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Receiver operating characteristic

A **receiver operating characteristic curve**, or **ROC curve**, is a graphical plot that illustrates the diagnostic ability of a binary classifier system as its discrimination threshold is varied. The method was originally developed for operators of military radar receivers starting in 1941, which led to its name.

The ROC curve is created by plotting the true positive rate (TPR) against the false positive rate (FPR) at various threshold settings. The true-positive rate is also known as sensitivity, recall or *probability of detection*.[10] The false-positive rate is also known as *probability of false alarm*[10] and can be calculated as (1 – specificity). The ROC can also be thought of as a plot of the power as a function of the Type I Error of the decision rule (when the performance is calculated from just a sample of the population, it can be thought of as estimators of these quantities). The ROC curve is thus the sensitivity or recall as a function of fall-out. In general, if the probability distributions for both detection and false alarm are known, the ROC curve can be generated by plotting the cumulative distribution function (area under the probability distribution from $-\infty$ to the discrimination threshold) of the detection probability in the y-axis versus the cumulative distribution function of the false-alarm probability on the x-axis.

ROC analysis provides tools to select possibly optimal models and to discard suboptimal ones independently from (and prior to specifying) the cost context or the class distribution. ROC analysis is related in a direct and natural way to cost/benefit analysis of diagnostic decision making.

The ROC curve was first developed by electrical engineers and radar engineers during World War II for detecting enemy objects in battlefields and was soon introduced to psychology to account for perceptual detection of stimuli. ROC analysis since then has been used in medicine, radiology, biometrics, forecasting of natural hazards,[11] meteorology,[12] model performance assessment,[13] and other areas for many decades and is increasingly used in machine learning and data mining research.

The ROC is also known as a relative operating characteristic curve, because it is a comparison of two operating characteristics (TPR and FPR) as the criterion changes.[14]

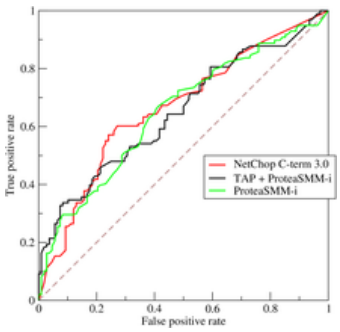
Contents

Basic concept

A classification model (classifier or diagnosis[15]) is a mapping of instances between certain classes/groups. Because the classifier or diagnosis result can be an arbitrary real value (continuous output), the classifier boundary between classes must be determined by a threshold value (for instance, to determine whether a person has hypertension based on a blood pressure measure). Or it can be a discrete class label, indicating one of the classes.

Consider a two-class prediction problem (binary classification), in which the outcomes are labeled either as positive (p) or negative (n). There are four possible outcomes from a binary classifier. If the outcome from a prediction is p and the actual value is also p, then it is called a *true positive* (TP); however if the actual value is n then it is said to be a *false positive* (FP). Conversely, a *true negative* (TN) has occurred when both the prediction outcome and the actual value are n, and *false negative* (FN) is when the prediction outcome is n while the actual value is p.

To get an appropriate example in a real-world problem, consider a diagnostic test that seeks to determine whether a person has a certain disease. A false positive in this case occurs when the person tests positive, but does not actually have the disease. A false negative, on the other hand, occurs when the person tests negative, suggesting they are healthy, when they actually do have the disease.



ROC curve of three predictors of peptide cleaving in the proteasome.

Let us define an experiment from **P** positive instances and **N** negative instances for some condition. The four outcomes can be formulated in a 2×2 *contingency table* or *confusion matrix*, as follows:

Terminology and derivations from a confusion matrix

condition positive (P)
the number of real positive cases in the data
condition negative (N)
the number of real negative cases in the data

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true positive (TP)
A test result that correctly indicates the presence of a condition or characteristic
true negative (TN)
A test result that correctly indicates the absence of a condition or characteristic
false positive (FP)
A test result which wrongly indicates that a particular condition or attribute is present
false negative (FN)
A test result which wrongly indicates that a particular condition or attribute is absent

