German University in Cairo
Media Engineering and Technology
DMET 502 Computer Graphics, Winter 2023
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Practice assignment 4

Solid Modeling

Q 1: Suppose that you are asked to build a 3D modeling software; this software can create only one type of solid primitives, which is cubes. Every primitive instance is initially created at the origin of the 3D coordinate space with lengths equal to 1 meter, as shown in Figure 1. You will use these primitives to build a 3D model for the GUC B buildings, B1 to B7.

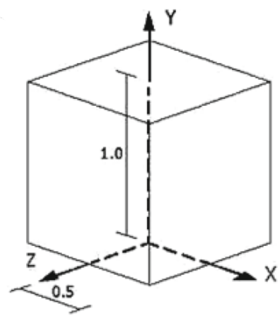


Figure 1: Unit Cube

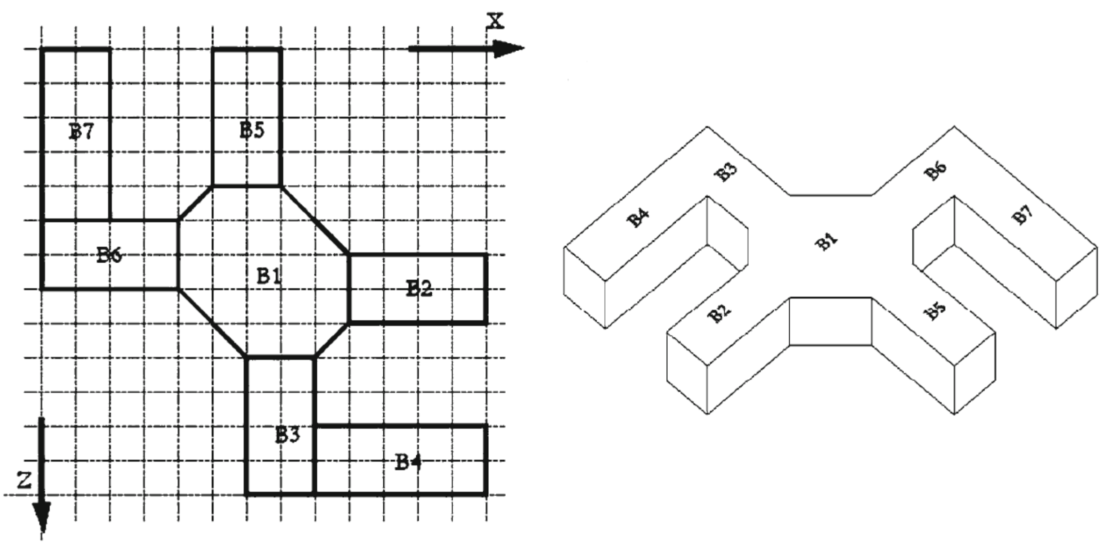


Figure 2: GUC Model

Shown in Figure 2 is an aerial view of the GUC B buildings. These buildings are placed on a grid of 10 $\times$ 10 m, where each side of each dotted square has a length of 10 meters. The x and z axes are shown on the grid, while the y-axis is pointing outwards. The origin is at the upper left corner of B7.

If the height of each building is 16 meters, construct each of these buildings as a solid, and transform and unionize them showing the CSG (constructive solid geometry) expression including the transformations. You are also required to show the CSG tree, as well as the transformation matrices for B1 and B2.

Solution: There are 7 cubes created to represent the 7 B buildings; each one of these cubes is transformed, and in the end, they will all be unionized together. Therefore, the CSG expression without the transformations is:

$trans(B_1) \cup trans(B_2) \cup trans(B_3) \cup trans(B_4) \cup trans(B_5) \cup trans(B_6) \cup trans(B_7)$

According to the aerial view in Figure 2, all buildings are scaled and translated with the exception of B_1 which is also rotated.

