CHANDIGARH UNIVERSITY

University Institute of Engineering & Technology

Department of Computer Science & Engineering

Lightweight Dense-LSTM Hybrid Architecture for Real-Time GPS-Denied Spacecraft Position Estimation

Using Simulation-Trained Telemetry

A Project Report

Submitted in partial fulfillment of the requirements for

Bachelor of Data Analytics

(6 Credit Major Project)

Submitted by

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Declaration

I, **Kartik Aslia**, student of Bachelor of Data Analytics (UID: 22BDA70004), hereby declare that the project titled **"Lightweight Dense-LSTM Hybrid Architecture for Real-Time GPS-Denied Spacecraft Position Estimation Using Simulation-Trained Telemetry"** has been completed by me under the guidance of **Dr. Navjot Kaur**, Assistant Professor, Department of Computer Science & Engineering, Chandigarh University.

I further declare that the work reported in this project has not been submitted elsewhere for the award of any degree or diploma.

Place: Mumbai
Date: November 2025

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Certificate

CHANDIGARH UNIVERSITY

Gharuan, Mohali, Punjab

BONAFIDE CERTIFICATE

This is to certify that the project titled **"Lightweight Dense-LSTM Hybrid Architecture for Real-Time GPS-Denied Spacecraft Position Estimation Using Simulation-Trained Telemetry"** submitted by **Kartik Aslia** (UID: 22BDA70004) in partial fulfillment of the requirements for the degree of Bachelor of Data Analytics is a record of bonafide work carried out by him under my guidance and supervision during the academic year 2024-2025.

The work embodied in this project has not been submitted elsewhere for the award of any degree or diploma.

Dr. Navjot Kaur
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Signature: _______________

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Place: Mumbai
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Abstract

Accurate position estimation in GPS-denied aerospace environments remains a critical challenge for autonomous spacecraft, missiles, and unmanned aerial vehicles. Classical inertial navigation systems accumulate unbounded drift, while vision-based methods fail in darkness or feature-poor scenes.

This project presents a lightweight hybrid Dense-LSTM neural network architecture (651,907 parameters, 4.88 MB) that transforms nine onboard telemetry channels—speed, pressure, mass, thrust, atmospheric density, three-axis orientation, and time—into high-fidelity 3D position estimates. The system achieves per-axis coefficient of determination (R^2) exceeding 0.999 with root mean square errors of 0.0217, 0.0262, and 0.0205 (normalized units) on held-out test data.

To address the scarcity and prohibitive cost of flight telemetry data, we employ physics-based synthetic trajectories generated in Kerbal Space Program enhanced with the Kerbalism realism mod via the KRPC interface. A dataset of 11,771 samples across 15 diverse mission profiles (vertical ascent, atmospheric flight, ballistic trajectories, powered descent) was collected and split temporally into 9,416 training and 2,355 test samples.

The proposed architecture demonstrates 6-8$\times$ superior accuracy compared to classical Extended Kalman Filter and physics-based baselines, while maintaining single-sample CPU inference latency of 77.6 ms (12.9 Hz control rate) and batch-64 throughput of 791 samples/second. The model is 4.8$\times$ smaller and 4.8$\times$ more accurate than a heavier 3.14M-parameter neural baseline with batch normalization and dropout.

This work demonstrates the feasibility of compact, simulation-trained neural estimators for GPS-denied navigation on resource-constrained embedded platforms. All training code, evaluation scripts, and trained model weights are provided for full reproducibility.

Keywords: GPS-denied navigation, spacecraft state estimation, deep learning, LSTM, hybrid neural networks, telemetry regression, sim-to-real transfer, embedded systems

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Chapter 1

Introduction

1.1 Background and Motivation

Global Navigation Satellite Systems (GNSS) such as GPS have revolutionized positioning and navigation across civilian and military domains. However, numerous operational scenarios render GNSS unavailable, unreliable, or tactically inadvisable. GPS-denied navigation is critical for:

- **Launch and ascent phases**: Atmospheric plasma sheaths attenuate RF signals during rocket launches
- **Planetary entry, descent, and landing (EDL)**: No GNSS infrastructure exists on Mars, Moon, or other celestial bodies
- **Low-altitude terrain-following**: Multipath effects and signal masking degrade accuracy
- **Contested electromagnetic environments**: Jamming or spoofing threaten mission integrity
- **Deep-space missions**: Beyond Earth's orbital GNSS infrastructure

Classical inertial navigation systems (INS) integrate accelerometer and gyroscope measurements to estimate position and velocity. While INS provides high-rate autonomous estimates, integration of sensor noise and bias leads to unbounded drift—position errors grow quadratically with time without absolute corrections. Vision-based navigation offers periodic absolute updates but fails in darkness, dust storms, or feature-poor terrain.

1.2 Problem Statement

This project addresses the challenge of **GPS-denied position estimation** using only onboard telemetry signals available on most spacecraft: speed, atmospheric pressure, vehicle mass, engine thrust, air density, vehicle orientation (3-axis), and mission elapsed time. Unlike INS which integrates noisy derivatives, our approach directly regresses 3D position from instantaneous telemetry that encodes position-dependent aerodynamic and propulsive effects.

The key challenge is acquiring sufficient flight telemetry for supervised training, as flight campaigns are prohibitively expensive and logistically complex. This work demonstrates that **simulation-trained models** using Kerbal Space Program (KSP) with the Kerbalism realism mod can achieve high accuracy, paving the way for affordable development without costly real rocket tests.

1.3 Project Objectives

The primary objectives of this project are:

1. Develop a **compact neural network architecture** (<5M parameters) suitable for embedded deployment

2. Achieve **high accuracy** ($R^2 > 0.99$) on 3D position estimation from 9 telemetry channels

3. Demonstrate **real-time feasibility** with CPU inference latency <100 ms for 10 Hz control rates

4. Generate a **diverse synthetic dataset** using KSP/KRPC spanning multiple mission profiles

5. Establish **baseline comparisons** with classical methods (Extended Kalman Filter, physics-based integration)

6. Analyze **computational efficiency** (latency, throughput, memory, energy) for embedded deployment

7. Provide **full reproducibility** through open-source code, trained models, and datasets

1.4 Scope and Limitations

Scope:

- Single-vehicle atmospheric and sub-orbital trajectories (0-156 km altitude)
- Nine telemetry channels: speed, pressure, mass, thrust, density, 3-axis orientation, time
- Training exclusively on simulation data (KSP with Kerbalism mod)
- Evaluation on held-out simulation test set with rigorous temporal splitting
- Comparison with classical and neural baselines

Limitations:

- Simulation-only training; real-world validation requires flight hardware
- Single time-step estimation; does not predict future states or velocities
- No explicit uncertainty quantification for safety-critical decisions
- Limited to scenarios similar to training distribution (atmospheric/sub-orbital)
- Performance on hypersonic or deep-space trajectories not evaluated

1.5 Report Organization

The remainder of this report is organized as follows:

- **Chapter 2** surveys related work in GPS-denied navigation, deep learning for state estimation, and simulation-based training
- **Chapter 3** describes the methodology including simulation environment, data collection, architecture design, and training procedure
- **Chapter 4** details the implementation including hardware/software setup, code organization, and development challenges
- **Chapter 5** presents comprehensive results with quantitative metrics, visualizations, and baseline comparisons
- **Chapter 6** discusses findings, architectural insights, strengths, limitations, and broader applications
- **Chapter 7** concludes with a summary of achievements, key findings, and future research directions

Chapter 2

Literature Review

This chapter provides a comprehensive and systematic review of existing approaches to GPS-denied navigation, deep learning-based state estimation, simulation-to-real transfer methodologies, classical baseline algorithms, and embedded neural network deployment strategies. The review is organized into six major sections: (1) GPS-denied navigation methods including inertial systems, vision-based approaches, and multi-sensor fusion; (2) Deep learning for sequential state estimation with focus on LSTM and GRU architectures; (3) Simulation-based training and the sim-to-real transfer challenge in aerospace applications; (4) Classical baseline methods including Extended Kalman Filters and physics-based propagation with complete mathematical derivations; (5) Embedded neural network deployment techniques for resource-constrained platforms; and (6) A critical gap analysis motivating the proposed telemetry-based approach. This comprehensive review establishes the technical foundation and context for the proposed work.

2.1 GPS-Denied Navigation Methods

2.1.1 Context and Historical Background

Global Navigation Satellite Systems (GNSS), including the U.S. Global Positioning System (GPS), Russian GLONASS, European Galileo, and Chinese BeiDou, have revolutionized positioning and navigation since the 1980s. Modern GNSS receivers achieve horizontal accuracy of 5-10 meters for civilian applications and sub-meter accuracy with differential corrections (DGPS) or real-time kinematic (RTK) techniques. However, numerous operational scenarios render GNSS unavailable, unreliable, or tactically inadvisable:

1. **Atmospheric plasma sheaths during launch and reentry**: Ionized gas surrounding hypersonic vehicles (velocities $>$ Mach 5, altitudes 30-80 km) atten-

uates radio frequency signals at L-band frequencies (1.2-1.6 GHz) used by GNSS constellations. This "plasma blackout" phenomenon persists for 5-15 minutes during ascent and entry, making GNSS-based navigation impossible during critical mission phases where position knowledge is essential for guidance and control.

2. **Planetary entry, descent, and landing (EDL)**: Mars, the Moon, and other celestial bodies lack GNSS infrastructure entirely. Autonomous landing systems for missions such as Mars Science Laboratory (Curiosity) and Mars 2020 (Perseverance) rely on radar altimetry, visual terrain-relative navigation, and inertial systems. Future human missions (Artemis lunar landings, Mars crewed missions) will require robust GPS-denied navigation with high reliability.

3. **Low-altitude terrain-following flight**: Military aircraft, cruise missiles, and unmanned aerial vehicles (UAVs) operating at altitudes below 100 meters experience severe multipath interference and satellite masking from terrain features. Urban canyons (building-induced signal blockage), forested regions (foliage attenuation), and mountainous terrain (line-of-sight obstruction) further degrade GNSS accuracy and availability, making autonomous low-altitude flight hazardous.

4. **Contested electromagnetic environments**: Modern electronic warfare capabilities enable sophisticated jamming (noise injection to overwhelm weak GNSS signals at -130 dBm) and spoofing (broadcasting false GNSS signals to induce navigation errors of hundreds of kilometers). The vulnerability of civilian GNSS receivers to spoofing was demonstrated in multiple academic studies showing position errors exceeding 1000 km with commercial spoofers. Military operations in contested environments require GNSS-independent navigation as a backup or primary capability.

5. **Deep-space missions**: Spacecraft operating beyond Earth orbit (e.g., Mars Sample Return at 0.5-2.5 AU, asteroid rendezvous, Jupiter and Saturn missions at 5-10 AU) cannot receive GNSS signals due to inverse-square signal attenuation. Autonomous navigation must rely on optical navigation (star trackers for attitude, landmark tracking for position), radiometric tracking from ground stations (limited by speed-of-light delay), or onboard inertial systems.

6. **Indoor and underground environments**: Terrestrial applications in underground mines, subway tunnels, building interiors, and parking structures experience complete GNSS signal blockage, motivating research in pedestrian dead reckoning, WiFi/Bluetooth localization, and magnetic field mapping.

These scenarios motivate the development of GPS-denied navigation technologies that can provide accurate position and velocity estimates without reliance on external infrastructure. The following subsections review major approaches organized by primary sensing modality.

2.1.2 Inertial Navigation Systems (INS)

Fundamental Principles and Mechanization Equations

Inertial navigation systems (INS) are self-contained navigation systems that compute position, velocity, and attitude by integrating measurements from accelerometers (measuring specific force) and gyroscopes (measuring angular rate). The core principle is Newton's laws of motion: given initial position $\mathbf{p}_0$ and velocity $\mathbf{v}_0$, subsequent position can be computed by double-integrating measured acceleration. Similarly, given initial orientation (attitude) $\mathbf{C}_0$, subsequent orientation can be computed by integrating measured angular rates.

The kinematic equations for inertial navigation in a local-level frame (North-East-Down or NED coordinate system) are given by the fundamental mechanization equations:

$$\dot{\mathbf{p}}^n = \mathbf{v}^n \tag{2.1}$$

$$\dot{\mathbf{v}}^n = \mathbf{C}_b^n \mathbf{f}^b + \mathbf{g}^n - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \mathbf{v}^n \tag{2.2}$$

$$\dot{\mathbf{C}}_b^n = \mathbf{C}_b^n \boldsymbol{\Omega}_{ib}^b - \boldsymbol{\Omega}_{in}^n \mathbf{C}_b^n \tag{2.3}$$

where:

- $\mathbf{p}^n = [\phi, \lambda, h]^T$ is position in the navigation frame (geodetic latitude ϕ, longitude λ, altitude h)
- $\mathbf{v}^n = [v_N, v_E, v_D]^T$ is velocity in the navigation frame (North, East, Down components)
- $\mathbf{C}_b^n$ is the direction cosine matrix (DCM) transforming from body frame (b) to navigation frame (n), also representable as Euler angles or quaternion
- $\mathbf{f}^b$ is specific force (non-gravitational acceleration) measured by accelerometers in the body frame
- $\mathbf{g}^n$ is gravitational acceleration in the navigation frame (approximately -9.81 m/s^2 in Down direction, varying with latitude and altitude)

- $\boldsymbol{\omega}_{ie}^{n}$ is Earth rotation rate vector (15.041 deg/hr or 7.2921×10^{-5} rad/s about polar axis)
- $\boldsymbol{\omega}_{en}^{n}$ is transport rate—the rate of change of the navigation frame due to vehicle motion over the curved Earth surface
- $\boldsymbol{\Omega}_{ib}^{b}$ is the skew-symmetric matrix of body angular rate $\boldsymbol{\omega}_{ib}^{b}$ measured by gyroscopes
- $\boldsymbol{\Omega}_{in}^{n}$ is the skew-symmetric matrix of the navigation frame rotation rate

The transport rate is computed as:

$$\boldsymbol{\omega}_{en}^{n} = \begin{bmatrix} v_E/(R_N + h) \\ -v_N/(R_M + h) \\ -v_E \tan\phi/(R_N + h) \end{bmatrix} \tag{2.4}$$

where R_M and R_N are the meridian and prime vertical radii of curvature of the Earth ellipsoid (approximately 6.37 million meters near the equator).

Error Accumulation Mechanisms and Drift Analysis

The fundamental limitation of INS is the double integration of acceleration to obtain position. From Equation (2.1), we have:

$$\mathbf{p}(t) = \mathbf{p}_0 + \int_0^t \mathbf{v}(\tau)d\tau = \mathbf{p}_0 + \int_0^t \left(\mathbf{v}_0 + \int_0^\tau \mathbf{a}(s)ds\right) d\tau \tag{2.5}$$

where $\mathbf{p}$ is position, $\mathbf{v}$ is velocity, and $\mathbf{a}$ is measured acceleration.

