In [1]:
```
from math import sqrt,pi
import numpy as np
import pandas as pd
from scipy.stats import multivariate_normal as mvnorm
import matplotlib.pyplot as plt

%matplotlib inline
```

Q3

In [2]:
```
def grid_gen(xlim,ylim,step):
    return np.mgrid[-xlim:xlim:step, -ylim:ylim:step]
```

In [3]:
```
def f_eval(grid,mu,cov):
    pos = np.dstack((grid[0],grid[1]))
    return mvnorm.pdf(pos,mu,cov)
```

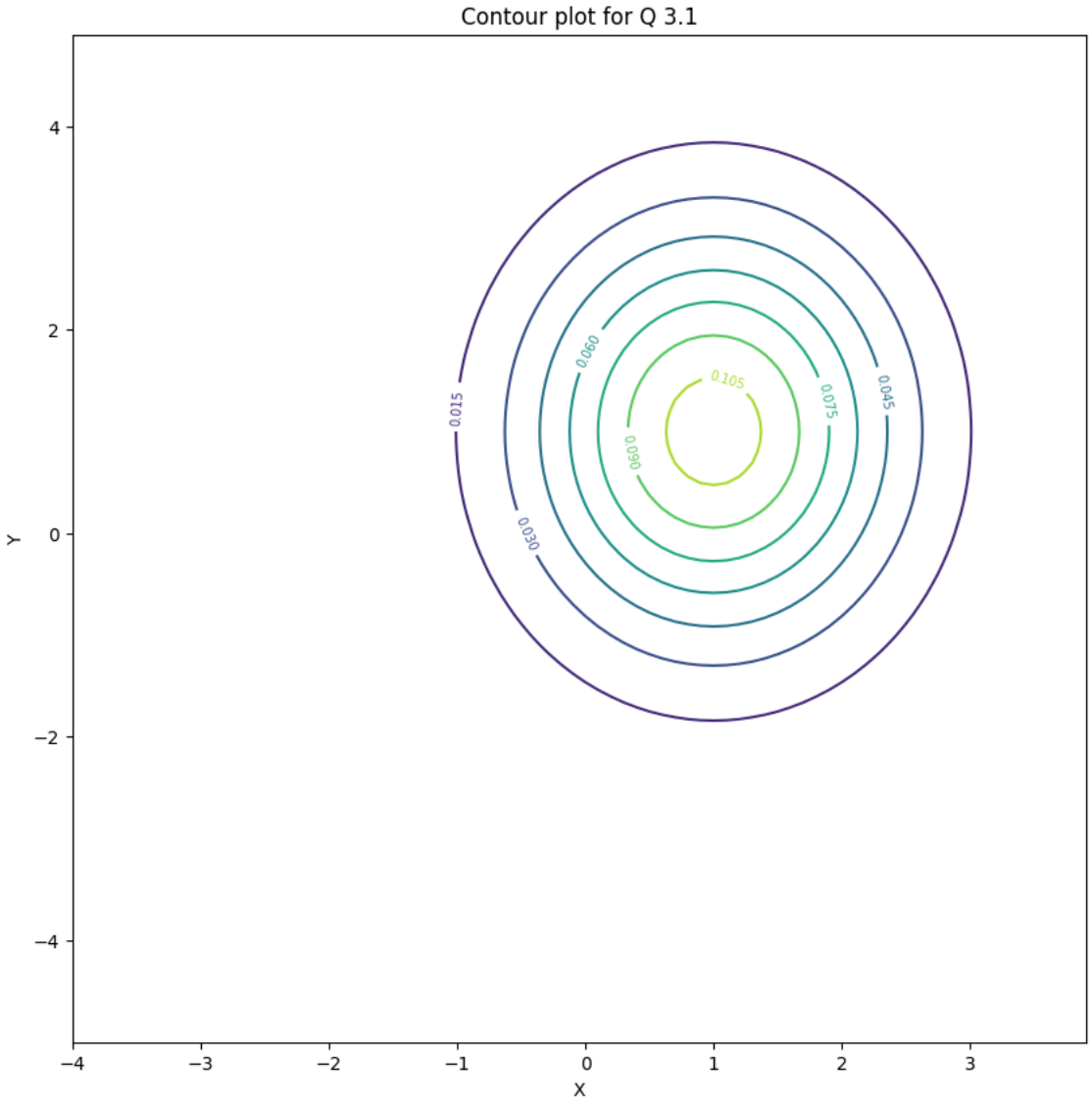
In [4]:
```
def contour(prob,mu=None,cov=None,xlim=None,ylim=None,step=None,f=None,grid=None,flag=True):

    if flag==True:
        grid = grid_gen(xlim,ylim,step)
        z = f_eval(grid,mu,cov)

    else:
        z=f

    fig = plt.figure(figsize=(10,10))
    cs = plt.contour(grid[0],grid[1],z)
    plt.clabel(cs,inline_spacing=5, fontsize=7)
    plt.xlabel('X')
    plt.ylabel('Y')
    plt.title('Contour plot for {}'.format(prob))
#     plt.savefig('{}_plot.png'.format(prob), dpi=300)
```

3.1

In [149]:
```
mu1 = np.array([1,1])
```

In [150]:
```
cov1 = np.array([[1, 0],[0, 2]])
```

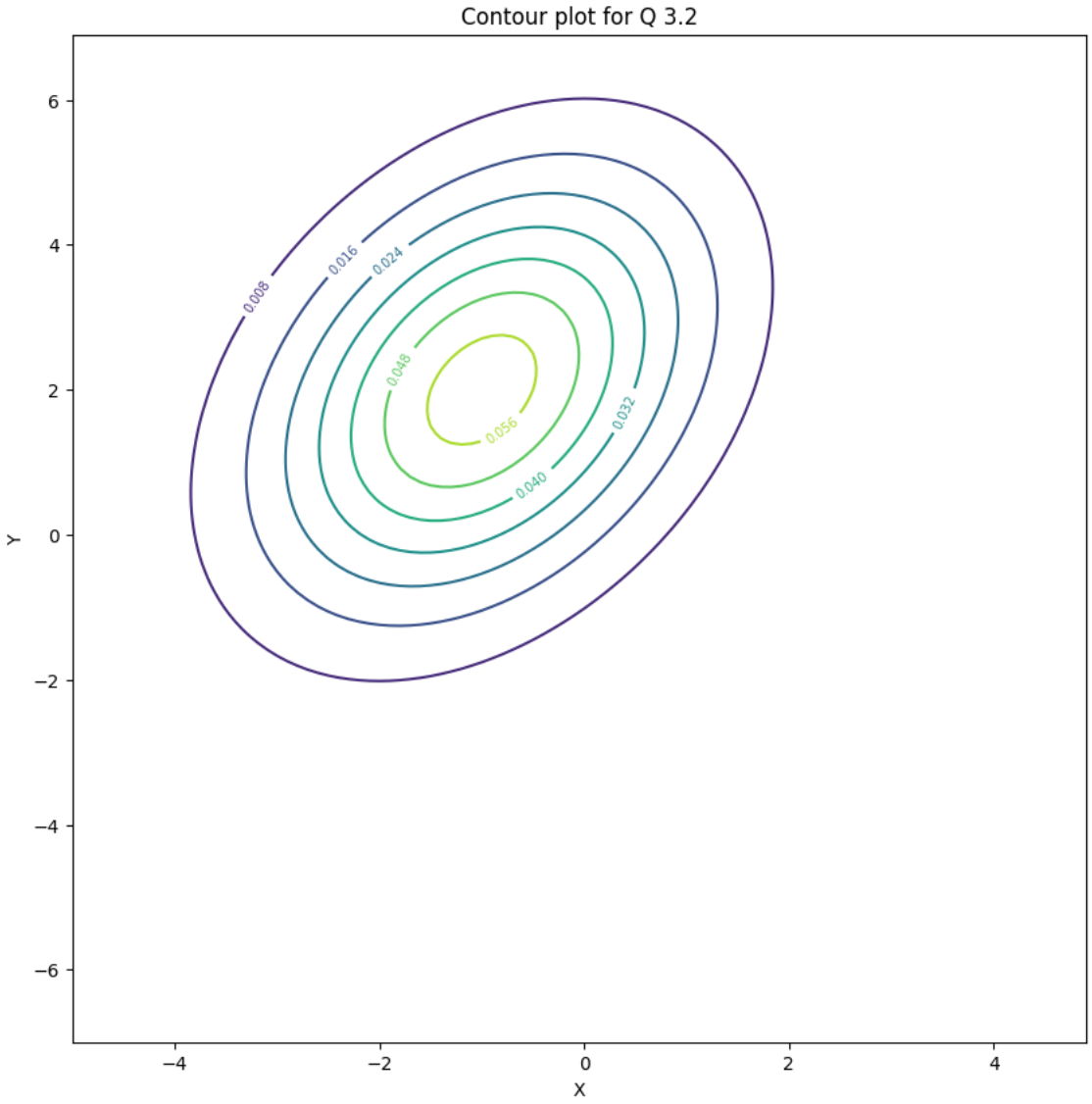
In [151]:
```
contour('Q 3.1',mu1,cov1,4,5,0.1)
```



