CS 763: Problem Set: Due: 10:00 PM, 18-Feb

- Please write (only if true) the honor code. If you used any source (person or thing) explicitly state it.
- Important: This is an INDIVIDUAL assignment.
- Always provide a brief explanation. (The length of the explanation required has been forecasted with the amount of space provided.)
- Submit following files in folder name lab03_roll_XX :
  1. readme.txt (case sensitive name). This text file contains identifying information, honor code, links to references used
  2. ReflectionEssay.pdf is optional but a brief one would be nice.
  3. lab03_roll_XX.pdf (includes all solutions).
  4. All relevant tex source (and images only if necessary). No other junk files, please.

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1. State whether or not the following points are the same and explain why.

   (a) $A[2, -1, 3]$, $B[4, -2, 6]$ (b) $A[\sqrt{2}/2, -1, 0]$, $B[1, -\sqrt{2}, 0]$

   Answer:

   (a) As points lie in homogeneous coordinate system.
   $A = [2/3, -1/3, 1]$, $B = [4/6, -2/6, 1] = [2/3, -1/3, 1]$
   Therefore the two points are same.

   (b) Points lie in homogeneous coordinate system.
   $A = [\sqrt{2}/2, -1, 0] = [x1, y1, w1]$, $B = [1, -\sqrt{2}, 0] = [x2, y2, w2]$
   As $w1$ and $w2$ are 0, the points A and B are ideal points.
   $(x1, y1)$ represent the direction for the point A at infinity.
   $(x2, y2)$ represent the direction for the point B at infinity.
   Direction of point A unit normalised $= [1/3, -\sqrt{2}/3]$
   Direction of point B unit normalised $= [1/3, -\sqrt{2}/3]$
   As the unit normalised direction for both points are equal, therefore they represent the same point.

2. In projective three-space, what are the standard homogeneous coordinates of (a) the origin and (b) ideal points determined by the intersections of the extensions of the coordinate axes and the ideal plane?

   Answer:

   (a) Origin $= [0, 0, 0, 1]$

   (b) Intersection of ideal plane p with x-axes is $[x, 0, 0, 0]$
   Intersection of ideal plane p with y-axes is $[0, y, 0, 0]$
   Intersection of ideal plane p with z-axes is $[0, 0, z, 0]$

3. Write standard homogeneous coordinates for the points specified in uppercase characters. (Use left and right to distinguish.)

images/image2.png

Answer:
Assuming the points are in P^2 space.
The points in homogeneous coordinate system are :
$A = [0, 0, 1] \quad B = [2, 0, 1]$
$C = [3, 1, 1] \quad L = [1.5, -0.5, 1]$
$K = [1, -4, 1] \quad I = [-1, 1, 1]$
$J = [-4, -2, 1] \quad H = [-4, 3, 1]$
$G = [-3, 4, 1] \quad E = [-1, 4.5, 1]$

4. Which of the following points lie on the line $3p_1 - 2p_2 + 5p_3 = 0$? Why?

   (a) $A[1, 1, 2]$
   (b) $B[4, 1, -2]$

Answer:

(a) Put $p_1 = 1, p_2 = 1, p_3 = 2$ in equation of the line $3p_1 - 2p_2 + 5p_3$
$LHS = 3 * 1 - 2 * 1 + 5 * 2 = 11$
$RHS = 0$
$LHS \neq RHS$
Therefore, the point does not lie on the line.

(b) Put $p_1 = 4, p_2 = 1, p_3 = -2$ in equation of the line $3p_1 - 2p_2 + 5p_3$
$LHS = 3 * 4 - 2 * 1 + 5 * (-2) = 0$
$RHS = 0$
$LHS = RHS$
Therefore, the point lies on the line.

5. Write the coordinates of the lines that are the extensions to the projective plane of the following Euclidean lines.

(a) $3x + 2y = 6$

(b) $4x + 5y + 7 = 0$

Answer:

(a) $\begin{bmatrix} 3 & 2 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

Therefore, coordinates of the line that is extension to the projective plane are $[3, 2, -6]$.

(b) $\begin{bmatrix} 4 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$

Therefore, coordinates of the line that is extension to the projective plane are $[4, 5, 7]$.

