Матан 6

№6.1

$$y'' + 2y' + 2y = e^{x}\sin x$$

$$\lambda^2 + 2\lambda + 2 = 0 \qquad y_0 = C_1 e^{(-1+i)x} + C_2 e^{(-1-i)x}$$

$$(\lambda+1)^2 = -1 \qquad y_{00} = C_1 e^{-x}\cos x + C_2 e^{-x}\sin x$$

$$\lambda + 1 = \pm i$$

$$\lambda = -1 \pm i \qquad C_1' e^{-x}\cos x + C_2' e^{-x}\sin x = 0$$

$$C_1'\left(-e^{-x}\cos x + e^{-x}(-\sin x)\right) + C_2'\left(-e^{-x}\sin x + e^{-x}\cos x\right) = e^{x}\sin x$$

$$\begin{pmatrix} e^{-x}\cos x & e^{-x}\sin x \\ e^{-x}(-\cos x - \sin x) & e^{-x}(\cos x - \sin x) \end{pmatrix} \begin{pmatrix} 0 \\ e^{x}\sin x \end{pmatrix}$$

$$\Delta = e^{-2x}(\cos x - \sin x)\cos x + e^{-2x}(\cos x + \sin x)\sin x =$$

$$= e^{-2x}\left(\cos^2 x - \cos x\sin x + \cos x\sin x + \sin^2 x\right) =$$

$$= e^{-2x}$$

$$C_1' = \frac{-\sin^2 x}{e^{-2x}}$$

$$\Delta_1 = -\sin^2 x$$

$$\Delta_2 = \cos x \sin x \qquad C_2' = \frac{\cos x\sin x}{e^{-2x}}$$

$$y_{00} = C_1 e^{-x}\cos x + C_2 e^{-x}\sin x$$

$$f_1 = e^{x}\sin x$$

f	α	β	$\alpha + i\beta$	r	n	
$e^{x}\sin x$	1	1	$1+i$	0	0	$e^{x}(A\cos x + B\sin x)$

$$y' = (B-A)e^{x}\sin x + (B+A)e^{x}\cos x$$

$y' = (B \cdots) e^{x} \cdots (B \cdots) e^{x} \cos x$

$y'' = 2Be^{x} \cos x - 2Ae^{x} \sin x$

$(4B - 4A) e^{x} \sin x + (4B + 4A) e^{x} \cos x = e^{x} \sin x$

$4B - 4A = 1 \qquad A = -\frac{1}{8} \quad B = \frac{1}{8}$

$4B + 4A = 0 \qquad y_{2н} = e^{x}\left(\frac{\sin x}{8} - \frac{\cos x}{8}\right)$

$y_{он} = \frac{C_1 \sin x}{e^{x}} + e^{x}\left(\frac{\sin x}{8} - \frac{\cos x}{8}\right) + \frac{C_2 \cos x}{e^{x}}$

№6.2

$y'' - 6y' + 13y = 4\cos 3x$

$\lambda^2 - 6\lambda + 13 = 0 \qquad y_{оо} = C_1 e^{3x} \cos 2x + C_2 e^{3x} \sin 2x$

$(\lambda - 3)^2 + 4 = 0$

f	α	β	$\alpha + i\beta$	r	n	$y_{2н}$
$4\cos 3x$	0	3	$3i$	0	0	$A\cos 3x + B\sin$

$\lambda - 3 = \pm 2i$

$\lambda = 3 \pm 2i \qquad y_{2н} = A\cos 3x + B\sin 3x$

$y'_{2н} = -3A\sin 3x + 3B\cos 3x$

$y''_{2н} = -9A\cos 3x - 9B\sin 3x$

$-9A\cos 3x - 9B\sin 3x + 18A\sin 3x - 18B\cos 3x + 13A\cos 3x + 13B\sin 3x =$

$= 4\cos 3x$

$\cos 3x\,(-9A - 18B + 13A) + \sin 3x\,(-9B + 18A + 13B) = 4\cos 3x$

$\begin{cases} 4A - 18B = 4 \end{cases} \qquad B = \frac{-18}{4}A \qquad A = \frac{4}{85}$

$$\begin{cases} 4B + 18A = 0 \end{cases} \qquad B = -\frac{18}{85}$$

$$y_{oн} = -\frac{18}{85}\sin 3x + \frac{4}{85}\cos 3x + C_1 e^{3x}\sin 2x + C_2 e^{3x}\cos 2x$$