3.2

In [152]:
```
mu2 = np.array([-1,2])
cov2 = np.array([[2, 1],[1, 4]])
```

In [153]:
```
contour('Q 3.2',mu2,cov2,5,7,0.1)
```



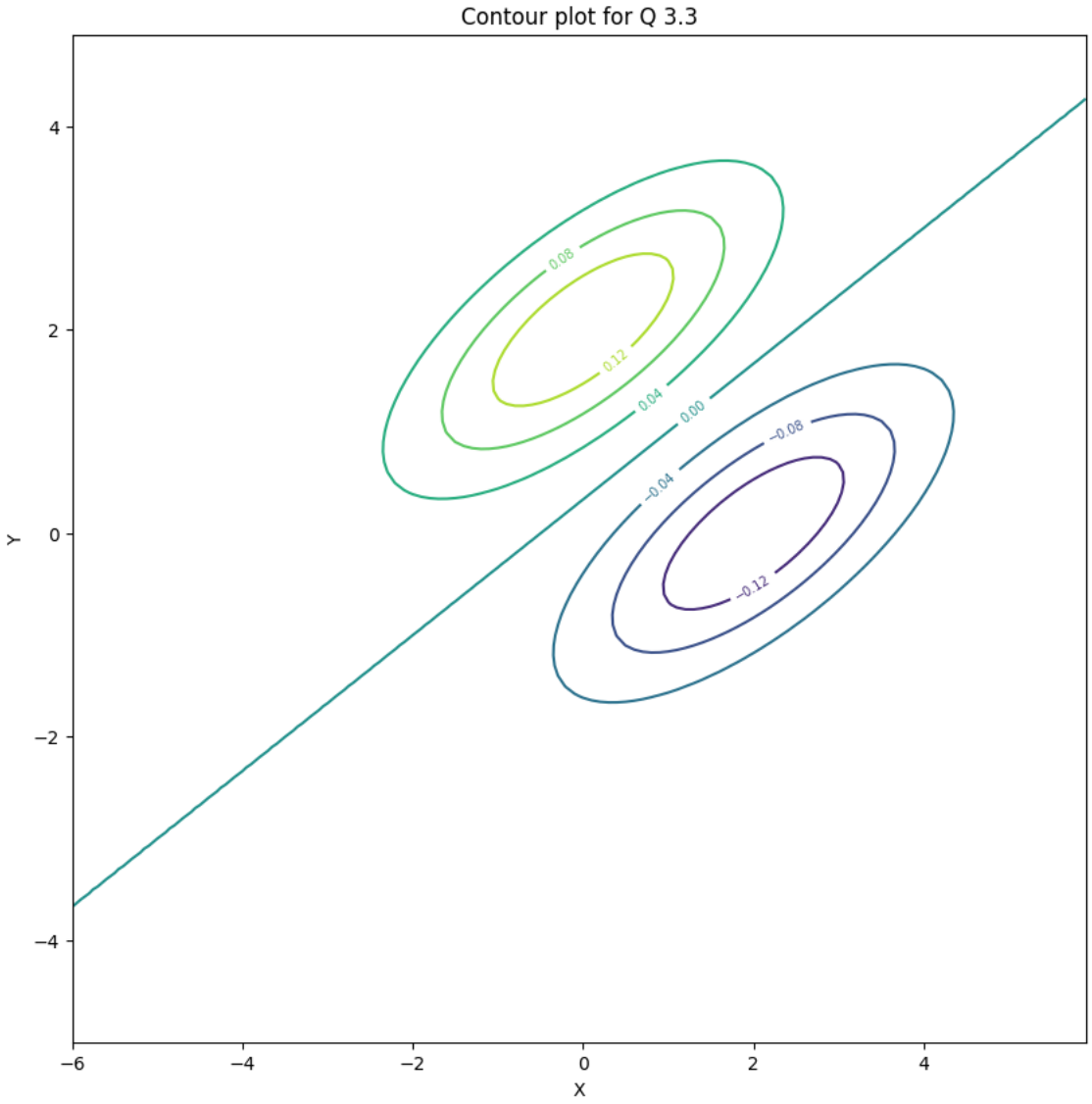
3.3

In [154]:
```
mu1 = np.array([0,2])
mu2 = np.array([2,0])
cov = np.array([[2, 1],[1, 1]])
```

In [155]:
```
mgrid = grid_gen(6,5,0.1)
```

In [156]:
```
z = f_eval(mgrid,mu1,cov)-f_eval(mgrid,mu2,cov)
```

In [157]:
```
contour('Q 3.3',f=z,grid=mgrid,flag=False)
```

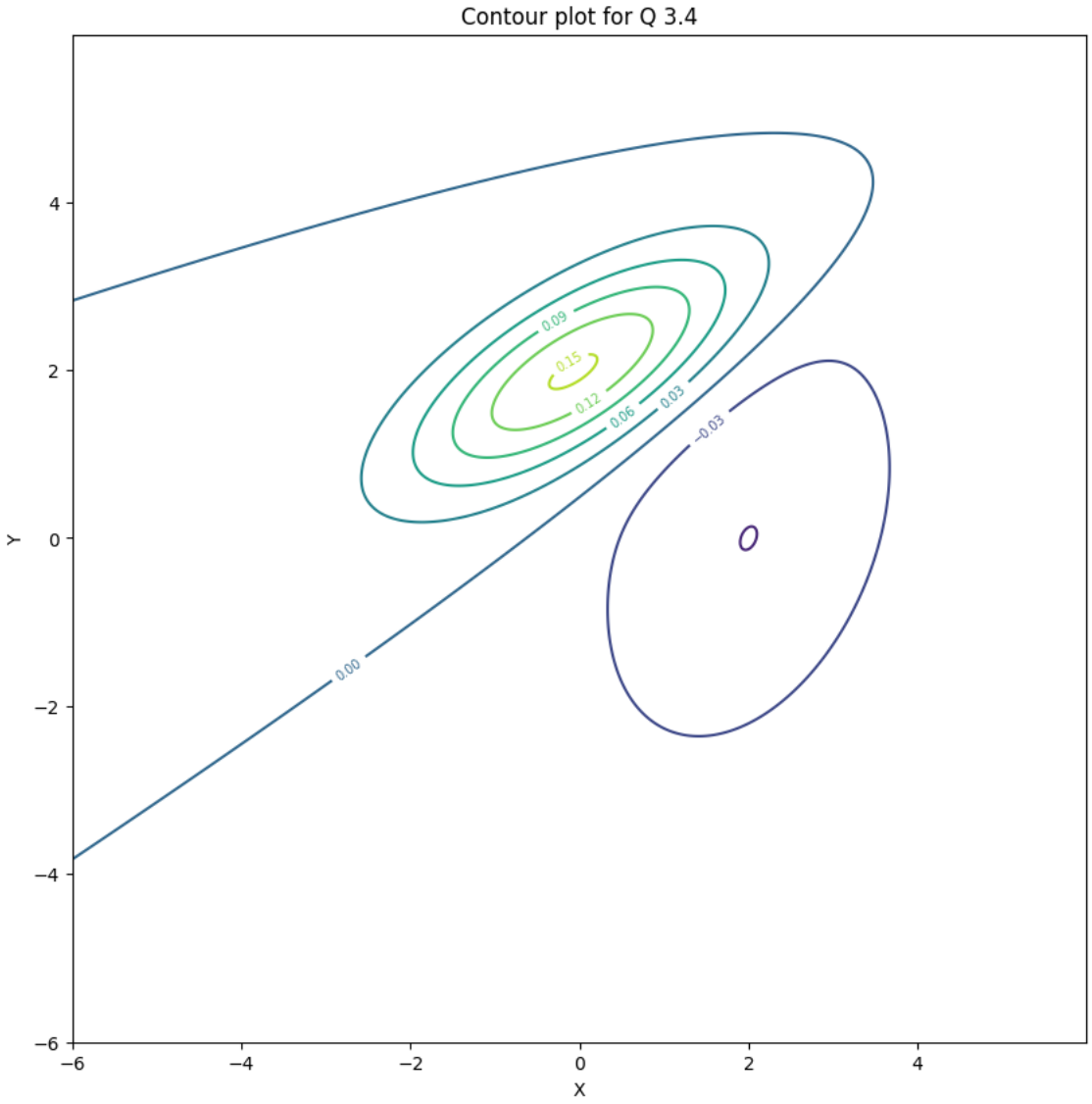


3.4

In [183]:
```
mu1 = np.array([0,2])
mu2 = np.array([2,0])
cov1 = np.array([[2, 1],[1, 1]])
cov2 = np.array([[2, 1],[1, 4]])
```

In [181]:
```
mgrid = grid_gen(6,6,0.01)
z = f_eval(mgrid,mu1,cov1)-f_eval(mgrid,mu2,cov2)
```

