Deep Learning Explained

Module 2: Logistic Regression

Sayan D. Pathak, Ph.D., Principal ML Scientist, Microsoft
Roland Fernandez, Senior Researcher, Microsoft

Module Outline

Application:

OCR with MNIST data

Model:

Logistic Regression

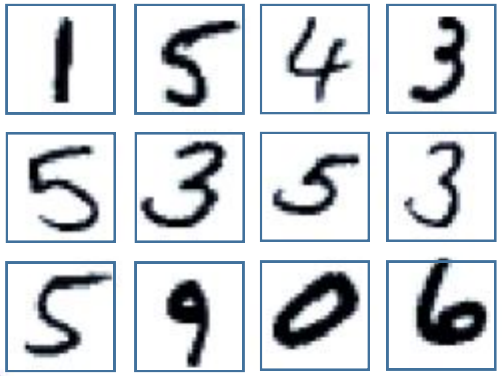
Concepts:

Loss, Minibatch

Train-Test-Predict workflow

MNIST Handwritten Digits (OCR)

Handwritten Digits

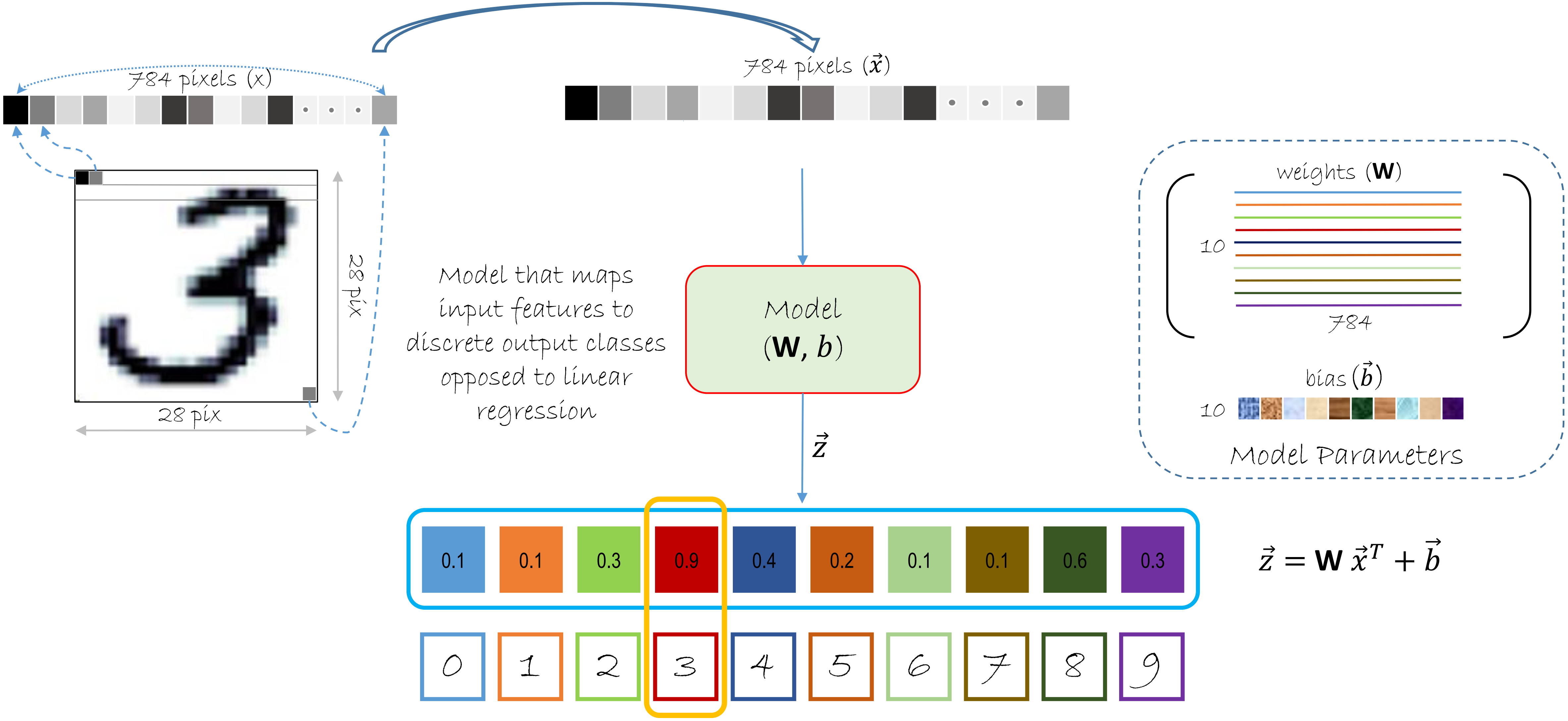


$$\left\{\begin{matrix} 1 & 5 & 4 & 3 \\ 5 & 3 & 5 & 3 \\ 5 & 9 & 0 & 6 \end{matrix}\right\}$$

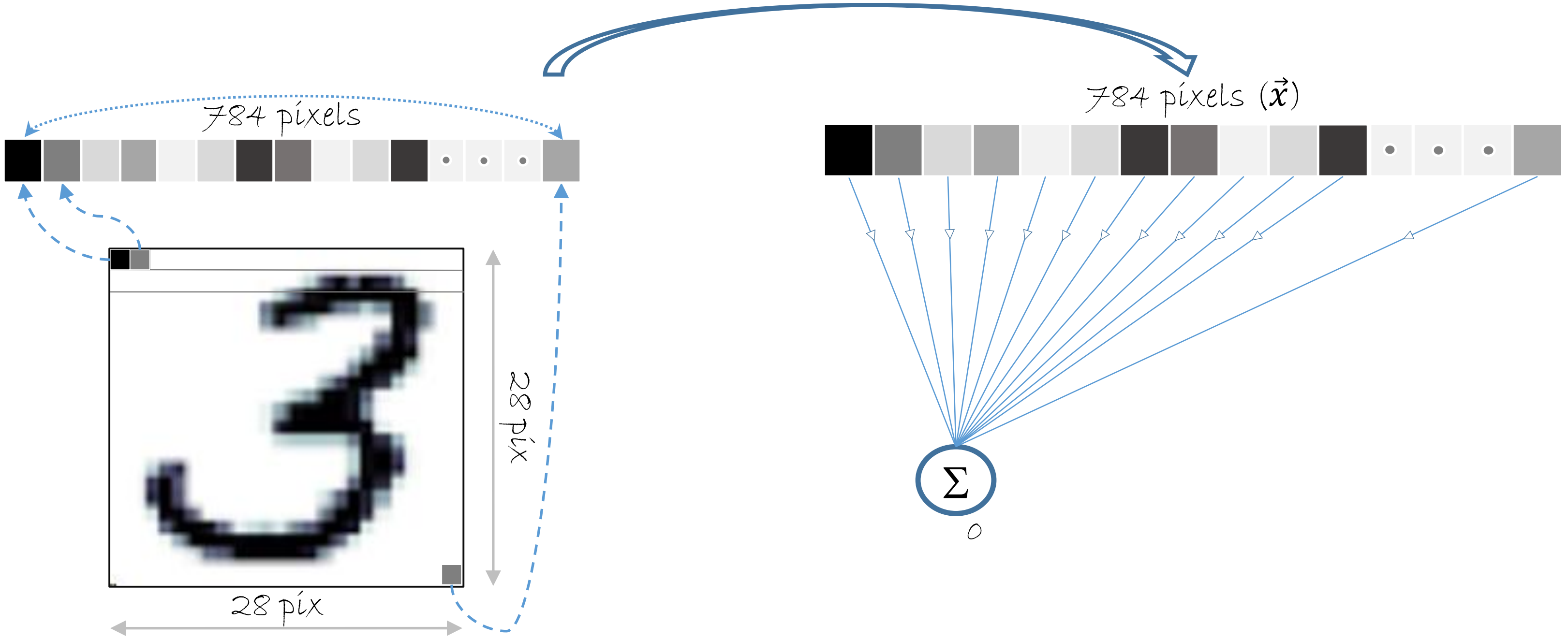
Corresponding Labels

- Data set of hand written digits (0-9) with
  - ✓60,000 training images
  - ✓10,000 test images
- Each image is: 28 x 28 pixels