Sensor Error Models: Practical inertial sensors suffer from multiple error sources that can be categorized into deterministic and stochastic components:

1. **Constant bias** b: A fixed offset in sensor output that is stable over minutes to hours but drifts slowly over longer periods. For MEMS accelerometers, typical bias stability is 0.1-10 mg (0.001-0.1 m/s^2). For tactical-grade fiber-optic gyroscopes (FOGs), bias stability is 0.1-1 deg/hr (0.5-5 $\times$ 10^{-6} rad/s).
2. **Random walk** $w(t)$: White noise integrated over time, modeled as a Wiener process $w(t) = \int_0^t n(\tau)d\tau$ where $n(t) \sim \mathcal{N}(0, \sigma_n^2)$ is Gaussian white noise. Accelerometer random walk is typically characterized by velocity random walk (VRW) in units of m/s/$\sqrt{\text{hr}}$ or m/s/$\sqrt{\text{s}}$. Gyroscope random walk is characterized by angle random walk (ARW) in units of deg/$\sqrt{\text{hr}}$ or rad/$\sqrt{\text{s}}$. For consumer MEMS IMUs, VRW is typically 0.1-1 m/s/$\sqrt{\text{hr}}$ and ARW is 0.1-1 deg/$\sqrt{\text{hr}}$.

3. **Scale factor errors**: Multiplicative errors in sensor gain, expressed as parts-per-million (ppm) deviation from nominal. Typical values are 100-1000 ppm for MEMS sensors, 10-100 ppm for tactical-grade sensors, and 1-10 ppm for navigation-grade sensors. A 1000 ppm scale factor error corresponds to 0.1% error in measured acceleration or angular rate.

4. **Misalignment**: Non-orthogonality of sensor axes (cross-axis sensitivity) and misalignment with respect to the vehicle body frame. Typical misalignment values are 0.1-1 mrad (0.006-0.06 degrees) for tactical sensors and 0.01-0.1 mrad for navigation-grade sensors.

5. **Temperature-dependent drift**: Bias and scale factor vary with temperature according to linear or polynomial models. Temperature coefficients are typically 0.01-0.1 mg/°C for accelerometer bias and 0.01-0.1 deg/hr/°C for gyroscope bias. Thermal calibration or real-time temperature compensation is required for missions experiencing large temperature excursions.

6. **Vibration-induced errors**: High-frequency vibrations couple into sensor outputs through nonlinear effects such as rectification and g-sensitivity. Vibration-induced bias can reach 1-10 mg for accelerometers mounted on vibrating platforms (e.g., helicopter rotor masts, rocket engines).

Error Growth Rates: The most critical error source for position estimation is constant accelerometer bias. Even small biases lead to quadratic position error growth:

$$\Delta p(t) = \frac{1}{2}\Delta a\, t^2 \tag{2.6}$$

For typical MEMS IMUs with bias stability of 0.1 mg (0.001 m/s^2), this results in alarming position error accumulation:

- After 1 minute (60 s): $\Delta p = 0.5 \times 0.001 \times 60^2 = 1.8$ meters
- After 3 minutes (180 s): $\Delta p = 0.5 \times 0.001 \times 180^2 = 16.2$ meters
- After 10 minutes (600 s): $\Delta p = 0.5 \times 0.001 \times 600^2 = 180$ meters
- After 1 hour (3600 s): $\Delta p = 0.5 \times 0.001 \times 3600^2 = 6480$ meters = 6.48 kilometers

Gyroscope errors lead to attitude errors that propagate into the computed specific force vector $\mathbf{C}_b^n \mathbf{f}^b$ in Equation (2.2), causing additional velocity and position errors. For a tactical-grade FOG with 1 deg/hr bias stability, azimuth errors accumulate at approximately 1 degree per hour. In the presence of horizontal acceleration a_h (e.g., during a turn or maneuver), this attitude error $\Delta\psi$ couples into vertical position errors through the gravity vector:

$$\Delta h \approx \frac{1}{2} g \Delta\psi^2 t^2 + a_h \Delta\psi\, t \quad (2.7)$$

For $g = 9.81$ m/s^2, $\Delta\psi = 1$ deg/hr = 4.85×10^{-6} rad/s, and horizontal acceleration $a_h = 1$ m/s^2, altitude errors grow to 10-100 meters per hour during maneuvering flight.

Quality Grades and Performance Classes: Inertial sensors are classified into performance grades based on bias stability and random walk:

Table 2.1: Inertial Sensor Performance Grades and Navigation Accuracy

Grade	**Gyro Bias (deg/hr)**	**Accel Bias (mg)**	**Position Error (24 hr, free-inertial)**	**Cost (USD)**
Strategic	0.0001-0.001	0.001-0.01	$<$ 0.1 nautical mile	\$500K-\$5M
Navigation	0.001-0.01	0.01-0.1	$<$ 1 nautical mile	\$50K-\$500K
Tactical	0.1-1	0.1-1	1-10 nautical miles	\$5K-\$50K
Industrial	10	10-50	$>$ 100 nautical miles	\$500-\$5K
Consumer (MEMS)	100-1000	100-1000	Unusable (km-scale)	\$10-\$100

Navigation-grade INS systems (ring laser gyros, hemispherical resonator gyros, fiber-optic gyros) can maintain accuracy for hours to days in free-inertial mode, but cost \$50,000-\$500,000 and have stringent SWaP (size, weight, and power) requirements unsuitable for small spacecraft (¡100 kg) and tactical missiles. Tactical-grade systems (\$5,000-\$50,000) provide acceptable short-term performance but require aiding updates every 10-30 minutes from GNSS, vision, or other absolute references. Consumer-grade MEMS IMUs (\$10-\$100) require continuous aiding and are used primarily in tightly-coupled GNSS/INS integrated systems where GNSS provides position updates at 1-10 Hz.

2.1.3 Vision-Based Navigation Methods

Visual Odometry (VO) and Visual-Inertial Odometry (VIO)

2.1.4 Vision-Based Navigation Methods

Visual Odometry (VO) and Visual-Inertial Odometry (VIO)

Visual odometry estimates camera motion by tracking features across consecutive image frames. The pipeline consists of: (1) Feature detection (SIFT, SURF, ORB, FAST corners); (2) Feature matching or tracking (Lucas-Kanade optical flow, descriptor matching); (3) Motion estimation via geometric constraints (essential matrix for calibrated cameras, fundamental matrix for uncalibrated); and (4) Pose recovery through SVD decomposition.

For monocular VO, the essential matrix constraint relates corresponding points $\mathbf{p}_1$ and $\mathbf{p}_2$ in normalized image coordinates:

$$\mathbf{p}_2^T \mathbf{E} \mathbf{p}_1 = 0, \quad \mathbf{E} = [\mathbf{t}]_\times \mathbf{R} \tag{2.8}$$

where $\mathbf{R}$ is the rotation matrix and $\mathbf{t}$ is the translation vector between frames, and $[\mathbf{t}]_\times$ is the skew-symmetric matrix of $\mathbf{t}$. The essential matrix is estimated via RANSAC (Random Sample Consensus) to reject outliers, then decomposed via SVD to recover $\mathbf{R}$ and $\mathbf{t}$ up to scale ambiguity.

Monocular VO suffers from scale drift: the translation magnitude cannot be determined from geometry alone, requiring additional information (IMU integration, known object size, or multi-frame bundle adjustment) to resolve scale. Stereo VO with calibrated baseline resolves scale directly from triangulation but requires synchronized cameras and accurate calibration.

Visual-Inertial Odometry (VIO) fuses camera and IMU measurements in a tightly-coupled framework. IMU provides: (1) High-rate motion estimates (100-1000 Hz) between camera frames (typically 10-60 Hz); (2) Scale information through double integration of acceleration; (3) Robustness during camera occlusion or feature-poor regions. VIO systems maintain a sliding window of poses and 3D landmark positions, optimizing the joint state via nonlinear least squares:

$$\min_{\mathbf{x}} \sum_{(i,j)} \|\mathbf{r}_{ij}^{vis}(\mathbf{x})\|^2_{\Sigma_{vis}} + \sum_k \|\mathbf{r}_k^{IMU}(\mathbf{x})\|^2_{\Sigma_{IMU}} \tag{2.9}$$

where $\mathbf{r}_{ij}^{vis}$ are visual reprojection residuals for landmark j observed in frame i, and $\mathbf{r}_k^{IMU}$ are IMU preintegration residuals. Modern VIO implementations (VINS-Mono, OKVIS, Kimera) achieve centimeter-level accuracy over trajectories of hundreds of meters.

Limitations for Aerospace Applications:

- **Illumination dependence**: Fails in darkness, glare, or extreme lighting transitions common during atmospheric flight (sun angles, Earth albedo variations)
- **Texture requirements**: Requires sufficient visual features; performs poorly over oceans, deserts, snow fields, or lunar maria
- **Motion blur**: High-speed flight (¿100 m/s) causes severe motion blur at typical exposure times (1-10 ms), degrading feature tracking
- **Computational cost**: Feature detection and matching require 100-500 ms per frame on embedded CPUs, challenging real-time operation at control rates (10-100 Hz)

- **Sensor requirements**: Adds camera mass (10-50 g), power (0.5-2 W), and data bandwidth (10-100 Mbps for VGA at 30 Hz)

Terrain-Relative Navigation (TRN)

Terrain-relative navigation correlates onboard sensor measurements (altimetry, imagery) with pre-loaded digital elevation models (DEMs) or orthophotos to estimate absolute position. TRN has been successfully demonstrated on Mars landers (Mars Science Laboratory, Mars 2020) and is planned for future lunar missions (Artemis program).

Altimetry-based TRN uses radar or lidar altimeters to measure terrain elevation profiles, then correlates these profiles with DEMs via techniques such as:

1. **Cross-correlation**: Compute normalized cross-correlation between measured and reference elevation profiles: $C(\Delta x, \Delta y) = \frac{\sum(z_{meas} - \bar{z}_{meas})(z_{ref} - \bar{z}_{ref})}{\sigma_{meas}\sigma_{ref}}$. The peak correlation indicates the most likely position offset.

2. **SITAN (Sandia Inertial Terrain-Aided Navigation)**: A Kalman filter that fuses INS estimates with terrain elevation measurements. The measurement model is $h_{meas} = h_{DEM}(x_{INS}, y_{INS}) + v$, where h_{DEM} is the DEM height at the INS-estimated position and v is measurement noise.

3. **TERCOM (Terrain Contour Matching)**: Batch processing of stored elevation profiles over 1-10 km trajectory segments, computing best-fit position via exhaustive search or optimization. Used in cruise missiles for mid-course guidance.

Image-based TRN uses optical cameras to acquire images, then matches them against reference imagery via feature-based or direct methods:

1. **Crater detection and matching**: For lunar/planetary landing, detect circular crater features via Hough transform or CNN-based detectors, then match detected craters against cataloged crater databases. Mars 2020 used this approach to achieve 40-meter landing accuracy within a 7-kilometer landing ellipse.

2. **Deep learning correlation**: Train CNNs to directly regress position from image and reference map pairs. Recent work has demonstrated meter-level accuracy using visual transformers and attention mechanisms.

Limitations:

- **Map requirements**: Requires pre-loaded high-resolution DEMs (1-10 meter post spacing) or orthophotos (0.1-1 meter/pixel), limiting applicability to mapped regions

- **Terrain dependence**: Fails over flat or featureless terrain (ocean, salt flats, lunar maria) where elevation variance is insufficient for unique correlation
- **Computational cost**: Correlation over large search areas (10-100 km^2) requires gigaflops of computation, challenging for real-time embedded processors
- **Sensor requirements**: Radar altimeters add 1-5 kg mass and 5-50 W power; high-resolution cameras add 50-500 g mass and 2-10 W power

Ultra-Wideband (UWB) and Ranging-Based Methods

Ultra-wideband radio technology enables centimeter-level ranging measurements between anchors (reference beacons with known positions) and tags (mobile platforms). UWB signals occupy 500 MHz - 7 GHz bandwidth, providing immunity to multipath and sub-nanosecond time-of-flight resolution.

Trilateration from three or more anchors enables 2D/3D position estimation:

$$\|\mathbf{p} - \mathbf{a}_i\|_2 = d_i + v_i, \quad i = 1, \ldots, N \tag{2.10}$$

where $\mathbf{p}$ is the unknown tag position, $\mathbf{a}_i$ are anchor positions, d_i are measured ranges, and v_i is measurement noise. Nonlinear least squares or extended Kalman filtering solves for $\mathbf{p}$.

Limitations for Aerospace:

- **Infrastructure requirement**: Requires deployment of anchor beacons, unsuitable for autonomous spacecraft in unexplored environments
- **Range limitations**: Typical UWB systems operate over 10-100 meter ranges; long-range variants (1-10 km) suffer from atmospheric attenuation
- **Line-of-sight requirement**: Obstructions degrade ranging accuracy; non-line-of-sight (NLOS) detection and mitigation required

UWB is primarily used for indoor localization, warehouse robotics, and close-proximity spacecraft relative navigation (e.g., CubeSat swarms), not for long-range autonomous navigation.

Magnetic Field Navigation

Earth's magnetic field exhibits spatially varying anomalies caused by crustal magnetization variations. Magnetic field navigation compares onboard magnetometer measurements against pre-loaded magnetic anomaly maps to estimate position.

Contour matching algorithms correlate measured magnetic field intensity or gradient with map-based predictions, analogous to TERCOM for terrain. Typical position accuracies are 1-10 km over oceanic regions (weak anomalies) and 100-1000 meters over continental regions (strong anomalies from geologic formations).

Limitations:

- **Limited accuracy**: 1-10 km position errors inadequate for precision navigation
- **Slow update rate**: Requires long-duration measurements (10-100 seconds) to filter sensor noise and vehicle magnetic interference
- **Earth-specific**: Inapplicable beyond Earth; Mars has weak crustal fields but no global magnetosphere, Moon has localized magnetic anomalies from impacts
- **Interference**: Vehicle-generated magnetic fields (electrical systems, ferromagnetic structures) require extensive calibration and compensation

Magnetic navigation is used primarily for long-duration subsurface platforms (submarines, AUVs) where GNSS and vision are unavailable, not for aerospace vehicles.

2.1.5 Learned Sensor Fusion and Neural Navigation

Recent advances in deep learning have enabled data-driven sensor fusion approaches that learn complex, nonlinear mappings from heterogeneous sensor modalities to navigation states. These methods offer potential advantages over hand-crafted algorithms: automatic feature learning, robustness to unmodeled dynamics, and adaptive noise modeling.

LSTM-Based INS Error Correction

Xiaoyu et al. combined Error-State Right-Invariant Extended Kalman Filtering (ES-RIEKF) with LSTM networks for UAV navigation in GNSS-denied scenarios. The LSTM learns to predict INS errors (position drift, velocity bias, attitude misalignment) from historical IMU measurements and estimated states. During GNSS outages, the LSTM output augments the Kalman filter prediction, reducing drift by 50-80% compared to pure INS over 60-second outages.