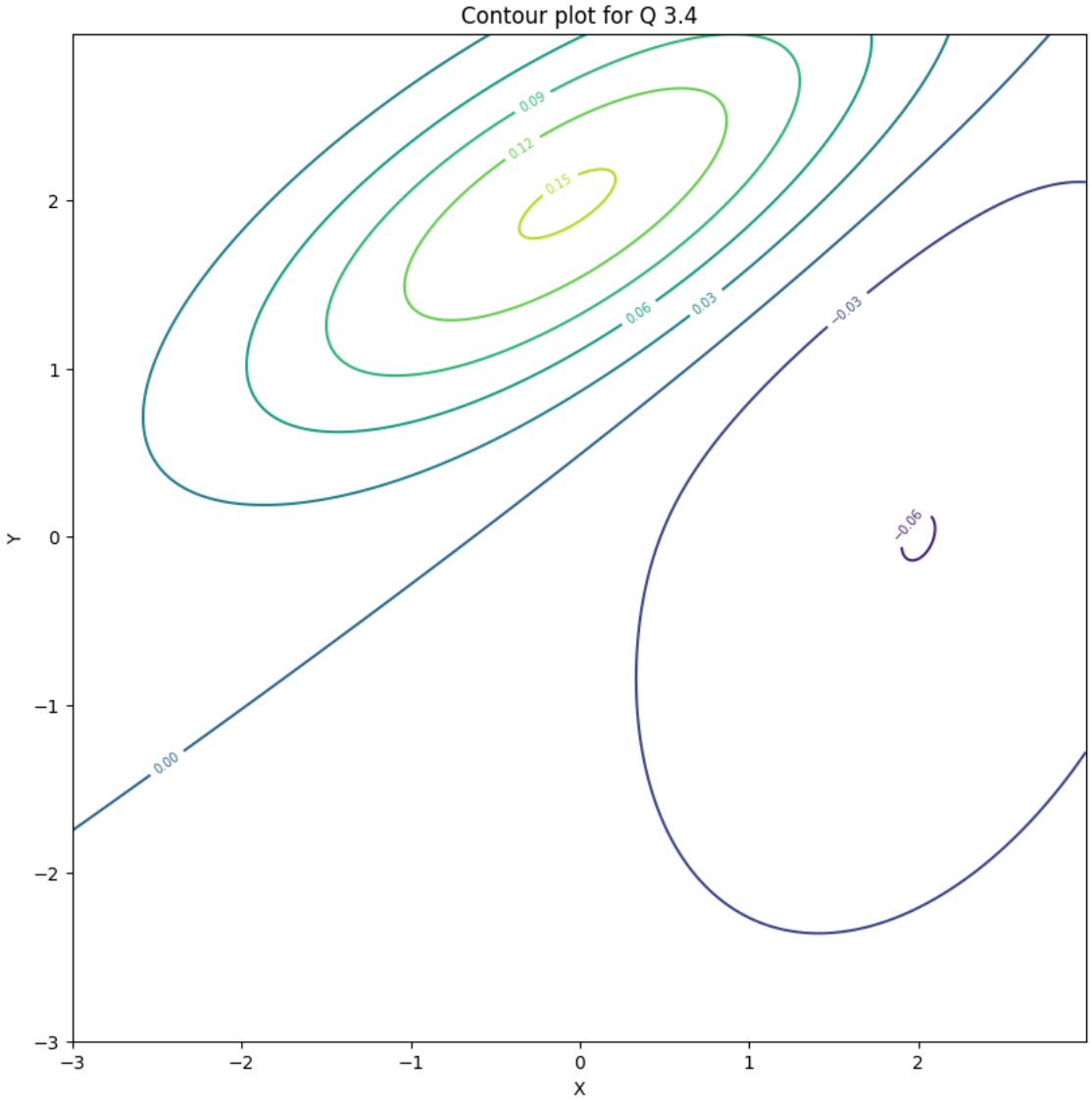
In [182]:
```
contour('Q 3.4',f=z,grid=mgrid,flag=False)
```



Zoom in to get the value for the one unlabeled contour

In [192]:
```
mgrid = grid_gen(3,3,0.01)
z = f_eval(mgrid,mu1,cov1)-f_eval(mgrid,mu2,cov2)
```

In [194]:
```
fig = plt.figure(figsize=(10,10))
cs = plt.contour(mgrid[0],mgrid[1],z)
plt.clabel(cs,inline_spacing=5, fontsize=7)
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Contour plot for Q 3.4')
# plt.savefig('Q 3.4_plot_zoom.png', dpi=300)
```

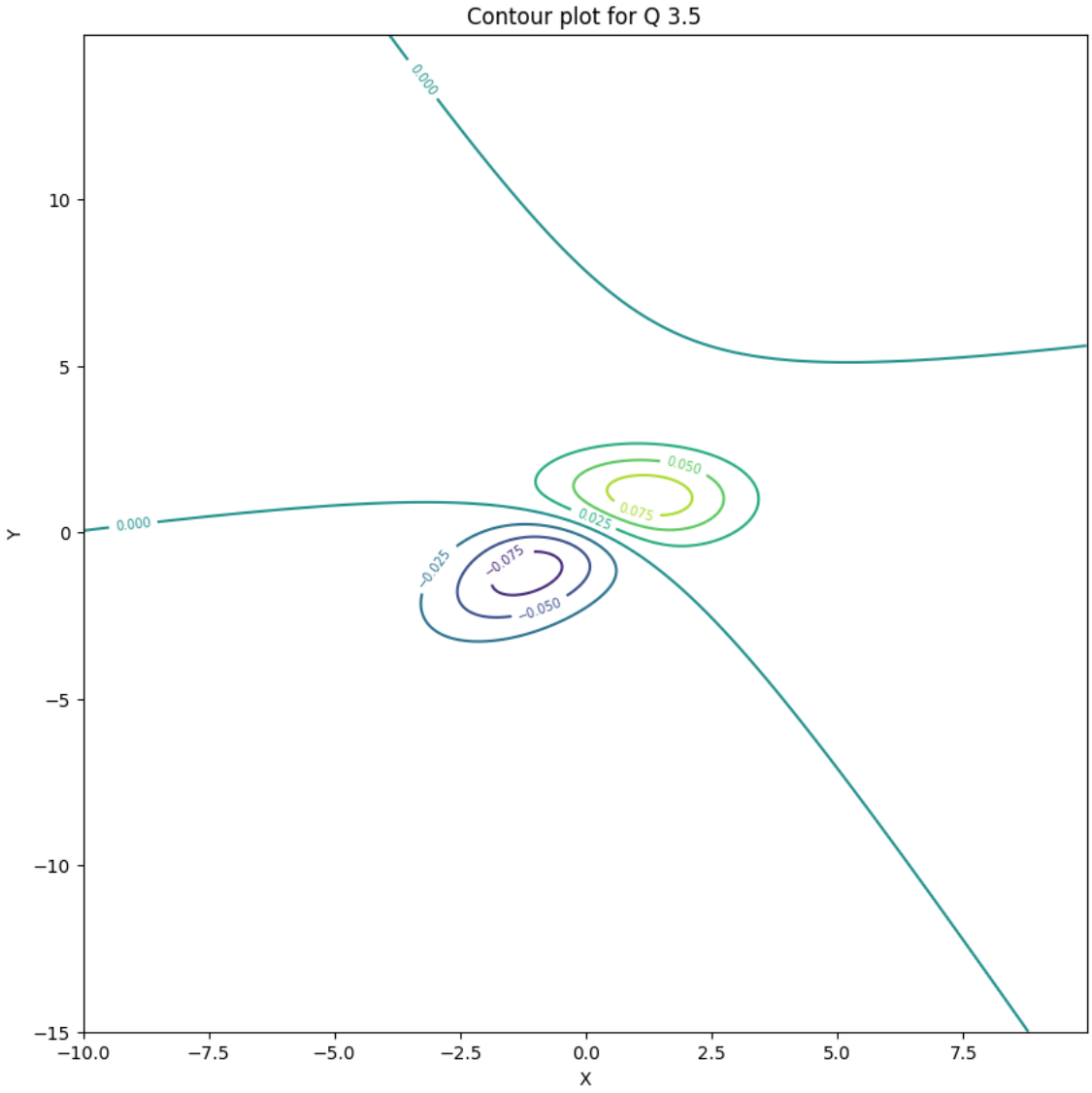


3.5

In [161]:
```
mu1 = np.array([1,1])
mu2 = np.array([-1,-1])
cov1 = np.array([[2, 0],[0, 1]])
cov2 = np.array([[2, 1],[1, 2]])
```

In [162]:
```
mgrid = grid_gen(10,15,0.05)
z = f_eval(mgrid,mu1,cov1)-f_eval(mgrid,mu2,cov2)
```