Logistic Regression



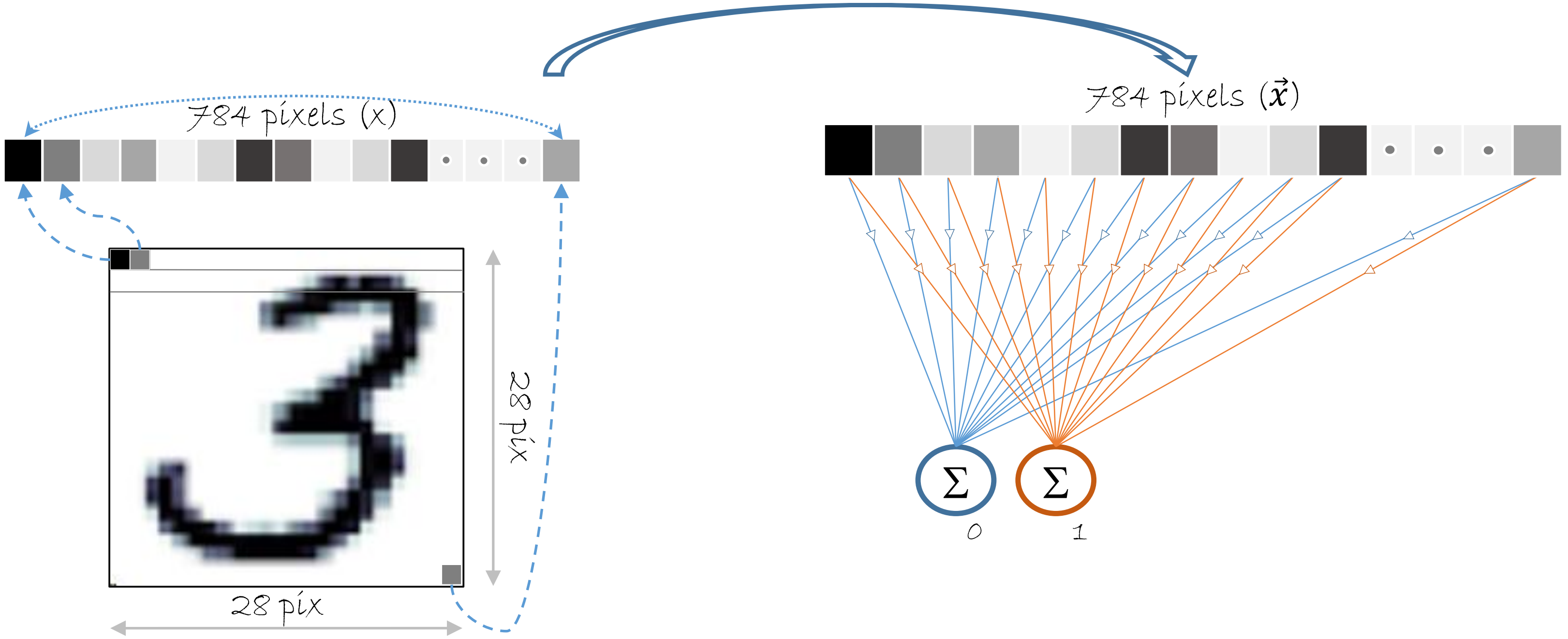
Logistic Regression



$\vec{w}_0$

$$\Sigma = \text{Sum}\,(\underbrace{\text{weights}}_{784} \times \underbrace{\text{pixels}}_{784}) = \vec{w}_0 \cdot \vec{x}^T$$

Logistic Regression

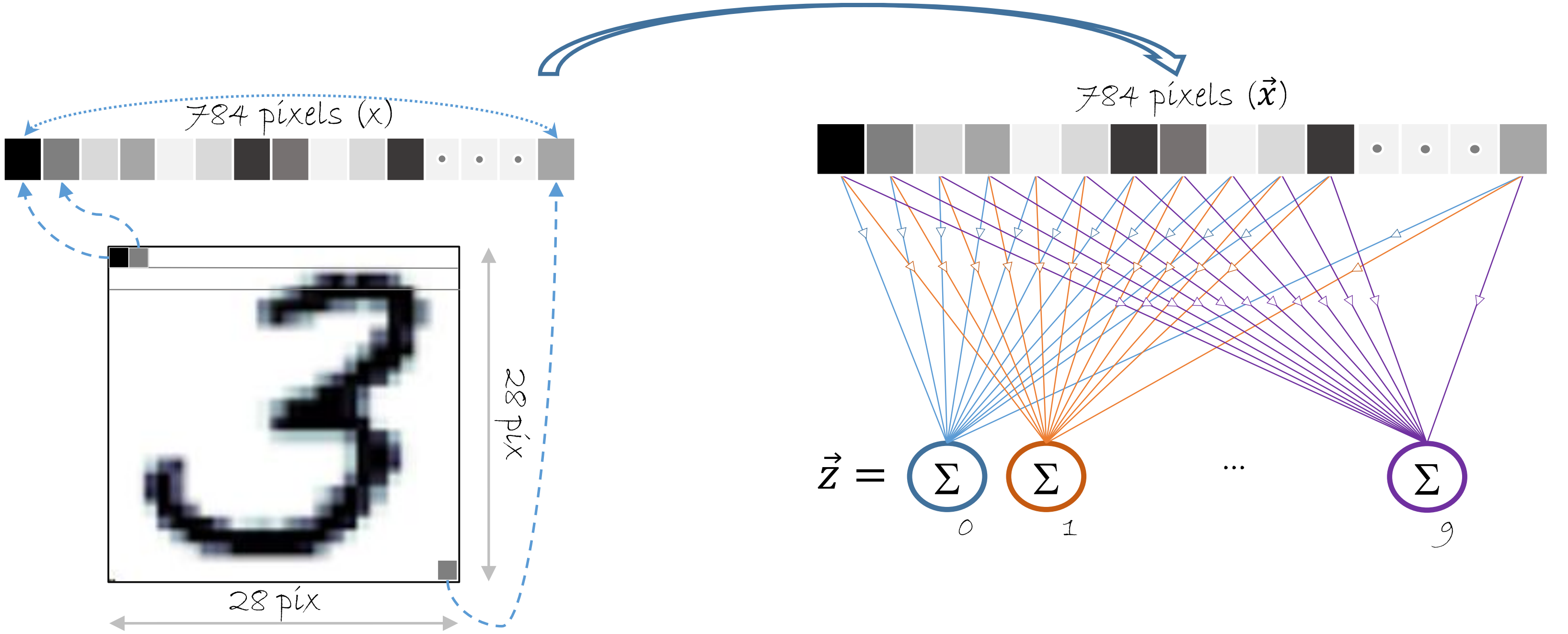


$\vec{w}_1$

$\Sigma = \text{Sum}\,(\underset{784}{\text{weights}} \times \underset{784}{\text{pixels}}) = \vec{w}_0 \cdot \vec{x}^T$

$\Sigma = \text{Sum}\,(\underset{784}{\text{weights}} \times \underset{784}{\text{pixels}}) = \vec{w}_1 \cdot \vec{x}^T$

Logistic Regression



Σ = Sum ($\underset{784}{\text{weights}}$ x $\underset{784}{\text{pixels}}$) = $\vec{w}_0 \cdot \vec{x}^T$

