Матан 4

$$2yy'' - 3(y')^2 = 4y^2$$

$$y' = p(y)$$

$$y'' = pp'$$

$$2yp'p - 3p^2 = 4y^2 \quad \Big| \frac{1}{2yp}$$

$$p' - \frac{3p}{2y} = \frac{2y}{p}$$

$$pp' - \frac{3p^2}{2y} = 2y$$

$$z = p^2$$

$$z' = 2p\ p'$$

$$\frac{1}{2}z' - \frac{3z}{2y} = 2y$$

$$z' - 3\frac{z}{y} = 4y \quad \text{лин.}$$

1) $z' = 3\frac{z}{y}$

$$z = Cy^3$$

2) $C = C(y)$

$$C'(y)y^3 + C(y)3y^2 - \frac{3Cy^3}{y} = 4y$$

$$C'(y)y^3 = 4y$$

$$C = -\frac{4}{y} + \tilde{C} \qquad z = \left(C - \frac{4}{y}\right)y^3$$

$$p^2 = -4y^2 + Cy^3 \qquad C = 8$$

$$p = 2\sqrt{2y^3 - y^2}$$

$$y' = 2\sqrt{2y^3 - y^2}$$

$$\int \frac{dy}{2\sqrt{2y^3-y^2}} = \int dx$$

$$\int \frac{dy}{2\sqrt{2y^3-y^2}} = \left| \begin{array}{ll} u = \sqrt{2y-1} & y = \frac{u^2+1}{2} \\ \frac{du}{dy} = \frac{1}{\sqrt{2y-1}} & dy = \sqrt{2y-1}\, du \end{array} \right| =$$

$$= \int \frac{\sqrt{2y-1}\, du}{2 \cdot \frac{u^2+1}{2} \sqrt{2y-1}} = \int \frac{du}{u^2+1} = \operatorname{arctg} u + C = \operatorname{arctg} \sqrt{2y-1}$$

$$\operatorname{arctg}\sqrt{2y-1} + C = x$$

$$C = -\frac{\pi}{4}$$

$$x = \operatorname{arctg}\sqrt{2y-1} - \frac{\pi}{4}$$

$$y = 0$$

№4.2 $y'' + (y')^2 = 4y'(e^y+1)^3$

$$y' = p$$

$$y'' = p'p$$

$$p'p + p^2 = 4p(e^y+1)^3 \qquad p = 0 \quad \underline{y = \text{const}}$$

$p' + p = 4(e^y+1)^3$ линейное

$$\frac{dp}{dy} + p = 0$$

$$\frac{dp}{p} = -dy$$

$$\ln|p| = -y + C$$

$\ln|p| = -y + C$

$p = Ce^{-y}$

$p = C(y)e^{-y}$

$C'(y)e^{-y} = 4(e^y+1)^3$

$\frac{dC}{dy} = 4(e^y+1)^3 e^y$

$C = (e^y+1)^4 + \tilde{C}$

$p = ((e^y+1)^4 + C)e^{-y} = y'$ $\qquad y(0)=0$, $y'(0)=16$

$16 = (1+1)^4 + C \quad C=0$

$p = e^{-y}(e^y+1)^4$

$\int \frac{dy}{e^{-y}(e^y+1)^4} = \int dx$

$\int \frac{du}{(u+1)^4} = \int dx \qquad -\frac{1}{3}(u+1)^{-3} = x + C$

$-\frac{1}{3}(e^y+1)^{-3} = x + C$

$-\frac{1}{3}(1+1)^{-3} = 0 + C$

$-\frac{1}{24} = C$

$x = -\frac{1}{3}(e^y+1)^4 + \frac{1}{24}$

№4.3

$y'' = \frac{y'}{x} - \frac{1}{2y'}$

$p = y'(x) \qquad p' = \frac{p}{x} - \frac{1}{2p} \qquad p \neq 0$

$y'' = (y'(x))' = p' \qquad pp' - \frac{p^2}{x} - \frac{1}{2} = 0$

$z = p^2$

$\frac{1}{2} z' - \frac{z}{x} - \frac{1}{2} = 0$

$z' = \frac{2z}{x} - 1 \qquad$ лин.