In [163]:
```
contour('Q 3.5',f=z,grid=mgrid,flag=False)
```



Q4

In [5]:
```
class tupl(object):
    def __init__(self,x,y):
        self.x = x
        self.y = y

    def __add__(self, other):
        return tupl(self.x+other.x,self.y+other.y)

    def __sub__(self, other):
        return tupl(self.x - other.x, self.y - other.y)

    def __rmul__(self, other):
        if isinstance(other, int) or isinstance(other, float):
            return tupl(other*self.x, other*self.y)

    def __repr__(self):
        return f"({self.x},{self.y})"
```

In [6]:
```
np.random.seed(42)
```

4.1

In [7]:
```
points = []

for i in range(100):
    x1=np.random.normal(3,3)
    points+=[tupl(x1 ,0.5*x1+np.random.normal(4,2))]
```

In [8]:
```
temp_sum=tupl(0,0)

for point in points:
    temp_sum+=point

points_mean = (1/len(points))*temp_sum
```

In [9]:
```
print('The mean in R2 is: ', points_mean)
```

```
The mean in R2 is:  (2.6533072356348306,5.394698266705852)
```

4.2

In [10]:
```
x1=[]
x2=[]

for point in points:
    x1+=[point.x]
    x2+=[point.y]

temp_matrix = np.array([x1,x2])
```

In [11]:
```
sigma = np.cov(temp_matrix)
```

In [12]:
```
print("The covariance matrix based off the samples is: \n\n ", sigma)
```

```
The covariance matrix based off the samples is:

  [[6.59945084 3.46560266]
 [3.46560266 5.80661362]]
```

4.3

In [633]:
```
eig_vals, eig_vecs = np.linalg.eig(sigma)
```

In [634]:
```
eig_vecs
```

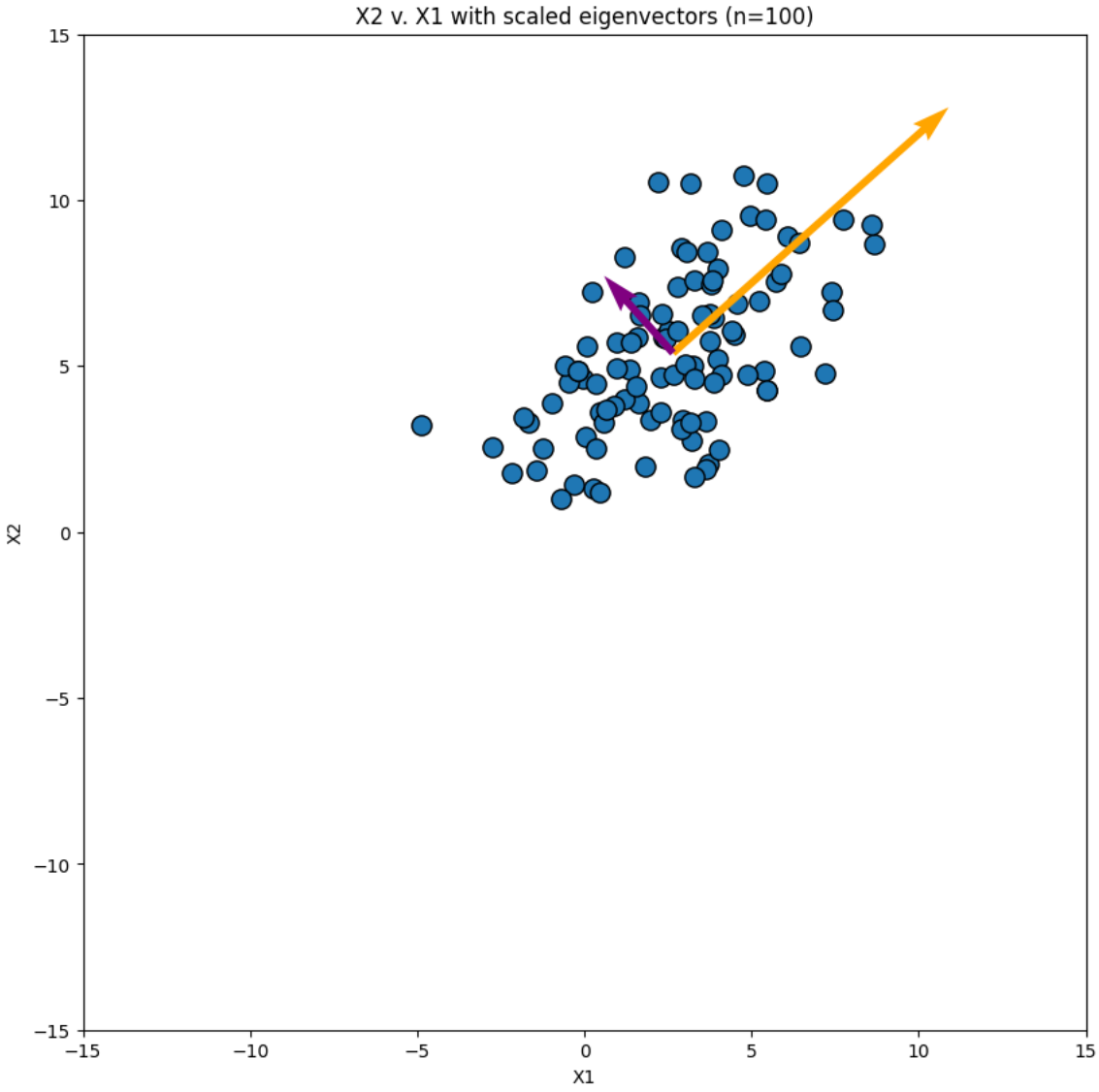
Out[634]:
```
array([[ 0.74620559, -0.66571557],
       [ 0.66571557,  0.74620559]])
```

In [638]:
```
eig_vals
```

Out[638]:
```
array([9.6912337 , 2.71483076])
```

4.4

In [694]:
```
plt.figure(figsize=(10,10))
xlim = [-15,15]
ylim = [-15,15]

plt.scatter(x1,x2,s=120, edgecolor='k')
plt.quiver(points_mean.x,points_mean.y, eig_vecs[0][0],eig_vecs[1][0],color='orange',scale=eig_vals[1])
plt.quiver(points_mean.x,points_mean.y, eig_vecs[0][1],eig_vecs[1][1],color='purple',scale=eig_vals[0])
plt.xlim(xlim)
plt.ylim(ylim)
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('X2 v. X1 with scaled eigenvectors (n=100)')
# plt.savefig('Q4.4_plot.png', dpi=300)
```

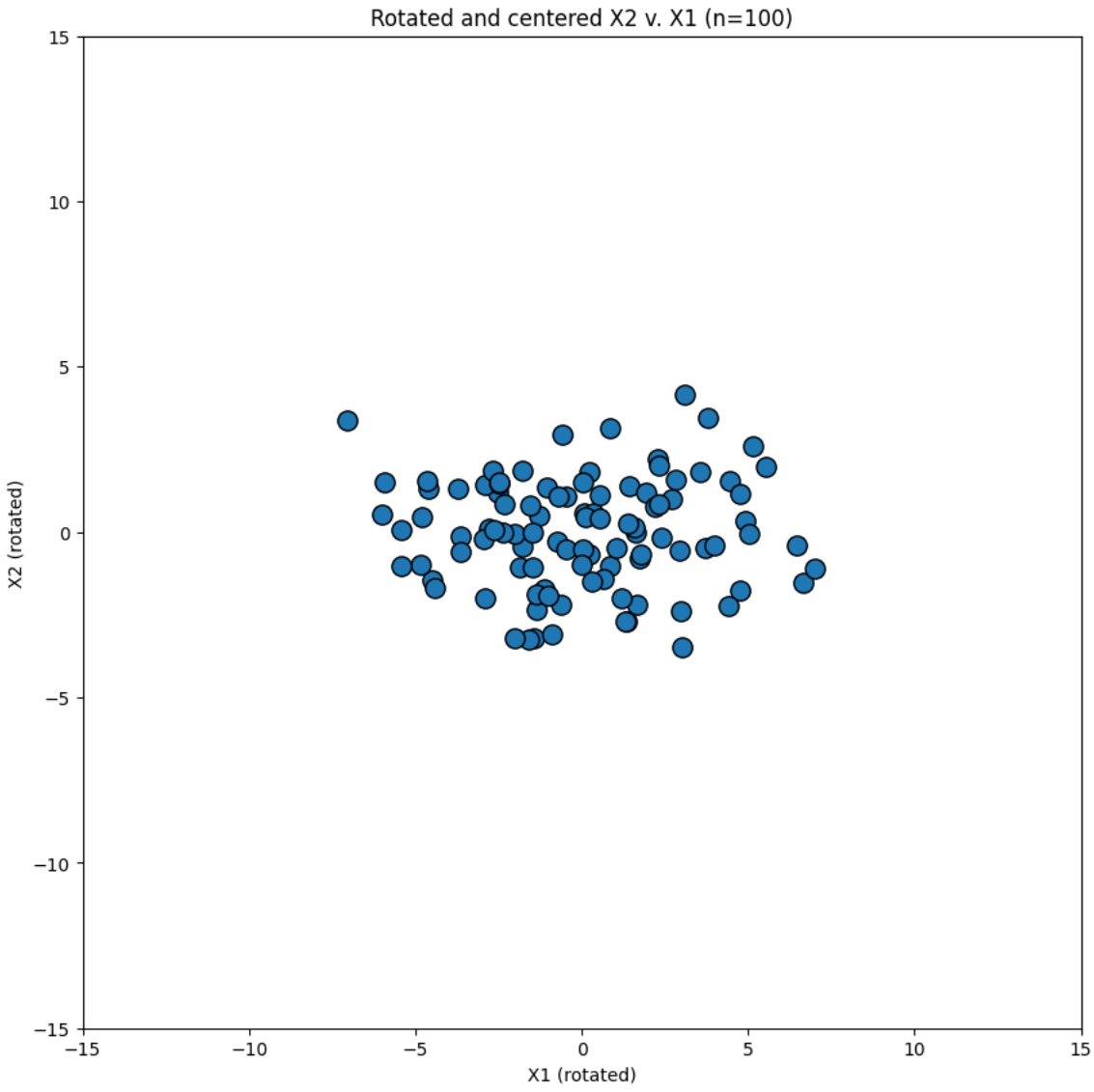


4.5

In [659]:
```
x_temp=[]

for point in points:

    x_temp+=[point-points_mean]
```

