Th	14
104	.531	- 3.281	Th	7
105	- 1.625	- .906	Th	16
106	2.781	-17.593	Th	5
107	3.469	-11.375	Th	13
108	7.219	-13.219	Th	16
109	7.469	-14.500	Th	15
110	- 8.625	-12.000	Th	10
111	- 9.344	- .813	Th	5
112	- 6.531	- 3.969	Th	9
113	- 9.625	- 9.813	F	7
114	-14.250	-13.250	F	19
115	-14.438	-16.156	F	8
116	- 1.531	-10.625	F	6
117	3.188	-12.000	F	8
118	4.969	- 9.031	F	6
119	2.813	- 6.156	F	4
120	6.875	- .625	F	3
121	- 5.781	- 6.281	F	5
122	- 4.969	- 4.469	F	10
123	- 4.875	- 3.250	F	3
124	- 6.875	- 3.375	F	16
125	2.281	- 6.844	S	3
126	-19.093	- 6.156	S	8

Depot coordinates are (0,0) and maximum daily load capacity is 235 for each of 4 available trucks. Spacing constraints are enforced only for points requiring service 2 or 3 times per week. In these two cases assignments cannot be totally consecutive, i.e., at least one day must elapse (without service) between the first and last day of collection. Only the six days Monday - Saturday are considered.

4. APPROACHES TO THE PROBLEM

We consider three heuristics in tackling the assignment routing problem. Some of our procedures have evolved from the initial work of Beltrami and Bodin [1] in their study of hoist compactor routing in New York City. They considered two practical approaches. In the first, routes were developed and then assigned to days of the week. In the second, accounts were randomly preassigned to days of the week and routes then developed. Their analysis was limited to accounts which require service either 3 or 6 times per week; we extend their analysis to consider all weekly levels of service.

Our first heuristic is devised to obtain a feasible solution to the assignment problem, that is, satisfy constraints (1)-(6). The first heuristic attempts to generate clusters of points which are compact. The procedure begins by assigning any points requiring service 6 days per week or any fixed points requiring service on specific days. The resulting nuclei of clusters on each day of the week act as magnets in attracting other unassigned points. Classes of points are assigned sequentially in order of their frequency of service; thus, one day a week points are assigned last. Three statistics are calculated in order to determine the combination of days to which a particular point should be assigned. The three statistical measures are the average distance to the assigned nucleus of points on each day combination, the variance in this average, and the average distance to the nearest point in each nucleus on each day of the week. A point is assigned to a particular day combination if the average distance to that day combination is ten percent less than any other day combination. If the ten percent improvement does not hold, then the two day combinations having the smallest averages are compared based on the variance in their average distance statistics. If one variance measure is not at least forty percent less than the other, the tie is broken by the third statistic, the average of the shortest distance to the nuclei of the respective day combinations. The ten and forty percent decision rules were found to work well in empirical testing.

Once all points have been assigned in the initial pass, subsequent passes are employed in which each point is considered for possible reassignment. Improvement in the quality of the assignments is achieved using up to three or four additional passes. This heuristic is extremely fast and useful for generating feasible solutions and leveling work loads over the week. However, it is very sensitive to the configuration of the initial nuclei, and not very effective in reducing weekly travel distance (upon rerouting). This heuristic is used primarily to obtain a feasible starting solution for other heuristics to be discussed.

More effective heuristics must simultaneously consider the assignment and routing aspects of the problem in order to achieve significant distance reductions. A second heuristic was developed to improve the initial feasible solution of heuristic 1. The procedure is a modification of the MTOUR algorithm that Russell [13] developed for the vehicle dispatch problem with side conditions. MTOUR is a generalization of the highly successful traveling salesman heuristic of Lin and Kernighan [10]. The MTOUR algorithm generalizes the Lin and Kernighan single traveling salesman procedure to M salesmen.

To briefly describe the approach, let S denote the set of all links in a node routing problem. The vehicle routing problem with side conditions can be stated as "find from S a subset T that forms M distinct routes that satisfy all load, distance, side conditions, and minimizes the total length of the routes." The MTOUR algorithm proceeds by identifying links y_i in S-T to replace links x_i in T, the current feasible set of M routes. The link exchange process can include 2, 3 or up to k exchanges, where k is not predetermined. Open ended k exchanges are explored as long as the gain criterion $G_k = \sum_{i=1}^{k} \left\{|x_i| - |y_i|\right\} > 0$ is satisfied. The algorithm does swap links between tours, and empirical testing on vehicle dispatch problems [13] indicates that run time grows approximately as $n^{2.3}$, where n is the number of points.

