

R Notebook

Course Note from Week 2 - Advanced Programming in R

Functional Programming

Functional programming is a programming philosophy based on lambda calculus. Lambda calculus was created by Alonzo Church, the PhD adviser to Alan Turing who is known for his role in cracking the encryption of the Nazi's Enigma machine during World War Two. Functional programming has been a popular approach ever since it helped bring down the Third Reich.

Functional programming concentrates on four constructs: - Data (numbers, strings, etc) - Variables (function arguments) - Functions - Function Applications (evaluating functions given arguments and/or data)

By now you're used to treating variables inside of functions as data, whether they're values like numbers and strings, or they're data structures like lists and vectors. With functional programming you can also consider the possibility that you can provide a function as an argument to another function, and a function can return another function as its result. If you've used functions like `apply()` or `args()` then it's easy to imagine how functions as arguments to other functions can be used. In the case of `apply()` the provided function is applied to data, and in the case of `args()` information about the function is returned. What's rarer to see is a function that returns a function when it's evaluated. Let's look at a small example of how this can work:

```
adder_maker <- function(n){  
  function(x){  
    n + x  
  }  
}  
add2 <- adder_maker(2)  
add3 <- adder_maker(3)  
add2(5)
```

```
## [1] 7
```

```
add3(5)
```

```
## [1] 8
```

In the example above the function `adder_maker()` returns a function with no name. The function returned adds `n` to its only argument `x`.

Core Functional Programming Function in R

There are groups of functions that are essential for functional programming. In most cases they take a function and a data structure as arguments, and that function is applied to that data structure in some way. The `purrr` package contains many of these functions and we'll be using it throughout this section. Functional programming is concerned mostly with lists and vectors. I may refer to just lists or vectors, but you should know that what applies for lists generally applies for vectors and vice-versa.

`map()`

The `map` family of functions applies a function to the elements of a data structure, usually a list or a vector. The function is evaluated once for each element of the vector with the vector element as the first argument to the function. The return value is the same kind of data structure (a list or vector) but with every element replaced by the result of the function being evaluated with the corresponding element as the argument to

the function. In the purrr package the map() function returns a list, while the map_lgl(), map_chr(), and map_dbl() functions return vectors of logical values, strings, or numbers respectively. Let's take a look at a few examples:

```
require(purrr)
```

```
## Loading required package: purrr
```

```
map_chr(c(5, 4, 3, 2, 1), function(x){  
  c("one", "two", "three", "four", "five")[x]  
})
```

```
## [1] "five" "four" "three" "two" "one"
```

```
map_lgl(c(1, 2, 3, 4, 5), function(x){  
  x > 3  
})
```

```
## [1] FALSE FALSE FALSE TRUE TRUE
```

Think about evaluating each function above with just one of the arguments in the specified numeric vector, and then combining all of those function results into one vector. The map_if() function takes as its arguments a list or vector containing data, a predicate function, and then a function to be applied. A predicate function is a function that returns TRUE or FALSE for each element in the provided list or vector. In the case of map_if(): if the predicate function evaluates to TRUE, then the function is applied to the corresponding vector element, otherwise if the predicate function evaluates to FALSE then the function is not applied. The map_if() function always returns a list, so I'm piping the result of map_if() to unlist() so it looks prettier

```
map_if(1:5, function(x){  
  x %% 2 == 0  
},  
function(y){  
  y^2  
}) %>% unlist()
```

```
## [1] 1 4 3 16 5
```

Notice how only the even numbers are squared, while the odd numbers are left alone. The map_at() function only applies the provided function to elements of a vector specified by their indexes. map_at() always returns a list so like before I'm piping the result to unlist():

```
map_at(seq(100, 500, 100), c(1, 3, 5), function(x){  
  x - 10  
}) %>% unlist()
```

```
## [1] 90 200 290 400 490
```

Like we expected to happen the provided function is only applied to the first, third, and fifth element of the vector provided. In each of the examples above we have only been mapping a function over one data structure, however you can map a function over two data structures with the map2() family of functions. The first two arguments should be two vectors of the same length, followed by a function which will be evaluated with an element of the first vector as the first argument and an element of the second vector as the second argument. For example:

```
map2_chr(letters, 1:26, paste)
```

```
## [1] "a 1" "b 2" "c 3" "d 4" "e 5" "f 6" "g 7" "h 8" "i 9" "j 10"  
## [11] "k 11" "l 12" "m 13" "n 14" "o 15" "p 16" "q 17" "r 18" "s 19" "t 20"  
## [21] "u 21" "v 22" "w 23" "x 24" "y 25" "z 26"
```

The `pmap()` family of functions is similar to `map2()`, however instead of mapping across two vectors or lists, you can map across any number of lists. The `list` argument is a list of lists that the function will map over, followed by the function that will be applied:

```
pmap_chr(list(
  list(1, 2, 3),
  list("one", "two", "three"),
  list("uno", "dos", "tres")
), paste)
```

```
## [1] "1 one uno"      "2 two dos"       "3 three tres"
```

Mapping is a powerful technique for thinking about how to apply computational operations to your data.