In [685]:
```
x_rot_x =[]
x_rot_y =[]

for i in range(len(x_temp)):
    x_rot_x+=[np.matmul(eig_vecs.T,np.array([x_temp[i].x,x_temp[i].y]))[0]]
    x_rot_y+=[np.matmul(eig_vecs.T,np.array([x_temp[i].x,x_temp[i].y]))[1]]
```

In [695]:
```
plt.figure(figsize=(10,10))
xlim = [-15,15]
ylim = [-15,15]

plt.scatter(x_rot_x,x_rot_y,s=120, edgecolor='k')
plt.xlim(xlim)
plt.ylim(ylim)
plt.xlabel('X1 (rotated)')
plt.ylabel('X2 (rotated)')
plt.title('Rotated and centered X2 v. X1 (n=100)')
# plt.savefig('Q4.5_plot.png', dpi=300)
```



Q8

8.1

In [144]:
```
mnist = np.load('data/mnist-data-hw3.npz')
```

In [145]:
```
mnist.files
```

Out[145]:
```
['training_data', 'training_labels', 'test_data']
```

In [146]:
```
mnist['training_data'].shape
```

Out[146]:
```
(60000, 1, 28, 28)
```

In [287]:
```
#Need to reshape the data to have the images be row vectors
mnist_data=mnist['training_data'].reshape(60000,784)
```

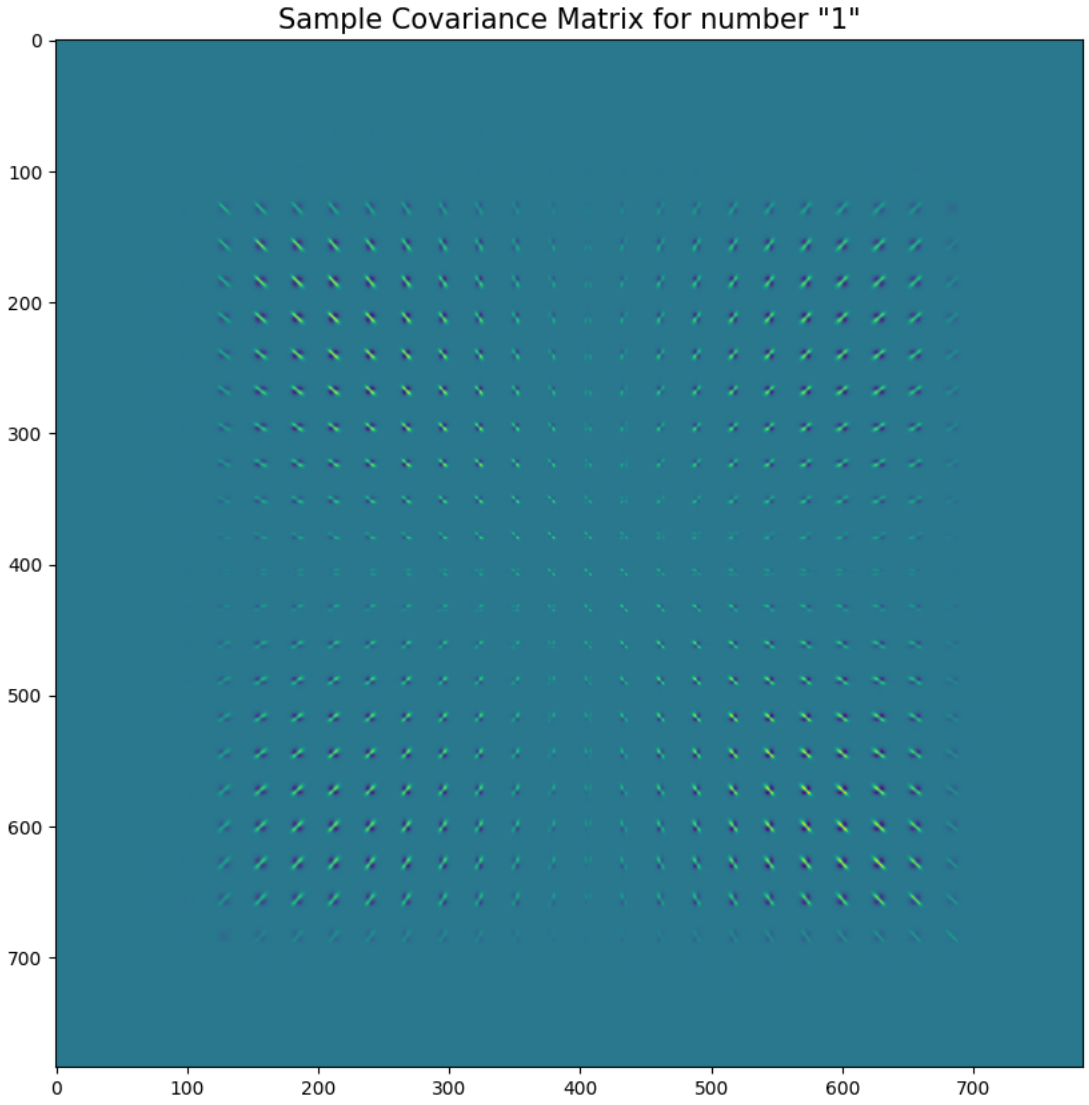
In [148]:
```
# contrast-normalize by the 2-norm of the pixels
for i in range(mnist_data.shape[0]):

    mnist_data[i] = 1/(np.linalg.norm(mnist_data[i])) * mnist_data[i]
```

In [299]:
```
#'data' needs to be an array with sample rows and column features
def fit_gaussians(data, labels):

    #Initialize dictionaries to hold the mean vectors and the covariance matrices for the different number classes
    mus = {}
    covs = {}

    #Step through each class
    for i in range(10):

        #create a temporary array with just the samples for class i; each sample is a row vector of pixels
        temp = data[labels==i,:]

        #get the sample covariance matrix for class i
        covs[i] = np.cov(temp, rowvar=False)

        #Initialize a list for the class's mean vector
        means = []

        #step through each column/feature, take its mean and add it to the list; the final list is the mean vector for the class
        for j in range(temp.shape[1]):
            means += [temp[:,j].mean()]

        #Add the class mean vector to the dictionary for the class means
        mus[i] = means

    return mus, covs
```

In [301]:
```
mus, covs = fit_gaussians(mnist_data,mnist['training_labels'])
```

8.2

In [303]:
```
plt.figure(figsize=(10,10))
plt.imshow(covs[1])
plt.title('Sample Covariance Matrix for number "1"', fontsize=15)
# plt.savefig('Q8.2_plot.png', dpi=300)
```



8.3

(a)

In [162]:
```
from sklearn.model_selection import train_test_split
from math import log,e
import random
```

In [452]:
```
def get_arrays(data,labels):

    # get the class means and the class covariance matrices
    mus, covs = fit_gaussians(data,labels)

    #Make a list of class priors calculate by dividing the number of class instances in the labels by the # of samples
    priors = []
    for i in range(10):

        priors += [sum(labels==i)/len(labels)]

    return mus, covs, priors
```

In [459]:
```
def get_pooled_inv(covs,labels):

    #calculate the class-pooled covariance matrix
    temp_matrix = np.zeros(covs[1].shape,dtype=float)
    for i in covs.keys():
        temp_matrix += covs[i]

    pooled_cov = 1/len(labels)*temp_matrix

    #Diagonally load pooled_cov to make it pd
    approx_cov = (1/len(labels)**4)*np.identity(covs[1].shape[0]) + pooled_cov

    #Get the inverse pooled pd covariance matrix
    inv_cov = np.linalg.inv(approx_cov)

    return inv_cov
```