It should be noted that all cube objects are scaled by 16 in the y direction. The scaling in x and z for all buildings but B_1 requires no calculations as their dimensions are multiples of 10. The unit with which B_1 is scaled could be calculated using the Pythagorean equation for right-angle triangles, which is:

$hypotenuse = \sqrt{unitX^2 + unitZ^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

The width of B1 consists of 3 units, which means the cube is scaled by $30\sqrt{2}$ along the x-axis. The height of B_1 consists of 4 units, which means that it is scaled along the z-axis by $40\sqrt{2}$.

The only object rotated is B_1, and the angle of rotation could be determined through the same triangle that was used to obtain the unit value with which B_1 was scaled. Since the triangle is a right-angled isosceles triangle, the angle between the hypotenuse and each other side is $45°$. B_1 is rotated in a counterclockwise direction about the y-axis, which means that it is rotated by $+45°$ about this axis and on the zx plane.

Another solution is scaling along the x-axis by $40\sqrt{2}$ and scaling along the z-axis by $30\sqrt{2}$. Then rotation about the y-axis is applied by an angle $-45°$ on the xz plane (clockwise direction).

All objects are initially centered at the origin, which means that the center of each object in the aerial view of Figure 2 is the value in x and z with which the object is translated.

With all transformations discussed, the complete CSG expression will look as follows:

trans(B_1)= $translated(rotated(scaled(cube1, \langle 30\sqrt{2}, 16, 40\sqrt{2}\rangle), \langle 45°, 0, 1, 0\rangle), \langle 65, 0, 65\rangle)$

$\cup$

trans(B_2)= $translated(scaled(cube2, \langle 40, 16, 20\rangle), \langle 110, 0, 70\rangle)$

$\cup$

trans(B_3)= $translated(scaled(cube3, \langle 20, 16, 40\rangle), \langle 70, 0, 110\rangle)$

$\cup$

trans(B_4)$= translated(scaled(cube4, \langle 50, 16, 20\rangle), \langle 105, 0, 120\rangle)$

$\cup$

trans(B_5)$= translated(scaled(cube5, \langle 20, 16, 40\rangle), \langle 60, 0, 20\rangle)$

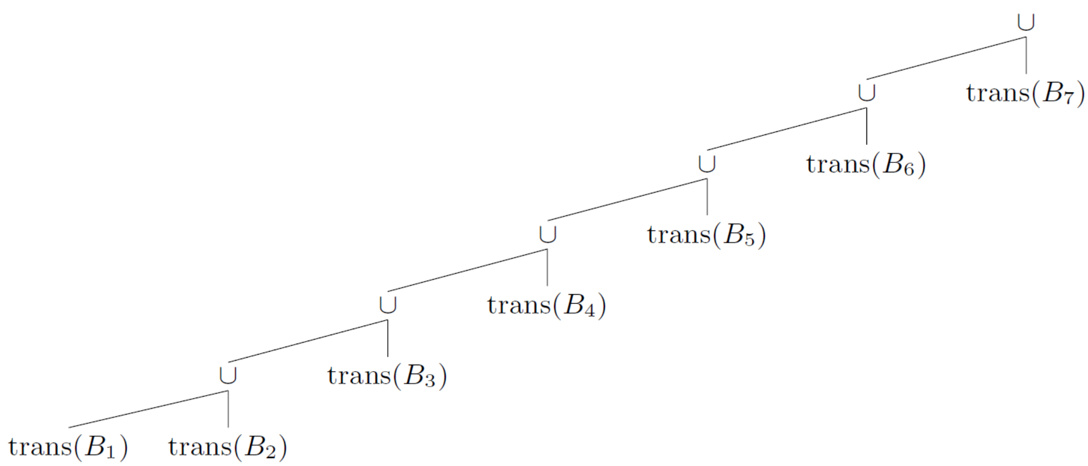
$\cup$

trans(B_6)$= translated(scaled(cube6, \langle 40, 16, 20\rangle), \langle 20, 0, 60\rangle)$