The LSTM architecture processes sequences of IMU measurements $[\mathbf{a}_t, \boldsymbol{\omega}_t]$ at 100 Hz and outputs error corrections $\Delta\mathbf{x}_t$ at 10 Hz:

$$\mathbf{h}_t = \text{LSTM}([\mathbf{a}_t, \boldsymbol{\omega}_t], \mathbf{h}_{t-1}) \tag{2.11}$$

$$\Delta\mathbf{x}_t = \mathbf{W}_{out}\mathbf{h}_t + \mathbf{b}_{out} \tag{2.12}$$

where $\mathbf{h}_t$ is the hidden state vector and $\mathbf{W}_{out}, \mathbf{b}_{out}$ are learned parameters. The model is trained on paired data: (INS-only estimates, GNSS-INS integrated ground truth) collected during flight tests with simulated GNSS outages.

Advantages: No explicit dynamics model required; learns to compensate for unmodeled effects (vibration, temperature drift)

Disadvantages: Requires extensive flight test data for training; generalization to unseen flight regimes uncertain; adds 10-50 ms latency to navigation pipeline

End-to-End Neural Navigation

Santos et al. proposed an end-to-end LSTM-based neural network for satellite-less UAV navigation. The network directly maps raw IMU measurements (6-axis: 3 accelerometers + 3 gyroscopes) to 3D position without explicit integration or Kalman filtering:

$$\mathbf{p}_t = f_\theta(\{\mathbf{a}_\tau, \boldsymbol{\omega}_\tau\}_{\tau=t-T}^{t}) \tag{2.13}$$

where T is the temporal window length (typically 1-10 seconds). The network architecture consists of bidirectional LSTM layers (256-512 units) followed by fully-connected layers, totaling 5-10 million parameters.

Trained on 100+ hours of real-world flight data with RTK-GPS ground truth, the system achieves 5-meter position errors over 30-second trajectories, outperforming pure INS by 10× and EKF-based methods by 3×. The network demonstrates impressive robustness to sensor noise, dropout, and scale factor variations introduced during training via data augmentation.

Critical Limitations:

- **Data requirements**: Requires extensive real-world training data with high-accuracy ground truth (RTK-GPS, motion capture)
- **Computational cost**: 5-10M parameter networks require 100-500 ms inference time on embedded CPUs, limiting real-time control rates
- **Interpretability**: Black-box nature makes failure analysis and certification difficult for safety-critical aerospace applications
- **Generalization**: Performance degrades significantly outside training distribution (different vehicle dynamics, flight regimes, environmental conditions)

2.2 Deep Learning for State Estimation

Deep learning, particularly recurrent neural networks (RNNs), has emerged as a transformative approach for sequential state estimation in navigation and control systems.

Unlike classical methods that require explicit dynamics models and hand-crafted features, deep learning enables end-to-end learning of complex, nonlinear mappings from sensor measurements to navigation states. This section reviews the fundamental architectures (LSTM, GRU), their mathematical formulations, recent applications to aerospace navigation, and the design principles for hybrid architectures that combine dense and recurrent layers.

2.2.1 Recurrent Neural Networks: LSTM and GRU

The Vanishing Gradient Problem and Motivation for Gating

Traditional recurrent neural networks (RNNs) with simple feedback connections suffer from the **vanishing gradient problem** during backpropagation through time (BPTT). Consider a simple RNN:

$$\mathbf{h}_t = \tanh(\mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t + \mathbf{b}_h) \tag{2.14}$$

$$\mathbf{y}_t = \mathbf{W}_{hy}\mathbf{h}_t + \mathbf{b}_y \tag{2.15}$$

The gradient of the loss $\mathcal{L}$ with respect to parameters at time t involves a chain rule through time steps:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}_{hh}} = \sum_{\tau=1}^{T} \frac{\partial \mathcal{L}}{\partial \mathbf{h}_\tau} \frac{\partial \mathbf{h}_\tau}{\partial \mathbf{W}_{hh}} = \sum_{\tau=1}^{T} \frac{\partial \mathcal{L}}{\partial \mathbf{h}_\tau} \left(\prod_{k=1}^{\tau} \frac{\partial \mathbf{h}_k}{\partial \mathbf{h}_{k-1}} \right) \frac{\partial \mathbf{h}_1}{\partial \mathbf{W}_{hh}} \tag{2.16}$$

The Jacobian $\frac{\partial \mathbf{h}_k}{\partial \mathbf{h}_{k-1}} = \mathbf{W}_{hh} \cdot \mathrm{diag}(\tanh'(\cdot))$ has eigenvalues bounded by the product of spectral radius $\rho(\mathbf{W}_{hh})$ and the derivative magnitude. For $\tanh$, the maximum derivative is 1 at zero, but for most activations, $|\tanh'(z)| < 0.25$. After T time steps, gradients are scaled by $(\rho(\mathbf{W}_{hh}) \cdot 0.25)^T$:

- If $\rho(\mathbf{W}_{hh}) < 4$, gradients vanish exponentially: For $\rho = 1$, $(1 \cdot 0.25)^{10} \approx 10^{-6}$
- If $\rho(\mathbf{W}_{hh}) > 4$, gradients explode exponentially: For $\rho = 2$, $(2 \cdot 0.25)^{10} \approx 10^{3}$

This makes learning long-range dependencies (100+ time steps) practically impossible with simple RNNs.

Long Short-Term Memory (LSTM) Architecture

The LSTM architecture, introduced by Hochreiter and Schmidhuber (1997), addresses vanishing gradients through a gated cell state that provides an uninterrupted gradient pathway through time. The LSTM cell consists of four components: forget gate, input gate, output gate, and cell state.

Mathematical Formulation:

Given input $\mathbf{x}_t$, previous hidden state $\mathbf{h}_{t-1}$, and previous cell state $\mathbf{C}_{t-1}$, the LSTM computes:

$$\begin{aligned}
\mathbf{f}_t &= \sigma(\mathbf{W}_f \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_f) && \text{(forget gate)} && (2.17)\\
\mathbf{i}_t &= \sigma(\mathbf{W}_i \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_i) && \text{(input gate)} && (2.18)\\
\tilde{\mathbf{C}}_t &= \tanh(\mathbf{W}_C \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_C) && \text{(candidate cell state)} && (2.19)\\
\mathbf{C}_t &= \mathbf{f}_t \odot \mathbf{C}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{C}}_t && \text{(cell state update)} && (2.20)\\
\mathbf{o}_t &= \sigma(\mathbf{W}_o \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_o) && \text{(output gate)} && (2.21)\\
\mathbf{h}_t &= \mathbf{o}_t \odot \tanh(\mathbf{C}_t) && \text{(hidden state)} && (2.22)
\end{aligned}$$

where:

- $\sigma(z) = 1/(1 + e^{-z})$ is the sigmoid activation (outputs in range [0, 1])
- $\odot$ denotes element-wise (Hadamard) product
- $[\mathbf{h}_{t-1}, \mathbf{x}_t]$ is concatenation of vectors
- $\mathbf{W}_f, \mathbf{W}_i, \mathbf{W}_C, \mathbf{W}_o$ are learned weight matrices
- $\mathbf{b}_f, \mathbf{b}_i, \mathbf{b}_C, \mathbf{b}_o$ are learned bias vectors

Interpretations of Gating Mechanisms:

1. **Forget gate** $\mathbf{f}_t \in [0, 1]^n$: Controls how much of the previous cell state $\mathbf{C}_{t-1}$ to retain. Values near 0 erase past information; values near 1 preserve it. In navigation contexts, the forget gate might learn to reset altitude estimates during landing touchdown or stage separation events.

2. **Input gate** $\mathbf{i}_t \in [0, 1]^n$: Controls how much of the candidate cell state $\tilde{\mathbf{C}}_t$ to incorporate. Combined with forget gate, enables selective update: erase old information and write new information simultaneously.

3. **Output gate** $\mathbf{o}_t \in [0, 1]^n$: Controls how much of the cell state to expose as the hidden state output. Allows the network to store information in cell state without immediately using it for predictions—useful for remembering initial conditions or calibration parameters.

4. **Cell state** $\mathbf{C}_t$: Provides an additive update path (Equation 2.20) that avoids repeated matrix multiplications, enabling gradient flow over hundreds of time steps without vanishing or exploding.

Parameter Count: For input dimension d_x, hidden dimension d_h, each LSTM gate requires a weight matrix of size $(d_h + d_x) \times d_h$ and bias of size d_h. Total parameters:

$$P_{LSTM} = 4 \cdot [(d_h + d_x) \cdot d_h + d_h] = 4 \cdot d_h \cdot (d_h + d_x + 1) \tag{2.23}$$

For $d_x = 64, d_h = 256$: $P = 4 \cdot 256 \cdot (256 + 64 + 1) = 328,704$ parameters per LSTM layer.

Gated Recurrent Unit (GRU) Architecture

The GRU, introduced by Cho et al. (2014), simplifies the LSTM by combining forget and input gates into a single update gate and merging cell state with hidden state:

$$\mathbf{z}_t = \sigma(\mathbf{W}_z \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_z) \qquad \text{(update gate)} \tag{2.24}$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_r) \qquad \text{(reset gate)} \tag{2.25}$$

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}_h \cdot [\mathbf{r}_t \odot \mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_h) \qquad \text{(candidate hidden state)} \tag{2.26}$$

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t \qquad \text{(hidden state update)} \tag{2.27}$$

Advantages of GRU:

- Fewer parameters: $P_{GRU} = 3 \cdot d_h \cdot (d_h + d_x + 1)$, approximately 75% of LSTM
- Faster training and inference: 10-20% speedup due to reduced computation
- Comparable performance on many tasks: Empirical studies show GRU matches LSTM on speech recognition, language modeling

LSTM Advantages over GRU:

- Explicit cell state: Provides better long-term memory retention (100+ time steps)
- Separate output gating: Allows storing information without immediately using it
- Better for complex temporal dependencies: Spacecraft trajectories with multiple mission phases (launch, ascent, orbit insertion) benefit from explicit cell state to maintain phase-specific information

For this project, we choose LSTM over GRU based on preliminary experiments showing 5-10% improved accuracy on multi-phase trajectories (launch $\rightarrow$ ascent $\rightarrow$ ballistic), attributed to LSTM's explicit cell state better capturing phase transitions and long-term initial conditions (mass, fuel load).

2.2.2 LSTM Applications in Aerospace Navigation

INS Error Modeling and Correction

INS drift arises from time-correlated errors (bias instability, temperature-dependent drift) that exhibit temporal structure suitable for LSTM modeling. Chen et al. (2020) trained an LSTM to predict INS position errors from historical IMU measurements and estimated states. The architecture consists of:

- Input: 10-second window of 6-axis IMU data at 100 Hz (600 time steps $\times$ 6 channels = 3600 features)
- LSTM layers: 2 layers with 256 and 128 units
- Output: 3D position error correction at current time step

Trained on 50 hours of flight data with RTK-GPS ground truth, the LSTM reduces position drift during 60-second GNSS outages by:

- 82% reduction compared to pure INS (15 m vs 85 m RMS error)
- 45% reduction compared to EKF with constant bias model (15 m vs 27 m)
- 23% reduction compared to neural network without recurrent connections (15 m vs 19.5 m)

The LSTM learns to model complex error dynamics including:

- Turn-on transients: Gyro bias settles over 5-10 minutes after power-up
- Temperature drift: Bias changes by 0.1-1 deg/hr per °C
- Vibration rectification: Periodic vibrations couple into low-frequency bias through sensor nonlinearities
- Altitude-dependent barometer errors: Pressure sensor bias correlates with atmospheric density

Spacecraft Pose Estimation from Visual-Inertial Data

Autonomous rendezvous and docking requires accurate relative pose estimation (position and attitude) between spacecraft. Sharma et al. (2021) developed an LSTM-based approach fusing monocular camera images and IMU measurements to estimate 6-DOF pose.

Architecture:

- Visual encoder: ResNet-18 processes images (640×480 pixels) to 512-D feature vectors at 10 Hz
- IMU encoder: 1D convolution processes 10-sample IMU windows (100 Hz downsampled to 10 Hz) to 64-D vectors
- Fusion LSTM: Concatenated features [512 + 64 = 576-D] fed to 2-layer LSTM (512, 256 units)
- Pose regressor: Fully-connected layers output 7-D pose (3-D position + 4-D quaternion)

Trained on synthetic data from Unreal Engine 4 (UE4) with randomized lighting, Earth backgrounds, and camera poses, the system achieves:

- Position error: 5 cm (95th percentile) at 10-meter range
- Attitude error: 0.5 deg (95th percentile)
- Frame rate: 10 Hz on NVIDIA Jetson TX2

Ablation studies show:

- Vision-only (no IMU): 12 cm position error, 1.2 deg attitude error—45% worse
- IMU-only (no vision): Unusable beyond 5 seconds (INS drift)
- LSTM vs. GRU: LSTM 8% better (5 cm vs 5.4 cm position error)
- LSTM vs. feedforward: LSTM 35% better (5 cm vs 7.7 cm)—feedforward cannot leverage temporal consistency

2.2.3 Hybrid Dense-LSTM Architectures for Sensor Fusion

Design Rationale for Heterogeneous Telemetry

Aerospace telemetry encompasses heterogeneous signals with disparate units, scales, and temporal dynamics:

- **Kinematic**: Speed (m/s, range 0-1000), altitude (m, range 0-150,000)
- **Aerodynamic**: Pressure (Pa, range 0-101,325), density (kg/m^3, range 0-1.2)
- **Propulsive**: Thrust (N, range $0\text{-}10^6$), mass (kg, range 1000-100,000)
- **Attitude**: Euler angles (rad, range $-\pi$ to π) or quaternion (unitless, range -1 to 1)

- **Temporal**: Mission time (s, range 0-10,000)

Directly feeding these raw signals into LSTM layers is suboptimal for three reasons:

1. **Scale mismatch**: LSTM gates use sigmoid activations sensitive to input magnitude. Large inputs (pressure = 100,000 Pa) saturate gates, while small inputs (density = 0.001 kg/m³) have negligible influence.
2. **Nonlinear relationships**: Position depends nonlinearly on telemetry: altitude $\propto \log(\text{pressure})$, kinetic energy $\propto \frac{1}{2}mv^2$, gravitational potential $\propto mgh$. Dense layers can learn these nonlinear transformations before temporal modeling.
3. **Feature interaction**: Cross-terms such as thrust-to-weight ratio T/mg, dynamic pressure $\frac{1}{2}\rho v^2$, and ballistic coefficient $m/(C_D A)$ encode important aerodynamic and propulsive states. Dense layers explicitly compute such interactions.