In [460]:
```
def lda_model(training_data, training_labels, validation):

    mus, covs, priors = get_arrays(training_data, training_labels)

    inv_cov = get_pooled_inv(covs,training_labels)

    preds = []

    for i in range(validation.shape[0]):

        class_scores = []

        for j in range(len(priors)):

            proj = np.matmul(np.array(mus[j]),inv_cov)

            class_scores += [np.matmul(proj,validation[i])-0.5*np.matmul(proj,mus[j]) + log(priors[j])]

        preds+=[class_scores.index(max(class_scores))]

    return preds
```

In [548]:
```
def get_error(predictions, labels):
    score = 0

    dig_scores = dict.fromkeys(np.linspace(0,9,10,dtype=int),0)

    for i in range(len(predictions)):
        if predictions[i] == labels[i]:
            score+=1
            dig_scores[labels[i]]+=1

    dig_errors = {}
    for key in dig_scores.keys():

        dig_errors[key] = 1-dig_scores[key]/sum(labels==key)

    return (1 - (score/len(predictions))), dig_errors
```

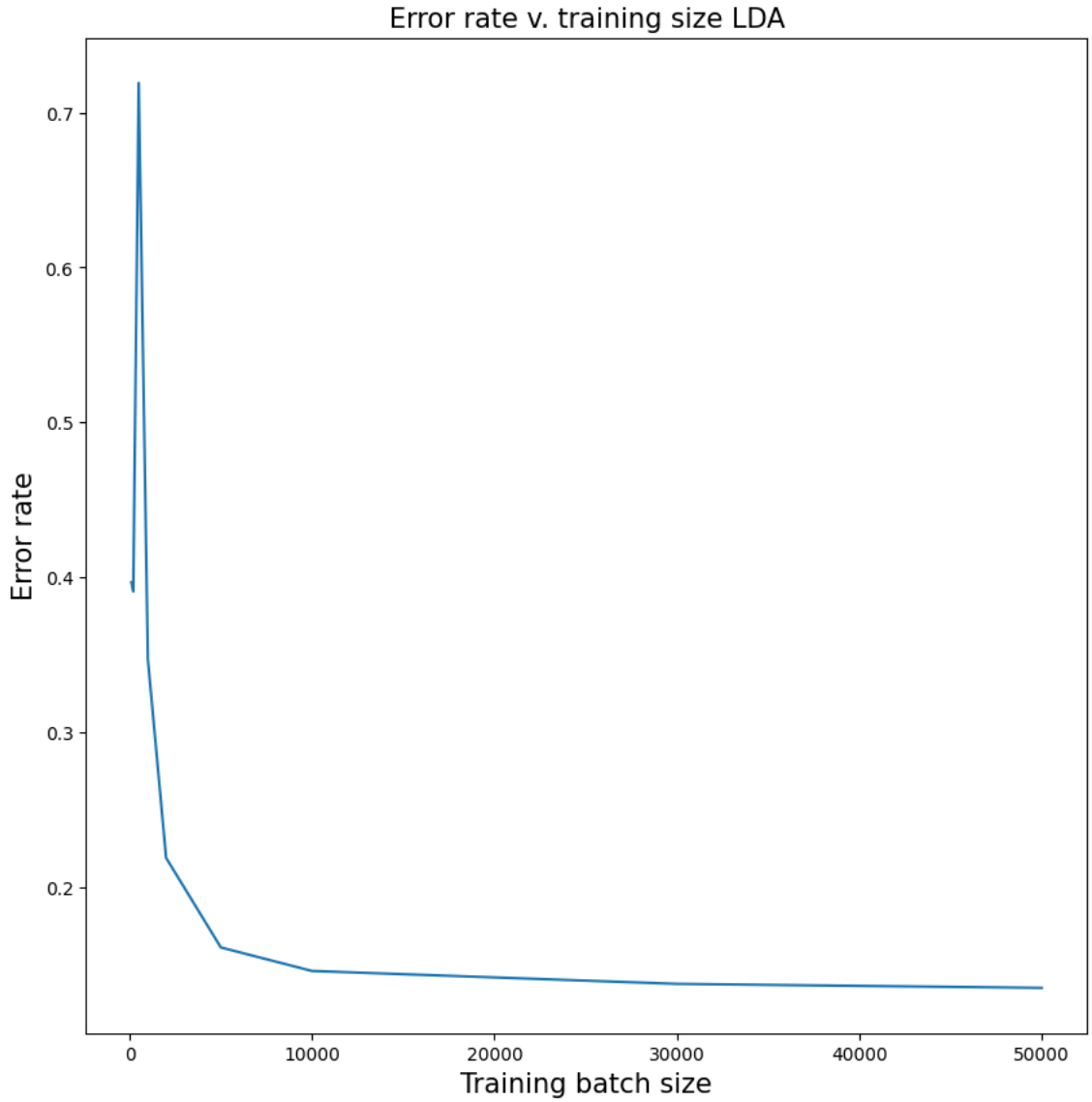
In [379]:
```
m_train, m_val, ml_train, ml_val = train_test_split(mnist_data,
                                                    mnist['training_labels'], test_size=1/6, random_state=42)
```

In [461]:
```
test = lda_model(m_train[:100],ml_train[:100], m_val)
```

In [416]:
```
training_n = [100,200,500,1000,2000,5000,10000,30000,50000]
```

In [549]:
```
# reset the random seed
random.seed(42)

# train on the different training set sizes
errors = []
dig_errors = []

for n in training_n:
    rlist=random.sample(range(0, len(m_train)), n)

    temp_train = m_train[rlist]
    temp_labels = ml_train[rlist]

    preds = lda_model(temp_train, temp_labels, m_val)

    model_error, dig_error = get_error(preds, ml_val)

    errors +=[model_error]
    dig_errors +=[dig_error]
```

In [550]:
```
plt.figure(figsize=(10,10))
plt.plot(training_n, errors)
plt.title('Error rate v. training size LDA', fontsize=15)
plt.xlabel('Training batch size', fontsize=15)
plt.ylabel('Error rate', fontsize=15)
# plt.savefig('Q8.3_plot.png', dpi=300)
```



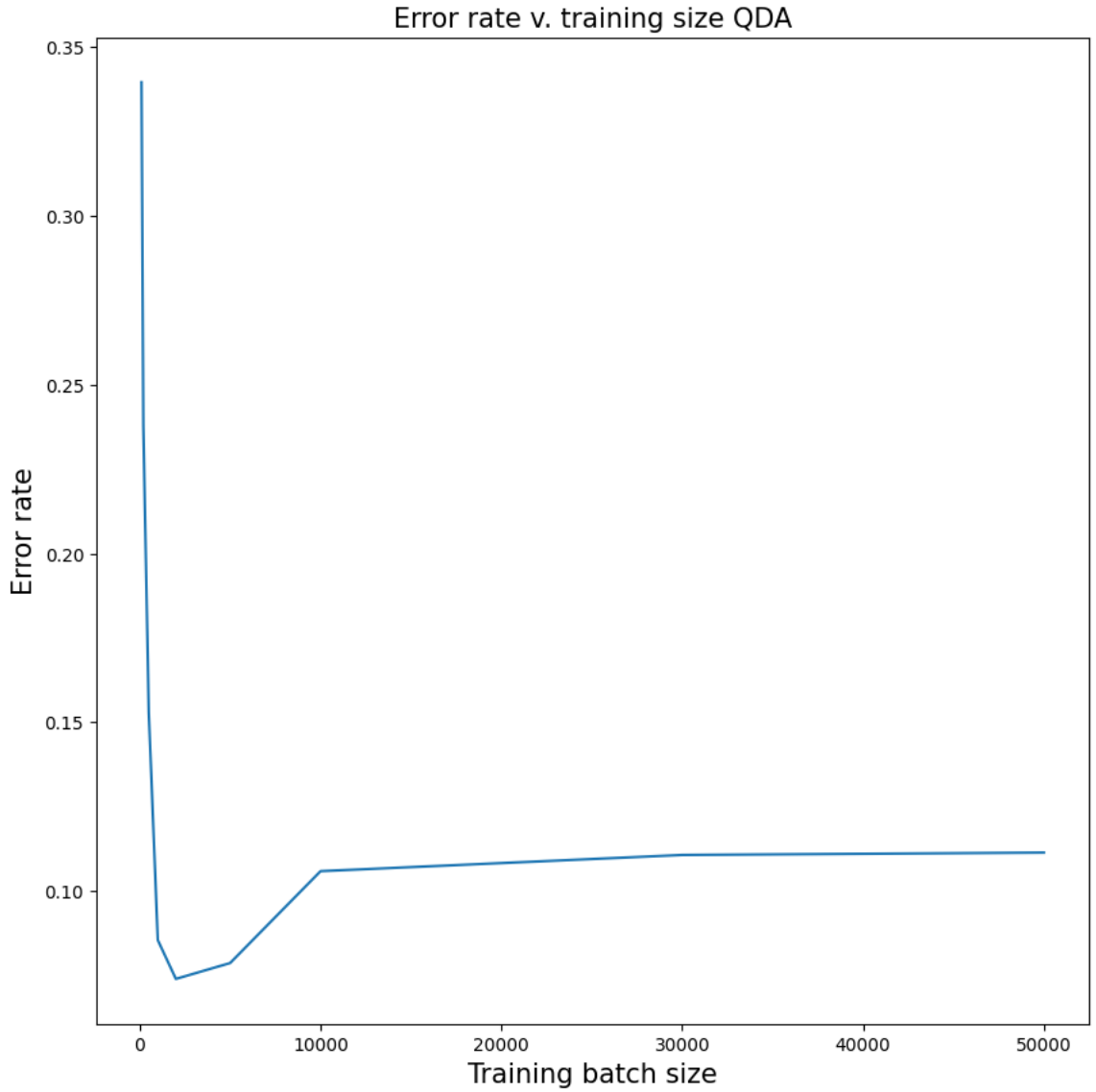
(b)