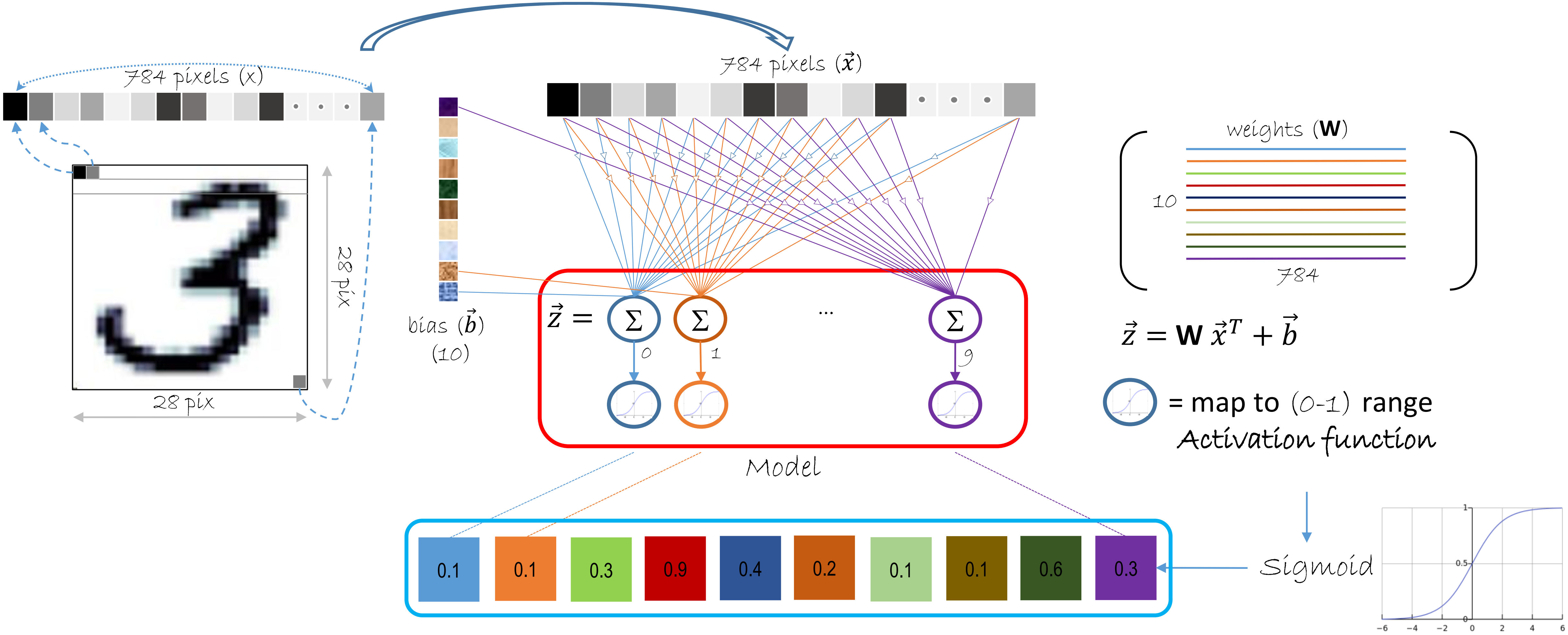
Σ = Sum ($\underset{784}{\text{weights}}$ x $\underset{784}{\text{pixels}}$) = $\vec{w}_1 \cdot \vec{x}^T$

$\vdots$

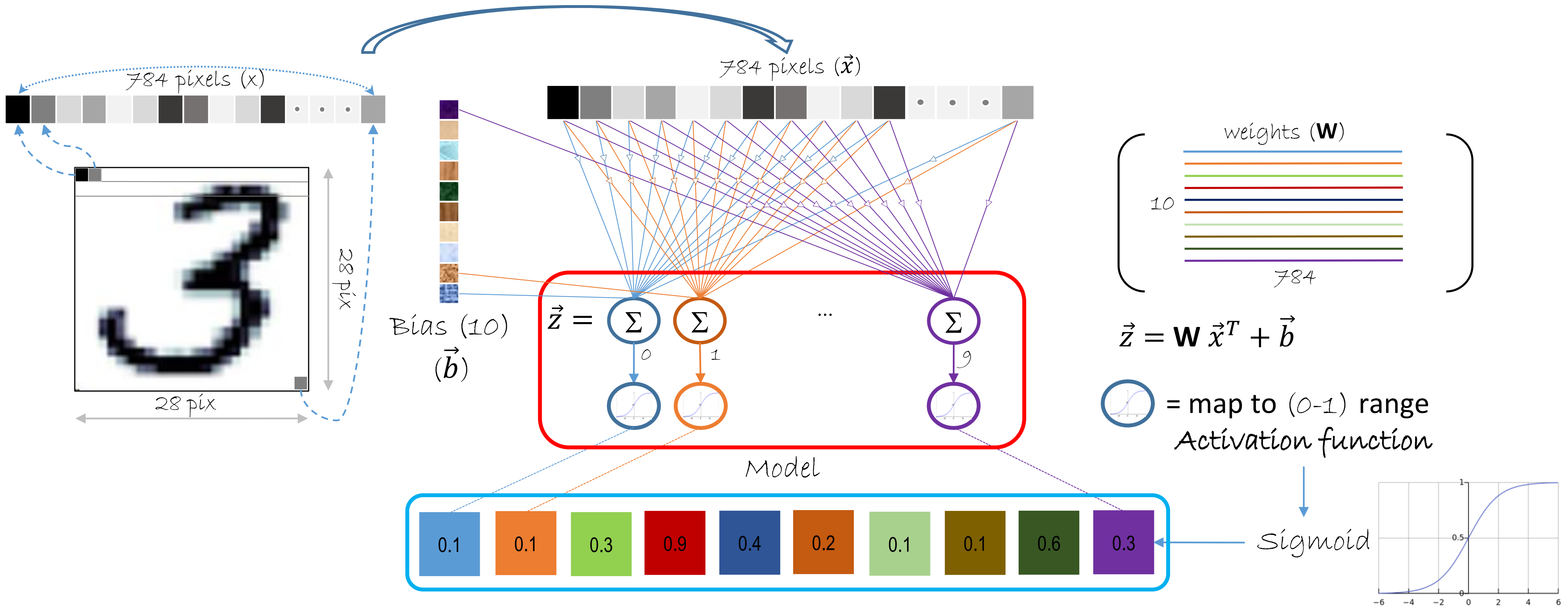
Σ = Sum ($\underset{784}{\text{weights}}$ x $\underset{784}{\text{pixels}}$) = $\vec{w}_9 \cdot \vec{x}^T$