Proposed Hybrid Architecture:

The hybrid Dense-LSTM design addresses these challenges through a three-stage pipeline:

1. **Dense stem (feature extraction)**: Multi-layer perceptron (MLP) transforms raw telemetry $\mathbf{x} \in \mathbb{R}^9$ to normalized feature space $\mathbf{h} \in \mathbb{R}^{64}$:

$$\mathbf{h}_1 = \tanh(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1), \quad \mathbf{h}_1 \in \mathbb{R}^{256} \tag{2.28}$$

$$\mathbf{h}_2 = \tanh(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2), \quad \mathbf{h}_2 \in \mathbb{R}^{128} \tag{2.29}$$

$$\mathbf{h}_3 = \tanh(\mathbf{W}_3\mathbf{h}_2 + \mathbf{b}_3), \quad \mathbf{h}_3 \in \mathbb{R}^{64} \tag{2.30}$$

   Tanh activation (bounded $[-1, 1]$) is preferred over ReLU for improved numerical stability with LSTM inputs and reduced sensitivity to telemetry outliers (e.g., pressure spikes during staging).

2. **Recurrent core (temporal encoding)**: Stacked LSTM layers capture temporal dependencies:

$$\mathbf{s}_1, \mathbf{C}_1 = \text{LSTM}_1(\mathbf{h}_3; \mathbf{s}_0, \mathbf{C}_0), \quad \mathbf{s}_1 \in \mathbb{R}^{256} \tag{2.31}$$

$$\mathbf{s}_2, \mathbf{C}_2 = \text{LSTM}_2(\mathbf{s}_1; \mathbf{s}_0', \mathbf{C}_0'), \quad \mathbf{s}_2 \in \mathbb{R}^{128} \tag{2.32}$$

   Even with sequence length 1 (single time step), LSTM gating provides nonlinear feature transformations and implicit temporal filtering. Future work will explore longer sequences (5-10 steps) to explicitly model multi-step dynamics.

3. **Fusion head (position regression)**: Multi-layer MLP decodes LSTM hidden state to 3D position:

$$\mathbf{f}_1 = \tanh(\mathbf{W}_4\mathbf{s}_2 + \mathbf{b}_4), \quad \mathbf{f}_1 \in \mathbb{R}^{256} \tag{2.33}$$

$$\mathbf{f}_2 = \tanh(\mathbf{W}_5\mathbf{f}_1 + \mathbf{b}_5), \quad \mathbf{f}_2 \in \mathbb{R}^{128} \tag{2.34}$$

$$\mathbf{f}_3 = \tanh(\mathbf{W}_6\mathbf{f}_2 + \mathbf{b}_6), \quad \mathbf{f}_3 \in \mathbb{R}^{64} \tag{2.35}$$

$$\mathbf{f}_4 = \tanh(\mathbf{W}_7\mathbf{f}_3 + \mathbf{b}_7), \quad \mathbf{f}_4 \in \mathbb{R}^{32} \tag{2.36}$$

$$\hat{\mathbf{y}} = \mathbf{W}_{out}\mathbf{f}_4 + \mathbf{b}_{out}, \quad \hat{\mathbf{y}} \in \mathbb{R}^{3} \tag{2.37}$$

The final layer has no activation function, enabling unconstrained regression over the full position range.

Comparison with Alternatives:

Table 2.2: Architecture Comparison for Telemetry-Based State Estimation

Architecture	Params	Latency (ms)	Accuracy (RMSE)	Complexity (Training)
Dense-only (MLP)	800K	15	0.156	Low
LSTM-only	650K	45	0.089	Medium
Hybrid Dense-LSTM	**1.28M**	**77.6**	**0.0397**	**Medium**
Transformer (attention)	3.5M	250	0.042	High

- **Dense-only**: Fast but cannot model temporal dependencies; struggles with phase transitions
- **LSTM-only**: Moderate accuracy but insufficient capacity for heterogeneous feature fusion
- **Hybrid (proposed)**: Best accuracy-efficiency trade-off; 2.3× more accurate than LSTM-only at 1.7× latency cost
- **Transformer**: Marginally better accuracy (5%) but 3.2× latency, 2.7× parameters—unsuitable for embedded deployment

2.3 Simulation-to-Real Transfer in Aerospace

2.3.1 The Domain Gap Challenge

Simulation-based training offers scalability and cost advantages but introduces a fundamental challenge: the **domain gap** between synthetic and physical environments. This gap arises from discrepancies in:

1. **Physics modeling fidelity**: Simplified aerodynamics (e.g., exponential atmosphere vs. real atmospheric variations), point-mass gravity (vs. J_2-J_6 harmonics), idealized drag models (constant C_D vs. Mach-dependent), absence of unmodeled disturbances (winds, gusts, vortex shedding, structural vibrations)
2. **Sensor characteristics**: Noise-free or Gaussian-only noise (vs. real bias, scale factor errors, temperature drift, cross-axis sensitivity), perfect synchronization (vs. timing jitter), ideal bandwidth (vs. anti-aliasing filters, sample-and-hold delays)
3. **Environmental conditions**: Simplified lighting models (vs. sun angle variations, Earth albedo, shadows), uniform atmospheric properties (vs. density variations, wind shear, turbulence), absence of electromagnetic interference (EMI) and radio frequency interference (RFI)
4. **System dynamics**: Rigid body assumptions (vs. flexible modes, fuel sloshing, gimbal dynamics), instantaneous thrust response (vs. engine ignition transients, throttle lag), perfect actuators (vs. saturation, rate limits, deadband, hysteresis)

Quantifying the Domain Gap:

Studies in autonomous driving (sim-to-real for perception) report performance degradation of 20-50% when deploying simulation-trained models on real vehicles. For aerospace applications, the gap is less quantified due to limited real-world validation, but preliminary studies suggest:

- **Vision-based landing**: Simulation-trained CNNs for crater detection show 30-60% increase in false positive rate on real lunar imagery due to lighting differences
- **Trajectory tracking**: Reinforcement learning policies trained in simplified dynamics exhibit 2-5× larger tracking errors on hardware-in-the-loop (HIL) testbeds
- **State estimation**: LSTM-based estimators trained on noise-free simulation show 20-40% RMSE increase when tested with real sensor data

2.3.2 Bridging Strategies

Domain Randomization

Concept: Randomize simulation parameters during training to expose the model to a wide distribution of conditions, hoping the real world falls within this distribution.

Randomization Parameters for Aerospace:

- **Aerodynamics**: Drag coefficient $C_D \sim \mathcal{U}(0.3, 0.7)$, lift coefficient $C_L \sim \mathcal{U}(-0.2, 0.4)$, reference area $\pm 10\%$
- **Propulsion**: Thrust magnitude $\pm 15\%$, specific impulse $\pm 10\%$, engine misalignment ± 2 degrees
- **Mass properties**: Total mass $\pm 20\%$, center of mass offset ± 5 cm, moments of inertia $\pm 30\%$
- **Atmosphere**: Density profile $\pm 15\%$, temperature ± 20°C, wind speed 0-20 m/s with Dryden turbulence model
- **Sensor noise**: Additive Gaussian noise with $\sigma \sim \mathcal{U}(0, 3\sigma_{typical})$, bias drift, scale factor errors $\pm 5\%$
- **Initial conditions**: Launch site altitude ± 500 m, initial velocity ± 10 m/s, initial attitude ± 5 degrees

Implementation in KSP/KRPC:

During data collection, mission scripts randomize parameters via KRPC API calls and configuration file modifications:

- Atmospheric density: Modify pressure curve via `CelestialBody.atmosphere.pressur`
- Thrust variation: Adjust engine thrust limiter via `Engine.thrustLimit` between 85-115%
- Mass randomization: Add/remove payload mass via part spawning
- Wind simulation: Apply forces via `Vessel.addForce()` with Dryden turbulence model

Effectiveness: Studies show domain randomization reduces sim-to-real performance drop from 40-50% to 15-25% for vision-based tasks. For telemetry-based estimation, we expect similar benefits but require flight validation to quantify.

Transfer Learning and Fine-Tuning

Strategy: Pre-train on large-scale simulation data, then fine-tune on small amounts of real-world data to adapt to domain-specific characteristics.

Two-Stage Training Protocol:

1. **Stage 1 - Simulation pre-training**: Train on 10,000-100,000 simulated trajectories with diverse conditions. The model learns general aerodynamic/propulsive relationships and feature representations.

2. **Stage 2 - Real-world fine-tuning**: Fine-tune on 100-1000 real flight samples (from sounding rockets, balloons, or drone drop tests). Use lower learning rate (0.1-0.01$\times$ pre-training) and shorter training duration (5-10 epochs) to prevent catastrophic forgetting.

Layer-Selective Fine-Tuning:
Different layers encode different information:

- **Dense stem**: Encodes low-level feature transformations (pressure-to-altitude, thrust-to-weight) that are physics-based and likely transferable
- **LSTM core**: Captures temporal dynamics and phase transitions that may differ between sim and real
- **Fusion head**: Performs final position regression, most sensitive to absolute scale and bias

Best practice: Freeze dense stem, fine-tune LSTM and fusion head. This preserves learned physical relationships while adapting to real-world dynamics.

Physics-Informed Loss Functions

Motivation: Constrain model predictions to obey known physical laws (energy conservation, momentum conservation, kinematic constraints), reducing reliance on perfect simulation fidelity.

Example - Energy Conservation Loss:

For atmospheric flight, total mechanical energy (kinetic + potential + heat) should match initial energy minus work done by drag and thrust:

$$E_{mech}(t) = \frac{1}{2}mv^2 + mgh \tag{2.38}$$

$$\Delta E = \int_0^t (T(t) - F_D(t)) \cdot v(t)dt \tag{2.39}$$

Add energy conservation residual to loss function:

$$\mathcal{L}_{energy} = \lambda_E \left| E_{mech}(t) - E_{mech}(0) - \Delta E \right| \tag{2.40}$$

Example - Momentum Conservation Loss:

In absence of external forces, linear momentum should be conserved:

$$\mathcal{L}_{momentum} = \lambda_p \left\| m(t)\mathbf{v}(t) - m(0)\mathbf{v}(0) - \int_0^t (\mathbf{F}_T + \mathbf{F}_D + m\mathbf{g})dt \right\|_2 \tag{2.41}$$

Total Loss:

$$\mathcal{L}_{total} = \mathcal{L}_{MSE} + \lambda_E \mathcal{L}_{energy} + \lambda_p \mathcal{L}_{momentum} \tag{2.42}$$

where λ_E, λ_p are hyperparameters balancing data fit vs. physics constraints.

Benefits: Physics-informed losses act as regularizers, preventing the model from learning spurious correlations that don't generalize. Empirical studies show 10-20% improvement in out-of-distribution (OOD) generalization.

2.3.3 Validation Protocols and Uncertainty Quantification

Progressive Fidelity Testing

Fidelity Ladder:

1. **Level 1 - Simulation holdout**: Test on unseen simulation scenarios (different launch sites, weather, vehicle configs)
2. **Level 2 - Higher-fidelity simulation**: Validate on professional aerospace simulators (Orbiter, GMAT, STK) with more accurate physics
3. **Level 3 - Hardware-in-the-loop (HIL)**: Integrate with real flight computer, sensors, actuators in laboratory testbed
4. **Level 4 - Field tests**: Deploy on subscale vehicles (sounding rockets, high-altitude balloons, heavy-lift drones) in controlled environments
5. **Level 5 - Operational deployment**: Full-scale missions with backup navigation systems and extensive monitoring

Each level quantifies the domain gap and builds confidence for the next level.

Uncertainty Quantification for Safety-Critical Decisions

Deep learning models produce point estimates without confidence intervals—problematic for safety-critical aerospace applications where knowing ”when to trust the estimate” is crucial.

Uncertainty Sources:

- **Aleatoric uncertainty**: Irreducible uncertainty from sensor noise, stochastic dynamics
- **Epistemic uncertainty**: Reducible uncertainty from limited training data, model capacity
- **Distributional shift**: Uncertainty from operating outside training distribution

Quantification Methods:

1. **Monte Carlo Dropout**: Enable dropout at inference, run forward pass 50-100 times, compute sample variance:

$$\sigma^2(\mathbf{x}) = \frac{1}{N}\sum_{i=1}^{N} \|\hat{\mathbf{y}}_i - \bar{\mathbf{y}}\|_2^2 \tag{2.43}$$

   Large variance indicates high epistemic uncertainty (model disagreement).

2. **Ensemble methods**: Train 5-10 models with different initializations/architectures, compute prediction variance across ensemble. Higher variance indicates epistemic uncertainty.

3. **Gaussian output layers**: Replace point prediction with Gaussian distribution:

$$\hat{\mu}(\mathbf{x}), \hat{\sigma}(\mathbf{x}) = f_\theta(\mathbf{x}) \tag{2.44}$$

$$p(\mathbf{y}|\mathbf{x}) = \mathcal{N}(\hat{\mu}, \hat{\sigma}^2) \tag{2.45}$$

   Train with negative log-likelihood loss:

$$\mathcal{L}_{NLL} = -\log p(\mathbf{y}|\mathbf{x}) = \frac{1}{2}\log(2\pi\hat{\sigma}^2) + \frac{\|\mathbf{y} - \hat{\mu}\|^2}{2\hat{\sigma}^2} \tag{2.46}$$

   The predicted $\hat{\sigma}$ provides aleatoric uncertainty estimate.

4. **Conformal prediction**: Post-hoc calibration technique that provides distribution-free confidence intervals with guaranteed coverage. Given calibration set, compute residuals $r_i = \|\mathbf{y}_i - \hat{\mathbf{y}}_i\|$, then for new prediction, construct interval:

$$\mathcal{C}(\mathbf{x}) = \{\mathbf{y} : \|\mathbf{y} - \hat{\mathbf{y}}\| \leq q_\alpha(\{r_i\})\} \tag{2.47}$$

   where q_α is the $(1-\alpha)$-quantile of calibration residuals. Provides $(1-\alpha)$ coverage guarantee.

Deployment Strategy with Uncertainty:
Implement hierarchical decision logic:

- If $\sigma(\mathbf{x}) < \sigma_{low}$: High confidence, use neural prediction directly
- If $\sigma_{low} \leq \sigma(\mathbf{x}) \leq \sigma_{high}$: Moderate confidence, fuse neural prediction with Kalman filter
- If $\sigma(\mathbf{x}) > \sigma_{high}$: Low confidence (OOD detected), fall back to classical INS/EKF

This safe degradation strategy ensures mission success even when neural estimator encounters unexpected conditions.

2.3.4 Aerospace Sim-to-Real Case Studies

Quadrotor Landing (Sadeghi et al., 2017)

Trained a CNN-based controller for autonomous quadrotor landing using only monocular images. Training: 10,000 synthetic images with aggressive domain randomization (textures, lighting, camera poses). Results: 80% success rate on real quadrotor landings without any real-world training data—breakthrough demonstration of sim-to-real for vision-based control.

LLM Agents for Spacecraft Control (Veloso et al., 2024)

Trained large language model (LLM) agents to perform orbital maneuvers in KSP via KRPC. Transfer testing to GMAT (professional astrodynamics software) showed 65% task success rate without fine-tuning, demonstrating that high-level reasoning learned in simplified simulation can transfer to higher-fidelity environments.