In [512]:
```
def diag_loading(covs,labels):

    #initialize libraries for the 'kludged' covariance matrices and their inverses for the different classes
    k_covs={}
    inv_k_covs={}

    for i in range(len(covs)):

        approx_cov = (1e-3*np.linalg.eig(covs[i])[0][0])*np.identity(covs[1].shape[0]) + covs[i]

        k_covs[i] = approx_cov

        inv_k_covs[i] = np.linalg.inv(approx_cov)

    return k_covs, inv_k_covs
```

In [496]:
```
def qda_model(data,labels,validation):

    mus, covs, priors = get_arrays(data, labels)

    covs, inv_covs = diag_loading(covs, labels)

    # Make a dictionary of the logs of the determinants of the covariance matrices so the calculations are done once
    dets = {}
    for i in range(len(covs)):
        dets[i] = np.linalg.slogdet(covs[i])

    preds = []

    for i in range(validation.shape[0]):

        class_scores = []

        for j in range(len(priors)):

            diff = validation[i] - mus[j]

            class_scores += [-0.5*np.matmul(np.matmul(diff,inv_covs[j]),diff) -0.5*dets[j][1] + log(priors[j])]

        preds+=[class_scores.index(max(class_scores))]

    return preds
```

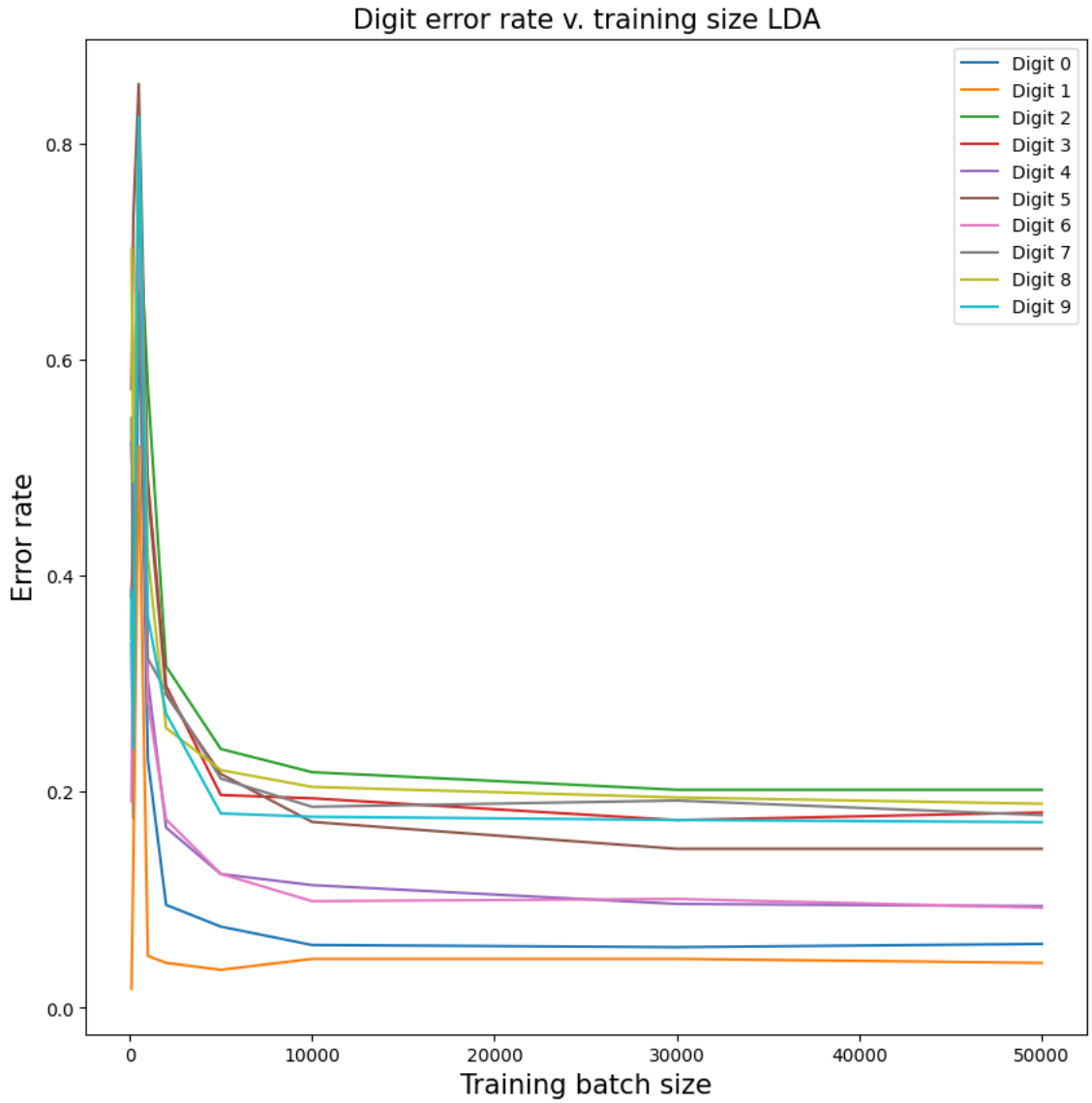
In [572]:
```
# reset the random seed
random.seed(42)

# train on the different training set sizes
qda_errors = []
qda_dig_errors = []

for n in training_n:
    rlist=random.sample(range(0, len(m_train)), n)

    temp_train = m_train[rlist]
    temp_labels = ml_train[rlist]

    preds = qda_model(temp_train, temp_labels, m_val)

    model_error, dig_error = get_error(preds, ml_val)

    qda_errors +=[model_error]
    qda_dig_errors +=[dig_error]
```

In [514]:
```
plt.figure(figsize=(10,10))
plt.plot(training_n, qda_errors)
plt.title('Error rate v. training size QDA', fontsize=15)
plt.xlabel('Training batch size', fontsize=15)
plt.ylabel('Error rate', fontsize=15)
# plt.savefig('Q8.3b_plot.png', dpi=300)
```