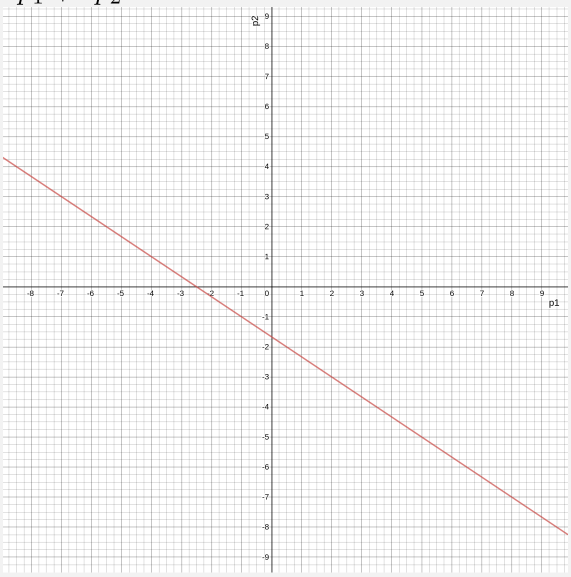
6. Sketch each line in the projective plane whose equation is given.

(a) $2p_1 + 3p_2 + 5p_3 = 0$

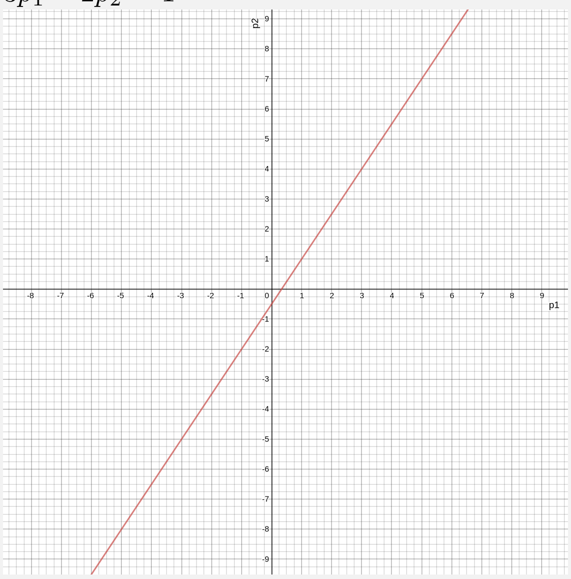
(b) $3p_1 - 2p_2 - p_3 = 0$

Answer:

(a) Put $p_3 = 1$ in the equation. After that equation becomes
$2p_1 + 3p_2 = -5$



(b) Put $p_3 = 1$ in the equation. After that equation becomes
$3p_1 - 2p_2 = 1$

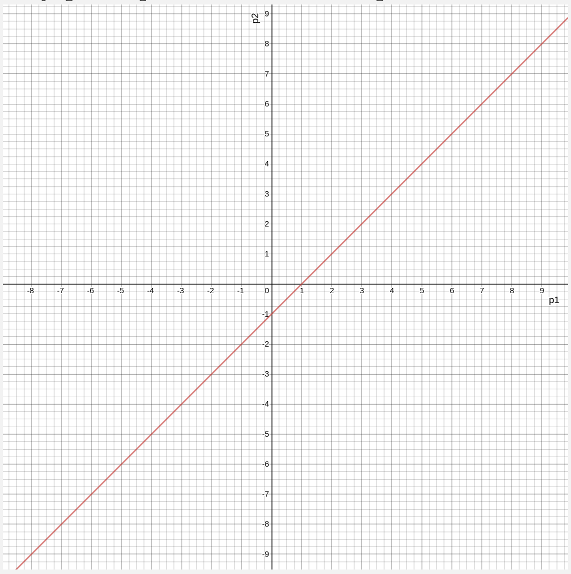


7. In each of the following cases, sketch the line determined by the two given points; then find the equation of the line.