$$\vec{z} = \mathbf{W}\, \vec{x}^T$$

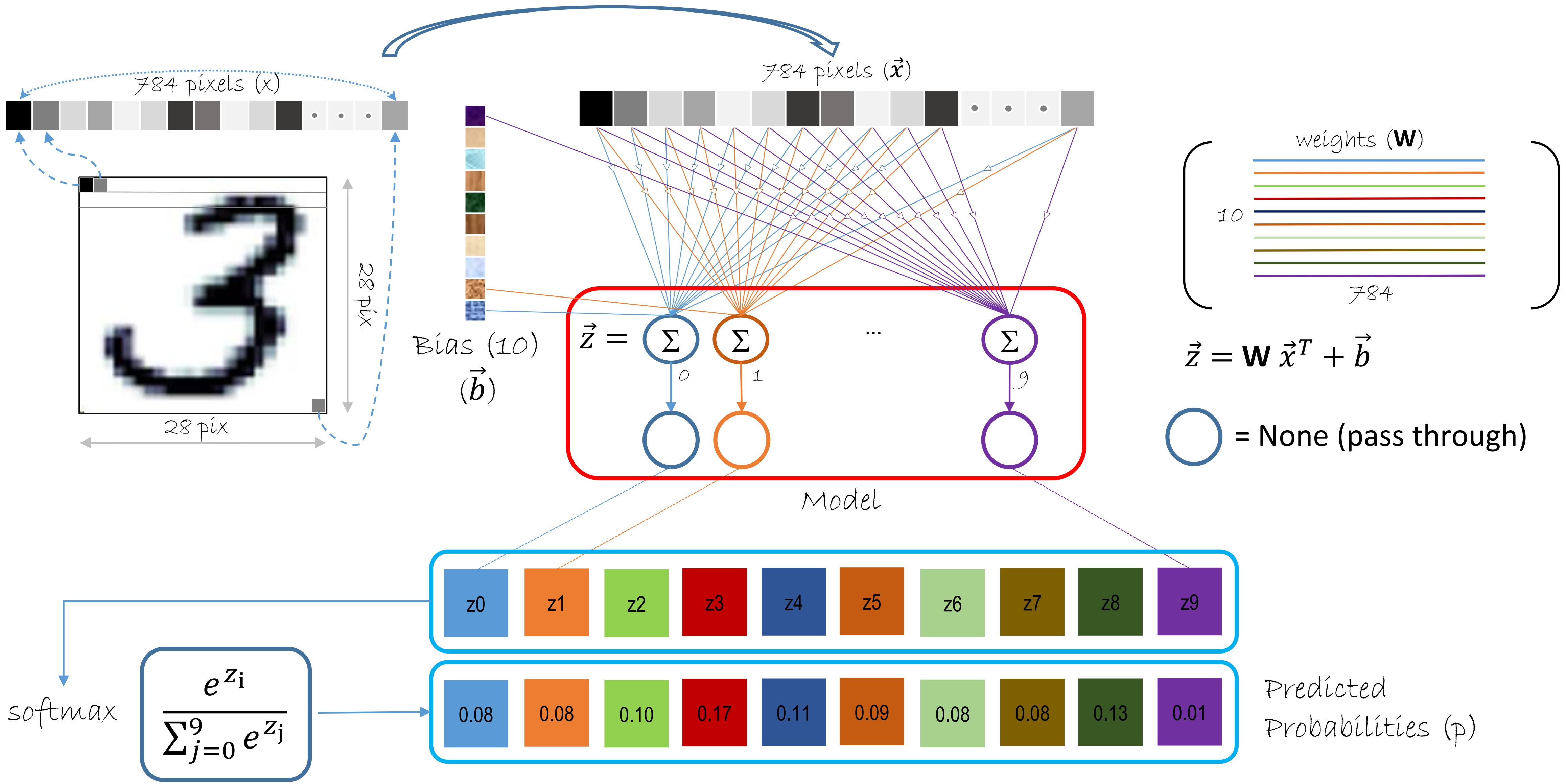
Logistic Regression



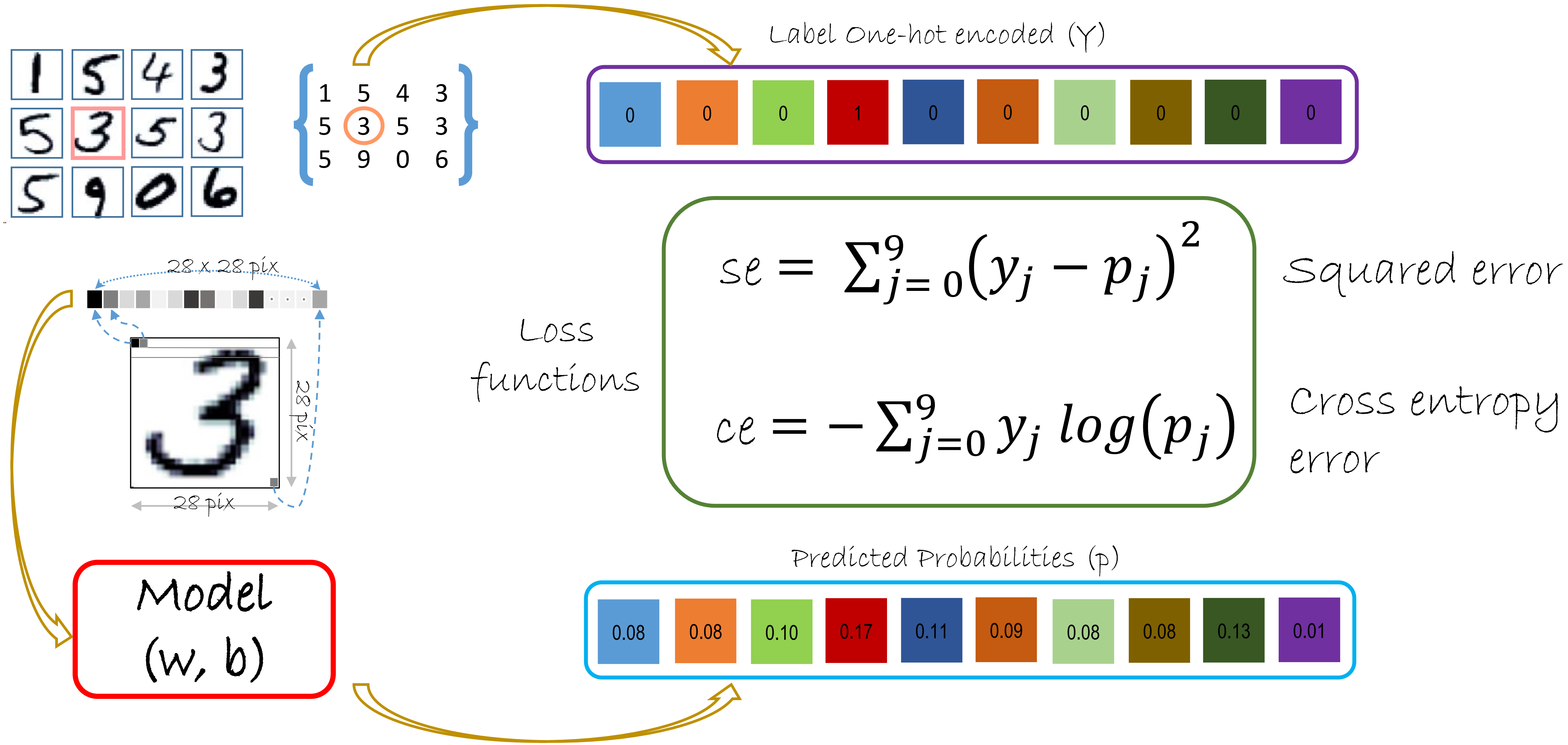
Logistic Regression



Logistic Regression with Softmax



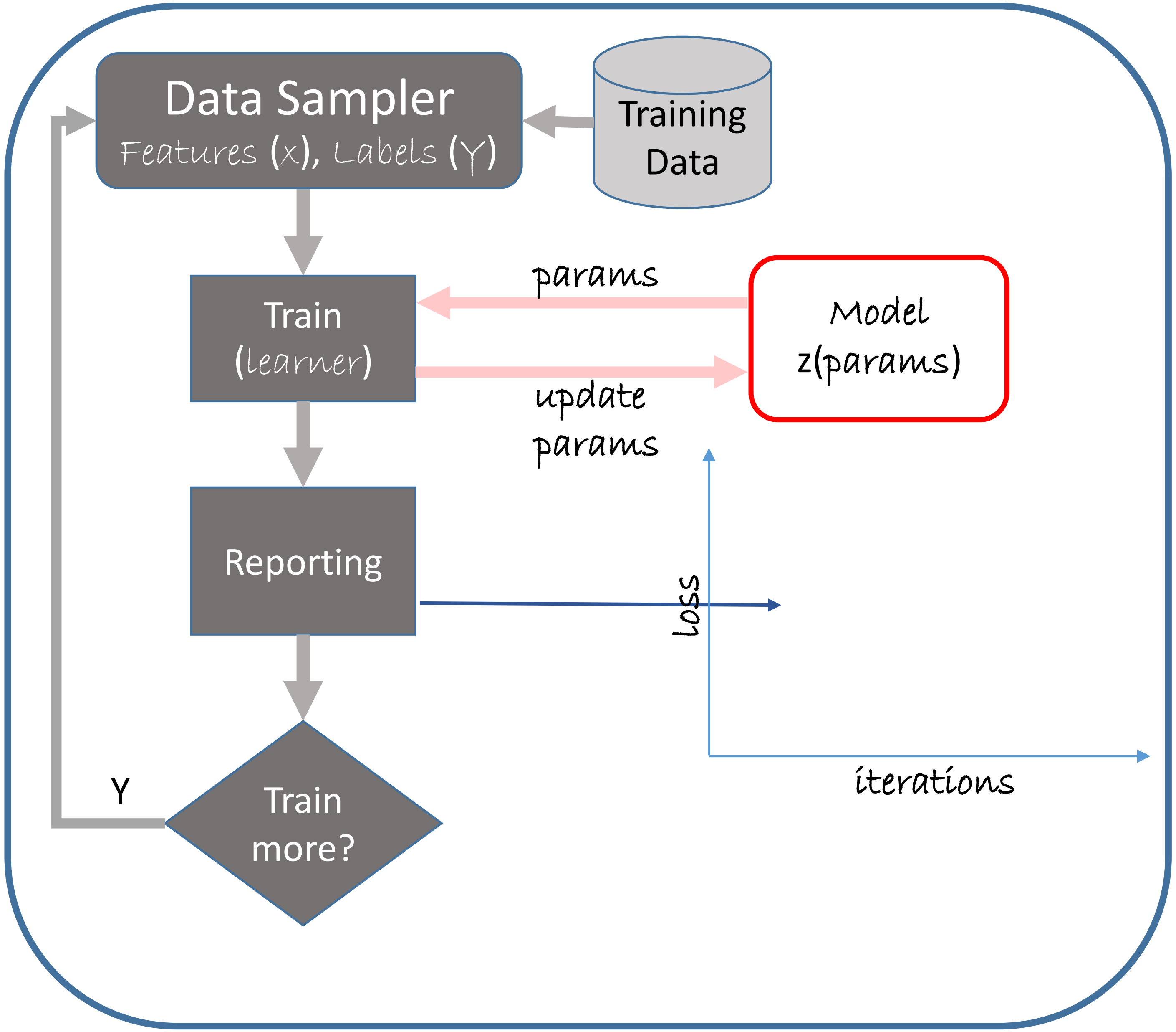
Loss Function



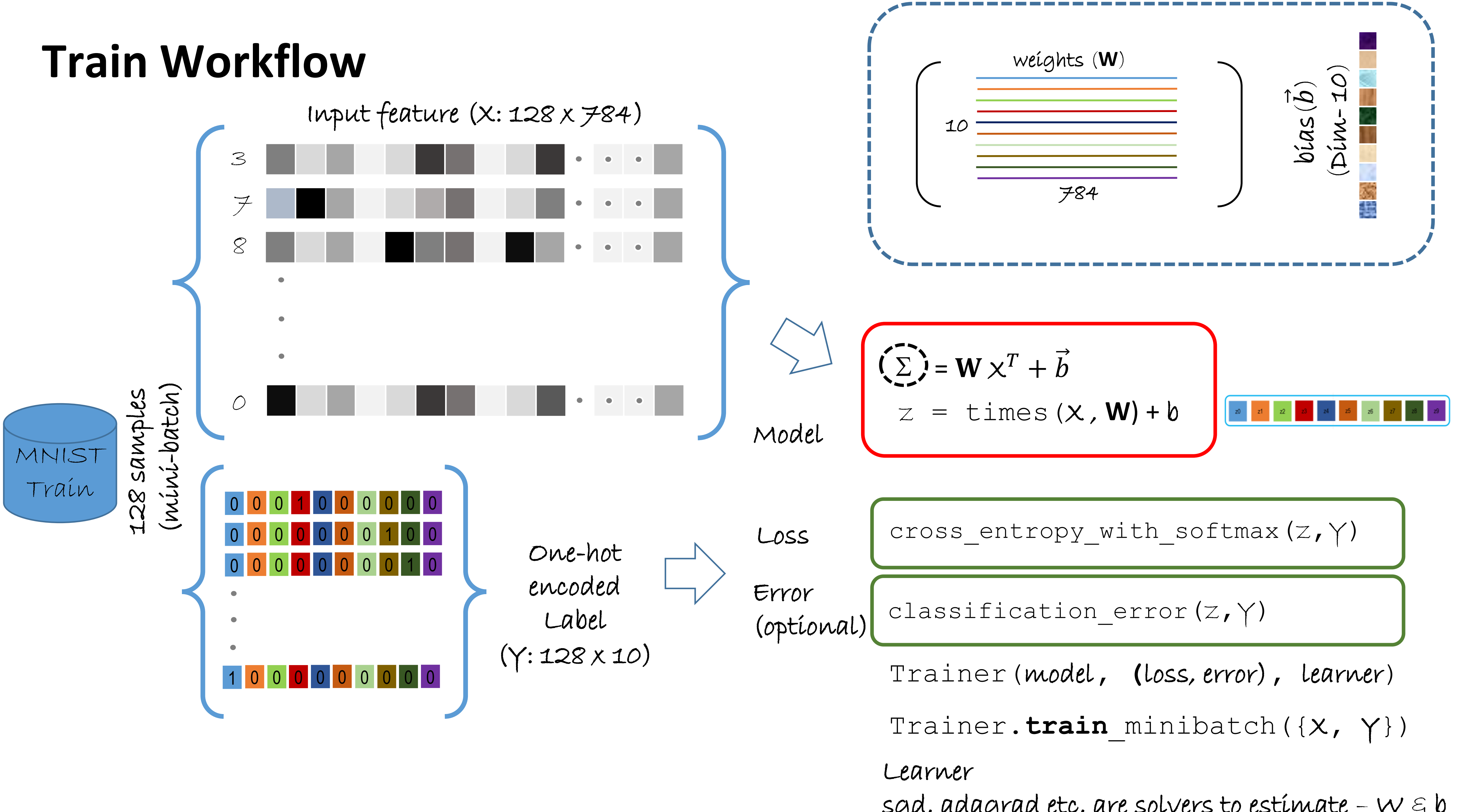
$$se = \sum_{j=0}^{9}\left(y_j - p_j\right)^2$$

$$ce = -\sum_{j=0}^{9} y_j \, log\left(p_j\right)$$

Train Workflow



Train Workflow



Learn the weights: Learners / Optimizers / Solvers