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sensitivity, recall, hit rate, or true positive rate (TPR)

$$\mathrm{TPR} = \frac{\mathrm{TP}}{\mathrm{P}} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN}} = 1 - \mathrm{FNR}$$

specificity, selectivity or true negative rate (TNR)

$$\mathrm{TNR} = \frac{\mathrm{TN}}{\mathrm{N}} = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FP}} = 1 - \mathrm{FPR}$$

precision or positive predictive value (PPV)

$$\mathrm{PPV} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FP}} = 1 - \mathrm{FDR}$$

negative predictive value (NPV)

$$\mathrm{NPV} = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FN}} = 1 - \mathrm{FOR}$$

miss rate or false negative rate (FNR)

$$\mathrm{FNR} = \frac{\mathrm{FN}}{\mathrm{P}} = \frac{\mathrm{FN}}{\mathrm{FN} + \mathrm{TP}} = 1 - \mathrm{TPR}$$

fall-out or false positive rate (FPR)

$$\mathrm{FPR} = \frac{\mathrm{FP}}{\mathrm{N}} = \frac{\mathrm{FP}}{\mathrm{FP} + \mathrm{TN}} = 1 - \mathrm{TNR}$$

false discovery rate (FDR)

$$\mathrm{FDR} = \frac{\mathrm{FP}}{\mathrm{FP} + \mathrm{TP}} = 1 - \mathrm{PPV}$$

false omission rate (FOR)

$$\mathrm{FOR} = \frac{\mathrm{FN}}{\mathrm{FN} + \mathrm{TN}} = 1 - \mathrm{NPV}$$

Positive likelihood ratio (LR+)

$$\mathrm{LR+} = \frac{\mathrm{TPR}}{\mathrm{FPR}}$$

Negative likelihood ratio (LR-)

$$\mathrm{LR-} = \frac{\mathrm{FNR}}{\mathrm{TNR}}$$

prevalence threshold (PT)

$$\mathrm{PT} = \frac{\sqrt{\mathrm{FPR}}}{\sqrt{\mathrm{TPR}} + \sqrt{\mathrm{FPR}}}$$

threat score (TS) or critical success index (CSI)

$$\mathrm{TS} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN} + \mathrm{FP}}$$

Prevalence

$$\frac{\mathrm{P}}{\mathrm{P} + \mathrm{N}}$$

accuracy (ACC)

$$\mathrm{ACC} = \frac{\mathrm{TP} + \mathrm{TN}}{\mathrm{P} + \mathrm{N}} = \frac{\mathrm{TP} + \mathrm{TN}}{\mathrm{TP} + \mathrm{TN} + \mathrm{FP} + \mathrm{FN}}$$

balanced accuracy (BA)

$$\mathrm{BA} = \frac{TPR + TNR}{2}$$

F1 score

is the harmonic mean of precision and sensitivity:

$$\mathrm{F_1} = 2 \times \frac{\mathrm{PPV} \times \mathrm{TPR}}{\mathrm{PPV} + \mathrm{TPR}} = \frac{2\mathrm{TP}}{2\mathrm{TP} + \mathrm{FP} + \mathrm{FN}}$$

phi coefficient (φ or r_φ) or Matthews correlation coefficient (MCC)

$$\mathrm{MCC} = \frac{\mathrm{TP} \times \mathrm{TN} - \mathrm{FP} \times \mathrm{FN}}{\sqrt{(\mathrm{TP} + \mathrm{FP})(\mathrm{TP} + \mathrm{FN})(\mathrm{TN} + \mathrm{FP})(\mathrm{TN} + \mathrm{FN})}}$$

Fowlkes–Mallows index (FM)

$$\mathrm{FM} = \sqrt{\frac{TP}{TP + FP} \times \frac{TP}{TP + FN}} = \sqrt{PPV \times TPR}$$

informedness or bookmaker informedness (BM)

$$\mathrm{BM} = \mathrm{TPR} + \mathrm{TNR} - 1$$

markedness (MK) or deltaP (Δp)

$$\mathrm{MK} = \mathrm{PPV} + \mathrm{NPV} - 1$$

Diagnostic odds ratio (DOR)

$$\mathrm{DOR} = \frac{\mathrm{LR+}}{\mathrm{LR-}}$$

Sources: Fawcett (2006),[1] Piryonesi and El-Diraby (2020),[2] Powers (2011),[3] Ting (2011),[4] CAWCR,[5] D. Chicco & G. Jurman (2020, 2021),[6][7] Tharwat (2018).[8] Balayla (2020)[9]