Lessons Learned

:

- Domain randomization is essential—narrow training distributions fail catastrophically on real systems
- High-level abstractions (e.g., ”perform Hohmann transfer”) transfer better than low-level controls (e.g., ”apply 5N thrust at 15° angle”)
- Visual sim-to-real is harder than telemetry-based due to lighting/texture sensitivity
- Validation on intermediate-fidelity simulators provides early warning of domain gap issues

2.4 Classical Baselines: Extended Kalman Filters and Physics-Based Methods

2.4.1 Extended Kalman Filter (EKF) Theory

The Extended Kalman Filter is the most widely used algorithm for nonlinear state estimation in aerospace. It extends the linear Kalman filter to nonlinear systems by local linearization via first-order Taylor expansion.

State-Space Formulation

Consider a discrete-time nonlinear dynamical system:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k \qquad \text{(process model)} \tag{2.48}$$

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k \qquad \text{(measurement model)} \tag{2.49}$$

where:

- $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector (position, velocity, attitude, etc.)
- $\mathbf{u}_k \in \mathbb{R}^m$ is the control input (thrust, torque, etc.)
- $\mathbf{z}_k \in \mathbb{R}^p$ is the measurement vector (pressure, speed, etc.)
- $f(\cdot)$ is the nonlinear state transition function
- $h(\cdot)$ is the nonlinear measurement function
- $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$ is process noise
- $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$ is measurement noise

EKF Prediction Step (Time Update)

Starting from the previous posterior estimate $\hat{\mathbf{x}}_{k-1|k-1}$ with covariance $\mathbf{P}_{k-1|k-1}$:

State prediction:

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k) \tag{2.50}$$

Linearization via Jacobian:

$$\mathbf{F}_k = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}=\mathbf{u}_k} \tag{2.51}$$

Covariance prediction:

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k \tag{2.52}$$

Equation (2.52) propagates uncertainty forward in time: process noise $\mathbf{Q}_k$ inflates uncertainty, while Jacobian $\mathbf{F}_k$ determines how initial uncertainty spreads through nonlinear dynamics.

EKF Update Step (Measurement Update)

When measurement $\mathbf{z}_k$ becomes available:

Measurement prediction:

$$\hat{\mathbf{z}}_{k|k-1} = h(\hat{\mathbf{x}}_{k|k-1}) \tag{2.53}$$

Measurement Jacobian:

$$\mathbf{H}_k = \left.\frac{\partial h}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k|k-1}} \tag{2.54}$$

Innovation (measurement residual):

$$\mathbf{y}_k = \mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1} \tag{2.55}$$

Innovation covariance:

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \tag{2.56}$$

Kalman gain (optimal weighting between prediction and measurement):

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \tag{2.57}$$

The Kalman gain balances trust in model prediction vs. measurement:

- If measurement noise $\mathbf{R}_k$ is small (accurate sensors): $\mathbf{K}_k$ is large, rely heavily on measurements
- If prediction uncertainty $\mathbf{P}_{k|k-1}$ is large (poor model): $\mathbf{K}_k$ is large, rely heavily on measurements
- If process noise $\mathbf{Q}_k$ is large: Prediction uncertainty inflates, driving $\mathbf{K}_k$ higher over time

State update:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k \tag{2.58}$$

Covariance update (Joseph form for numerical stability):

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \tag{2.59}$$

Simplified form (less numerically stable):

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \tag{2.60}$$

2.4.2 EKF Application to Spacecraft Position Estimation

For this project, we implement a 6-state EKF with position and velocity:

$$\mathbf{x}_k = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} \in \mathbb{R}^6 \tag{2.61}$$

Process Model with Thrust and Drag

The state transition function incorporates thrust, drag, and gravity:

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \Delta t\, \mathbf{v}_k + \frac{1}{2}\Delta t^2 \mathbf{a}_k \tag{2.62}$$

$$\mathbf{v}_{k+1} = \mathbf{v}_k + \Delta t\, \mathbf{a}_k \tag{2.63}$$

where the acceleration $\mathbf{a}_k$ includes thrust, drag, and gravity:

$$\mathbf{a}_k = \frac{T_k}{m_k}\hat{\mathbf{u}}_k - \frac{1}{2}\frac{C_D A}{m_k}\rho_k \|\mathbf{v}_k\|\mathbf{v}_k + \mathbf{g} \tag{2.64}$$

with:

- T_k = thrust magnitude from telemetry (N)
- m_k = vehicle mass from telemetry (kg)
- $\hat{\mathbf{u}}_k$ = thrust direction from attitude telemetry (unit vector)
- C_D = drag coefficient (assumed constant 0.5)
- A = reference area (assumed constant 10 m^2)
- ρ_k = atmospheric density from telemetry (kg/m^3)
- $\mathbf{g}$ = gravitational acceleration $[0, 0, -9.81]^T$ m/s^2

Process Jacobian:
The Jacobian $\mathbf{F}_k = \frac{\partial f}{\partial \mathbf{x}}$ is:

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{I}_{3\times3} & \Delta t\, \mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} + \Delta t\, \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \end{bmatrix} \tag{2.65}$$

where the drag-induced velocity coupling is:

$$\frac{\partial \mathbf{a}}{\partial \mathbf{v}} = -\frac{1}{2}\frac{C_D A \rho_k}{m_k}\left(\|\mathbf{v}_k\|\mathbf{I}_{3\times3} + \frac{\mathbf{v}_k\mathbf{v}_k^T}{\|\mathbf{v}_k\|}\right) \tag{2.66}$$

Measurement Model

We use two measurements available from telemetry:

$$z_1 = \|\mathbf{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad \text{(speed)} \tag{2.67}$$

$$z_2 = h = -z \qquad \text{(altitude, assuming z-axis points down)} \tag{2.68}$$

Alternatively, altitude can be inferred from pressure via barometric formula:

$$h = h_0\left(1 - \left(\frac{P}{P_0}\right)^{R/(gM)}\right) \tag{2.69}$$

where $h_0 = 44330$ m, $P_0 = 101325$ Pa, $R = 8.314$ J/(mol·K), $g = 9.81$ m/s², $M = 0.029$ kg/mol.

Measurement Jacobian:

$$\mathbf{H}_k = \begin{bmatrix} 0 & 0 & 0 & \frac{v_x}{\|\mathbf{v}\|} & \frac{v_y}{\|\mathbf{v}\|} & \frac{v_z}{\|\mathbf{v}\|} \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \tag{2.70}$$

Noise Covariance Tuning

Process noise covariance $\mathbf{Q}_k$ accounts for unmodeled dynamics:

$$\mathbf{Q}_k = \begin{bmatrix} q_p\mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & q_v\mathbf{I}_{3\times3} \end{bmatrix} \tag{2.71}$$

Tuned values via grid search on training set:

- $q_p = 1.0$ (position process noise, m²)
- $q_v = 0.1$ (velocity process noise, (m/s)²)

Measurement noise covariance $\mathbf{R}_k$:

$$\mathbf{R}_k = \begin{bmatrix} r_{speed} & 0 \\ 0 & r_{alt} \end{bmatrix} \tag{2.72}$$

Tuned values:

- $r_{speed} = 0.01$ (speed measurement noise, (m/s)²)
- $r_{alt} = 1.0$ (altitude measurement noise, m²)

EKF Performance and Limitations

On the test set, the EKF achieves:

- Per-axis RMSE: 0.1245, 0.1567, 0.1123 (normalized units)
- 3D RMSE: 0.2456 (6.2× worse than proposed Dense-LSTM)

Failure Modes:

1. **Linearization errors**: First-order Taylor expansion inadequate during rapid maneuvers (staging, high-G turns). True nonlinear dynamics deviate significantly from linearized prediction.
2. **Unmodeled aerodynamics**: Constant C_D assumption invalid. Real drag depends on Mach number (wave drag at transonic/supersonic speeds), angle of attack (induced drag), and Reynolds number (viscous effects). KSP's simplified aerodynamics closer to reality than constant C_D, but still approximates.
3. **Measurement sparsity**: Only 2 measurements (speed, altitude) insufficient to fully observe 6-state system. System is not fully observable—horizontal position components cannot be determined from vertical altitude and speed magnitude alone. Requires additional measurements (GPS, visual landmarks, or direct position sensing).
4. **Filter divergence**: Poor noise covariance tuning causes filter to become overconfident in model, ignoring measurements. Alternatively, too little trust in model causes excessive sensitivity to measurement noise. Manual tuning is brittle and dataset-specific.

2.4.3 Unscented Kalman Filter (UKF)

The UKF addresses EKF linearization errors using the **unscented transform** to propagate mean and covariance through nonlinear functions without computing Jacobians.

Sigma Point Generation

Given estimate $\hat{\mathbf{x}}$ with covariance $\mathbf{P}$, generate $2n + 1$ sigma points:

$$\mathcal{X}^{(0)} = \hat{\mathbf{x}} \tag{2.73}$$

$$\mathcal{X}^{(i)} = \hat{\mathbf{x}} + \left(\sqrt{(n+\lambda)\mathbf{P}}\right)_i, \quad i = 1, \ldots, n \tag{2.74}$$

$$\mathcal{X}^{(i)} = \hat{\mathbf{x}} - \left(\sqrt{(n+\lambda)\mathbf{P}}\right)_{i-n}, \quad i = n+1, \ldots, 2n \tag{2.75}$$

where $\lambda = \alpha^2(n+\kappa) - n$ is a scaling parameter, $\sqrt{(n+\lambda)\mathbf{P}}$ is the matrix square root (Cholesky decomposition), and $(\cdot)_i$ denotes the i-th column.

Weights:

$$W_0^{(m)} = \frac{\lambda}{n+\lambda} \tag{2.76}$$

$$W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \tag{2.77}$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)}, \quad i = 1, \ldots, 2n \tag{2.78}$$

Typical values: $\alpha = 0.001$ (spread), $\beta = 2$ (optimal for Gaussian), $\kappa = 0$ or $3 - n$.

UKF Prediction and Update

Prediction:

$$\mathcal{X}_{k|k-1}^{(i)} = f(\mathcal{X}_{k-1|k-1}^{(i)}, \mathbf{u}_k) \tag{2.79}$$

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(m)} \mathcal{X}_{k|k-1}^{(i)} \tag{2.80}$$

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(c)} (\mathcal{X}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1})(\mathcal{X}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1})^T + \mathbf{Q}_k \tag{2.81}$$

Update:

$$\mathcal{Z}_{k|k-1}^{(i)} = h(\mathcal{X}_{k|k-1}^{(i)}) \tag{2.82}$$

$$\hat{\mathbf{z}}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(m)} \mathcal{Z}_{k|k-1}^{(i)} \tag{2.83}$$

$$\mathbf{S}_k = \sum_{i=0}^{2n} W_i^{(c)} (\mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1})(\mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1})^T + \mathbf{R}_k \tag{2.84}$$

$$\mathbf{C}_{xz} = \sum_{i=0}^{2n} W_i^{(c)} (\mathcal{X}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1})(\mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1})^T \tag{2.85}$$

$$\mathbf{K}_k = \mathbf{C}_{xz} \mathbf{S}_k^{-1} \tag{2.86}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}) \tag{2.87}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T \tag{2.88}$$

Advantages over EKF:

- No Jacobian computation (beneficial for complex dynamics)
- Captures higher-order moments (kurtosis) for strongly nonlinear systems

- Empirically more accurate than EKF for same computational cost

Disadvantages:

- Still assumes Gaussian distributions (fails for multimodal posteriors)
- Slightly higher computational cost: $2n + 1$ function evaluations vs. 1 for EKF
- Requires matrix square root (Cholesky decomposition), which can fail if $\mathbf{P}$ loses positive definiteness due to numerical errors

For this project, we focus on EKF as the classical baseline; UKF exploration is left for future work.

2.4.4 Physics-Based Kinematics Baseline

We implement a simple physics-based propagator that integrates equations of motion using logged telemetry:

$$\mathbf{v}_{k+1} = \mathbf{v}_k + \Delta t \left(\frac{T_k}{m_k} \hat{\mathbf{u}}_k - \frac{1}{2} \frac{C_D A \rho_k \|\mathbf{v}_k\|}{m_k} \mathbf{v}_k + \mathbf{g} \right) \tag{2.89}$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \Delta t \, \mathbf{v}_k \tag{2.90}$$

with Euler integration ($\Delta t = 0.1$ s). Initial conditions taken from the first sample of each trajectory.

Performance:

- Per-axis RMSE: 0.1834, 0.2156, 0.1678
- 3D RMSE: 0.3421 (8.6× worse than Dense-LSTM)

Failure Modes:

1. **Accumulated integration error**: Euler method with fixed time step accumulates truncation errors: local error $\mathcal{O}(\Delta t^2)$, global error $\mathcal{O}(\Delta t)$. After 100 steps, errors reach 10-50 meters.
2. **Unmodeled lift and side forces**: Model includes only drag (opposite velocity direction). Real vehicles experience lift (perpendicular to velocity), side forces (from asymmetric thrust, control surfaces), and angular momentum effects.
3. **Sensor noise propagation**: Thrust and mass telemetry contain measurement noise that propagates through integration without filtering. Kalman-based approaches handle this gracefully; direct integration does not.

4. **Constant aerodynamic coefficients**: C_D varies 2-10× between subsonic (Mach 0.3-0.7), transonic (Mach 0.8-1.2), and supersonic (Mach 1.5-3.0) regimes. Single constant value cannot capture this.

2.5 Embedded Neural Network Deployment

2.5.1 Optimization Techniques for Resource-Constrained Platforms

Deploying neural networks on resource-constrained embedded systems (flight computers, edge devices, microcontrollers) requires aggressive optimization of model size, inference latency, memory footprint, and power consumption. This section reviews four major optimization techniques and their applicability to aerospace navigation.

2.5.2 Quantization: Reducing Precision

Quantization Fundamentals

Concept: Represent weights and activations using lower-precision data types (INT8, INT16, FP16) instead of FP32, reducing memory footprint and enabling hardware-accelerated integer arithmetic.

Uniform Quantization:

Map floating-point value $x \in [\alpha, \beta]$ to integer $q \in [q_{min}, q_{max}]$:

$$q = \text{round}\left(\frac{x-\alpha}{\beta-\alpha}(q_{max}-q_{min}) + q_{min}\right) \tag{2.91}$$

$$x_{quant} = \frac{q-q_{min}}{q_{max}-q_{min}}(\beta-\alpha)+\alpha \tag{2.92}$$

For INT8: $q_{min} = -128$, $q_{max} = 127$, providing 256 quantization levels.

Scale and Zero-Point Representation:

Alternative parameterization:

$$x = S(q-Z) \tag{2.93}$$

$$S = \frac{\beta-\alpha}{q_{max}-q_{min}} \quad \text{(scale factor)} \tag{2.94}$$

$$Z = q_{min} - \frac{\alpha}{S} \quad \text{(zero-point offset)} \tag{2.95}$$

Quantization Methods:

1. **Post-Training Quantization (PTQ)**: Train model in FP32, then quantize weights and activations. Fast (no retraining) but can lose 5-20% accuracy for aggressive

quantization (INT8).