For 1 sample:

$$\text{Loss } (L_i) = -\sum_{j=0}^{9} y_j^{(i)} log(p_j) \quad \text{where: } p_j = f(x^{(i)};\theta)_j$$
$$\theta \in (w,b)$$

For all samples (m = 60000 images):

$$\text{Total loss} = \sum_{i=1}^{m} L_i\ (\theta; (x^{(i)}, y^{(i)}))$$

Convex function:

There is 1 and only 1 minimum

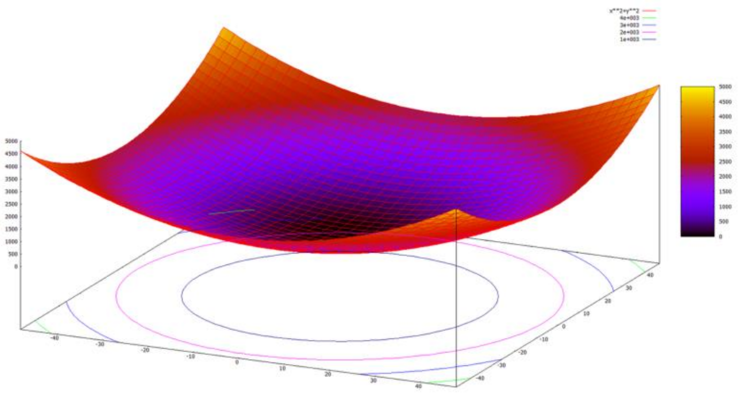
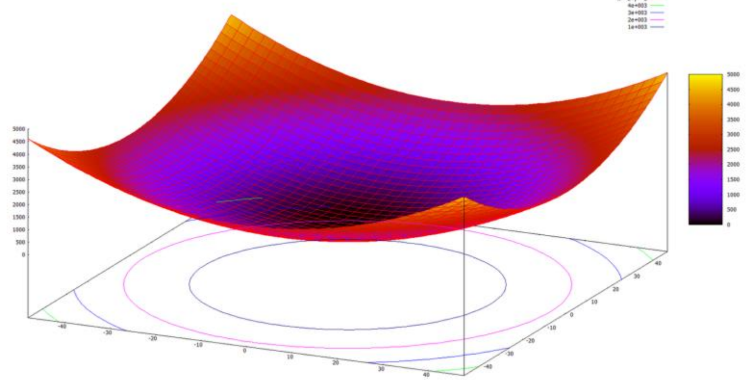
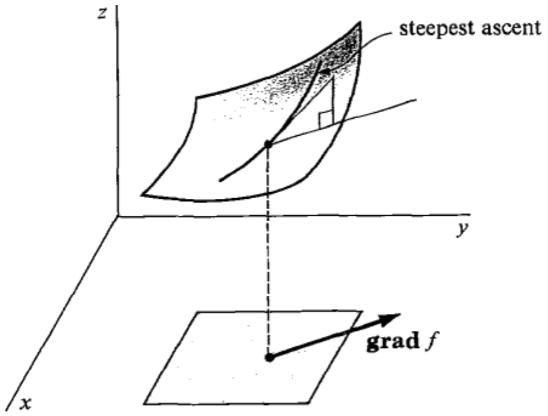
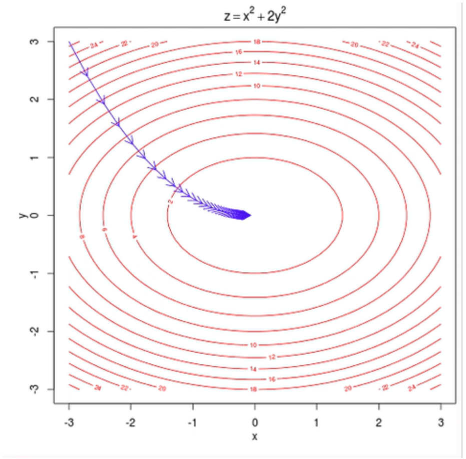