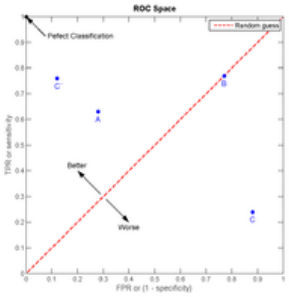
Sources: [16][17][18][19][20][21][22][23][24]

		Predicted condition			
	Total population = P + N	**Positive (PP)**	**Negative (PN)**	Informedness, bookmaker informedness (BM) = TPR + TNR − 1	Prevalence threshold (PT) = $\frac{\sqrt{\mathrm{TPR} \times \mathrm{FPR}} - \mathrm{FPR}}{\mathrm{TPR} - \mathrm{FPR}}$
Actual condition	**Positive (P)**	**True positive (TP), hit**	**False negative (FN), type II error, miss, underestimation**	True positive rate (TPR), recall, sensitivity (SEN), probability of detection, hit rate, power = $\frac{\mathrm{TP}}{\mathrm{P}}$ = 1 − FNR	False negative rate (FNR), miss rate = $\frac{\mathrm{FN}}{\mathrm{P}}$ = 1 − TPR
	Negative (N)	**False positive (FP), type I error, false alarm, overestimation**	**True negative (TN), correct rejection**	False positive rate (FPR), probability of false alarm, fall-out = $\frac{\mathrm{FP}}{\mathrm{N}}$ = 1 − TNR	True negative rate (TNR), specificity (SPC), selectivity = $\frac{\mathrm{TN}}{\mathrm{N}}$ = 1 − FPR
	Prevalence = $\frac{\mathrm{P}}{\mathrm{P+N}}$	Positive predictive value (PPV), precision = $\frac{\mathrm{TP}}{\mathrm{PP}}$ = 1 − FDR	False omission rate (FOR) = $\frac{\mathrm{FN}}{\mathrm{PN}}$ = 1 − NPV	Positive likelihood ratio (LR+) = $\frac{\mathrm{TPR}}{\mathrm{FPR}}$	Negative likelihood ratio (LR−) = $\frac{\mathrm{FNR}}{\mathrm{TNR}}$
	Accuracy (ACC) = $\frac{\mathrm{TP + TN}}{\mathrm{P + N}}$	False discovery rate (FDR) = $\frac{\mathrm{FP}}{\mathrm{PP}}$ = 1 − PPV	Negative predictive value (NPV) = $\frac{\mathrm{TN}}{\mathrm{PN}}$ = 1 − FOR	Markedness (MK), deltaP (Δp) = PPV + NPV − 1	Diagnostic odds ratio (DOR) = $\frac{\mathrm{LR+}}{\mathrm{LR-}}$
	Balanced accuracy (BA) = $\frac{\mathrm{TPR + TNR}}{2}$	$\mathrm{F_1}$ score = $\frac{2\mathrm{PPV} \times \mathrm{TPR}}{\mathrm{PPV + TPR}} = \frac{2\mathrm{TP}}{2\mathrm{TP + FP + FN}}$	Fowlkes–Mallows index (FM) = $\sqrt{\mathrm{PPV} \times \mathrm{TPR}}$	Matthews correlation coefficient (MCC) = $\sqrt{\mathrm{TPR} \times \mathrm{TNR} \times \mathrm{PPV} \times \mathrm{NPV}} - \sqrt{\mathrm{FNR} \times \mathrm{FPR} \times \mathrm{FOR} \times \mathrm{FDR}}$	Threat score (TS), critical success index (CSI), Jaccard index = $\frac{\mathrm{TP}}{\mathrm{TP + FN + FP}}$

ROC space

The contingency table can derive several evaluation "metrics" (see infobox). To draw a ROC curve, only the true positive rate (TPR) and false positive rate (FPR) are needed (as functions of some classifier parameter). The TPR defines how many correct positive results occur among all positive samples available during the test. FPR, on the other hand, defines how many incorrect positive results occur among all negative samples available during the test.