$\frac{dz}{dx} = \frac{2z}{x}$

$\frac{dz}{z} = 2\frac{dx}{x} \qquad z = C x^2$

$C'(x) x^2 + C(x) 2x = \frac{2 C x^2}{x} - 1$

$\frac{dC}{dx} = \frac{-1}{x^2}$

$dC = -\frac{dx}{x^2}$

$C = \frac{1}{x} + \tilde{C}$

$z = x + x^2 C \qquad y(1) = \frac{2}{3}$

$p^2 = x + x^2 C \qquad y'(1) = 1$

$1 = 1 + C \quad C = 0$

$y' = \sqrt{x}$

$\frac{dy}{dx} = x^{1/2}$

$y = \frac{2}{3} x^{3/2} + C \qquad \frac{2}{3} = \frac{2}{3} + C \quad C = 0$

$y = \frac{2}{3} x^{3/2}$

$y'' \quad (y')^2$

4.4. $y'' + \frac{(y')^2}{y} = y'(4+y^2)$

Нет

$p(y) = y'$

$y'' = pp'$

$pp' + \frac{p^2}{y} = p(4+y^2)$ $\qquad p=0 \quad y=const$

$\underbrace{p' + \frac{p}{y}} = 4+y^2$ - линейное

$\frac{dp}{dy} = -\frac{p}{y}$

$\frac{dp}{p} = -\frac{dy}{y}$ $\qquad p = \frac{C}{y}$ $\qquad p' = \frac{C'(y)y - C(y)}{y^2} =$

$= \frac{C'(y)}{y} - \frac{C(y)}{y^2}$

$\frac{C'(y)}{y} - \frac{C(y)}{y^2} + \frac{C(y)}{y^2} = 4+y^2$

$\frac{C'(y)}{y} = 4+y^2$

$dC = (4y+y^3)\,dy$

$C = 2y^2 + \frac{y^4}{4} + \tilde{C}$ $\qquad y(0)=1$

$y'(0) = \frac{25}{4}$

$p = \left(2y^2 + \frac{y^4}{4} + C\right) \cdot \frac{1}{y} = y'$

$\frac{25}{4} = \left(2 + \frac{1}{4} + C\right)$

$\frac{16}{4} = C = 4$

$p = \left(2y^2 + \frac{y^4}{4} + 4\right) \cdot \frac{1}{y} = 2y + \frac{y^3}{4} + \frac{4}{y}$

$$\int \frac{dy}{2y + \frac{y^3}{4} + \frac{4}{y}} = \int dx$$

$$\int \frac{4y\,dy}{8y^2 + y^4 + 16} = x + C$$

$$2\int \frac{du}{8u + u^2 + 16} = x + C$$

$$2\int \frac{du}{(u+4)^2} = x + C$$

$$-\frac{2}{y^2+4} = x + C \qquad y(0) = 1$$

$$\frac{-2}{5} = C \qquad C = -\frac{2}{5}$$

$$x = \frac{-2}{y^2+4} + \frac{2}{5}$$

№4.6

$$xy'' + y' = \ln x$$

$$p = y'$$

$$xp' + p = \ln x$$

$$p' + \frac{p}{x} = \frac{\ln x}{x}$$

$$p = u \cdot v$$

$$\frac{du}{dx}v + u\frac{dv}{dx} + \frac{u \cdot v}{x} = \frac{\ln x}{x}$$

$$v\left(\frac{du}{} + \frac{u}{}\right) + u\frac{dv}{} = \frac{\ln x}{}$$

$$\frac{du}{u} = -\frac{dx}{x} \qquad \frac{dv}{x\,dx} = \frac{\ln x}{x}$$

$$u = \frac{C}{x} \qquad u_1 = \frac{1}{x} \qquad dv = \ln x \, dx$$

$$v = x(\ln x - 1) + C$$

$$p = uv = \ln x - 1 + \frac{C}{x} \qquad p = y'$$

$$y(1) = 0$$
$$y'(1) = 1$$

$$1 = 0 - 1 + \frac{C}{1}$$

$$C = 2$$

$$y' = \ln x - 1 + \frac{2}{x}$$

$$y = x(\ln x - 1) - x - \frac{2}{x^2} + C \qquad y = x(\ln x - 2) - \frac{2}{x^2} + 4$$

