

What are the main theoretical barriers to resolving P vs NP?

An Extended Project Qualification Dissertation

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Defining the problem

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Big O-Notation

Big O Notation

When shopping, there are multiple ways to collect all the items on your list. You might start at one end of the store and work through it methodically, which is often the most efficient - but only if your list is already sorted by store layout. Without planning, you might find yourself doubling back or criss-crossing the store multiple times. These differences in method lead to drastically different total effort. Similarly, the time taken depends not just on the method, but also on who is doing the shopping and how many items are on the list. These human and environmental factors make it impossible to measure an algorithm's speed purely in seconds. To address this, computer scientists use **Big O notation**, a mathematical way to describe how the number of steps an algorithm takes grows with the size of the input. For example, an algorithm with complexity $O(n)$ will perform at most a number of operations proportional to the length of the input list (n). An algorithm with $O(n^2)$ complexity might check each item against every other, leading to significantly more steps. These expressions describe the algorithm's **asymptotic behaviour** - that is, how it scales as inputs grow very large. When this growth follows a polynomial pattern like $O(n)$ or $O(n^2)$, we say it runs in **polynomial time**, which is generally considered efficient and tractable.