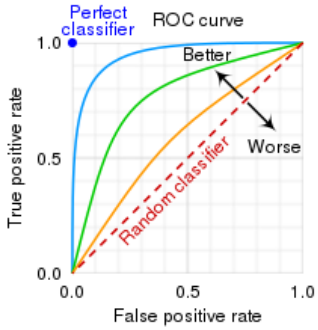
A ROC space is defined by FPR and TPR as *x* and *y* axes, respectively, which depicts relative trade-offs between true positive (benefits) and false positive (costs). Since TPR is equivalent to sensitivity and FPR is equal to 1 − specificity, the ROC graph is sometimes called the sensitivity vs (1 − specificity) plot. Each prediction result or instance of a confusion matrix represents one point in the ROC space.



The ROC space and plots of the four prediction examples.

The best possible prediction method would yield a point in the upper left corner or coordinate (0,1) of the ROC space, representing 100% sensitivity (no false negatives) and 100% specificity (no false positives). The (0,1) point is also called a *perfect classification*. A random guess would give a point along a diagonal line (the so-called *line of no-discrimination*) from the bottom left to the top right corners (regardless of the positive and negative base rates).[25] An intuitive example of random guessing is a decision by flipping coins. As the size of the sample increases, a random classifier's ROC point tends towards the diagonal line. In the case of a balanced coin, it will tend to the point (0.5, 0.5).

The diagonal divides the ROC space. Points above the diagonal represent good classification results (better than random); points below the line represent bad results (worse than random). Note that the output of a consistently bad predictor could simply be inverted to obtain a good predictor.



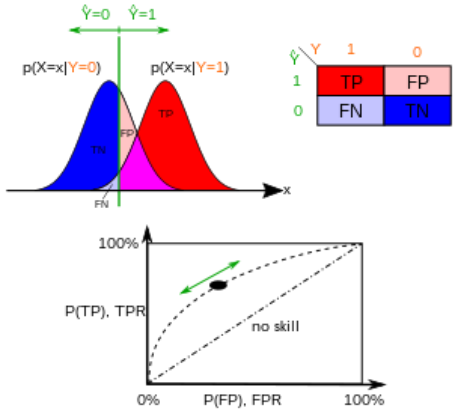
The ROC space for a "better" and "worse" classifier.

Let us look into four prediction results from 100 positive and 100 negative instances:

A	B	C	C′
TP=63 FN=37 100 FP=28 TN=72 100 91 109 200	TP=77 FN=23 100 FP=77 TN=23 100 154 46 200	TP=24 FN=76 100 FP=88 TN=12 100 112 88 200	TP=76 FN=24 100 FP=12 TN=88 100 88 112 200
TPR = 0.63	TPR = 0.77	TPR = 0.24	TPR = 0.76
FPR = 0.28	FPR = 0.77	FPR = 0.88	FPR = 0.12
PPV = 0.69	PPV = 0.50	PPV = 0.21	PPV = 0.86
F1 = 0.66	F1 = 0.61	F1 = 0.23	F1 = 0.81
ACC = 0.68	ACC = 0.50	ACC = 0.18	ACC = 0.82

Plots of the four results above in the ROC space are given in the figure. The result of method **A** clearly shows the best predictive power among **A**, **B**, and **C**. The result of **B** lies on the random guess line (the diagonal line), and it can be seen in the table that the accuracy of **B** is 50%. However, when **C** is mirrored across the center point (0.5,0.5), the resulting method **C′** is even better than **A**. This mirrored method simply reverses the predictions of whatever method or test produced the **C** contingency table. Although the original **C** method has negative predictive power, simply reversing its decisions leads to a new predictive method **C′** which has positive predictive power. When the **C** method predicts **p** or **n**, the **C′** method would predict **n** or **p**, respectively. In this manner, the **C′** test would perform the best. The closer a result from a contingency table is to the upper left corner, the better it predicts, but the distance from the random guess line in either direction is the best indicator of how much predictive power a method has. If the result is below the line (i.e. the method is worse than a random guess), all of the method's predictions must be reversed in order to utilize its power, thereby moving the result above the random guess line.