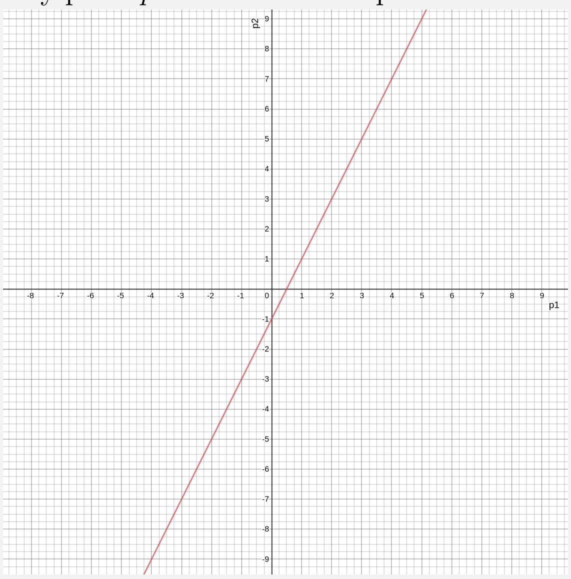
(a) $A[3, 1, 2]$, $B[1, 2, -1]$ (b) $A[2, 1, 3]$, $B[1, 2, 0]$

Answer:

(a) Let the line be represented as $L = [l_1, l_2, 1]$.
The equations are $3l_1 + l_2 + 2 = 0, l_1 + 2l_2 - 1 = 0$.
After solving the above equations, $L = [-1, 1, 1]$.
Any point p on line can be represented in homogeneous coordinate as $p = [p_1, p_2, 1]$



(b) Let the line be represented as $L = [l_1, l_2, 1]$.
The equations are $2l_1 + l_2 + 3 = 0, l_1 + 2l_2 = 0$.
After solving the above equations, $L = [-2, 1, 1]$.
Any point p on line can be represented in homogeneous coordinate as $p = [p_1, p_2, 1]$



8. Find the standard homogeneous coordinates of the point of intersection for each pair of lines.

   (a) $p_1 + p_2 - 2p_3 = 0, 3p_1 + p_2 + 4p_3 = 0$

   (b) $p_1 + p_2 = 0, 4p_1 - 2p_2 + p_3 = 0$

   Answer:

   (a) For intersection of points in homogeneous coordinate system, put $p_3 = 1$ in the both equations and solve for p_1 and p_2.
   $p_1 + p_2 - 2 * 1 = 0$
   $3p_1 + p_2 + 4 * 1 = 0$
   After solving the above two equations, $p_1 = -3$ and $p_2 = 5$.
   Therefore, the intersection of lines in homogeneous coordinates is $[-3, 5, 1]$.

   (b) For intersection of points in homogeneous coordinate system, put $p_3 = 1$ in the both equations and solve for p_1 and p_2.
   $p_1 + p_2 = 0$
   $4p_1 - 2p_2 + 1 * 1 = 0$
   After solving the above two equations, $p_1 = -3$ and $p_2 = 5$.
   Therefore, the intersection of lines in homogeneous coordinates is $[-1/6, 1/6, 1]$.

9. Determine which of the following sets of three points are collinear.

   (a) $A[1, 2, 1]$, $B[0, 1, 3]$, $[2, 1, 1]$

   (b) $A[1, 2, 3]$, $B[2, 4, 3]$, $[1, 2, -2]$

   Answer:

   (a) $AB = [-1, -1, 2], BC = [2, 0, -2]$.
   As $AB \neq k * BC, k$ being a real number.
   Therefore, the points are not collinear.

   (b) $AB = [1, 2, 0], BC = [-1, -2, -5]$.
   As $AB \neq k * BC, k$ being a real number.
   Therefore, the points are not collinear.

10. Determine which of the following sets of three lines meet in a point.

(a) $l[1, 0, 1], m[1, 1, 0], n[0, 1, -1]$

(b) $l[1, 0, -1], m[1, -2, 1], n[3, -2, -1]$

Answer:

(a) Let $L = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

Let x be the point which is on all three lines. Then $Lx = 0$

If $|L| = 0$, then all three lines intersect in one point.

$L = 1 * (1 * (-1) - 0 * 1) - 0 * (1 * (-1) - 0 * 0) + 1 * (1 * 1 - 0 * 1) = 0$

Therefore, the lines intersect in one point.

(b) Let $L = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 3 & -2 & -1 \end{bmatrix}$

Let x be the point which is on all three lines. Then $Lx = 0$

If $|L| = 0$, then all three lines intersect in one point.

$L = 1 * ((-2) * (-1) - (-2) * 1) - 0 + (-1) * (1 * (-2) - (-2) * (3)) = 0$

Therefore, the lines intersect in one point.