№6.3

$$y'' - 4y' + 3y = \sin 2x$$

$$\lambda^2 - 4\lambda + 3 = 0 \qquad y_{oo} = C_1 e^{x} + C_2 e^{3x}$$

$$(\lambda - 3)(\lambda - 1) = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = 3 \qquad \begin{cases} C_1' e^{x} + C_2' e^{3x} = 0 \\ C_1' e^{x} + C_2' 3e^{3x} = \sin 2x \end{cases}$$

$$2C_2' e^{3x} = \sin 2x$$

$$C_2' = \frac{\sin 2x}{2e^{3x}}$$ сука

f	α	β	$\alpha + i\beta$	r	n	y
$\sin 2x$	0	2	$2i$	0	0	$A\cos 2x + B\sin 2x$

$$y_2 = A\cos 2x + B\sin 2x \qquad -4A\cos 2x - 4B\sin 2x + 8A\sin 2x - 8B\cos 2x$$

$$y_2' = -2A\sin 2x + 2B\cos 2x \qquad + 3A\cos 2x + 3B\sin 2x = \sin 2x$$

$$y_2'' = -4A\cos 2x - 4B\sin 2x \qquad \begin{cases} -4B + 8A + 3B = 1 \\ -4A - 8B + 3A = 0 \end{cases}$$

$$A = \frac{8}{65} \qquad B = -\frac{1}{65}$$

$$y = \frac{8}{} \cos 2x - \frac{1}{} \sin 2x + C_1 e^{3x} + C_2 e^{x}$$

$y_{чн} = \frac{\ }{65}\cos 2x - \frac{\ }{65} \ldots$

№6.4 $\quad y'' - 3y' = 18x - 10\cos x$

$\lambda^2 - 3\lambda = 0 \qquad y_{oo} = C_1 + C_2 e^{3x}$

$\lambda(\lambda - 3) \qquad \begin{cases} C_1' + C_2' e^{3x} = 0 \\ 3C_2' e^{3x} = 18x - 10\cos x \end{cases}$

f	α	β	$\alpha + i\beta$	r	n	y
$18x$	0	0	0	1	1	$x(Ax+B)$
$-10\cos x$	0	1	i	0	0	$C\cos x + D\sin x$

$y_1 = x(Ax+B) \qquad y_2 = C\cos x + D\sin x$

$y_1' = 2Ax + B \qquad y_2' = D\cos x - C\sin x$

$y_1'' = 2A \qquad y_2'' = -D\sin x - C\cos x$

$\begin{cases} -6A = 18 \\ 2A - 3B = 0 \end{cases} \quad \begin{cases} A = -3 \\ B = -2 \end{cases} \qquad \begin{cases} -3B - A = -10 \\ 3A - B = 0 \end{cases} \quad \begin{cases} A = 1 \\ B = 3 \end{cases}$

$y = 3\sin x + \cos x + C_1 e^{3x} + (-3x - 2)x + C_2$

№6.5

$y'' + y = \sec x$

$\lambda^2 + 1 = 0 \qquad y_{oo} = C_1 \cos x + C_2 \sin x$

$\lambda = \pm i$

$\begin{cases} C_1' \cos x + C_2' \sin x = 0 \\ C_1'(-\sin x) + C_2' \cos x = \sec x \end{cases}$

$\Delta = \begin{vmatrix} \cos x & \sin x \end{vmatrix} = 1$

$$\begin{vmatrix} -\sin x & \cos x \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix} = -\operatorname{tg} x \qquad C_1' = -\operatorname{tg} x \qquad C_1 = \ln|\cos x| + \tilde{C}_1$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix} = 1 \qquad C_2' = 1 \qquad C_2 = x + \tilde{C}_2$$

$$y_{on} = (\ln|\cos x| + C_1)\cos x + (x + C_2)\sin x$$

16.6

$$y'' - 5y' = 1 - 75x^2 + 10\sin 5x$$

$$\lambda^2 - 5\lambda = 0 \qquad y_{oo} = C_1 + C_2 e^{5x}$$

$$\lambda_1 = 0 \quad \lambda_2 = 5$$

f	α	β	$\alpha + i\beta$	r	n	y
$1 - 75x^2$	0	0	0	1	2	$x(Ax^2 + Bx + C)$
$10\sin 5x$	0	5	$5i$	0	0	$D\cos 5x + E\sin 5x$

$$y_1 = Ax^3 + Bx^2 + Cx \qquad 6Ax + 2B + 1 - 15Ax^2 - 10Bx - 5C = 1 - 75x^2$$

$$y_1' = 3Ax^2 + 2Bx + C \qquad \begin{cases} 2B - 5C = 1 \\ -15A = -75 \\ 6A - 10B = 0 \end{cases} \qquad \begin{aligned} A &= 5 \\ B &= 3 \\ C &= 1 \end{aligned}$$

$$y_1'' = 6Ax + 2B + 1$$

$$y_1 = x(5x^2 + 3x + 1)$$

$$y_2 = A\cos 5x + B\sin 5x \qquad (25A - 25B)\sin 5x + (-25B - 25A)\cos 5x = 10\sin 5x$$

$$y_2' = -5A\sin 5x + 5B\cos 5x \qquad A = \frac{1}{5} \quad B = -\frac{1}{5}$$

$$y_2'' = -25A\cos 5x - 25B\sin 5x$$

$$y_{он} = C_1 + C_2 e^{5x} + x(5x^2 + 3x + 1) + \frac{1}{5}\cos 5x - \frac{1}{5}\sin 5x$$