In order to accommodate assignment routing problems, the MTOUR algorithm is modified to handle spacing conditions (6) as well as load and distance constraints. (MTOUR can be coded to explicitly handle a variety of side conditions.) Thus, a promising link exchange is implemented only if all side conditions will be satisfied. The algorithm requires a feasible starting solution, and feasible link exchanges are continued until no further improvement can be obtained. Even though MTOUR is very effective, it is not practical for really large scale problems where n > 300.

Our final approach is undertaken to gain efficiency in solving large scale problems. The third heuristic is a modification of the widely used Clarke and Wright savings approach [3]. The Clarke and Wright algorithm is another exchange algorithm in which links are deleted and added to existing tours at each iteration. Initially the Clarke and Wright procedure assumes that all nodes are connected directly to the depot, or home city, as in Figure 1.

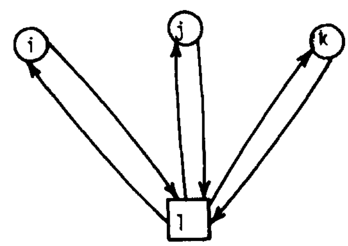


Fig. 1 Initial tours.

For every possible pair of demand nodes i and j, there is a corresponding savings s_{ij}. The savings measures the distance saved if nodes i and j are joined on the same tour. For example, if nodes i and j are connected in Figure 1, the resulting savings $s_{ij} = d_{i1} + d_{1j} - d_{ij}$. Figure 2 illustrates the resulting tours.

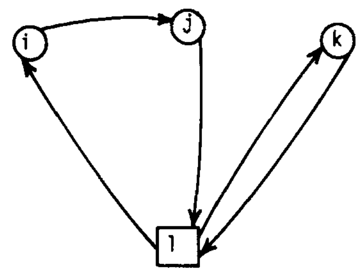


Fig. 2 Tour configuration after nodes i and j have been connected.

The original Clarke and Wright algorithm connects two nodes i and j with largest savings s_{ij} only if the capacities pertaining to load and distance are not exceeded. Our modification of the Clarke and Wright algorithm embeds an extra check for the spacing conditions (6). Thus, a savings link is implemented only if spacing conditions are satisfied. In order to ensure a feasible solution we had to initially assign each point to a day of the week in accordance with (6); this was done using the simple procedure of heuristic 1 discussed previously. Figure 3 illustrates a hypothetical starting point for the modified Clarke and Wright algorithm. In Figure 3, we assume that node i requires service 3 times per week and node j requires service twice a week. Node i is assigned to M W F and node j is assigned to T F.

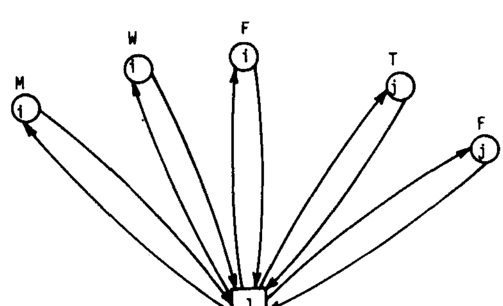


Fig. 3 Starting point for modified Clarke and Wright algorithm.

In order to solve problems as large as 776 points, some simplifications are required in the Clarke and Wright algorithm. Golden, Magnanti, and Nguyen [7] incorporated three modifications which reduce computer core requirements and computation time. Along these lines, we modified the storage requirements of the algorithm by limiting the possible links for a point to a specified L nearest neighbors. Both reduced core storage and reduced computation time have resulted. We experimented with the route shape parameter as discussed by Yellow [15] and Golden et al. [7], but found the standard value of 1 to perform best on this particular problem. Significantly reduced computation time can further be realized if the list processing and heapsort modifications devised by Golden et al. [7] are implemented. They report solving a 600 node problem in 20 seconds of IBM 370/168 C.P.U. time.

5. COMPUTATIONAL RESULTS

The results of the computational experiments are summarized in Tables IV, V, and VI. In Table IV we report the results of 3 approaches on the 192 point data. The original assignment procedure refers to management's current assignment of accounts to days of the week. The solution for this assignment is derived by using MTOUR to solve the resulting node routing problem on each of the six days of the week. The comparisons then represent the best achievable routing scheme under each assignment procedure. The solution column represents the total distance traveled by the 4 trucks during the course of a week. MTOUR proved to be the most effective and computationally most costly procedure, achieving a distance reduction of almost 11%.

Table IV

Results on 192 Point Problem

Assignment Technique	Weekly Route Distance	% Reduction	Computation Time*
Management	1070	-----	1.5
Modified Clarke & Wright	998	6.9%	.6
MTOUR (with spacing enforced)	956	10.6%	6
MTOUR (without spacing)	935	12.6%	6

*All Computation times in minutes on IBM 370/158

In Table V we present the results of the complete 776 point problem. Since it is impractical to use MTOUR on such a large problem, we reduced the 776 data points to only 192 points by clustering points together.

