EPQ Proposal

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Working Title

Understanding the Difficulty of P vs NP: A Study of Relativization, Natural Proofs, and Algebrization

Aims and Objectives

The aim of this project is to understand why the P vs NP problem remains unsolved, despite being one of the most studied open problems in theoretical computer science and mathematics. I will focus on three major theoretical frameworks—relativization, natural proofs, and algebrization—that have been shown to act as barriers to current proof techniques. My objectives are to:

- Define and explain the classes P, NP, and NP-completeness.

- Introduce and explain each of the three major barriers in detail.

- Evaluate their impact on attempts to resolve the P vs NP problem.

- Reflect on what these barriers suggest about the future direction of complexity theory.

Rationale

I have chosen this topic because I am applying to study Computer Science at the University of Cambridge, and I want to demonstrate genuine engagement with the kind of deep theoretical thinking that the course requires. This topic allows me to explore advanced concepts in computational complexity, logic, and meta-mathematics, while producing a structured and focused research project. Rather than speculating on a solution to P vs NP, I will analyse why a solution has proved so elusive, which is a far more realistic and academically rigorous goal for an EPQ.

Research Methods and Approach

My research will consist of a structured theoretical investigation using primary and secondary sources. I will:

- Read and interpret foundational academic papers in complexity theory.

- Consult textbooks and lectures to support my understanding of complex proofs.

- Organise my findings into thematic chapters corresponding to each barrier.

- Include critical commentary and comparisons between approaches.

This will be a literature-based project with analytical commentary, not an experimental or practical investigation.

Resources

- *Computational Complexity: A Modern Approach* by Arora and Barak

- *Introduction to the Theory of Computation* by Michael Sipser

- "Relativizations of the P=?NP Question" by Baker, Gill, and Solovay (1975)

- "Natural Proofs" by Razborov and Rudich (1994)

- "Algebrization: A New Barrier in Complexity Theory" by Aaronson and Wigderson (2008)

- Blog posts and lecture materials by Scott Aaronson and Ryan O'Donnell

- Selected undergraduate-level lecture notes from MIT and CMU

Expected Outcomes

By the end of the project, I will:

- Produce a 5,000–6,000 word report suitable for a mathematically literate audience.

- Demonstrate deep understanding of theoretical computer science topics.

- Critically assess the limits of current mathematical techniques in resolving the P vs NP question.

- Reflect on how these limitations shape the future of computational complexity.