`reduce()`

List or vector reduction iteratively combines the first element of a vector with the second element of a vector, then that combined result is combined with the third element of the vector, and so on until the end of the vector is reached. The function to be applied should take at least two arguments. Where mapping returns a vector or a list, reducing should return a single value. Some examples using `reduce()` are illustrated below:

```
reduce(c(1, 3, 5, 7), function(x, y){
  message("x is ", x)
  message("y is ", y)
  message("")
  x + y
})
```

```
## x is 1
## y is 3
##
## x is 4
## y is 5
##
## x is 9
## y is 7
##
## [1] 16
```

On the first iteration `x` has the value 1 and `y` has the value 3, then the two values are combined (they're added together). On the second iteration `x` has the value of the result from the first iteration (4) and `y` has the value of the third element in the provided numeric vector (5). This process is repeated for each iteration. Here's a similar example using string data:

```
reduce(letters[1:4], function(x, y){
  message("x is ", x)
  message("y is ", y)
  message("")
  paste0(x, y)
})
```

```
## x is a
```

```
## y is b
##
## x is ab
## y is c
##
## x is abc
## y is d
##
## [1] "abcd"
```

By default `reduce()` starts with the first element of a vector and then the second element and so on. In contrast the `reduce_right()` function starts with the last element of a vector and then proceeds to the second to last element of a vector and so on:

```
reduce_right(letters[1:4], function(x, y){
  message("x is ", x)
  message("y is ", y)
  message("")
  paste0(x, y)
})
```

```
## x is d
## y is c
##
## x is dc
## y is b
##
## x is dcb
## y is a
##
## [1] "dcba"
```

search()

You can search for specific elements of a vector using the `contains()` and `detect()` functions. `contains()` will return `TRUE` if a specified element is present in a vector, otherwise it returns `FALSE`:

```
contains(letters, "a")
```

```
## [1] TRUE
```

```
contains(letters, "A")
```

```
## [1] FALSE
```

The `detect()` function takes a vector and a predicate function as arguments and it returns the first element of the vector for which the predicate function returns `TRUE`:

```
detect(20:40, function(x){
  x > 22 && x %% 2 == 0
})
```

```
## [1] 24
```

The `detect_index()` function takes the same arguments, however it returns the index of the provided vector which contains the first element that satisfies the predicate function:

```
detect_index(20:40, function(x){
  x > 22 && x %% 2 == 0
})
```

```
## [1] 5
```

Filter

The group of functions that includes `keep()`, `discard()`, `every()`, and `some()` are known as filter functions. Each of these functions takes a vector and a predicate function. For `keep()` only the elements of the vector that satisfy the predicate function are returned while all other elements are removed:

```
keep(1:20, function(x){
  x %% 2 == 0
})
```

```
## [1] 2 4 6 8 10 12 14 16 18 20
```

The `discard()` function works similarly, it only returns elements that don't satisfy the predicate function:

```
discard(1:20, function(x){
  x %% 2 == 0
})
```

```
## [1] 1 3 5 7 9 11 13 15 17 19
```

The `every()` function returns TRUE only if every element in the vector satisfies the predicate function, while the `some()` function returns TRUE if at least one element in the vector satisfies the predicate function:

```
every(1:20, function(x){
  x %% 2 == 0
})
```

```
## [1] FALSE
```

```
some(1:20, function(x){
  x %% 2 == 0
})
```

```
## [1] TRUE
```

Compose

Finally, the `compose()` function combines any number of functions into one function:

```
n_unique <- compose(length, unique)
# The composition above is the same as:
# n_unique <- function(x){
#   length(unique(x))
# }
rep(1:5, 1:5)
```

```
## [1] 1 2 2 3 3 3 4 4 4 4 5 5 5 5 5
n_unique(rep(1:5, 1:5))

## [1] 5
```

Partial Application

Partial application of functions can allow functions to behave a little like data structures. Using the `partial()` function from the `purrr` package you can specify some of the arguments of a function, and then `partial()` will return a function that only takes the unspecified arguments. Let's take a look at a simple example:

```
require(purrr)
mult_three_n <- function(x, y, z){
  x * y * z
}
mult_by_15 <- partial(mult_three_n, x = 3, y = 5)
mult_by_15(z = 4)

## [1] 60
```

By using partial application you can bind some data to the arguments of a function before using that function elsewhere.