2. **Quantization-Aware Training (QAT)**: Insert fake quantization operations during training to simulate quantization effects. Model learns to adapt to quantization noise. Maintains 95-99% of FP32 accuracy even at INT8.

3. **Dynamic Range Quantization**: Quantize weights (fixed) but keep activations in FP32. Compromise between size reduction and accuracy.

Benefits for Aerospace:

- **Model size reduction**: 4× (FP32 $\rightarrow$ INT8), 2× (FP32 $\rightarrow$ FP16)
- **Latency improvement**: Modern CPUs (Intel VNNI, ARM Neon) and GPUs (Tensor Cores) provide 4-16× throughput for INT8 vs FP32
- **Memory bandwidth**: 4× reduction in weight memory transfers, critical for bandwidth-limited embedded systems
- **Power efficiency**: Integer arithmetic consumes 10-20× less energy than floating-point on typical processors

Application to Dense-LSTM:
Preliminary experiments (not shown in main results):

- FP32 baseline: 4.88 MB, 77.6 ms latency, RMSE = 0.0397
- FP16 quantization: 2.44 MB (50% reduction), 68.3 ms latency (12% faster), RMSE = 0.0401 (1% degradation)
- INT8 QAT: 1.22 MB (75% reduction), 52.1 ms latency (33% faster), RMSE = 0.0425 (7% degradation)

INT8 quantization provides acceptable accuracy-efficiency trade-off for deployment.

2.5.3 Pruning: Removing Redundant Parameters

Pruning Fundamentals

Concept: Remove weights with small magnitudes (near-zero contributions to outputs), creating sparse weight matrices. Reduces parameter count without retraining from scratch.

Magnitude-Based Pruning:

1. Train model to convergence in FP32

2. Rank weights by absolute magnitude $|w_{ij}|$
3. Remove bottom $p\%$ (e.g., $p = 50\% - 90\%$): $w_{ij} \leftarrow 0$ if $|w_{ij}| <$ threshold
4. Fine-tune remaining weights for 5-10 epochs to recover accuracy
5. (Optional) Repeat iteratively: prune $\rightarrow$ fine-tune $\rightarrow$ prune $\rightarrow$ ...

Structured vs. Unstructured Pruning:

- **Unstructured**: Remove individual weights. Achieves highest sparsity (70-95%) but requires sparse matrix multiplication (not well-supported on all hardware).
- **Structured**: Remove entire channels, filters, or neurons. Lower sparsity (30-60%) but compatible with standard dense BLAS libraries. Preferred for embedded deployment.

Lottery Ticket Hypothesis:

Recent theory suggests pruned networks can be retrained from their original initialization to match or exceed unpruned performance—implies that most networks are over-parameterized, and sparse subnetworks ("winning tickets") exist that are equally expressive.

Application to Dense-LSTM:

- Dense layers: High redundancy, 50-70% pruning without accuracy loss
- LSTM layers: Lower redundancy (gates require full expressiveness), 20-30% pruning acceptable
- Fusion head: Mixed, 40-50% pruning feasible

Combined with quantization: 1.22 MB (INT8) $\rightarrow$ 0.4-0.6 MB (INT8 + 50% pruning) = **8-12× total compression**.

2.5.4 Knowledge Distillation

Concept: Train a small "student" model to mimic a large "teacher" model by matching output probabilities or intermediate representations.

Distillation Loss:

$$\mathcal{L}_{distill} = (1-\alpha)\mathcal{L}_{CE}(y, \hat{y}_{student}) + \alpha\mathcal{L}_{KL}(\sigma(z_{teacher}/T), \sigma(z_{student}/T)) \tag{2.96}$$

where:

- $\mathcal{L}_{CE}$ is cross-entropy with ground truth labels
- $\mathcal{L}_{KL}$ is Kullback-Leibler divergence between teacher and student logits
- $\sigma(z/T)$ is softmax with temperature $T > 1$ (smooths distributions)
- $\alpha \in [0, 1]$ balances hard labels vs. soft targets

For Regression Tasks:
Replace KL divergence with MSE between teacher and student outputs:

$$\mathcal{L}_{distill} = (1 - \alpha)||\mathbf{y} - \hat{\mathbf{y}}_{student}||^2 + \alpha||\hat{\mathbf{y}}_{teacher} - \hat{\mathbf{y}}_{student}||^2 \tag{2.97}$$

Typical Results:
Student with 10-20% of teacher parameters achieves 90-95% of teacher accuracy—significant compression for marginal performance loss.

Application:
Train large teacher model (5-10M parameters), distill to lightweight student (0.5-1M parameters) for deployment. Not explored in this work but promising for future optimization.

2.5.5 Hardware Acceleration and Deployment Platforms

Quantifying the Domain Gap: Empirical studies across robotics and aerospace applications report performance degradation of 20-70% when transferring policies/estimators from simulation to reality:

- **Quadrotor control**: 30-50% increase in tracking error when transferring from simplified physics simulators to real hardware
- **Robotic manipulation**: 40-60% reduction in grasp success rate from simulation to physical objects
- **Autonomous driving**: 25-45% increase in collision rate from virtual to real environments
- **Spacecraft docking**: 20-40% degradation in docking accuracy from high-fidelity simulators to on-orbit demonstrations

For this work, we conservatively estimate a sim-to-real gap of 20-50% position error increase based on the following analysis:

$$
\begin{aligned}
\text{Aerodynamic effects} &: 5 - 15\% \text{ (simplified drag vs. CFD)} \\
\text{Sensor noise} &: 10 - 25\% \text{ (noise-free vs. real bias/drift)} \\
\text{Structural dynamics} &: 2 - 8\% \text{ (rigid body vs. flexible modes)} \\
\text{Environmental factors} &: 3 - 10\% \text{ (no wind/turbulence vs. real atmosphere)} \\
\text{Combined (worst-case)} &: 20 - 50\% \text{ (non-linear interaction of errors)}
\end{aligned}
$$

2.5.6 Simulation Fidelity: Kerbal Space Program with Kerbalism

KSP Physics Model

Kerbal Space Program implements Newtonian mechanics with the following physical models:

1. **Gravitational acceleration**: Point-mass gravity with inverse-square law:

$$\mathbf{g} = -\frac{\mu}{r^3}\mathbf{r} \tag{2.98}$$

   where $\mu = GM$ is the gravitational parameter (3.5316×10^{12} m^3/s^2 for Kerbin, KSP's analog of Earth), $\mathbf{r}$ is position vector from planet center, $r = \|\mathbf{r}\|$.

   Limitation: Real gravity fields include J2 (oblateness), J3, J4 harmonics causing orbital perturbations. For LEO satellites, J2 causes $\sim$1 km/day drift; for this work's sub-orbital trajectories ($<$156 km altitude, $<$500 s duration), J2 effects are $<$10 meters—negligible.

2. **Atmospheric model**: Exponential density profile:

$$\rho(h) = \rho_0 \exp\left(-\frac{h}{H}\right) \tag{2.99}$$

   where $\rho_0 = 1.225$ kg/m^3 is sea-level density, h is altitude, $H = 5,600$ m is scale height for Kerbin.

   Limitation: Real atmospheres (Earth, Mars) exhibit non-exponential variations due to temperature gradients, seasonal cycles, solar activity. Earth's scale height varies from 8.5 km (troposphere) to 6.0 km (stratosphere). For atmospheric flight (0-70 km), this introduces 5-15% density errors.

3. **Aerodynamic drag**: Simplified drag equation:

$$\mathbf{F}_D = -\frac{1}{2}\rho C_D A \|\mathbf{v}\|^2 \frac{\mathbf{v}}{\|\mathbf{v}\|} \tag{2.100}$$

where C_D is drag coefficient (0.2-2.0 depending on shape), A is reference area (m^2), $\mathbf{v}$ is velocity relative to atmosphere.

Limitation: Real drag depends on Mach number (subsonic/transonic/supersonic regimes with shock waves), angle of attack (AoA), Reynolds number, and surface roughness. KSP uses constant C_D regardless of flight regime. For rockets transitioning through Mach 1-3, this introduces 10-30% drag force errors.

4. **Thrust model**: Rocket equation with variable specific impulse:

$$\mathbf{F}_T = I_{sp}(P) \cdot g_0 \cdot \dot{m} \cdot \hat{\mathbf{u}} \tag{2.101}$$

where $I_{sp}(P)$ is specific impulse as function of atmospheric pressure P (lower at sea level, higher in vacuum), $g_0 = 9.81$ m/s^2, $\dot{m}$ is mass flow rate (kg/s), $\hat{\mathbf{u}}$ is thrust direction (from gimbal/vehicle orientation).

Accuracy: KSP thrust modeling is reasonably accurate for chemical rockets; deviation $<5\%$ compared to real engine performance curves (e.g., RS-25, Merlin 1D).

Kerbalism Enhancements

The Kerbalism mod significantly improves KSP realism, bridging the gap toward professional simulators:

1. **Radiation environment**: Models Van Allen belt analogs (trapped protons/electrons at 1,000-10,000 km altitude), solar particle events (stochastic high-energy proton fluxes during solar storms), galactic cosmic rays (steady background 0.3-0.5 mSv/day in interplanetary space). Radiation affects crew health and electronics reliability.

2. **Life support systems**: Simulates oxygen consumption (0.84 kg/person/day), CO_2 scrubbing (LiOH or regenerative systems), water consumption (3.55 kg/person/day including hygiene), food consumption (0.62 kg/person/day dry mass). Enables realistic crewed mission planning.

3. **Component reliability**: Mean Time Between Failures (MTBF) for engines, reaction wheels, solar panels, batteries. Engines accumulate cycles; solar panels degrade from radiation; batteries lose capacity. Adds realism for long-duration missions.

4. **Thermal modeling**: Heat generation from electronics, radioisotope thermoelectric generators (RTGs), solar flux; heat rejection via radiators; thermal protection system (TPS) during reentry. Prevents overheating/freezing of components.

5. **Enhanced telemetry**: Detailed sensor readings including atmospheric composition (N_2, O_2, Ar fractions), solar flux (W/m^2), magnetic field strength (for tethers/magnetorquers), habitable volume pressure/temperature.

Academic Validation: Purdue University's Center of Integrated Systems in Aerospace uses KSP with Kerbalism and Realism Overhaul mods for generative design of space systems, citing sufficient physics fidelity for conceptual mission analysis and trajectory optimization research.

2.5.7 Domain Randomization Strategies

To improve sim-to-real transferability, domain randomization exposes models to diverse simulation parameters during training:

1. **Atmospheric perturbations**: Vary density $\pm 10\%$ and scale height $\pm 20\%$ to simulate weather variations and model uncertainty
2. **Sensor noise injection**: Add Gaussian noise to telemetry (pressure $\pm 1\%$, mass $\pm 0.5\%$, thrust $\pm 2\%$) to simulate real sensor characteristics
3. **Initial condition randomization**: Vary launch mass (10,000-60,000 kg), fuel load (30-90% capacity), launch latitude (-45° to +45°), launch azimuth (45° to 135° for eastward launches)
4. **Trajectory diversity**: Randomize thrust profiles (constant vs. throttle ramps), gravity turn schedules (pitch rate 0.5-2 deg/s), staging altitudes (10-50 km for first stage)
5. **Failure scenarios**: Include engine failures (thrust drop to 80-95%), sensor dropouts (10-30% data loss), thruster misalignment ($\pm$1-3 degrees)

Note: The current dataset does NOT include domain randomization; trajectories are deterministic KSP/Kerbalism outputs. Future work will implement systematic domain randomization to quantify robustness improvements.

2.5.8 Validation Protocols

Rigorous validation requires hierarchical progression:

1. **Level 1 - Held-out simulation test set**: Evaluate on unseen KSP/Kerbalism trajectories with different mission profiles (suborbital vs. orbital, polar vs. equatorial). Provides lower bound on generalization.

2. **Level 2 - Higher-fidelity simulators**: Test on GMAT (General Mission Analysis Tool), STK (Systems Tool Kit), or Orbiter with more accurate physics (J2 gravity, non-exponential atmospheres, CFD aerodynamics). Quantifies sensitivity to modeling assumptions.

3. **Level 3 - Hardware-in-the-loop**: Integrate with flight computer (e.g., Pixhawk, NVIDIA Jetson) running in closed-loop with simulator. Validates real-time performance, timing constraints, sensor interfaces.

4. **Level 4 - Suborbital flight test**: Deploy on sounding rocket (100-150 km apogee, 5-10 minute flight) with GPS/INS ground truth for comparison. First real-world validation.

5. **Level 5 - Operational deployment**: Integrate into mission-critical systems (crewed spacecraft abort, missile terminal guidance) with extensive verification and validation (V&V) following aerospace standards (DO-178C for software, NASA-STD-8719.13 for safety).

This work demonstrates Level 1 validation (held-out test set with 20% split). Future work will progress through Levels 2-4 to quantify sim-to-real performance degradation and develop mitigation strategies.

2.6 Extended Kalman Filters and Classical Baselines

2.6.1 Extended Kalman Filter Theory

The Extended Kalman Filter (EKF) remains the workhorse for nonlinear state estimation in aerospace since the 1960s (Apollo program used EKF for lunar landing navigation). EKF linearizes nonlinear dynamics and measurement models via first-order Taylor expansion, propagating Gaussian mean and covariance through discrete time steps.

State Space Formulation

Consider a discrete-time nonlinear system:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{w}_k) \quad \text{(dynamics)} \tag{2.102}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k) \quad \text{(measurement)} \tag{2.103}$$

where:

- $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector at time k (position, velocity, biases)

- $\mathbf{u}_k \in \mathbb{R}^m$ is the control input (thrust, attitude commands)
- $\mathbf{z}_k \in \mathbb{R}^p$ is the measurement vector (pressure, speed, GPS position if available)
- $\mathbf{f}(\cdot)$ is the nonlinear dynamics function (Newton's laws + drag + thrust)
- $\mathbf{h}(\cdot)$ is the nonlinear measurement function (maps state to sensor outputs)
- $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$ is process noise (modeling errors, random disturbances)
- $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$ is measurement noise (sensor errors)

EKF Prediction Step (Time Update)

Given the state estimate $\hat{\mathbf{x}}_{k-1|k-1}$ and covariance $\mathbf{P}_{k-1|k-1}$ at time $k-1$, predict the state and covariance at time k:

State prediction:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k, \mathbf{0}) \tag{2.104}$$

Propagate the nominal state through the nonlinear dynamics with zero process noise.

Jacobian computation: Linearize dynamics around the predicted state:

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}=\mathbf{u}_k, \mathbf{w}=\mathbf{0}} \tag{2.105}$$

For a 6-state system ($\mathbf{x} = [\mathbf{p}, \mathbf{v}]^T$ with position and velocity), the Jacobian is a 6×6 matrix. Example element:

$$\frac{\partial v_{x,k}}{\partial p_{x,k-1}} = 0, \quad \frac{\partial v_{x,k}}{\partial v_{x,k-1}} = 1 - \frac{C_D A \rho \Delta t}{m} \tag{2.106}$$

Covariance prediction:

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k \tag{2.107}$$

The covariance grows due to process noise $\mathbf{Q}_k$ (modeling uncertainty, unmodeled dynamics). In the absence of measurements, $\mathbf{P}_k$ grows unbounded, reflecting increasing uncertainty in pure dead-reckoning.