Curves in ROC space



In binary classification, the class prediction for each instance is often made based on a continuous random variable X, which is a "score" computed for the instance (e.g. the estimated probability in logistic regression). Given a threshold parameter T, the instance is classified as "positive" if $X > T$, and "negative" otherwise. X follows a probability density $f_1(x)$ if the instance actually belongs to class "positive", and $f_0(x)$ if otherwise. Therefore, the true positive rate is given by $\mathrm{TPR}(T) = \int_T^\infty f_1(x)\,dx$ and the false positive rate is given by $\mathrm{FPR}(T) = \int_T^\infty f_0(x)\,dx$. The ROC curve plots parametrically $\mathrm{TPR}(T)$ versus $\mathrm{FPR}(T)$ with T as the varying parameter.

For example, imagine that the blood protein levels in diseased people and healthy people are normally distributed with means of 2 g/dL and 1 g/dL respectively. A medical test might measure the level of a certain protein in a blood sample and classify any number above a certain threshold as indicating disease. The experimenter can adjust the threshold (green vertical line in the figure), which will in turn change the false positive rate. Increasing the threshold would result in fewer false positives (and more false negatives), corresponding to a leftward movement on the curve. The actual shape of the curve is determined by how much overlap the two distributions have.

Further interpretations

Sometimes, the ROC is used to generate a summary statistic. Common versions are:

- the intercept of the ROC curve with the line at 45 degrees orthogonal to the no-discrimination line - the balance point where Sensitivity = 1 - Specificity
- the intercept of the ROC curve with the tangent at 45 degrees parallel to the no-discrimination line that is closest to the error-free point (0,1) - also called Youden's J statistic and generalized as Informedness
- the area between the ROC curve and the no-discrimination line multiplied by two is called the *Gini coefficient*. It should not be confused with the measure of statistical dispersion also called Gini coefficient.
- the area between the full ROC curve and the triangular ROC curve including only (0,0), (1,1) and one selected operating point (tpr, fpr) - Consistency[26]
- the area under the ROC curve, or "AUC" ("area under curve"), or A' (pronounced "a-prime"),[27] or "c-statistic" ("concordance statistic").[28]
- the sensitivity index *d′* (pronounced "d-prime"), the distance between the mean of the distribution of activity in the system under noise-alone conditions and its distribution under signal-alone conditions, divided by their standard deviation, under the assumption that both these distributions are normal with the same standard deviation. Under these assumptions, the shape of the ROC is entirely determined by *d′*.

However, any attempt to summarize the ROC curve into a single number loses information about the pattern of tradeoffs of the particular discriminator algorithm.

Probabilistic interpretation

When using normalized units, the area under the curve (often referred to as simply the AUC) is equal to the probability that a classifier will rank a randomly chosen positive instance higher than a randomly chosen negative one (assuming 'positive' ranks higher than 'negative').[29] In other words, when given one randomly selected positive instance and one randomly selected negative instance, AUC is the probability that the classifier will be able to tell which one is which.

This can be seen as follows: the area under the curve is given by (the integral boundaries are reversed as large threshold T has a lower value on the x-axis)

$$\begin{aligned}&\mathrm{TPR}(T) : T \mapsto y(x)\\&\mathrm{FPR}(T) : T \mapsto x\\&A = \int_{x=0}^{1} \mathrm{TPR}(\mathrm{FPR}^{-1}(x))\,dx = \int_{\infty}^{-\infty} \mathrm{TPR}(T)\mathrm{FPR}'(T)\,dT = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(T' > T) f_1(T') f_0(T)\,dT'\,dT = P(X_1 > X_0)\end{aligned}$$

where X_1 is the score for a positive instance and X_0 is the score for a negative instance, and f_0 and f_1 are probability densities as defined in previous section.