$$0 = 1(0-1) - 1 - 2 + C$$

$$C = 4$$

№4.7

$$y'' = \frac{y'}{x}\left(1 + \ln\frac{y'}{x}\right) \qquad p = y'$$

$$p' = \frac{p}{x}\left(1 + \ln\frac{p}{x}\right)$$

$$t = \frac{p}{x} \qquad p = tx$$
$$p' = t'x + t$$

$$t'x + \cancel{t} = \cancel{t} + t\ln t$$

$$t'x = t\ln t$$

$$\frac{dt}{dx}x = t\ln t$$

dx

$$\frac{dt}{t\ln t} = \frac{dx}{x}$$

$$\int \frac{d\ln t}{\ln t} = \ln x\, C \qquad \ln\ln t = \ln x\, C$$

$$x = C \ln t$$

$$Cx = \ln \frac{p}{x}$$

$$e^{Cx} = \frac{p}{x}$$

$$y(1) = \frac{1}{2}$$

$$y'(1) = 1$$

$$e^{C} = 1$$

$$C = 0$$

$$1 = \frac{p}{x} \qquad p = x$$

$$y = \frac{x^2}{2} + C$$

$$y = \frac{x^2}{2}$$

№ 4.9

$$y'' - (y')^2 \operatorname{tg} y = 2y' \sin y \qquad y\left(\frac{2}{3}\right) = 0$$

$$y'\left(\frac{2}{3}\right) = 1$$

$$y' = p(y)$$

$$y'' = pp'$$

$$y = const$$

$$pp' - p^2 \operatorname{tg} y = 2p \sin y$$

$$p' - p \operatorname{tg} y = 2 \sin y \quad \text{- линейное}$$

$$\frac{dp}{dy} = p \operatorname{tg} y \qquad p = C(x) \sec x$$

$$C'(x) \sec x - C(x) \operatorname{tg} x \sec x - C(x) \sec x \operatorname{tg} y = 2 \sin y$$

$$\frac{dp}{p} = \operatorname{tg} y\, dy$$

$$C'(x) \sec x = 2 \sin y$$

$\ln|pC| = \ln|\sec y|$

$p = C \sec y$

$\frac{dC}{dy} = \sin 2y$

$C = -\frac{1}{2} \cos 2y + C$

$p = \left(-\frac{1}{2}\cos 2y + C\right) \sec y \qquad C = \frac{3}{2}$

$y' = -\frac{1}{2} \cos 2y \sec y + \frac{3}{2} \sec y$

$x = \operatorname{arctg} \sin y + \frac{2}{3}$

~ 4.10

$y'y^2 + yy'' - (y')^2 = 0 \qquad y(0) = 1$

$y'(0) = 2$

$p(y) = y'$

$py^2 + y\,pp' - p^2 = 0 \qquad y = Const$

$y^2 + yp' - p = 0$

$p' - \frac{p}{y} + y = 0$

$p = u \cdot v$

$\frac{du}{dy} v + u \frac{dv}{dy} - \frac{u \cdot v}{y} + y = 0$

$v \underbrace{\left(\frac{du}{dy} - \frac{u}{y}\right)} + u \frac{dv}{dy} + y = 0$