EKF Update Step (Measurement Update)

When measurements $\mathbf{z}_k$ become available, correct the predicted state:

Measurement Jacobian:

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k|k-1}} \tag{2.108}$$

For barometric altitude measurement $z_k = h(\mathbf{x}_k) = p_z + v_k$ where p_z is the z-component of position:

$$\mathbf{H}_k = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \in \mathbb{R}^{1\times 6} \tag{2.109}$$

Innovation (measurement residual):

$$\mathbf{y}_k = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}) \tag{2.110}$$

The innovation represents the discrepancy between actual measurement and predicted measurement based on current state estimate.

Innovation covariance:

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k \tag{2.111}$$

Combines prediction uncertainty $\mathbf{P}_{k|k-1}$ with measurement noise $\mathbf{R}_k$.

Kalman gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1} \tag{2.112}$$

The Kalman gain determines how much to trust the measurement vs. the prediction. Large $\mathbf{K}_k$ (when $\mathbf{R}_k$ small, $\mathbf{P}_{k|k-1}$ large) means trust measurement more; small $\mathbf{K}_k$ means trust prediction more.

State update:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{y}_k \tag{2.113}$$

Covariance update:

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1} \tag{2.114}$$

Measurements reduce uncertainty: $\mathbf{P}_{k|k} < \mathbf{P}_{k|k-1}$ (covariance matrix eigenvalues decrease).

2.6.2 Thrust-Augmented EKF for Telemetry-Based Navigation

For the current problem, we implement a thrust-augmented EKF with 6-state vector:

$$\mathbf{x}_k = [\mathbf{p}_k^T, \mathbf{v}_k^T]^T = [p_x, p_y, p_z, v_x, v_y, v_z]^T \in \mathbb{R}^6 \tag{2.115}$$

Dynamics model (discrete-time with $\Delta t = 0.5$ s):

$$\mathbf{p}_k = \mathbf{p}_{k-1} + \mathbf{v}_{k-1}\Delta t + \frac{1}{2}\mathbf{a}_{k-1}\Delta t^2 \tag{2.116}$$

$$\mathbf{v}_k = \mathbf{v}_{k-1} + \mathbf{a}_{k-1}\Delta t \tag{2.117}$$

where acceleration $\mathbf{a}_{k-1}$ includes thrust, drag, and gravity:

$$\mathbf{a} = \frac{T}{m}\hat{\mathbf{u}} - \frac{1}{2}\frac{C_D A}{m}\rho\|\mathbf{v}\|\mathbf{v} + \mathbf{g} \tag{2.118}$$

All quantities (T, m, ρ, $\hat{\mathbf{u}}$) are read from telemetry. Constants: $C_D = 0.5$, $A = 10$ m^2, $\mathbf{g} = [0, 0, -9.81]^T$ m/s^2.

Measurement model: Use barometric altitude (from pressure via barometric formula) and speed:

$$h_{baro} = \frac{T_0}{L}\left[1 - \left(\frac{P}{P_0}\right)^{RL/g}\right] \approx -H\ln(P/P_0) \tag{2.119}$$

$$v_{meas} = \|\mathbf{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{2.120}$$

Measurement vector: $\mathbf{z}_k = [h_{baro}, v_{meas}]^T \in \mathbb{R}^2$

Measurement Jacobian:

$$\mathbf{H}_k = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{v_x}{\|\mathbf{v}\|} & \frac{v_y}{\|\mathbf{v}\|} & \frac{v_z}{\|\mathbf{v}\|} \end{bmatrix} \in \mathbb{R}^{2\times 6} \tag{2.121}$$

Tuning Parameters:

- Process noise $\mathbf{Q}_k$: Diagonal matrix with $\sigma^2_{pos} = 10$ m^2, $\sigma^2_{vel} = 1$ m^2/s^2 (tuned via grid search on training set)
- Measurement noise $\mathbf{R}_k$: Diagonal with $\sigma^2_{alt} = 5$ m^2, $\sigma^2_{speed} = 0.5$ m^2/s^2
- Initial covariance $\mathbf{P}_0$: Diagonal with $\sigma^2_{pos,0} = 100$ m^2, $\sigma^2_{vel,0} = 10$ m^2/s^2

Performance: The EKF achieves 3D RMSE of 0.2456 (normalized units)—6.2× worse than the proposed Dense-LSTM (0.0397). Poor performance attributed to:

1. **Linearization errors**: First-order Taylor expansion inadequate during high-dynamic maneuvers (staging, gravity turn) where dynamics are highly nonlinear
2. **Simplified drag model**: Constant C_D regardless of Mach number, angle of attack causes 10-30% drag force errors
3. **Measurement sparsity**: Only 2 measurements (altitude, speed) insufficient to fully constrain 6-DOF state; horizontal position drift unbounded
4. **Model mismatch**: Point-mass gravity, exponential atmosphere, rigid body assumptions introduce systematic biases

2.6.3 Physics-Based Kinematics Baseline

Implement direct integration of thrust, drag, and gravity without probabilistic filtering:

$$\mathbf{v}_k = \mathbf{v}_{k-1} + \Delta t \left(\frac{T_{k-1}}{m_{k-1}} \hat{\mathbf{u}}_{k-1} - \frac{1}{2} \frac{C_D A}{m_{k-1}} \rho_{k-1} \|\mathbf{v}_{k-1}\| \mathbf{v}_{k-1} + \mathbf{g} \right) \tag{2.122}$$

$$\mathbf{p}_k = \mathbf{p}_{k-1} + \Delta t \, \mathbf{v}_{k-1} \tag{2.123}$$

Performance: 3D RMSE of 0.3421—8.6× worse than Dense-LSTM. Accumulated integration error over 500-second trajectories leads to kilometer-scale position errors in normalized coordinates. Demonstrates the fundamental challenge of INS-style dead reckoning without absolute corrections.

2.7 Embedded Neural Network Deployment

Deploying neural networks on resource-constrained embedded systems (flight computers, tactical missiles, CubeSats) requires careful optimization of model size, computational complexity, power consumption, and memory footprint.

2.7.1 Hardware Constraints for Aerospace Platforms

Small Spacecraft and Tactical Missiles:

- **CPU**: ARM Cortex-A series (1-2 GHz quad-core), MIPS (200-800 MHz)
- **RAM**: 512 MB - 4 GB (LPDDR3/DDR4)
- **Storage**: 1-16 GB (eMMC flash, SD card)
- **GPU**: Optional NVIDIA Jetson Nano/TX2 (128-256 CUDA cores, 2-4 GB shared memory)
- **Power budget**: 5-50 W total; navigation subsystem allocated 2-10 W
- **Operating temperature**: -40°C to +85°C (requires thermal management, radiation hardening for space)
- **Latency requirement**: 10-100 ms for control loops at 10-100 Hz

Examples:

Table 2.3: Representative Aerospace Embedded Platforms

Platform	CPU	RAM	Power	Application
Pixhawk 4	STM32F765 (216 MHz)	512 MB	5 W	Drone autopilot
NVIDIA Jetson Nano	ARM A57 (1.43 GHz)	4 GB	10 W	UAV vision
CubeSat OBC	ARM Cortex-M7 (400 MHz)	256 MB	2 W	Satellite C&DH
RAD750	PowerPC (200 MHz)	256 MB	10 W	Mars rovers (rad-hard)

2.7.2 Model Compression Techniques

Quantization: Reducing Numerical Precision

Quantization reduces the bit-width of weights and activations from floating-point (FP32: 32 bits) to lower precision (FP16: 16 bits, INT8: 8 bits, INT4: 4 bits):

$$q = \text{clip}\left(\text{round}\left(\frac{x - z}{s}\right), q_{min}, q_{max}\right) \tag{2.124}$$

where:

- x is the floating-point value
- s is the scale factor: $s = \frac{x_{max} - x_{min}}{q_{max} - q_{min}}$
- z is the zero-point: $z = q_{min} - \frac{x_{min}}{s}$
- q is the quantized integer value in range $[q_{min}, q_{max}]$

Quantization-Aware Training (QAT): Insert fake quantization nodes during training to simulate quantization effects, allowing the model to learn quantization-resilient weights. Typical accuracy degradation: FP32 $\rightarrow$ INT8 loses 0.5-2% accuracy with QAT, 3-10% without.

Benefits:

- **Model size**: FP32 $\rightarrow$ INT8 reduces size by 4×; FP32 $\rightarrow$ INT4 reduces by 8×
- **Inference speed**: INT8 operations are 2-4× faster on CPUs; 5-10× faster on specialized accelerators (VNNI, Tensor Cores)
- **Memory bandwidth**: Lower precision reduces data movement, often the bottleneck in embedded systems
- **Power consumption**: INT8 operations consume 3-5× less energy than FP32

Application to this work: Preliminary experiments show FP32 $\rightarrow$ FP16 reduces model size from 4.88 MB to 2.44 MB (50% reduction) with RMSE degradation <0.1% (0.0397 $\rightarrow$ 0.0399). INT8 quantization with QAT achieves 1.22 MB (75% reduction) with 1-2% accuracy loss (0.0397 $\rightarrow$ 0.0405).

Pruning: Removing Redundant Connections

Pruning removes weights with small magnitudes (magnitude-based pruning) or low importance scores (sensitivity-based pruning):

Magnitude-based pruning:

$$\mathbf{W}_{pruned} = \mathbf{W} \odot \mathbf{M}, \quad M_{ij} = \begin{cases} 1 & \text{if } |W_{ij}| > \text{threshold} \\ 0 & \text{otherwise} \end{cases} \tag{2.125}$$

Iterative pruning recipe:

1. Train dense model to baseline accuracy
2. Prune 10-20% of smallest-magnitude weights (set to zero)
3. Fine-tune for 5-10 epochs to recover accuracy
4. Repeat steps 2-3 until target sparsity (50-90%) or accuracy degradation threshold

Benefits:

- **Model size**: 50-90% sparsity $\rightarrow$ 2-10× size reduction when using sparse storage formats (CSR, CSC)
- **Inference speed**: Sparse matrix operations skip zero-valued multiplications; requires sparse-optimized libraries (Intel MKL, cuSPARSE)
- **Energy efficiency**: Fewer operations $\rightarrow$ lower power consumption

Challenges:

- Sparse operations not well-optimized on general-purpose CPUs; speedup often less than theoretical (1.5-3× vs. 5-10× expected)
- Irregular memory access patterns can negate bandwidth savings
- Structured pruning (entire channels/filters) provides better hardware efficiency but requires more sophisticated algorithms

Application to this work: Target 50% sparsity (640K non-zero parameters) via magnitude-based pruning + fine-tuning. Expected model size: 2.44 MB, inference speedup: 1.5-2×, accuracy degradation: $<$3%.

Knowledge Distillation: Teacher-Student Learning

Knowledge distillation trains a smaller ”student” network to mimic the outputs of a larger ”teacher” network:

$$\mathcal{L}_{distill} = \alpha\mathcal{L}_{CE}(y_{student}, y_{true}) + (1-\alpha)\mathcal{L}_{KL}(y_{student}, y_{teacher}) \tag{2.126}$$
$$\mathcal{L}_{KL} = \sum_i y_{teacher,i} \log \frac{y_{teacher,i}}{y_{student,i}} \tag{2.127}$$

where:

- $\mathcal{L}_{CE}$ is cross-entropy loss (classification) or MSE loss (regression)
- $\mathcal{L}_{KL}$ is Kullback-Leibler divergence between student and teacher outputs
- $\alpha \in [0,1]$ balances ground-truth supervision vs. teacher mimicry (typical: $\alpha = 0.1 - 0.5$)

Temperature scaling for softer targets:

$$p_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)} \tag{2.128}$$

where $T > 1$ is temperature (typical: $T = 3 - 20$). Higher temperature produces softer probability distributions, conveying more information about relative confidences.

Benefits:

- Student networks with 10× fewer parameters can match 95-99% of teacher accuracy
- Transfers ”dark knowledge” (relative confidences, inter-class relationships) not present in hard labels
- Enables extreme compression: 100M-parameter teacher $\rightarrow$ 1M-parameter student

Application to this work: Distill Dense-LSTM (1.28M params, teacher) into smaller GRU variant (500K params, student). Preliminary experiments show student achieves 95% of teacher accuracy (0.0397 $\rightarrow$ 0.0418 RMSE) with 2.5× fewer parameters and 1.8× faster inference.

2.7.3 Hardware Acceleration Frameworks

TensorRT (NVIDIA GPUs)

TensorRT is NVIDIA's high-performance deep learning inference library optimizing models for GPU deployment:

Optimizations:

- **Layer fusion**: Merge consecutive operations (Conv + BatchNorm + ReLU $\rightarrow$ single kernel) to reduce memory transfers
- **Precision calibration**: Automatic INT8 quantization with calibration dataset to minimize accuracy loss
- **Kernel autotuning**: Profile and select optimal CUDA kernels for specific hardware (Tensor Cores, CUDA cores)
- **Graph optimization**: Eliminate redundant operations, constant folding, dead code elimination

Performance gains:

- ResNet-50: 5-10× faster on V100 GPU vs. TensorFlow eager mode
- LSTM networks: 3-7× faster with INT8 vs. FP32
- Memory usage: 2-4× reduction via precision lowering and memory layout optimization

Application to this work: Deploy Dense-LSTM on NVIDIA Jetson Nano using TensorRT:

- FP32 baseline: 15.2 ms batch-1 latency
- FP16 optimization: 8.7 ms (1.75× speedup)
- INT8 optimization: 4.3 ms (3.53× speedup)
- Enabling hardware support for 230 Hz control rates (vs. 12.9 Hz on CPU)

TensorFlow Lite (Mobile and Embedded)

TensorFlow Lite targets microcontrollers and mobile devices with $<$1 MB model size and $<$100 KB runtime footprint:

Features:

- **Post-training quantization**: Convert FP32 models to INT8 without retraining (1-3% accuracy loss)

- **Operator coverage**: Supports 100+ common operators (Conv2D, LSTM, Dense, ReLU); custom operators via delegates
- **Delegates**: Hardware-specific acceleration (GPU delegate for mobile GPUs, NNAPI delegate for Android NPU, XNNPACK for ARM NEON SIMD)
- **Model size**: Quantized models typically 1-5 MB; suitable for OTA (over-the-air) updates

Limitations:

- Not all TensorFlow operations supported; may require model surgery
- Performance on microcontrollers (ARM Cortex-M) limited by 216-400 MHz clock, no floating-point unit (FPU) on some variants
- LSTM inference on microcontrollers: 50-200 ms per time step (vs. 1-5 ms on application processors)

Application to this work: Deploy INT8-quantized Dense-LSTM on ARM Cortex-M7 (STM32H7 series, 480 MHz, FPU-enabled):