Area under the curve

It can be shown that the AUC is closely related to the Mann–Whitney U,[30][31] which tests whether positives are ranked higher than negatives. It is also equivalent to the Wilcoxon test of ranks.[31] For a predictor f, an unbiased estimator of its AUC can be expressed by the following *Wilcoxon-Mann-Whitney* statistic:[32]

$$AUC(f) = \frac{\sum_{t_0 \in \mathcal{D}^0} \sum_{t_1 \in \mathcal{D}^1} \mathbf{1}[f(t_0) < f(t_1)]}{|\mathcal{D}^0| \cdot |\mathcal{D}^1|},$$

where, $\mathbf{1}[f(t_0) < f(t_1)]$ denotes an *indicator function* which returns 1 iff $f(t_0) < f(t_1)$ otherwise return 0; $\mathcal{D}^0$ is the set of negative examples, and $\mathcal{D}^1$ is the set of positive examples.

The AUC is related to the Gini impurity index (G_1) by the formula $G_1 = 2\mathrm{AUC} - 1$, where:

$$G_1 = 1 - \sum_{k=1}^{n} (X_k - X_{k-1})(Y_k + Y_{k-1})$$[33]

In this way, it is possible to calculate the AUC by using an average of a number of trapezoidal approximations. G_1 should not be confused with the measure of statistical dispersion that is also called Gini coefficient.

It is also common to calculate the Area Under the ROC Convex Hull (ROC AUCH = ROCH AUC) as any point on the line segment between two prediction results can be achieved by randomly using one or the other system with probabilities proportional to the relative length of the opposite component of the segment.[34] It is also possible to invert concavities – just as in the figure the worse solution can be reflected to become a better solution; concavities can be reflected in any line segment, but this more extreme form of fusion is much more likely to overfit the data.[35]

The machine learning community most often uses the ROC AUC statistic for model comparison.[36] This practice has been questioned because AUC estimates are quite noisy and suffer from other problems.[37][38][39] Nonetheless, the coherence of AUC as a measure of aggregated classification performance has been vindicated, in terms of a uniform rate distribution,[40] and AUC has been linked to a number of other performance metrics such as the Brier score.[41]

Another problem with ROC AUC is that reducing the ROC Curve to a single number ignores the fact that it is about the tradeoffs between the different systems or performance points plotted and not the performance of an individual system, as well as ignoring the possibility of concavity repair, so that related alternative measures such as Informedness or DeltaP are recommended.[26][42] These measures are essentially equivalent to the Gini for a single prediction point with DeltaP' = Informedness = 2AUC-1, whilst DeltaP = Markedness represents the dual (viz. predicting the prediction from the real class) and their geometric mean is the Matthews correlation coefficient.

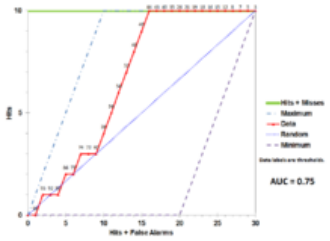
Whereas ROC AUC varies between 0 and 1 — with an uninformative classifier yielding 0.5 — the alternative measures known as Informedness, Certainty [26] and Gini Coefficient (in the single parameterization or single system case) all have the advantage that 0 represents chance performance whilst 1 represents perfect performance, and −1 represents the "perverse" case of full informedness always giving the wrong response.[43] Bringing chance performance to 0 allows these alternative scales to be interpreted as Kappa statistics. Informedness has been shown to have desirable characteristics for Machine Learning versus other common definitions of Kappa such as Cohen Kappa and Fleiss Kappa.[44]

Sometimes it can be more useful to look at a specific region of the ROC Curve rather than at the whole curve. It is possible to compute partial AUC.[45] For example, one could focus on the region of the curve with low false positive rate, which is often of prime interest for population screening tests.[46] Another common approach for classification problems in which P ≪ N (common in bioinformatics applications) is to use a logarithmic scale for the x-axis.[47]