Table V

Results on 776 Point Problem

Assignment Technique	Weekly Route Distance	% Reduction	Computation Time (Minutes)
Management	1412	---	----
MTOUR (expanded 192 solution)	1344	4.8	6
Modified Clarke & Wright (L = 40)	1337	5.3	17.9
Modified Clarke & Wright (L = 60)	1316	6.8	20.6
Modified Clarke & Wright (L = 70)	1306	7.5	22.0

The 192 points are precisely those analyzed in Table IV. Thus, the 192 point problem is analyzed not only as a test problem in itself, but also as a clustered subproblem of the 776 point data; note, however, that the other four techniques in Table V are applied to the full 776 data points. The clustering/expansion procedure was less successful than solving the full 776 point problem with the modified Clarke and Wright algorithm improved by MTOUR. In Table V the L in parenthesis defines the limit on the number of nearest neighbors considered for linking to each point. As we would expect, larger values of L yield better solution values, but the point of diminishing returns is quickly reached taking both computer core and C.P.U. time into account. We should note that the computation times of the modified Clarke and Wright algorithm could be reduced to approximately 1 or 2 minutes by implementing the streamlined procedures suggested by Golden et al. [7].

From Tables IV and V it appears that significant but not remarkable reduction in distance traveled can be achieved by augmenting a routing analysis by reassigning points to days of the week. However, the results of Tables IV and V are data dependent. The points which require service 5 and 6 times per week tend to be widely dispersed geographically, thus limiting the potential savings that could be realized through reassigning points to different days of the week. We examined a problem with more inherent flexibility than the 776 point problem by considering the 321 sub-points that required service only 1 day per week. The results of our analysis are presented in Table VI.

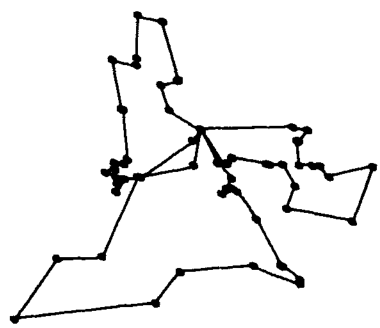
Table VI

Results on 321 Point Problem

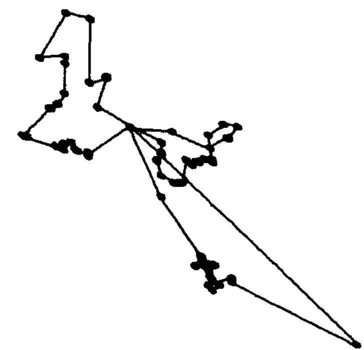
Assignment Technique	Weekly Route Distance	% Reduction	Computation Time
Management	924	----	---
Modified Clarke & Wright (L = 90)	618	33.1	2.5

For this particular problem a dramatic improvement is realized through the reassigning of points to days of the week--a 33.1% reduction in distance traveled. A visual illustration of the compactness of the resulting routes is shown in Figure 4 (a)-(k). The graphs depict the best attainable routing on management's assignment of points (the before) and the best attainable routing after the modified Clarke and Wright reassignment. Most notable is the distance reduction for Wednesday which decreased from 219.3 to 78.4. We should note, however, that management's assignment of points is conditioned by the existence of the other 169 points requiring service two to six times per week. Since we extracted the 321 point management assignment from their associated 490 point assignment scheme, it is reasonable to assume that their 321 point assignment would be slightly improved had only the 321 points been involved. Thus, the 33.1% reduction only indicates the algorithm's ability to improve a hypothetical problem with maximal flexibility.

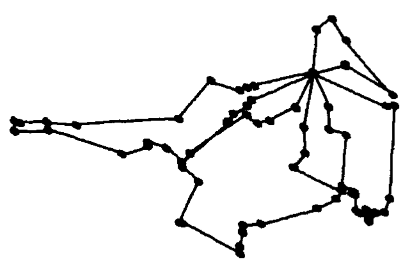
In Figure 4 (k) the six points of the Saturday "before" assignment have been absorbed by the other days of the week in the reassignment analysis. Thus, no Saturday "after" graph appears. Finally we should note that even though the distance for the entire week has been reduced, the total distance for Friday has actually increased; the algorithm minimizes total weekly distance.



