Comments on binomial graph learning

Binary graph learning model

Version 1 (Eigenvector matrix from Graph Laplacian)

Let $Y_{i,j}$ denote the measurement on the node i at round j, where $j = 1, \ldots, M$, and $i = 1, \ldots, N$. $Y_{i,j}$ is a binomial signal that can be 1, or 0. Suppose the signals at round j denoted by $Y[, j]$ for all N nodes are independent of the signals at round k denoted by $Y[, k]$, for $i \neq k$. Let $p_{i,j}$ denote the probability of $Y_{i,j} = 1$. Our model assumes

$$\text{logit}(p_{i,j}) = \alpha_j + (\chi h)_i,$$

where χ is the eigenvector matrix from Graph Laplacian L, h is a vector of latent factors that governs $p_{i,j}$ through χ, and α_j is a round specific parameter at round j.

Version 2 (Adjacency matrix from Graph)

Graph We consider a weighted undirected graph $G = (V, E)$, with the vertices set $V = 1, 2, \ldots, N$, and edge set E. Let $\mathbf{A}$ denote the weighted adjacency matrix for the graph G. In the case of weighted undirected graph, $\mathbf{A}$ is a square and symmetric matrix.

Signals on the graph Let $Y_{i,j}$ denote the signal on the node i of graph G at round j, where $j = 1, \ldots, M$, and $i = 1, \ldots, N$. We assume that $Y_{i,j}$ is a binary signal that can be 1, or 0. Suppose the signals at round j denoted by $Y[, j]$ for all N nodes are independent of the signals at round (or strata) k denoted by $Y[, k]$, for $i \neq k$, borrowing the idea of conditional logistic regression. Let $p_{i,j}$ denote the probability of $Y_{i,j} = 1$. Our model assumes

$$\text{logit}(p_{i,j}) = \alpha_j + (\mathbf{A}h)_i, \tag{1}$$

where $\mathbf{A}$ is the adjacency matrix from the graph G, h is a vector of latent factors that governs $p_{i,j}$ through $\mathbf{A}$, and α_j is a round specific parameter at round or strata j. In the following, we will use

Method of Estimation

The conditional likelihood function for model (1) given is (I will update soon)

Miscellaneous

Constraints:

- Case χ: Here I imagine the constraints are on the Laplacian

$$\begin{aligned} &\text{tr}(L) = N, \\ &L_{i,j} = L_{j,i} \leq 0, \quad i \neq j, \\ &L \cdot \mathbf{1} = \mathbf{0} \end{aligned} \tag{2}$$

- Case with adjacency matrix:

$$
\begin{aligned}
&A_{i,j} = 0 \text{ if } i = j, \\
&A_{i,j} = L_{j,i} \leq 0, \quad i \neq j
\end{aligned} \tag{3}
$$

Sigmoid computation, Bound and Branch method of optimization

hint for factor analysis solution might be useful because there is another object function varimax.

https://conservancy.umn.edu/bitstream/handle/11299/95957/Choi_umn_0130E_11451.pdf?sequence=1&isAllowed=y

https://web.stanford.edu/~boyd/papers/pdf/max_sum_sigmoids.pdf

- complete separation do not exist. observable metric ?, it is a potential new topic to be able to identify whether complete separation exists. possible discuss it in the Discussion and future work.
- While working on this one, you may also consider implementing our regressor paper.

Model Specification

Our goal is to estimate the Graph Laplacian

- We do not have to use the eigenvector matrix χ. Instead, we may consider adjacency matrix A in place of χ. This is a viable direction.

Maximum Likelihood estimation, Quasi likelihood estimation,