Fig: courtesy http://codingwiththomas.blogspot.com/2012/09/particle-swarm-optimization.html

Gradient Descent







$$\theta' = \theta \; - \mu \, grad \, (L; \theta)$$

Where:

θ = model parameter

μ = learning rate

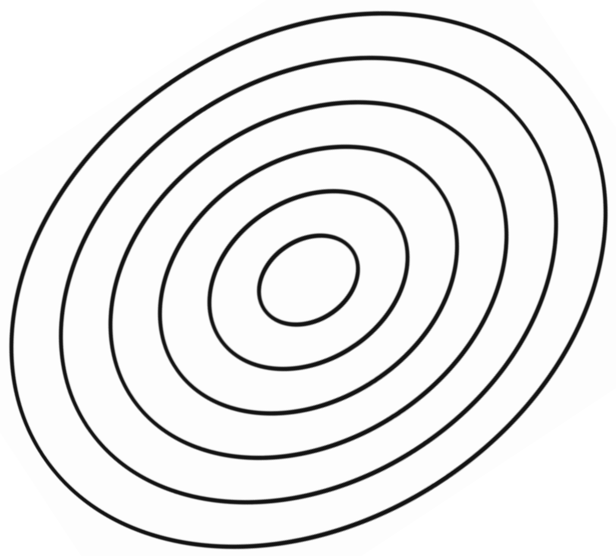
Computing "Total Loss" ($\sum_i^n L_i$) for large data set is expensive and often redundant

- refer to http://sebastianruder.com/optimizing-gradient-descent/ for details

Stochastic Gradient Descent (SGD)

SGD:

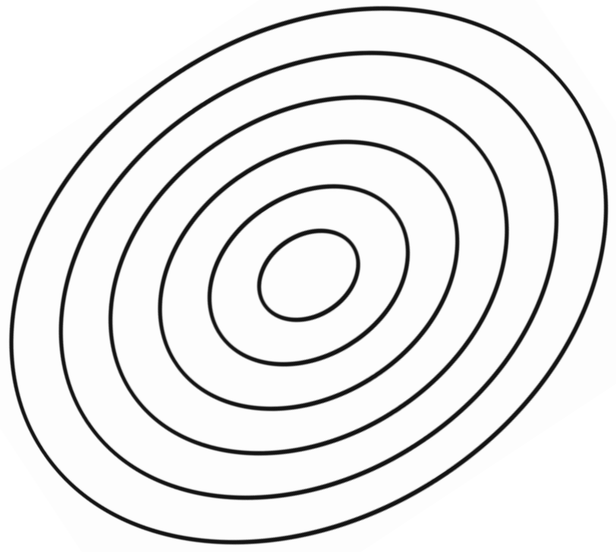
Update the parameters for each (data, label) pair



Mini-batch SGD:

Update the parameters for mini-batch set

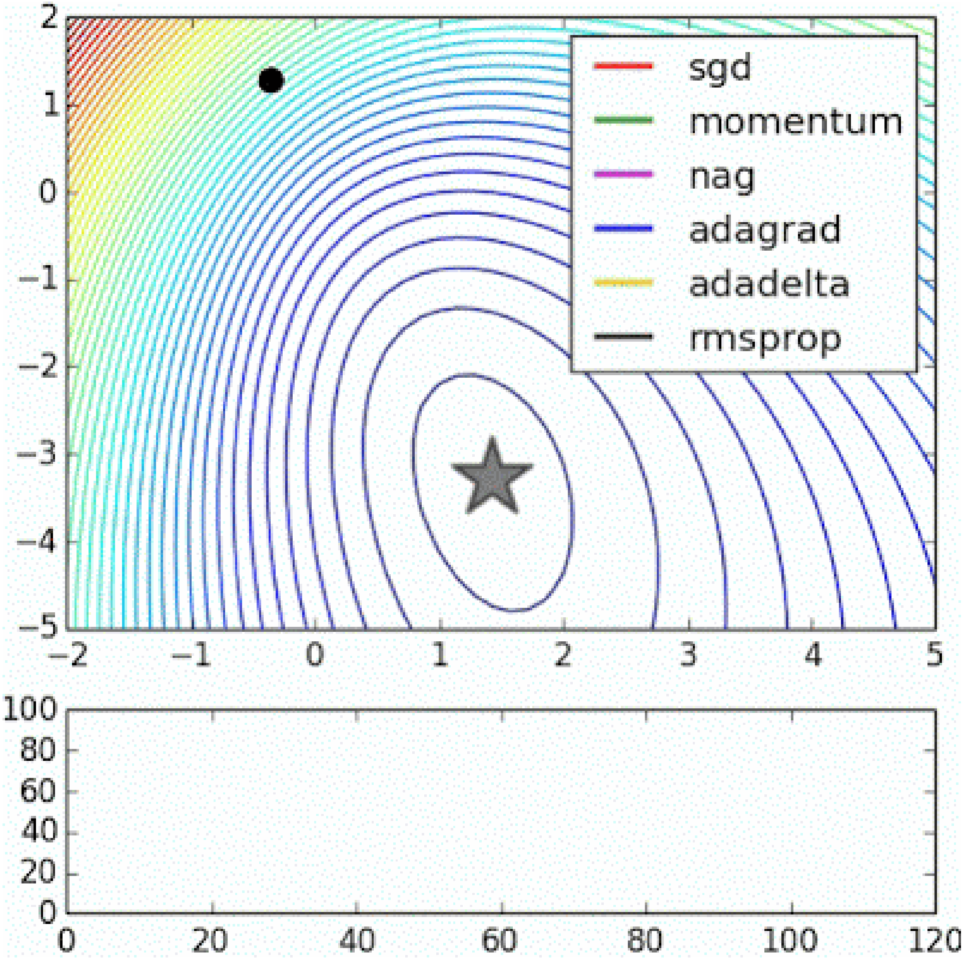
Set of (data, label) pairs



refer to http://sebastianruder.com/optimizing-gradient-descent/ for details on different learners

Other learners

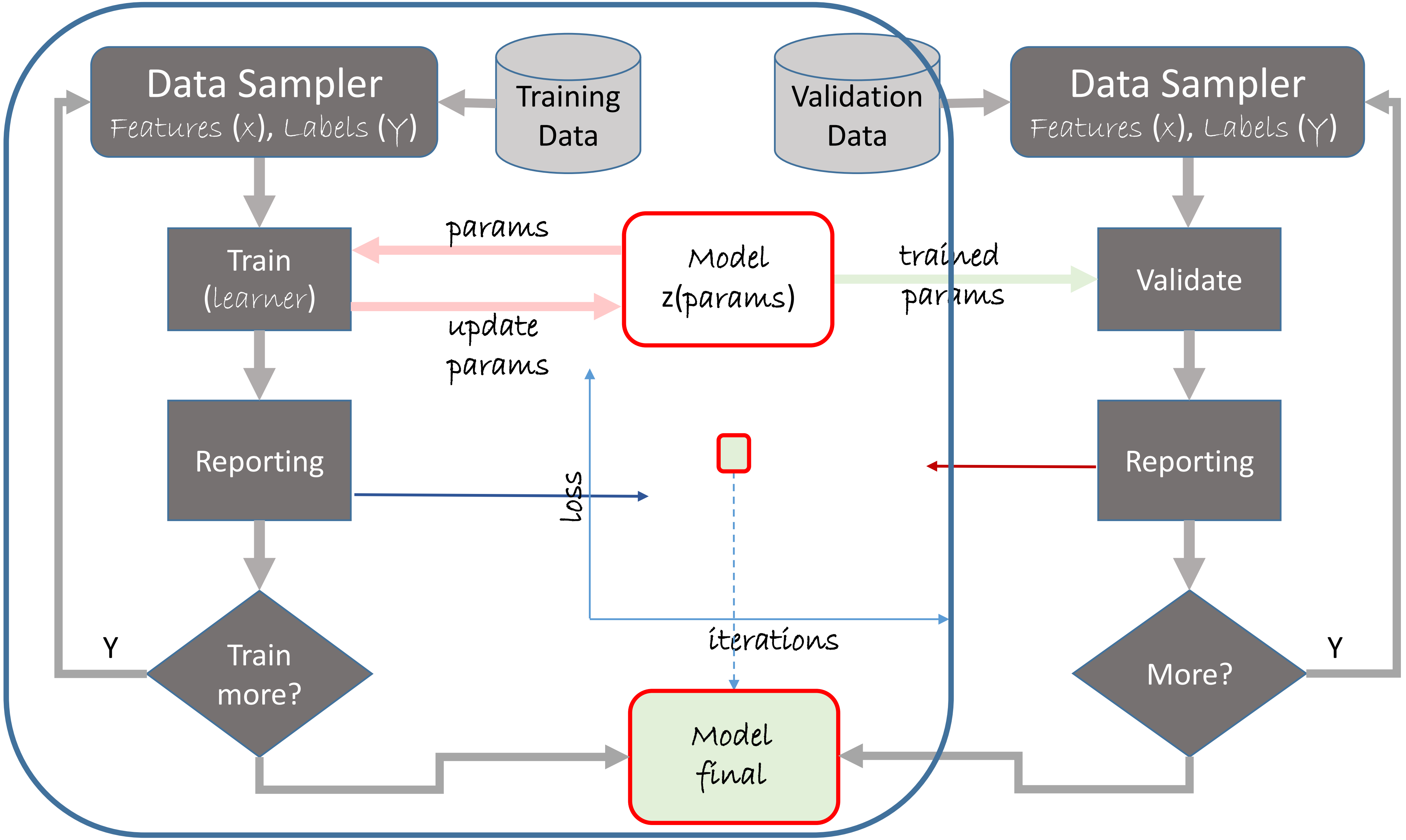
Momentum-SGD
Nestorov
Adagrad
Adsdelta
Adam