The ROC area under the curve is also called **c-statistic** or **c statistic**.[48]

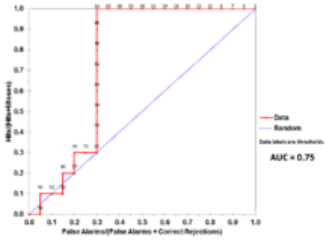
Other measures

The Total Operating Characteristic (TOC) also characterizes diagnostic ability while revealing more information than the ROC. For each threshold, ROC reveals two ratios, TP/(TP + FN) and FP/(FP + TN). In other words, ROC reveals $\frac{\textbf{hits}}{\textbf{hits} + \textbf{misses}}$ and $\frac{\textbf{false alarms}}{\textbf{false alarms} + \textbf{correct rejections}}$. On the other hand, TOC shows the total information in the contingency table for each threshold.[49] The TOC method reveals all of the information that the ROC method provides, plus additional important information that ROC does not reveal, i.e. the size of every entry in the contingency table for each threshold. TOC also provides the popular AUC of the ROC.[50]



TOC Curve

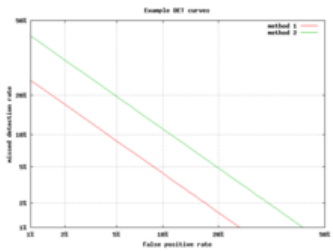
These figures are the TOC and ROC curves using the same data and thresholds. Consider the point that corresponds to a threshold of 74. The TOC curve shows the number of hits, which is 3, and hence the number of misses, which is 7. Additionally, the TOC curve shows that the number of false alarms is 4 and the number of correct rejections is 16. At any given point in the ROC curve, it is possible to glean values for the ratios of $\frac{\textbf{false alarms}}{\textbf{false alarms} + \textbf{correct rejections}}$ and $\frac{\textbf{hits}}{\textbf{hits} + \textbf{misses}}$. For example, at threshold 74, it is evident that the x coordinate is 0.2 and the y coordinate is 0.3. However, these two values are insufficient to construct all entries of the underlying two-by-two contingency table.



ROC Curve

Detection error tradeoff graph

An alternative to the ROC curve is the detection error tradeoff (DET) graph, which plots the false negative rate (missed detections) vs. the false positive rate (false alarms) on non-linearly transformed x- and y-axes. The transformation function is the quantile function of the normal distribution, i.e., the inverse of the cumulative normal distribution. It is, in fact, the same transformation as zROC, below, except that the complement of the hit rate, the miss rate or false negative rate, is used. This alternative spends more graph area on the region of interest. Most of the ROC area is of little interest; one primarily cares about the region tight against the y-axis and the top left corner – which, because of using miss rate instead of its complement, the hit rate, is the lower left corner in a DET plot. Furthermore, DET graphs have the useful property of linearity and a linear threshold behavior for normal distributions.[51] The DET plot is used extensively in the automatic speaker recognition community, where the name DET was first used. The analysis of the ROC performance in graphs with this warping of the axes was used by psychologists in perception studies halfway through the 20th century, where this was dubbed "double probability paper".[52]


Example DET graph

Z-score

If a standard score is applied to the ROC curve, the curve will be transformed into a straight line.[53] This z-score is based on a normal distribution with a mean of zero and a standard deviation of one. In memory strength theory, one must assume that the zROC is not only linear, but has a slope of 1.0. The normal distributions of targets (studied objects that the subjects need to recall) and lures (non studied objects that the subjects attempt to recall) is the factor causing the zROC to be linear.

