Exam Style Questions

Composite Functions
Inverse Functions

Corbettmaths

Ensure you have: Pencil, pen, ruler, protractor, pair of compasses and eraser

You may use tracing paper if needed

Guidance

1. Read each question carefully before you begin answering it.
2. Don't spend too long on one question.
3. Attempt every question.
4. Check your answers seem right.
5. Always show your workings

Revision for this topic

www.corbettmaths.com/contents

Video 369
Video 370

1. Given $f(x) = \dfrac{2x+1}{3}$

$f(7) = \dfrac{2 \times 7 + 1}{3}$

$= \dfrac{14 + 1}{3}$

$= \dfrac{15}{3}$

(a) Calculate the value of $f(7)$

............5............
(1)

(b) Find $f^{-1}(x)$

$y = \dfrac{2x+1}{3}$

$3y = 2x + 1$

$3y - 1 = 2x$

$\dfrac{3y-1}{2} = x$

$f^{-1}(x) = \dfrac{3x-1}{2}$

....
(2)

2. The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 3x - 1$$
$$g(x) = 2x + 4$$

(a) Calculate the value of $fg(2)$

$g(2) = 2 \times 2 + 4 = 8$

$f(8) = 3 \times 8 - 1 = 23$

$fg(2) = 23$

............23............
(2)

(b) Calculate the value of $ff(3)$

$f(3) = 3 \times 3 - 1 = 8$

$f(8) = 23$

............23............
(2)

(c) Find $gf(x)$

$gf(x) = 2(3x-1) + 4$

$gf(x) = 6x - 2 + 4$

$= 6x + 2$

$\underline{gf(x) = 6x+2}$
(2)

3. The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = x^2 - 3$$
$$g(x) = 2x + 1$$
$$h(x) = \frac{x}{2}$$

(a) Find $fg(x)$ $= (2x+1)^2 - 3$
$= (2x+1)(2x+1) - 3$
$= 4x^2 + 4x + 1 - 3$

$fg(x) = 4x^2 + 4x - 2$

.... (2)

(b) Find $gh(x)$
$= 2\left(\frac{x}{2}\right) + 1 = x + 1$

$gh(x) = x + 1$

.... (2)

(c) Find $f^{-1}(x)$ $y = x^2 - 3$
$y + 3 = x^2$
$\sqrt{y+3} = x$

$f^{-1}(x) = \sqrt{x+3}$ where $x \geq -3$

.... (2)

4. The function f is such that $f(x) = 4x - 7$

(a) Solve $f(x) = 17$ $4x - 7 = 17$
$4x = 24$
$x = 6$

$x = 6$

.... (2)

(b) Find $f^{-1}(x)$ $y = 4x - 7$
$y + 7 = 4x$
$\frac{y+7}{4} = x$

$f^{-1}(x) = \frac{x+7}{4}$

.... (2)

5. Given $f(x) = x^2 + 2$ and $g(x) = x + 14$

Find the values of a such that $f(a) = g(a)$

$$a^2 + 2 = a + 14$$
$$a^2 - a - 12 = 0$$
$$(a - 4)(a + 3) = 0$$

$\underline{a = 4 \text{ or } -3}$

(3)

6. The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 8 - 3x$$
$$g(x) = 4x$$

(a) Calculate the value of $gf(3)$

$$f(3) = 8 - (3 \times 3) = -1$$
$$g(-1) = 4 \times -1$$

$\underline{-4}$

(2)

(b) Solve the equation $gf(x) = 80$

$$gf(x) = 4(8 - 3x) = 32 - 12x$$

$$32 - 12x = 80$$
$$-48 = 12x$$

$\underline{x = -4}$

(4)

7. $f(x) = 2x^2 - 1$

Find $f^{-1}(x)$

$$y = 2x^2 - 1$$
$$y + 1 = 2x^2$$
$$\frac{y+1}{2} = x^2$$
$$\sqrt{\frac{y+1}{2}} = x$$

$$f^{-1}(x) = \sqrt{\frac{x+1}{2}}$$

(3)

where $x \geq -1$

8. Given $f(x) = x^2 + 3x - 5$

Express $f(2x - 1)$ in the form ax² + bx + c

$f(2x-1) = (2x-1)^2 + 3(2x-1) - 5$
$= (2x-1)(2x-1) + 6x - 3 - 5$
$= 4x^2 - 4x + 1 + 6x - 8$
$= 4x^2 + 2x - 7$

$4x^2 + 2x - 7$
....
(3)

9. The function f is such that $f(x) = kx + 3$

The function g is such that $g(x) = 2x - 4$

Given that $gf(2) = 34$

work out the value of k

$gf(x) = 2(kx+3) - 4 = 2kx + 6 - 4$
$= 2kx + 2$

$gf(2) = 4k + 2 = 34$
$k = 8$

8
....
(3)

10. For all values of x,

$$f(x) = x^2 + 4$$
$$g(x) = x - 9$$

$gf(x) = (x^2 + 4) - 9$
$= x^2 - 5$

Solve $fg(x) = gf(x)$

$fg(x) = (x-9)^2 + 4$
$= (x-9)(x-9) + 4$
$= x^2 - 18x + 81 + 4$
$= x^2 - 18x + 85$

$fg(x) = gf(x)$

$x^2 - 18x + 85 = x^2 - 5$
$-18x + 85 = -5$
$-18x = -90$

$x = 5$
....
(4)

11. $f(x) = x^2 + 2x + 1$

Show that $f(x+2) - f(x) = 4x + 8$

$$f(x) = x^2 + 2x + 1$$

$$f(x+2) = (x+2)^2 + 2(x+2) + 1$$

$$= x^2 + 4x + 4 + 2x + 4 + 1$$

$$= x^2 + 6x + 9$$

$$f(x+2) - f(x) = 4x + 8 \qquad QED$$

(3)