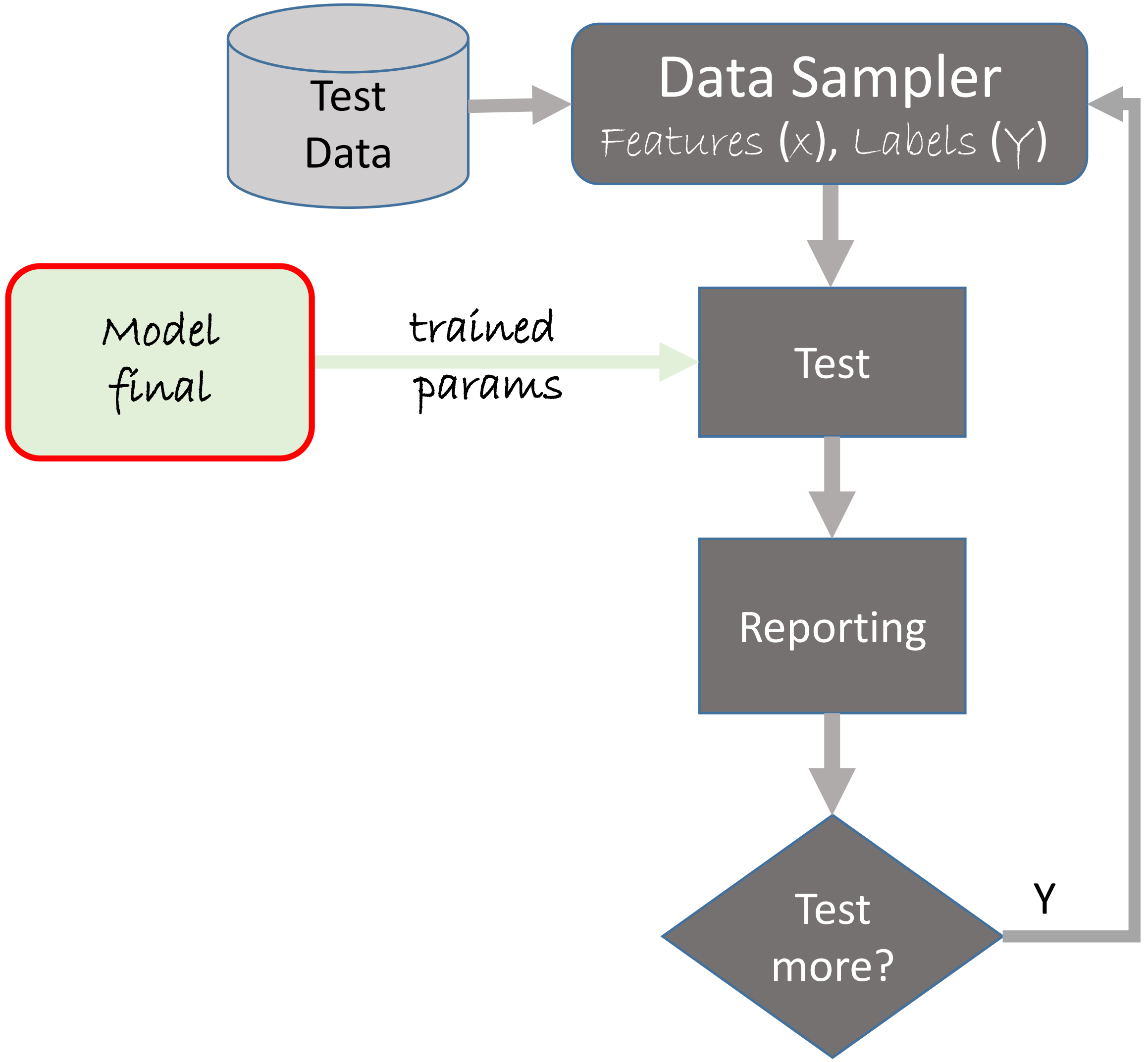
Refer to http://sebastianruder.com/optimizing-gradient-descent/ for details on different learners

