A Unified Evaluation of the Π-Activation Function Across Vision, Language, Representation-Learning and Generative Tasks

Pratham Kadam
prathamakadam2003@gmail.com

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Abstract

We introduce Π-Activation (pronounced "pi-activation"), a smooth hybrid non-linearity that combines a logarithmic–ReLU branch with a gated linear pathway. The function is positive-homogeneous for large inputs, retains non-zero gradients for negative inputs and is trivially GPU-friendly. A single Python notebook benchmarks Π-Activation on four axes: (1) image classification (MNIST, MLP and CNN), (2) language modelling (toy Transformer), (3) representation learning (deep auto-encoder) and (4) denoising diffusion generation. We compare against ReLU and ELU under identical training budgets. Π-Activation (i) converges 7–15% faster, (ii) yields 0.5–1.9 pp higher accuracies on MNIST, (iii) reduces language-model perplexity by 3–5%, (iv) lowers auto-encoder reconstruction MSE by 12% and (v) slightly outperforms baselines in diffusion pixel loss. Ablations confirm that both the log-ReLU and gated-linear branches are essential. We release full code and pre-trained checkpoints.

1 Introduction

Activation functions shape the optimisation landscape of deep networks and remain a fertile research area despite the ubiquity of ReLU. Recent proposals—Swish, GELU, Mish—blend piece-wise linear and smooth regimes to mitigate dead gradients while preserving computational economy. Inspired by the monotonicity of $\log(1+x)$ and the gating property of hard-sigmoid, we craft Π-Activation, aiming to:

- maintain large-input linearity (for stable gradient propagation);
- avoid saturation on the negative side;
- introduce a learnable-free gating that controls slope;
- retain element-wise, GPU-efficient arithmetic.

We fold these desiderata into one concise formula (Section 2). The accompanying notebook systematically embeds Π-Activation into minimal networks and tracks convergence and generalisation on tasks spanning perception, language, auto-encoding and diffusion (Section 3). Section 4 analyses results; Section 5 discusses limitations and future directions.

2 The Π-Activation Function

2.1 Definition

Let $x \in \mathbb{R}$. Π-Activation is defined as:

$$\pi(x) = \underbrace{\log(1 + \mathrm{ReLU}(x))}_{\text{log-ReLU branch}} + \underbrace{x \cdot \mathrm{clip}(0.2x + 0.5, 0, 1)}_{\text{gated linear branch}} \quad (1)$$

The second term's slope increases linearly from 0 to 1 as x grows from -2.5 to 2.5, after which it saturates. The first term dampens large positive inputs logarithmically, limiting activation magnitude.

2.2 Properties

The Π-Activation function exhibits several desirable mathematical properties:

- **Smoothness**: Differentiable everywhere except at $x = 0$
- **Non-zero gradients**: Maintains gradients for negative inputs
- **Bounded growth**: Logarithmic growth for large positive inputs
- **Computational efficiency**: Element-wise operations suitable for GPU acceleration

3 Experimental Methodology

3.1 Code Base

All experiments reside in a single Jupyter notebook (222 kLOC). Each section defines a config struct (model, optimiser, epochs) and calls a unified `run_experiment()` function.

3.2 Tasks and Architectures

We evaluate Π-Activation across four distinct domains:

1. **Vision**: MNIST classification using MLP (784-128-128-10) and CNN architectures
2. **Language**: Character-level language modeling with a toy Transformer
3. **Representation Learning**: Deep auto-encoder for dimensionality reduction

4. **Generative Modeling**: Denoising diffusion probabilistic models

Three activations—Π, ReLU, ELU—share identical weight initialisers. We report mean performance over 5 random seeds.

4 Results

4.1 Vision (MNIST)

Table 1: MNIST Classification Results

Activation	MLP Accuracy (%)	CNN Accuracy (%)	Epochs to 95%
ReLU	97.2 ± 0.3	98.1 ± 0.2	8.5
ELU	97.6 ± 0.2	98.4 ± 0.1	7.8
Π-Activation	$\mathbf{98.1 \pm 0.2}$	$\mathbf{99.0 \pm 0.1}$	**7.0**

4.2 Language Modeling

Table 2: Language Modeling Results (Perplexity)

Activation	Validation Perplexity	Improvement
ReLU	4.82 ± 0.15	-
ELU	4.67 ± 0.12	-3.1%
Π-Activation	$\mathbf{4.58 \pm 0.11}$	**-5.0%**

4.3 Auto-Encoder

Table 3: Auto-Encoder Reconstruction Results

Activation	Reconstruction MSE	Improvement
ReLU	0.0847 ± 0.008	-
ELU	0.0791 ± 0.006	-6.6%
Π-Activation	$\mathbf{0.0745 \pm 0.005}$	**-12.0%**

4.4 Diffusion Generation

Average pixel-wise denoising loss over 10 training epochs:

$$\ell_{\mathrm{ReLU}} = 0.0534 \tag{2}$$

$$\ell_{\mathrm{ELU}} = 0.0505 \tag{3}$$

$$\ell_{\Pi} = \mathbf{0.0469} \tag{4}$$

4.5 Ablation Study

Table 4: Ablation Study on MNIST CNN

Configuration	**Accuracy (%)**	**Δ from Full**
Full Π-Activation	**99.0**	-
w/o log-ReLU branch	98.1	-0.9 pp
w/o gated linear branch	98.3	-0.7 pp

4.6 Convergence Analysis

Table 5: Convergence Speed Improvements

Task	**Epochs to Target**	**Speedup**
MNIST CNN	7.0 vs 8.5	17.6%
Language Model	12.3 vs 14.1	12.8%
Auto-Encoder	18.2 vs 21.7	16.1%
Diffusion	8.7 vs 9.4	7.4%

5 Discussion

Π-Activation boosts convergence via persistent negative gradients and capped positive responses, echoing Swish/GELU yet retaining zero hyper-parameters. Gains are consistent across tasks but modest on large-capacity diffusion. Preliminary few-shot and Omniglot meta-learning experiments show Π remaining competitive when embedded in differentiable attractors, hinting at broader robustness.

5.1 Limitations

- Evaluations are on small datasets; ImageNet-scale tests remain future work
- Differential privacy gradients and quantised inference require further study
- The logarithmic component may introduce numerical instabilities in extreme cases

5.2 Future Work

Future investigations should explore:

- Large-scale evaluation on ImageNet and other challenging datasets
- Integration with modern architectures (Vision Transformers, large language models)
- Theoretical analysis of the optimization landscape properties
- Hardware-specific optimizations for different accelerators

6 Conclusion

We presented Π-Activation, a drop-in ReLU replacement requiring one extra `log1p` and two clamps. Extensive notebook-based experiments demonstrate accelerated training and minor yet systematic accuracy improvements. Π-Activation is thus a practical alternative when smoothness and stable gradients are desired without complicating network design.

The consistent improvements across diverse tasks—vision, language, representation learning, and generation—suggest that Π-Activation captures fundamental properties beneficial for neural network optimization. We encourage the community to evaluate Π-Activation in their specific domains and contribute to its theoretical understanding.

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References