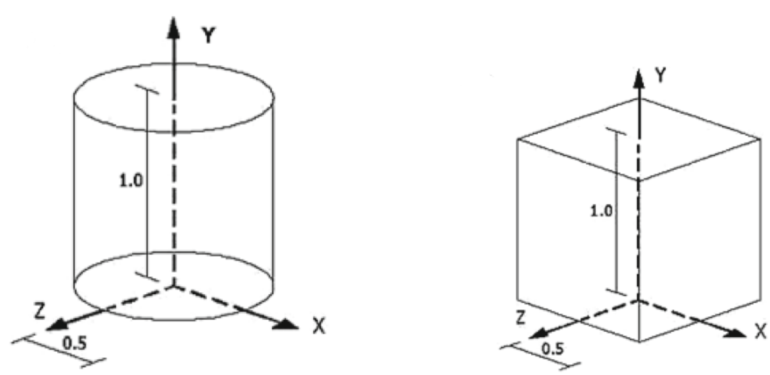
$\cup$

trans(B_7)$= translated(scaled(cube7, \langle 20, 16, 50\rangle), \langle 10, 0, 25\rangle)$

The CSG tree for the previously given CSG expression is:



Q 2: Suppose that you are asked to build a 3D modeling software. This software can create two types of solid primitives, which are cylinders and cubes. Every primitive instance is initially created at the origin of the 3D coordinate space with lengths equal to the units shown in Figure 3.



(a) (b)

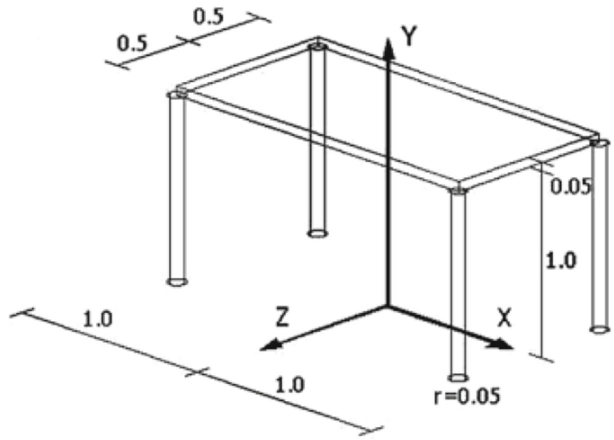


Figure 3: Solid Modeling

You will use the given primitives to build a 3D model for a table, which is also shown in Figure 3. Create the CSG expression required to represent this solid, and build the CSG tree representing this expression.

Solution: The resulting shape in Figure 3 contains 4 transformed cylinders and 1 transformed cube, all unionized together. Therefore, the CSG expression without transformation representing this solid is:

trans(box) $\cup$ trans($cylinder_1$) $\cup$ trans($cylinder_2$) $\cup$ trans($cylinder_3$) $\cup$ trans($cylinder_4$)

All shapes will be scaled and then translated, making the complete CSG expression looks as follows:

trans(box)$=$ $translated(scaled(cube, \langle 2, 0.05, 1\rangle), \langle 0, 1, 0\rangle)$

$\cup$

trans($cylinder_1$)$=$ $translated(scaled(cylinder1, \langle 0.1, 1, 0.1\rangle), \langle 1, 0, 0.5\rangle)$

$\cup$

trans($cylinder_2$)$=$ $translated(scaled(cylinder2, \langle 0.1, 1, 0.1\rangle), \langle 1, 0, -0.5\rangle)$