Side Effects

Side effects of functions occur whenever a function interacts with the “outside world” – reading or writing data, printing to the console, and displaying a graph are all side effects. The results of side effects are one of the main motivations for writing code in the first place! Side effects can be tricky to handle though, since the order in which functions with side effects are executed often matters and there are variables that are external to the program (the relative location of some data). If you want to evaluate a function across a data structure you should use the `walk()` function from `purrr`.

Here's a simple example:

Recursion

Recursion is very powerful tool, both mentally and in software development, for solving problems.

Recursive functions have two main parts: a few easy to solve problems called “base cases,” and then a case for more complicated problems where the function is called inside of itself. The central philosophy of recursive programming is that problems can be broken down into simpler parts, and then combining those simple answers results in the answer to a complex problem. Imagine you wanted to write a function that adds together all of the numbers in a vector. You could of course accomplish this with a loop:

```
vector_sum_loop <- function(v){
  result <- 0
  for(i in v){
    result <- result + i
  }
  result
}
vector_sum_loop(c(5, 40, 91))

## [1] 136
```

You could also think about how to solve this problem recursively.

First ask yourself:

- what's the base case of finding the sum of a vector? If the vector only contains one element, then the sum is just the value of that element.
- In the more complex case the vector has more than one element. We can remove the first element of the vector, but then what should we do with the rest of the vector?
- Thankfully we have a function for computing the sum of all of the elements of a vector because we're writing that function right now!

So we'll add the value of the first element of the vector to whatever the cumulative sum is of the rest of the vector. The resulting function is illustrated below:

```
vector_sum_rec <- function(v){  
  if(length(v) == 1){  
    v  
  } else {  
    v[1] + vector_sum_rec(v[-1])  
  }  
}
```

```
vector_sum_rec(c(5, 40, 91))
```

```
## [1] 136
```

Exercise “The Fibonnaci Sequence”

Another useful exercise for thinking about applications for recursion is computing the Fibonacci sequence.

The Fibonacci sequence is a sequence of integers that starts: 0, 1, 1, 2, 3, 5, 8 where each proceeding integer is the sum of the previous two integers. This fits into a recursive mental framework very nicely since each subsequent number depends on the previous two numbers.

Let's write a function to compute the nth digit of the Fibonacci sequence such that the first number in the sequence is 0, the second number is 1, and then all proceeding numbers are the sum of the n - 1 and the n - 2 Fibonacci number. It is immediately evident that there are three base cases: n must be greater than 0.

- When n is equal to 1, return 0.
- When n is equal to 2, return 1.

And then the recursive case: Otherwise return the sum of the n - 1 Fibonacci number and the n - 2 Fibonacci number.

Let's turn those words into code:

```
fib <- function(n){  
  stopifnot(n > 0)  
  if(n == 1){  
    0  
  } else if(n == 2){  
    1  
  } else {  
    fib(n - 1) + fib(n - 2)  
  }  
}
```

```
map_dbl(1:12, fib)
```

```
## [1] 0 1 1 2 3 5 8 13 21 34 55 89
```

Looks like it's working well! There is one optimization that we could apply here which comes up in recursive programming often. When you execute the function `fib(6)`, within that function you'll `execute` `fib(5)` and `fib(4)`. Then within the execution of `fib(5)`, `fib(4)` will be executed again.

An illustration of this phenomenon is below:

Insert Picture here

Figure 2.1: Memoization of `fib()` function

This duplication of computation slows down your program significantly as you calculate larger numbers in the Fibonacci sequence. Thankfully you can use a technique called **memoization** in order to speed this computation up.

Memoization stores the value of each calculated Fibonacci number in table so that once a number is calculated you can look it up instead of needing to recalculate it!

Below is an example of a function that can calculate the first 25 Fibonacci numbers.

- First we'll create a very simple table which is just a vector containing 0, 1, and then 23 NAs.
- The `fib_mem()` function will check if the number is in the table, and if it is then it is returned. Otherwise the Fibonacci number is recursively calculated and stored in the table.

Notice that we're using the **complex assignment operator** `<<-` in order to modify the table outside the scope of the function.

You'll learn more about the complex operator in the section titled Expressions & Environments.

```
fib_tbl <- c(0, 1, rep(NA, 23))
fib_mem <- function(n){
  stopifnot(n > 0)

  if(!is.na(fib_tbl[n])){
    fib_tbl[n]
  } else {
    fib_tbl[n - 1] <<- fib_mem(n - 1)
    fib_tbl[n - 2] <<- fib_mem(n - 2)
    fib_tbl[n - 1] + fib_tbl[n - 2]
  }
}
map_dbl(1:12, fib_mem)
```

```
## [1] 0 1 1 2 3 5 8 13 21 34 55 89
```

It works! But is it any faster than the original `fib()`?