Image by: Alec Radford

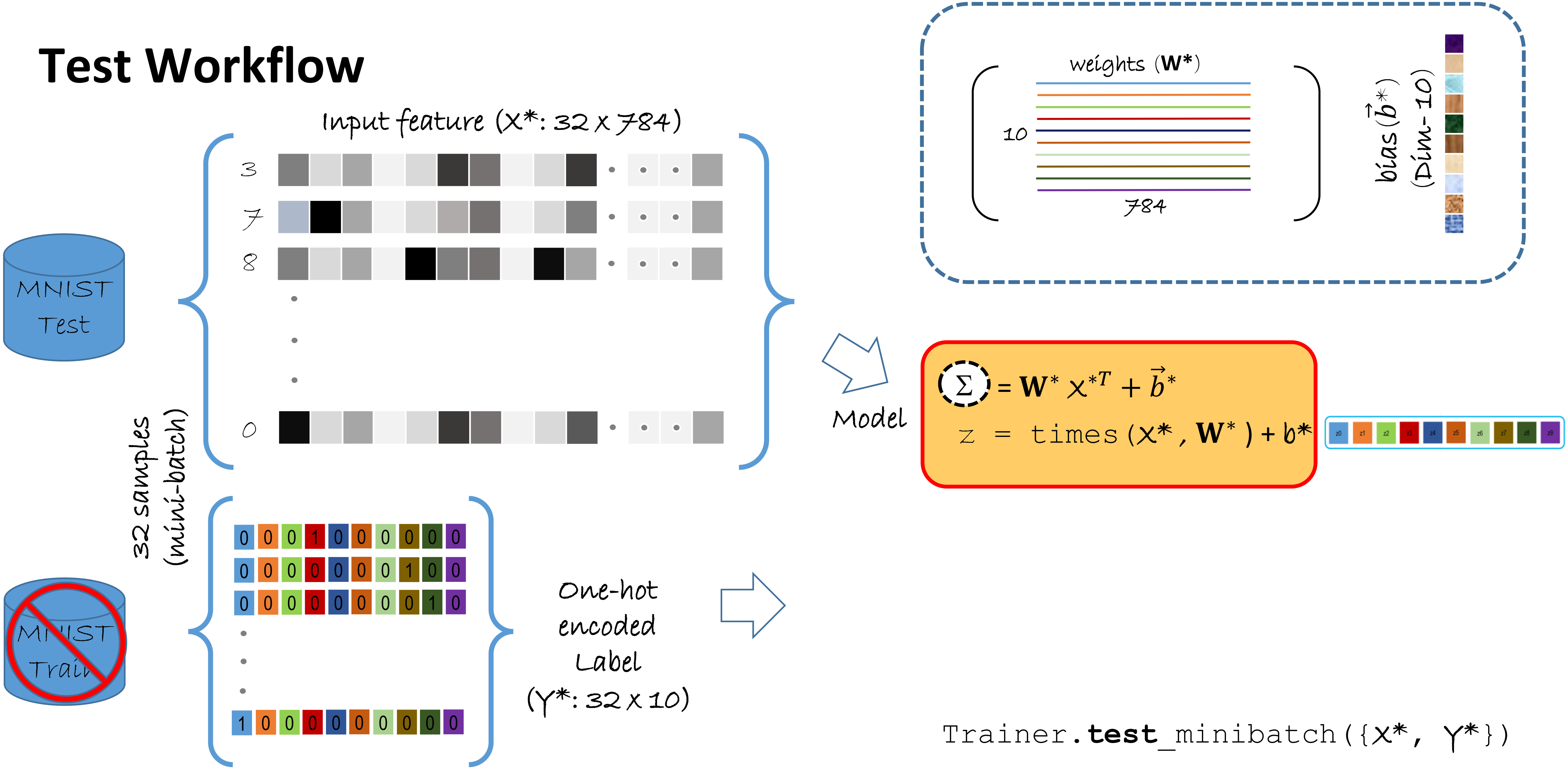
Validation Workflow



Test Workflow



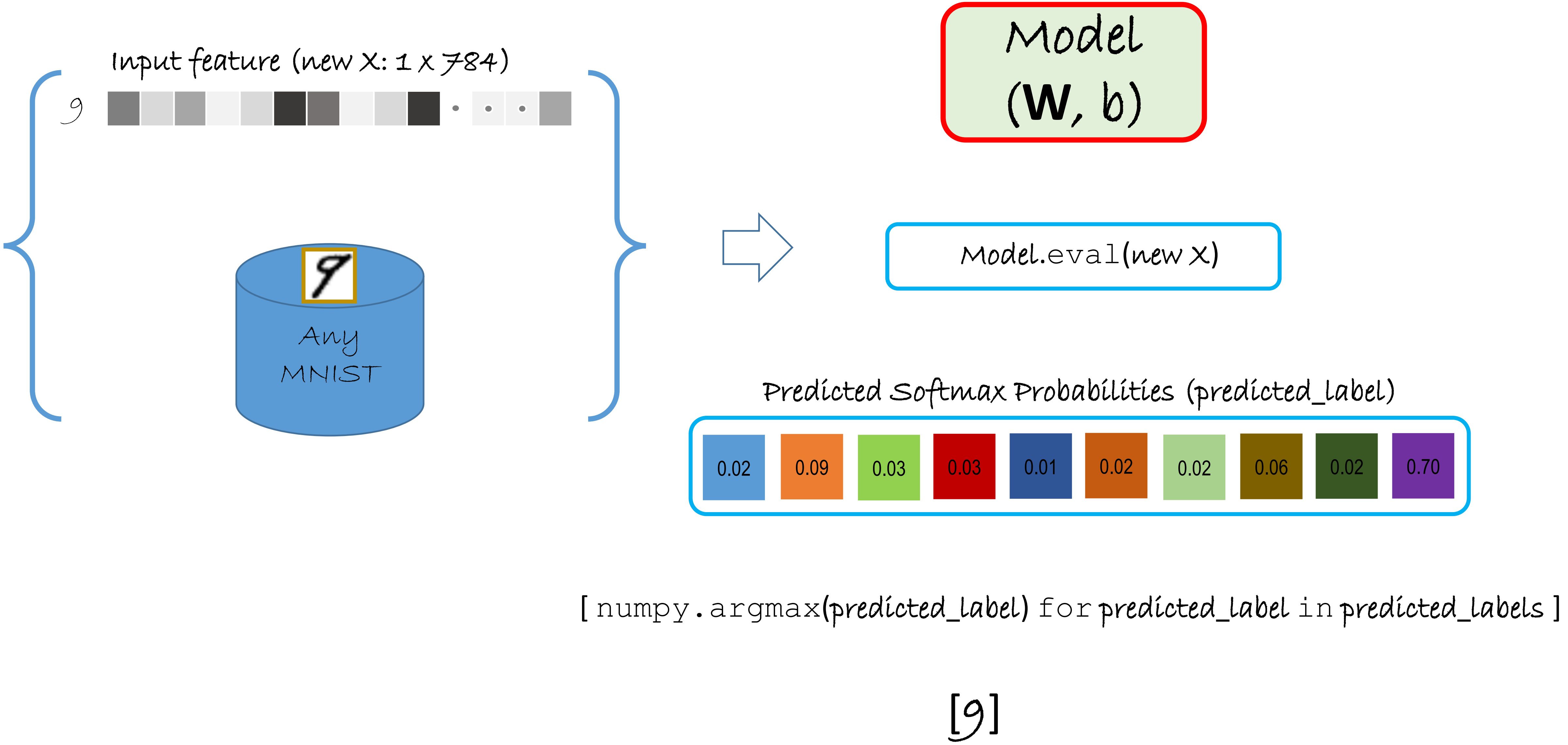
Test Workflow



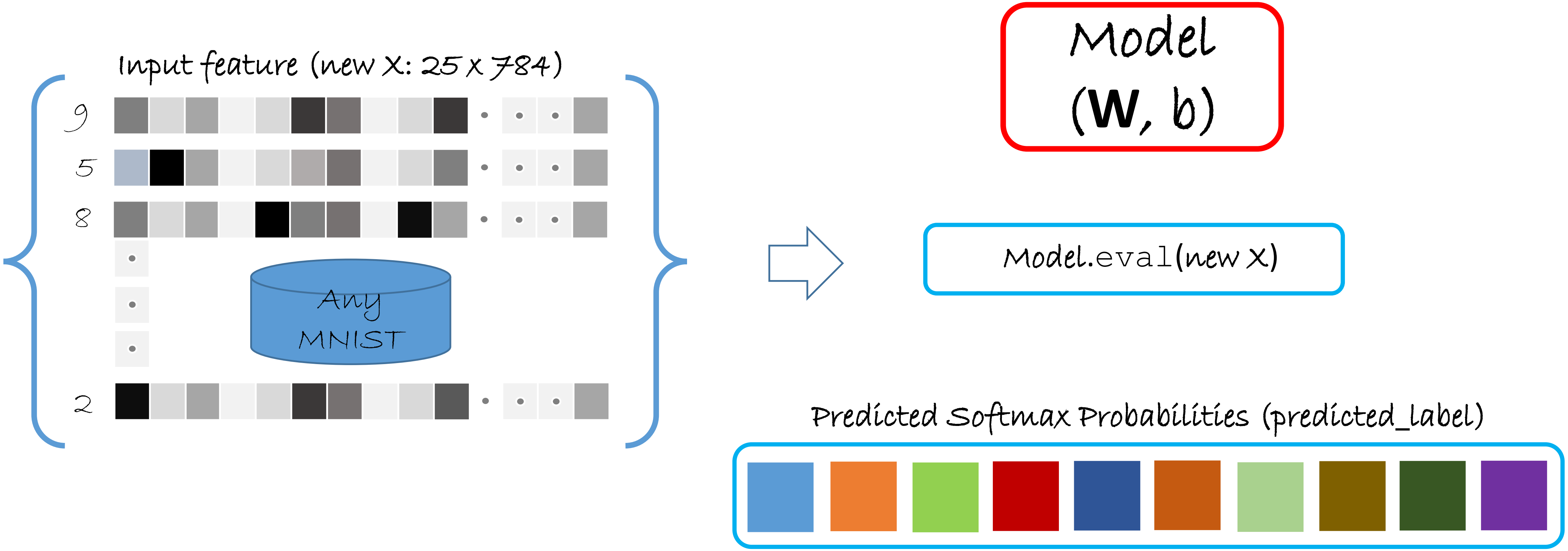
`Trainer.`**`test`**`_minibatch({X*, Y*})`

Returns the classification error as % incorrectly labeled MNIST image.

Prediction Workflow



Prediction Workflow



[numpy.argmax(predicted_label) for predicted_label in predicted_labels]

[9, 5, 8, ..., 2]