$\frac{du}{dy} = \frac{u}{y} \qquad \frac{y\, dv}{dy} + y = 0$

$u = Cy \quad u_1 = y \quad \frac{dv}{dy} = -1$

$$v = -y + C$$

$$p = y(-y + C) = -y^2 + Cy$$

$y(0) = 1 \quad y'(0) = 2$

$2 = -1 + C \quad C = 3$

$$y' = -y^2 + 3y$$

$$\int \frac{dy}{3y - y^2} = \int dx$$

$$\int \frac{dy}{y(1-y)} = \int x \qquad 1- \qquad = x\ C$$

$$\frac{1 - y + y}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}$$

№4.11

$yy'' + 2(y')^2 = y^2 \qquad y(0) = 1$

$y'(0) = 2$

$p(y) = y' \quad y'' = pp'$

$y = 0$ - реш

$$ypp' + 2p^2 = y^2$$

$$p' + 2\frac{p}{y} = \frac{y}{p} \qquad pp' + \frac{2p^2}{y} = y$$

$z = \frac{p}{y} \quad p = zy \qquad z = p^2$

$p' = z'y + z \qquad z' = 2pp'$

$z'y + z + 2z = \frac{1}{z} \qquad \frac{1}{2}z' + \frac{2z}{y} = y$

$z'y + 3z = \frac{1}{z} \qquad z' + \frac{4z}{y} = 2y$ лин.

$$\frac{dz}{z} = -4\frac{dy}{y}$$

$$\ln|zC| = \ln|y^{-4}|$$

$$z = C y^{-4}$$

$$C'(y) y^{-4} + C(y)(-4) y^{-3} + \frac{4 C(y) e^{-4}}{4} = 2y$$

$$z = \left(\frac{2}{7} y^7 + C\right) y^{-4}$$

$$p^2 = \left(\frac{2}{7} y^7 + C\right) y^{-4}$$

$$C'(y) = 2y^6$$

$$C = \frac{1}{3} y^6 + C$$

$$p^2 = \frac{y^2}{3} + \frac{C}{y^4} \qquad C = \frac{11}{3}$$

$$y' = \sqrt{\frac{y^2}{3} + \frac{11}{3y^4}}$$

$$\int \frac{dy}{\sqrt{\frac{y^2}{3} + \frac{11}{3y^4}}} = x$$

$$\int \frac{\sqrt{3}\, y^2\, dy}{\sqrt{y^6 + 11}} = \frac{\sqrt{3}}{3} \int \frac{dy^3}{\sqrt{y^6 + 11}} = \frac{\sqrt{3}}{3} \ln\left|y^3 + \sqrt{y^6 + 11}\right| + C$$

№ 4.12 $\quad y'' + (y')^2 = \frac{y'}{(e^y + 1)^2} \qquad y\left(\frac{1}{2}\right) = 0, \quad y'\left(\frac{1}{2}\right) = -\frac{1}{2}$

$$p = y'(y)$$

$$y'' = pp'$$

$$pp' + p^2 - \frac{p}{(e^y + 1)^2} = 0$$

$$p' + p - \frac{1}{(e^y + 1)^2} = 0 \quad \text{— линейное}$$

$$\int \frac{dp}{p} = \int -dy$$

$$\ln|Cp| = -y$$

$$p = Ce^{-y}$$

$$C'e^{-y} - Ce^{-y} + Ce^{-y} - \frac{1}{(e^y+1)^2} = 0$$

$$C'e^{-y} = \frac{1}{(e^y+1)^2}$$

$$\int dC = \int \frac{de^y}{(e^y+1)^2}$$

$$C = \frac{-1}{e^y+1} + \tilde{C}$$

$$\left(C - \frac{1}{e^y+1}\right)e^{-y} = p$$

$$C - \frac{1}{2} = -\frac{1}{2}$$

$$C = 0$$

$$y' = \frac{e^{-y}}{e^y+1}$$

$$\frac{dy}{dx} = \frac{e^{-y}}{e^y+1}$$

$$\int e^y(e^y+1)\,dy = \int dx$$

$$\int (e^y+1)\,de^y = x$$

$$\frac{e^{2y}}{2} + e^y + C = x \qquad y\left(\frac{1}{2}\right) = 0$$

$$\frac{1}{2} + 1 + C = \frac{1}{2}$$

$$C = -1$$

$$\frac{e^{2y}}{2} + e^y - 1 = x$$

№ 4.13

$$y''(2y+3) - 2(y')^2 = 0$$

$$p = y'$$

'' '