Below I'm going to use the `microbenchmark` package in order to assess whether `fib()` or `fib_mem()` is faster:

```
require(purrr)
require(microbenchmark)
```

```
## Loading required package: microbenchmark
```

```
require(tidyr)
```

```
## Loading required package: tidyr
```

```
require(magrittr)
```

```
## Loading required package: magrittr
```

```
##
```

```
## Attaching package: 'magrittr'
```



```

## The following object is masked from 'package:tidyr':
##
##   extract
## The following object is masked from 'package:purrr':
##
##   set_names
require(dplyr)

## Loading required package: dplyr
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:purrr':
##
##   contains, order_by
## The following objects are masked from 'package:stats':
##
##   filter, lag
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union
fib_data <- map(1:10, function(x){microbenchmark(fib(x), times = 100)$time})
names(fib_data) <- paste0(letters[1:10], 1:10)
fib_data <- as.data.frame(fib_data)
fib_data %<>%
  gather(num, time) %>%
  group_by(num) %>%
  summarise(med_time = median(time))
memo_data <- map(1:10, function(x){microbenchmark(fib_mem(x))$time})
names(memo_data) <- paste0(letters[1:10], 1:10)
memo_data <- as.data.frame(memo_data)
memo_data %<>%
  gather(num, time) %>%
  group_by(num) %>%
  summarise(med_time = median(time))
plot(1:10, fib_data$med_time, xlab = "Fibonacci Number", ylab = "Median Time
(Nanoseconds)",
     pch = 18, bty = "n", xaxt = "n", yaxt = "n")
axis(1, at = 1:10)
axis(2, at = seq(0, 350000, by = 50000))
points(1:10 + .1, memo_data$med_time, col = "blue", pch = 18)
legend(1, 300000, c("Not Memorized", "Memoized"), pch = 18,
      col = c("black", "blue"), bty = "n", cex = 1, y.intersp = 1.5)

```

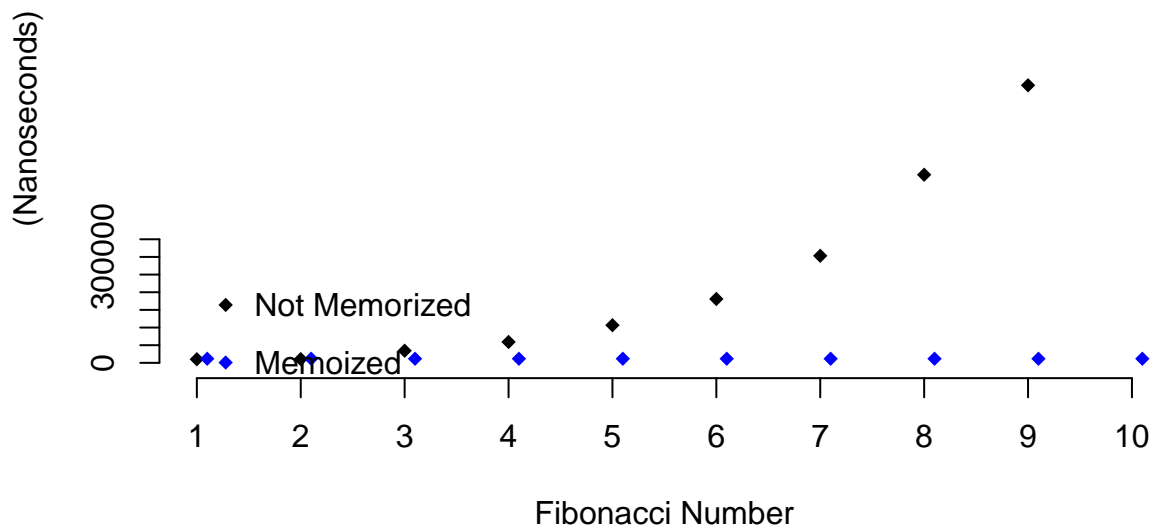


Figure 2.2: Speed comparison of memoization As you can see as higher Fibonacci numbers are calculated the time it takes to calculate a number with `fib()` grows exponentially, while the time it takes to do the same task with `fib_mem()` stays constant.

Summary

- Functional programming is based on lambda calculus.
- This approach concentrates on data, variables, functions, and function applications.
- It's possible for functions to be able to return other functions.
- The core functional programming concepts can be summarized in the following categories: map, reduce, search, filter, and compose.
- Partial application of functions allows functions to be used like data structures.
- Side effects are difficult to debug although they motivate a huge fraction of computer programming.
- The most important part of understanding recursion is understanding recursion.