The linearity of the zROC curve depends on the standard deviations of the target and lure strength distributions. If the standard deviations are equal, the slope will be 1.0. If the standard deviation of the target strength distribution is larger than the standard deviation of the lure strength distribution, then the slope will be smaller than 1.0. In most studies, it has been found that the zROC curve slopes constantly fall below 1, usually between 0.5 and 0.9.[54] Many experiments yielded a zROC slope of 0.8. A slope of 0.8 implies that the variability of the target strength distribution is 25% larger than the variability of the lure strength distribution.[55]

Another variable used is *d'* (d prime) (discussed above in "Other measures"), which can easily be expressed in terms of z-values. Although *d'* is a commonly used parameter, it must be recognized that it is only relevant when strictly adhering to the very strong assumptions of strength theory made above.[56]

The z-score of an ROC curve is always linear, as assumed, except in special situations. The Yonelinas familiarity-recollection model is a two-dimensional account of recognition memory. Instead of the subject simply answering yes or no to a specific input, the subject gives the input a feeling of familiarity, which operates like the original ROC curve. What changes, though, is a parameter for Recollection (R). Recollection is assumed to be all-or-none, and it trumps familiarity. If there were no recollection component, zROC would have a predicted slope of 1. However, when adding the recollection component, the zROC curve will be concave up, with a decreased slope. This difference in shape and slope result from an added element of variability due to some items being recollected. Patients with anterograde amnesia are unable to recollect, so their Yonelinas zROC curve would have a slope close to 1.0.[57]

History

The ROC curve was first used during World War II for the analysis of radar signals before it was employed in signal detection theory.[58] Following the attack on Pearl Harbor in 1941, the United States army began new research to increase the prediction of correctly detected Japanese aircraft from their radar signals. For these purposes they measured the ability of a radar receiver operator to make these important distinctions, which was called the Receiver Operating Characteristic.[59]

In the 1950s, ROC curves were employed in psychophysics to assess human (and occasionally non-human animal) detection of weak signals.[58] In medicine, ROC analysis has been extensively used in the evaluation of diagnostic tests.[60][61] ROC curves are also used extensively in epidemiology and medical research and are frequently mentioned in conjunction with evidence-based medicine. In radiology, ROC analysis is a common technique to evaluate new radiology techniques.[62] In the social sciences, ROC analysis is often called the ROC Accuracy Ratio, a common technique for judging the accuracy of default probability models. ROC curves are widely used in laboratory medicine to assess the diagnostic accuracy of a test, to choose the optimal cut-off of a test and to compare diagnostic accuracy of several tests.

ROC curves also proved useful for the evaluation of machine learning techniques. The first application of ROC in machine learning was by Spackman who demonstrated the value of ROC curves in comparing and evaluating different classification algorithms.[63]

ROC curves are also used in verification of forecasts in meteorology.[64]

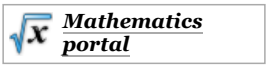
ROC curves beyond binary classification

The extension of ROC curves for classification problems with more than two classes is cumbersome. Two common approaches for when there are multiple classes are (1) average over all pairwise AUC values[65] and (2) compute the volume under surface (VUS).[66][67] To average over all pairwise classes, one computes the AUC for each pair of classes, using only the examples from those two classes as if there were no other classes, and then averages these AUC values over all possible pairs. When there are c classes there will be $c(c-1)/2$ possible pairs of classes.

The volume under surface approach has one plot a hypersurface rather than a curve and then measure the hypervolume under that hypersurface. Every possible decision rule that one might use for a classifier for c classes can be described in terms of its true positive rates (TPR_1, ..., TPR_c). It is this set of rates that defines a point, and the set of all possible decision rules yields a cloud of points that define the hypersurface. With this definition, the VUS is the probability that the classifier will be able to correctly label all c examples when it is given a set that has one randomly selected example from each class. The implementation of a classifier that knows that its input set consists of one example from each class might first compute a goodness-of-fit score for each of the c^2 possible pairings of an example to a class, and then employ the Hungarian algorithm to maximize the sum of the c selected scores over all $c!$ possible ways to assign exactly one example to each class.

Given the success of ROC curves for the assessment of classification models, the extension of ROC curves for other supervised tasks has also been investigated. Notable proposals for regression problems are the so-called regression error characteristic (REC) Curves [68] and the Regression ROC (RROC) curves.[69] In the latter, RROC curves become extremely similar to ROC curves for classification, with the notions of asymmetry, dominance and convex hull. Also, the area under RROC curves is proportional to the error variance of the regression model.

See also



- Brier score
- Coefficient of determination
- Constant false alarm rate