$y'' = pp'$

$pp'(2y+3) - 2p^2 = 0$

$p'(2y+3) - 2p = 0$

$p' = \frac{2p}{2y+3}$

$\frac{dp}{dy} = \frac{2p}{2y+3}$ $\quad \int \frac{dp}{2p} = \int \frac{dy}{2y+3}$

$\frac{1}{2} \ln |cp| = \frac{1}{2} \ln |2y+3|$

$p = C(2y+3)$

$y' = C_1 2y + C_1 3$

$\frac{dy}{dx} = C_1 2y + C_1 3$

$\frac{dy}{C_1 2y + C_1 3} = dx$

$\frac{1}{2} \ln |2y C_1 + 3C_1| = x + C_2$

№4.14

$xy'' - y' = 2\sqrt{xy'}$

$p = y'(x)$

$xp' - p = 2\sqrt{xp}$

$p' - \frac{p}{x} = 2\sqrt{\frac{p}{x}}$

$z = xp$

$\cancel{p = \frac{z}{x}}$

$z = \frac{p}{x}$

$p = zx$

$p' = z'x + z$

$z'x + z - z = 2\sqrt{z}$

$z'x = 2\sqrt{z}$

$$\frac{dz}{dx}x = 2\sqrt{z}$$

$$\frac{dz}{2\sqrt{z}} = \frac{dx}{x}$$

$$\sqrt{z} = \ln|xC|$$

$$z = \ln^2 xC$$

$$z = \frac{f}{x} = \ln^2 x$$

$$\int \sec^n(x)\,dx = \int \operatorname{ctg} x \sec^{n-1}x \; d\sec x =$$

$$d(\sec x) = \sec x \operatorname{tg} dx \qquad = \operatorname{ctg} x \sec^n x - \int \sec$$

$$d(\operatorname{ctg} x \sec^{n-1} x) =$$

$$= \left(\csc^2 x \sec^{n-1} x + \operatorname{ctg} x\,(n-1)\sec^{n-2} x \sec x \operatorname{tg} x\right) dx$$

$$= (\csc^2 x \sec^{n-1} x + (n-1)\sec^{n-1} x)\,dx$$

$$I_n = \operatorname{ctg} x \sec^n x - \int \sec^{n-1} d\operatorname{ctg} x - (n-1)\, I_{n-1}$$

$$\int \sec^{n-1} d\operatorname{ctg} x = \sec^{n-1} x \operatorname{ctg} x - \int \operatorname{ctg} x \; d\sec^{n-1} x =$$

$$= \sec^{n-1} x \operatorname{ctg} x - \int \operatorname{ctg} x\,(n-1)\sec^{n-2} x \sec x \operatorname{tg} x\,dx$$

$$= \sec^{n-1} x \operatorname{ctg} x - (n-1)\, I_{n-1}$$

$$I_n = \operatorname{ctg} x \sec^n x - \operatorname{ctg} x \sec^{n-1} x + (n-1) I_{n-1} - (n-1) I_{n-1}$$

$$\int \cos^n x\,dx = \int \cos^{n-1} x \; d\sin x =$$

$$= \sin x \cos^{n-1} x - \int \sin x \; d\cos^{n-1} x =$$

$$d\cos^{n-1} x = (n-1)\cos^{n-2} x\,(-\sin x)\,dx$$

$$(=) \sin x \cos x + (n-1) \int \cos x \sin x \, dx$$

$$\int \cos^{n-2} x \sin^2 x \, dx = \int \cos^{n-3} x \sin^2 x \, d\sin x =$$

$$= \cos^{n-3} x \sin^3 x - \int \sin x$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \left| \begin{array}{l} x = a\,\mathrm{tg}\, t \\ dx = a\sec^2 t \, dt \end{array} \right| =$$

$$= \int \frac{a\sec^2 t \, dt}{a\sec t} = \int \sec t \, dt$$

$$\int \frac{dx}{a^2 - x^2} = \left| \begin{array}{l} x = a \sin t \\ dx = a\cos t \, dt \end{array} \right| =$$

$$= \int \frac{a\cos t \, dt}{\cancel{a} \cos^2 t} = \int \sec t \, dt$$