№6.7 $y'' - 8y' + 17y = (2x - 13)e^{4x}$

$\lambda^2 - 8\lambda + 17 = 0$ $\quad y_{оо} = C_1 e^{4x}\cos x + C_2 e^{4x}\sin x$

$(\lambda - 4)^2 + 1 = 0$

$\lambda = 4 \pm i$

f	α	β	$\alpha + i\beta$	r	n	y
$(2x-13)e^{4x}$	4	0	4	0	1	$e^{4x}(Ax+B)$

$y = e^{4x}(Ax + B)$

$y' = 4e^{4x}Ax + Ae^{4x} + 4e^{4x} = e^{4x}(4Ax + 4B + A)$

$y'' = e^{4x}(16Ax + 16B + 8A)$

$Axe^{4x} + Be^{4x} = (2x - 13)e^{4x}$

$A = 2 \quad B = -13$

$$y_{он} = C_1 e^{4x}\cos x + C_2 e^{4x}\sin x + (2x - 13)e^{4x}$$

№6.8 $y'' + 3y' + 2y = \frac{1}{e^x + 1}$

$\lambda^2 + 3\lambda + 2 = 0$ $\quad y_{оо} = C_1 e^{-2x} + C_2 e^{-x}$

$(\lambda + 2)(\lambda + 1) = 0$

$$\begin{cases} C_1' e^{-2x} + C_2' e^{-x} = 0 \\ C_1'(-2e^{-2x}) + C_2'(-e^{-x}) = \frac{1}{e^x + 1} \end{cases}$$

$$A = \begin{pmatrix} e^{-2x} & e^{-x} \\ -2e^{-2x} & -e^{-x} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \frac{1}{e^x+1} \end{pmatrix}$$

$$\Delta = -e^{-3x} + 2e^{-3x} = e^{-3x}$$

$$\Delta_1 = -\frac{e^{-x}}{e^x+1} = \frac{-1}{e^x(e^x+1)} \qquad C_1' = \frac{\Delta_1}{\Delta} = \frac{-e^{3x}}{e^{2x}+e^x}$$

$$\Delta_2 = \frac{1}{e^{3x}+e^{2x}} \qquad C_2' = \frac{\Delta_2}{\Delta} = \frac{e^{3x}}{e^{3x}+e^{2x}}$$

$$C_1 = -\int \frac{e^{3x}}{e^{2x}+e^x}\,dx = \ln|e^x+1| - e^x + \tilde{C}_1$$

$$C_2 = \int \frac{e^{3x}}{e^{3x}+e^{2x}}\,dx = \ln|e^x+1| + \tilde{C}_2$$

$$y = (\ln|e^x+1| - e^x + C_1)e^{-2x} + (\ln(e^x+1) + C_2)e^{-2x}$$

$$\int \frac{e^{3x}}{e^{2x}+e^x}\,dx = \int \frac{u^2\,du}{u^2+u} = \int \left(1 - \frac{u}{u^2+u}\right)du = \int \left(1 - \frac{1}{u+1}\right)du =$$
$$= u - \ln|u+1| + C$$

$$\int \frac{e^{3x}}{e^{3x}+e^{2x}} = \int \frac{u^2\,du}{u^3+u^2} = \int \frac{du}{u+1} = \ln|u+1| + C$$

№ 6.9

$$y'' + 9y = \frac{1}{\sin^3 3x}$$

$$\lambda^2 + 9 = 0 \qquad y_{oo} = C_1\cos 3x + C_2 \sin 3x$$

$$\lambda = \pm 3i$$

$$\left| \; C_1'\cos 3x + C_2' \sin 3x = 0 \right.$$

$$\left[C_1'(-3\sin 3x) + C_2'(3\cos 3x) = \csc^3 3x \right.$$

$$\Delta = 3\cos^2 x + 3\sin^2 3x = 3$$

$$\Delta_1 = -\csc^2 3x \qquad C_1 = -\int \frac{1}{3}\csc^2 3x\,dx = \frac{\operatorname{ctg} 3x}{9} + \tilde{C}_1$$

$$\Delta_2 = \csc^2 3x \operatorname{ctg} 3x \qquad C_2 = \frac{1}{3}\int \csc^2 3x \operatorname{ctg} 3x\,dx =$$

$$= \tilde{C}_2 - \frac{1}{18}\csc^2 3x$$

$$y_{on} = \left(\frac{\operatorname{ctg} 3x}{9} + C_1\right)\cos 3x + \left(C_2 - \frac{1}{18}\csc^2 3x\right)$$

№6.10