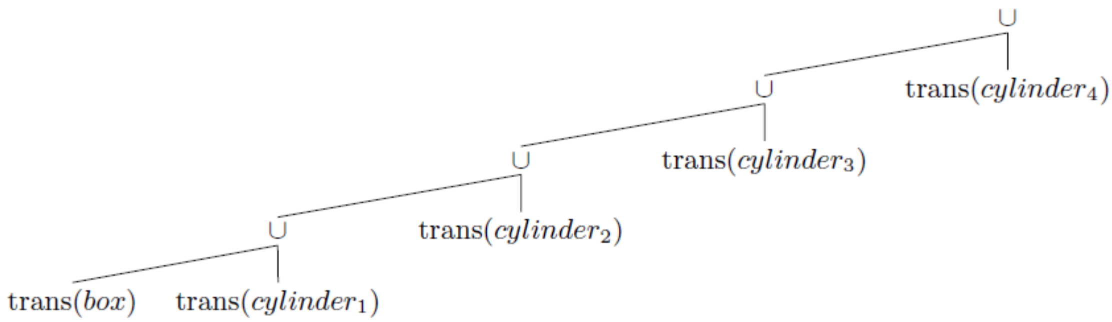
$\cup$

trans($cylinder_3$)$=$ $translated(scaled(cylinder3, \langle 0.1, 1, 0.1\rangle), \langle -1, 0, 0.5\rangle)$

$\cup$

trans($cylinder_4$)$=$ $translated(scaled(cylinder4, \langle 0.1, 1, 0.1\rangle), \langle -1, 0, -0.5\rangle)$

The CSG tree that represents the previously given CSG expression is:



Q 3: Consider the tetrahedron in Figure 4 where the vertices are indicated by the lowercase letters a, b, c, and d, while the faces are indicated by the uppercase letters A, B, C, and D. The edges in this figure are indicated by the digits 1, 2, 3, 4, 5, and 6. Write down for this solid the entries of the table of Baumgart's Winged-edge Data Structure.

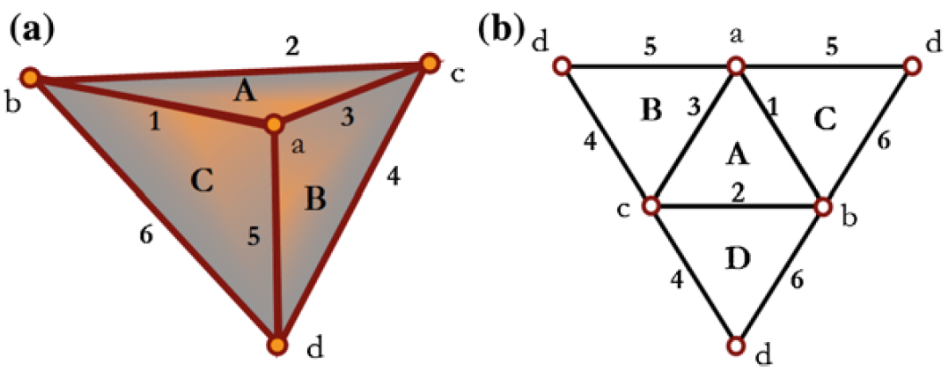


Figure 4: Tetrahedron

Solution: A clockwise direction is used to specify the vertices that start and end an edge. The start point of an edge is usually the bottom vertex of this edge, while the endpoint is its top vertex.

The start and end vertices of an edge specify which face lies to the left of this edge, and which face lies to its right. The left face of an edge is the one where the start point is at the top and the endpoint is at the bottom; this means that the right face is where the edge goes from bottom to top.

The clockwise direction of traversing edges also helps specify the predecessors and successors of these edges. The predecessor of an edge is the edge that leads to it, while the successor of this edge is the one that follows it.

Using the details given earlier, the edge table is:

Edge	Vertices		Faces		Left Traverse		Right Traverse	
Name	Start	End	Left	Right	Pred	Succ	Pred	Succ
1	a	b	C	A	6	5	3	2
2	b	c	D	A	4	6	1	3
3	c	a	B	A	5	4	2	1
4	c	d	D	B	6	2	3	5
5	a	d	B	C	4	3	1	6
6	d	b	D	C	2	4	5	1

Table 1: Table 1: Edge Table of Baumgart's Winged-edge Data Structure

Q 4: Write down the entries of the edge table for the tetrahedron in Figure 4 if it is to be represented as a wire-frame model.

Solution: In wire-frame models, each edge has a start and end vertex. The order of the start and end vertices is unimportant as directions are not taken into consideration in wire-frame models. The edge table for the shape in Figure 4 is:

Edge	Start Vertex	End Vertex
1	a	b
2	b	c
3	a	c
4	c	d
5	a	d
6	b	d

Table 2: Table 2: Wire-frame Edge Table