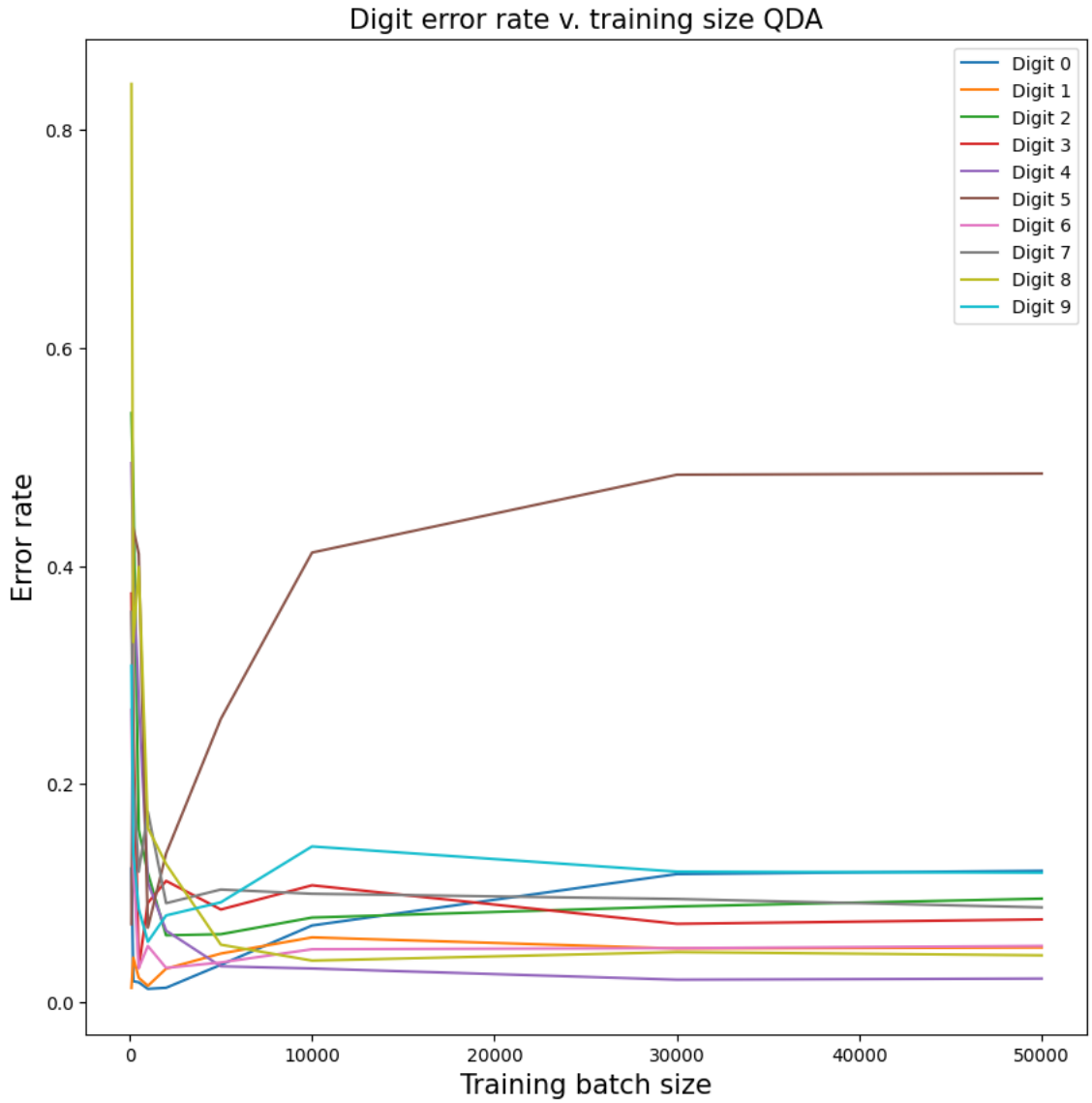


(d)

In [571]:
```
plt.figure(figsize=(10,10))
plt.plot(training_n, [dig_errors[i][0] for i in range(len(dig_errors))], label='Digit 0')
plt.plot(training_n, [dig_errors[i][1] for i in range(len(dig_errors))], label='Digit 1')
plt.plot(training_n, [dig_errors[i][2] for i in range(len(dig_errors))], label='Digit 2')
plt.plot(training_n, [dig_errors[i][3] for i in range(len(dig_errors))], label='Digit 3')
plt.plot(training_n, [dig_errors[i][4] for i in range(len(dig_errors))], label='Digit 4')
plt.plot(training_n, [dig_errors[i][5] for i in range(len(dig_errors))], label='Digit 5')
plt.plot(training_n, [dig_errors[i][6] for i in range(len(dig_errors))], label='Digit 6')
plt.plot(training_n, [dig_errors[i][7] for i in range(len(dig_errors))], label='Digit 7')
plt.plot(training_n, [dig_errors[i][8] for i in range(len(dig_errors))], label='Digit 8')
plt.plot(training_n, [dig_errors[i][9] for i in range(len(dig_errors))], label='Digit 9')
plt.title('Digit error rate v. training size LDA', fontsize=15)
plt.xlabel('Training batch size', fontsize=15)
plt.ylabel('Error rate', fontsize=15)
plt.legend()
# plt.savefig('Q8.3d_plot_LDA.png', dpi=300)
```



In [573]:
```
plt.figure(figsize=(10,10))
plt.plot(training_n, [qda_dig_errors[i][0] for i in range(len(qda_dig_errors))], label='Digit 0')
plt.plot(training_n, [qda_dig_errors[i][1] for i in range(len(qda_dig_errors))], label='Digit 1')
plt.plot(training_n, [qda_dig_errors[i][2] for i in range(len(qda_dig_errors))], label='Digit 2')
plt.plot(training_n, [qda_dig_errors[i][3] for i in range(len(qda_dig_errors))], label='Digit 3')
plt.plot(training_n, [qda_dig_errors[i][4] for i in range(len(qda_dig_errors))], label='Digit 4')
plt.plot(training_n, [qda_dig_errors[i][5] for i in range(len(qda_dig_errors))], label='Digit 5')
plt.plot(training_n, [qda_dig_errors[i][6] for i in range(len(qda_dig_errors))], label='Digit 6')
plt.plot(training_n, [qda_dig_errors[i][7] for i in range(len(qda_dig_errors))], label='Digit 7')
plt.plot(training_n, [qda_dig_errors[i][8] for i in range(len(qda_dig_errors))], label='Digit 8')
plt.plot(training_n, [qda_dig_errors[i][9] for i in range(len(qda_dig_errors))], label='Digit 9')
plt.title('Digit error rate v. training size QDA', fontsize=15)
plt.xlabel('Training batch size', fontsize=15)
plt.ylabel('Error rate', fontsize=15)
plt.legend()
# plt.savefig('Q8.3d_plot_QDA.png', dpi=300)
```



8.4

In [579]:
```
#Need to reshape the data to have the images be row vectors
test=mnist['test_data'].reshape(10000,784)
```

In [580]:
```
rlist=random.sample(range(0, len(m_train)), 200)
temp_train = m_train[rlist]
temp_labels = ml_train[rlist]
```

In [582]:
```
kaggle_preds = qda_model(temp_train, temp_labels, test)
```

In [585]:
```
df = pd.DataFrame({'Id': np.linspace(1,10000,10000,dtype=int), 'Category': kaggle_preds},dtype=np.int64)
df.to_csv('mnist_preds.csv',index=False)
```

8.5

In [588]:
```
spam = np.load('data/spam-data-hw3.npz')
```

In [589]:
```
spam.files[]
```

Out[589]:
```
['training_data', 'training_labels', 'test_data']
```

In [590]:
```
import os
```

In [591]:
```
def get_freqs(path):

    files = os.listdir(path)
    word_freqs = {}

    for file in files:
        with open('{}/{}'.format(path,file), encoding='utf8', errors='ignore') as f:
            try:
                lines = f.readlines() # Read in text from file
            except Exception as e:
                    # skip files we have trouble reading.
                continue

        for i in range(len(lines)):
            words=lines[i].split()

            for word in words:
                if word not in word_freqs.keys():
                    word_freqs[word] = 1
                elif word in word_freqs.keys():
                    word_freqs[word] += 1

    word_freqs = sorted(word_freqs.items(), key=lambda x:x[1],reverse=True)
    return word_freqs
```

In [597]:
```
spam_freqs=get_freqs('data/spam')
```

In [598]:
```
ham_freqs=get_freqs('data/ham')
```

In [599]:
```
spam_words=[spam_freqs[:200][i][0] for i in range(200)]
spam_freqs=[spam_freqs[:200][i][1] for i in range(200)]
```

In [600]:
```
ham_words=[ham_freqs[:200][i][0] for i in range(200)]
ham_freqs=[ham_freqs[:200][i][1] for i in range(200)]
```

In [601]:
```
# add these all as features
additions=list(set(spam_words)^set(ham_words))
```

In [602]:
```
additions
```

```
 '20',
 'more',
 'change',
 'free',
 'today',
 '11',
 'statements',
 'software',
 'without',
 'over',
 'office',
 '>',
 'daily',
 'o',
 'texas',
 '69',
 'april',
 'products',
 'cc',
 'robert',
```

In [611]:
```
for word in additions:
    print('def freq_{}_feature(text, freq):\n    return float(freq[\'{}\'])'.format(word,word))
```

```
    return float(freq['hpl'])
def freq_style_feature(text, freq):
    return float(freq['style'])
def freq_nd_feature(text, freq):
    return float(freq['nd'])
def freq_energy_feature(text, freq):
    return float(freq['energy'])
def freq_volume_feature(text, freq):
    return float(freq['volume'])
def freq_than_feature(text, freq):
    return float(freq['than'])
def freq_b_feature(text, freq):
    return float(freq['b'])
def freq_http_feature(text, freq):
    return float(freq['http'])
def freq_prices_feature(text, freq):
    return float(freq['prices'])
def freq_adobe_feature(text, freq):
    return float(freq['adobe'])
```

In [612]:
```
for word in additions:
    print('feature.append(freq_{}_feature(text, freq))'.format(word))
```

```
feature.append(freq_microsoft_feature(text, freq))
feature.append(freq_50_feature(text, freq))
feature.append(freq_size_feature(text, freq))
feature.append(freq_forward_feature(text, freq))
feature.append(freq_2001_feature(text, freq))
feature.append(freq_best_feature(text, freq))
feature.append(freq_align_feature(text, freq))
feature.append(freq_only_feature(text, freq))
feature.append(freq_forwarded_feature(text, freq))
feature.append(freq_contract_feature(text, freq))
feature.append(freq_31_feature(text, freq))
feature.append(freq_ticket_feature(text, freq))
feature.append(freq_its_feature(text, freq))
feature.append(freq_g_feature(text, freq))
feature.append(freq_xp_feature(text, freq))
feature.append(freq_let_feature(text, freq))
feature.append(freq_go_feature(text, freq))
feature.append(freq_xls_feature(text, freq))
feature.append(freq_other_feature(text, freq))
feature.append(freq_www_feature(text, freq))
```