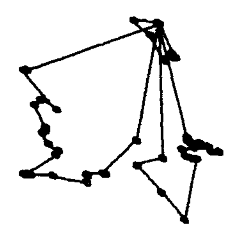
(a) Monday before - distance = 163.5



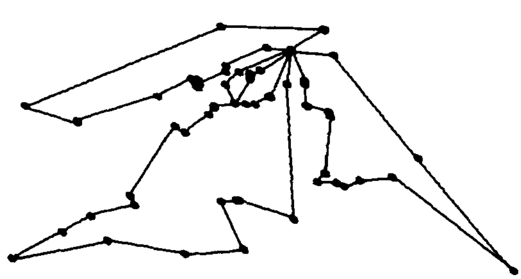
(b) Monday after - distance = 135.3



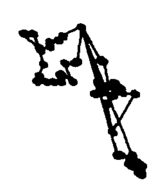
(c) Tuesday before - distance = 172.5



(d) Tuesday after - distance = 109.3

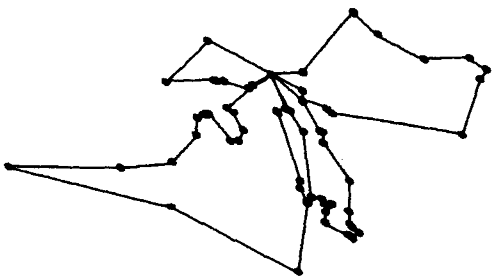


(e) Wednesday before - distance = 219.3

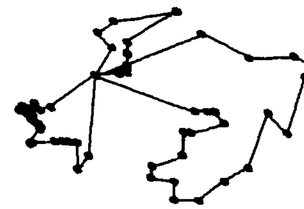


(f) Wednesday after - distance = 78.4

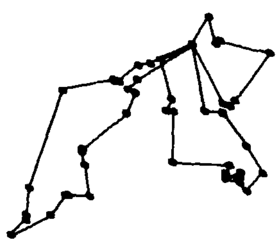
Fig. 4 Illustration of before and after routes.



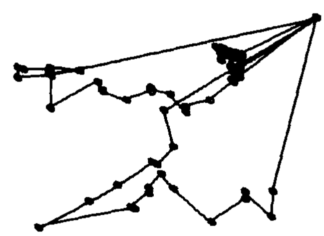
(g) Thursday before - distance = 186.2



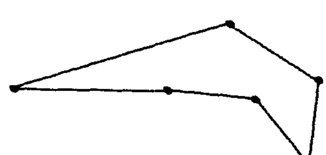
(h) Thursday after - distance = 121.4



(i) Friday before - distance = 116.9



(j) Friday after - distance = 173.7



(k) Saturday before - distance = 66.2

Fig. 4 Illustration of before and after routes.

6. CONCLUSIONS

From our analysis we can see that the benefits derived from an assignment routing analysis are data dependent. The results not only depend on the side conditions enforced but the relative composition of the points to be assigned. Problem sets with many points widely dispersed and requiring service 5 or 6 times per week have less flexibility and will benefit less from reassigning points to days of the week. Tables IV and V suggest 7 to 10% as a crude approximation to the expected improvement from a reassignment analysis applied to the real world data reported in this paper; other applications may result in more or less improvement. The 33.1% improvement in Table VI represents more of an upper bound as it reflects an ideal problem in terms of flexibility.

Both the MTOUR algorithm and the Clarke and Wright algorithm have been tested against standard vehicle dispatch problems in the literature [2,13]. MTOUR has proven to be the most accurate while the streamlined Clarke and Wright is reasonably accurate and very cost effective. As modified for the assignment routing problem, these algorithms have been effective as compared with managerial ad hoc solutions. Algorithm error is difficult to assess as no means presently exist for establishing an optimal solution with which to compare.

The solutions generated by both the modified MTOUR and Clarke and Wright algorithms are starting point dependent. Thus, for problems of moderate size, it may be advisable to generate several starting points via heuristic 1 and obtain more than one solution. The two algorithms also depend on the size of the spacing requirements sets P_j. For a given point j, a smaller set P_j will allow fewer day assignment combinations. Thus if the P_j decrease in size, we would expect a decrease in the efficiency of both algorithms as proportionally more attractive link exchanges would have to be rejected. For exceptionally large problems it may be necessary to cluster points together to effectively reduce the problem size. Another alternative would be to employ a less accurate but more efficient approach such as the sequential route building approach of Mole and Jameson [11].

The benefits of an assignment analysis cannot be measured purely in terms of percentage distance reduction. It may prove more useful in the initial design of a new set of routes. The analysis can also be used to smooth work loads over a particular planning horizon. In Table VI we not only reduced distance traveled but decreased the required number of vehicles from 4 to 3 per day. Capitalizing on recent computational advances [7], the assignment analysis can be performed simultaneously with a node routing analysis. Thus, the assignment routing algorithm can offer a low cost solution to this important distribution problem.

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