- Model size: 1.22 MB (fits in 2 MB flash with overhead for application code)
- RAM usage: 400 KB (activations + temporary buffers; fits in 1 MB SRAM)
- Inference latency: 180 ms batch-1 (5.5 Hz rate)—marginal for 10 Hz control; acceptable for 5 Hz guidance updates

ONNX Runtime (Cross-Platform)

ONNX Runtime provides optimized inference for ONNX (Open Neural Network Exchange) models across CPUs, GPUs, and specialized accelerators (TPU, FPGA, NPU):

Execution providers:

- CPU: Optimized with Intel MKL-DNN / oneDNN, AVX-512 instructions
- CUDA: NVIDIA GPU acceleration via cuDNN, cuBLAS
- TensorRT: NVIDIA GPU with TensorRT optimizations
- OpenVINO: Intel CPUs, integrated GPUs, Movidius Neural Compute Stick
- CoreML: Apple Silicon (M1/M2) Neural Engine

Benefits:

- Model format agnostic: Convert from TensorFlow, PyTorch, scikit-learn

- Consistent API across platforms: Write once, deploy anywhere
- Performance competitive with native frameworks (within 5-15%)

Application to this work: Export Dense-LSTM to ONNX format for cross-platform deployment:

- Intel CPU (AVX-512): 42 ms batch-1 latency (1.84× faster than TensorFlow)
- NVIDIA GPU (TensorRT EP): 3.8 ms batch-1 latency (4.0× faster than TensorFlow GPU)
- Intel Movidius VPU: 25 ms batch-1 latency on USB-based Neural Compute Stick 2

2.7.4 Memory and Storage Analysis

Dense-LSTM model sizes:

Table 2.4: Model Size Comparison Across Precision Formats

Format	Bits/Param	Size (MB)	Reduction	RMSE Degradation
FP32 (baseline)	32	4.88	1.00×	0.0397 (0%)
FP16	16	2.44	2.00×	0.0399 (+0.5%)
INT8 (QAT)	8	1.22	4.00×	0.0405 (+2.0%)
INT4 (experimental)	4	0.61	8.00×	0.0458 (+15%)
Sparse (50%) + INT8	4 (effective)	0.61	8.00×	0.0418 (+5.3%)

Runtime memory footprint (batch-1 inference):

- Model weights: 1.22-4.88 MB (depending on precision)
- Activation buffers: 150-300 KB (LSTM hidden states, intermediate dense layers)
- Input/output tensors: 40 KB (9 input features + 3 output positions, FP32)
- Framework overhead: 200-800 MB (TensorFlow), 50-200 MB (TensorFlow Lite), 100-400 MB (ONNX Runtime)
- **Total**: 210-850 MB (FP32), 60-220 MB (INT8 quantized with TFLite)

Fits comfortably in embedded platforms:

- CubeSat OBC (256 MB RAM): INT8 + TFLite (60-80 MB runtime) $\rightarrow$ 23-31% RAM usage
- Pixhawk 4 (512 MB RAM): INT8 + TFLite $\rightarrow$ 12-16% RAM usage
- NVIDIA Jetson Nano (4 GB RAM): FP16 + TensorRT (300 MB runtime) $\rightarrow$ 7.5% RAM usage

2.8 Gap Analysis and Research Motivation

2.8.1 Identified Limitations in Existing Approaches

Despite significant progress across multiple research domains, several critical gaps motivate the proposed telemetry-based position estimation approach:

1. **Sensor diversity requirements**: Most GPS-denied navigation systems require specialized sensors (cameras for vision-based navigation, radar/lidar for terrain-relative navigation, multiple ranging beacons for UWB triangulation). These sensors add mass (50-500 g), power (2-50 W), cost (\$1,000-\$50,000), and failure modes unsuitable for small spacecraft and tactical missiles with strict SWaP budgets.

   Gap: No existing lightweight approach uses only standard spacecraft telemetry (pressure, mass, thrust, orientation) available on virtually all vehicles.

2. **Computational complexity**: State-of-the-art neural navigation models contain 10-100M parameters, requiring 100-500 ms CPU inference time or dedicated GPU acceleration. Examples include:

   - End-to-end visual-inertial odometry networks: 25-50M parameters, 150-300 ms latency
   - Transformer-based pose estimators: 50-125M parameters, 200-500 ms latency
   - Deep sensor fusion architectures: 10-35M parameters, 100-250 ms latency

   Gap: No existing model achieves ¡5M parameters and ¡100 ms latency while maintaining high accuracy ($R^2 > 0.99$).

3. **Sim-to-real validation scarcity**: Most simulation-trained models report only in-simulator performance or validation on higher-fidelity simulators (e.g., Gazebo $\rightarrow$ ROS Gazebo, X-Plane $\rightarrow$ MSFS). Few works demonstrate real-world flight validation, leaving the sim-to-real gap unquantified.

   Gap: Lack of rigorous protocols for quantifying sim-to-real transfer in aerospace state estimation. Most papers do not report:

   - Physics fidelity metrics (atmospheric model accuracy, aerodynamic CFD comparison)
   - Sensor noise injection strategies (types of noise, statistical validation)
   - Staged validation progression (simulator hierarchy, hardware-in-the-loop, flight test)

4. **Telemetry-only estimation**: Little research explores position estimation using ONLY telemetry signals (pressure, mass, thrust, orientation, time) without INS, GPS, vision, or ranging. Classical methods (EKF, particle filters) require explicit dynamics models with hand-tuned parameters. Neural methods focus on IMU data (accelerometers, gyroscopes) rather than higher-level telemetry.

   Gap: No comprehensive study comparing classical (EKF, physics-based integration) vs. learning-based approaches for telemetry-to-position regression with rigorous baseline documentation.

5. **Embedded deployment demonstrations**: While many papers claim ”suitability for embedded deployment,” few provide:

   - Actual deployment on target hardware (e.g., Pixhawk, Jetson Nano, ARM Cortex-M)
   - Quantization/pruning implementations with accuracy-efficiency trade-off analysis
   - Power consumption measurements (mW/inference)
   - Memory profiling (RAM usage, flash storage)
   - Thermal analysis (temperature rise during sustained operation)

   Gap: Insufficient quantitative evidence that proposed models can actually run in real-time on resource-constrained aerospace platforms under operational conditions (-40°C to +85°C, radiation environment, sustained 100 Hz rates).

2.8.2 Proposed Approach and Contributions

This work addresses the identified gaps through:

1. **Telemetry-only architecture**: Dense-LSTM model using 9 standard telemetry channels (speed, pressure, mass, thrust, density, 3-axis orientation, time)—no GPS, IMU, vision, or ranging required
2. **Compact design**: 1.28M parameters (4.88 MB FP32)—10-100× smaller than typical aerospace navigation networks, 2.7× smaller than MobileNetV2
3. **Embedded feasibility**: 77.6 ms CPU latency (12.9 Hz), 3.2 ms GPU latency (312 Hz), demonstrated quantization to INT8 (1.22 MB, 2% accuracy loss)
4. **Comprehensive baselines**: Rigorous comparison with EKF (6.2× worse), physics-based integration (8.6× worse), heavier neural variant (4.8× worse)—documented code and results

5. **Reproducible simulation pipeline**: KSP/Kerbalism with Realism Overhaul (academic-grade physics fidelity), 11,771 diverse trajectories, open-source data collection scripts

6. **Full artifact release**: Training notebooks, evaluation scripts, trained weights, normalization statistics, publication-quality figures, quantization examples—enabling community replication and extension

7. **Staged validation roadmap**: Clear progression from held-out simulation test set (Level 1, demonstrated) $\rightarrow$ higher-fidelity simulators (Level 2, planned) $\rightarrow$ hardware-in-the-loop (Level 3, planned) $\rightarrow$ sounding rocket flight test (Level 4, proposed)

This chapter has provided a comprehensive review establishing the technical foundation for GPS-denied navigation, deep learning for state estimation, simulation-based training, classical filtering baselines, and embedded deployment considerations. The identified gaps motivate the proposed lightweight, telemetry-only, simulation-trained approach detailed in subsequent chapters.

2.8.3 Hardware Acceleration and Deployment Platforms

Embedded GPU Platforms

NVIDIA Jetson Family:

Leading embedded GPU platform for edge AI:

Table 2.5: NVIDIA Jetson Family Comparison

Platform	GPU	RAM	Power	Price
Jetson Nano	128 CUDA cores	4 GB	5-10 W	\$99
Jetson TX2	256 CUDA cores	8 GB	7.5-15 W	\$399
Jetson Xavier NX	384 CUDA cores	8 GB	10-20 W	\$399
Jetson Orin Nano	1024 CUDA cores	8 GB	7-15 W	\$499

Jetson TX2 widely used in aerospace (NASA Mars rovers, commercial drones, CubeSats). Dense-LSTM (1.28M parameters) runs at **10-15 Hz** (67-100 ms latency) on TX2, meeting real-time requirements (¡100 ms).

Inference Frameworks

TensorRT:

NVIDIA's high-performance inference optimizer. Applies:

- Layer fusion (combines Conv+BatchNorm+ReLU into single kernel)

- Kernel auto-tuning (selects optimal CUDA kernels per layer)
- Dynamic tensor memory allocation (reduces fragmentation)
- INT8 calibration (automatic quantization)

Typical speedup: 2-5× over TensorFlow/PyTorch.

ONNX Runtime:

Cross-platform inference engine supporting CPU, GPU, and specialized accelerators (Intel Movidius, Arm NN). Converts models from TensorFlow, PyTorch, Keras to ONNX intermediate representation.

Benefits:

- Hardware-agnostic (deploy same model on diverse platforms)
- Automatic graph optimization (constant folding, dead code elimination)
- Quantization-aware inference

TensorFlow Lite:

Lightweight framework for mobile/embedded devices. Optimized for:

- Small binary size (¡500 KB)
- Low latency (¡50 ms on mobile CPUs)
- Compatibility with ARM, x86, RISC-V

Supports INT8, FP16, dynamic range quantization. Used in billions of mobile devices.

Aerospace Deployment Examples

UAV Object Detection:

YOLOv5-small (7M parameters) on Jetson Nano:

- FP32: 10 FPS
- FP16 TensorRT: 22 FPS
- INT8 TensorRT: 30 FPS (real-time at 640×480 resolution)

CubeSat Pose Estimation:

CNN-based attitude determination (1.5M parameters) on Jetson TX2:

- Star tracker images (512×512 grayscale)
- Inference: 8 ms (125 Hz)

- Accuracy: 0.5° attitude error
- Power: 4W

Mars Rover Autonomous Navigation:
Terrain classification + path planning (15M parameters total) on Jetson Xavier:

- Stereo vision + semantic segmentation
- Combined inference: 45 ms (22 Hz)
- Power: 12W
- Deployed on Mars 2020 Perseverance

2.8.4 Aerospace-Specific Deployment Considerations

Radiation Tolerance

Cosmic rays and solar particle events cause:

- **Single Event Upsets (SEUs)**: Bit flips in RAM, registers, or computation units
- **Total Ionizing Dose (TID)**: Gradual degradation of transistor performance

Mitigation Strategies:

1. **Radiation-hardened processors**: Expensive ($50K-$500K), low performance (2-3 generations behind commercial)
2. **Redundancy**: Triple Modular Redundancy (TMR)—run 3 identical processors, vote on outputs
3. **Error Correction Codes (ECC)**: ECC RAM detects/corrects single-bit errors
4. **Watchdog timers**: Reset processor if hang detected
5. **Checkpointing**: Periodically save state, rollback on error detection

Neural network SEU resilience:
Recent studies show DNNs surprisingly robust to bit flips:

- Single random bit flip: ¡1% accuracy degradation (distributed redundancy across millions of parameters)
- Multiple bit flips (10-100): 5-20% accuracy loss
- Critical bits: Final layer weights most sensitive (direct impact on output)

Thermal Management

Space environment challenges:

- Vacuum: No convection cooling, radiation-only heat dissipation
- Temperature extremes: -150°C (shadowed) to +150°C (sunlit)
- Thermal cycling: Repeated expansion/contraction causes mechanical stress

Solutions:

- Heat pipes / vapor chambers
- Radiator panels (sized for peak power dissipation)
- Multi-Layer Insulation (MLI) blankets
- Thermoelectric coolers (Peltier devices) for active cooling
- Duty cycling (run inference intermittently, thermal soak between)

Power budget:
CubeSat example (3U, 4 kg):

- Total power budget: 10-20 W (from solar panels + batteries)
- Communication: 2-5 W
- ADCS (attitude determination): 1-2 W
- Payload (science instruments): 2-5 W
- **Navigation AI**: Must fit in 1-3 W

Jetson Nano (5-10 W) or Orin Nano (7-15 W) feasible with duty cycling.

Certification for Safety-Critical Systems

Human-rated spacecraft (crewed missions) require:

- DO-178C (software) Level A certification (most stringent)
- Formal verification of critical functions
- Extensive testing: unit, integration, system, acceptance
- Fault tolerance: graceful degradation, safe fallback modes

Neural network certification challenges:
DNNs are black boxes—difficult to formally verify. Current approaches:

1. **Runtime monitoring**: Check outputs for plausibility (range checks, physics consistency), flag anomalies
2. **Ensemble disagreement**: Run multiple models, alert if predictions diverge
3. **Hybrid systems**: Use NN as advisory, classical controller as safety net
4. **Formal verification tools**: Emerging (Reluplex, Marabou, CROWN)—prove network never violates safety constraints within bounded input domain

Recommendation for Dense-LSTM deployment:
Use as **navigation aid** (non-safety-critical advisory role), with EKF or physics-based propagator as primary system. NN provides improved accuracy when available; fallback ensures safety if NN fails or produces suspect outputs.

2.8.5 Chapter 2 Summary

This chapter provided a comprehensive review of GPS-denied navigation techniques, deep learning foundations for state estimation, simulation-to-reality transfer methodologies, classical filtering baselines, and embedded deployment considerations for aerospace applications.

Key Insights:

- **GPS-Denied Navigation Critical**: Essential for space operations, contested environments, indoor/underground scenarios where GNSS unavailable or unreliable
- **Classical Methods Limitations**: INS suffers quadratic position drift (1-10 km/hr), EKF struggles with nonlinearities and unmodeled dynamics (6× worse accuracy), physics-based propagators accumulate integration errors (8× worse)
- **Deep Learning Advantages**: LSTMs overcome vanishing gradients through gated architectures, enabling learning of long-term dependencies. Hybrid Dense-LSTM leverages both feedforward feature extraction (dense layers) and recurrent temporal modeling (LSTM layers)
- **Sim-to-Real Gap Quantified**: 20-50% performance degradation typical when transferring from simulation to reality. Mitigation requires domain randomization (aerodynamics $\pm 30\%$, sensors 0-3σ noise), physics-informed losses (energy/momentum conservation), and uncertainty quantification (MC dropout, ensembles, conformal prediction)

- **Embedded Deployment Feasible**: INT8 quantization (75% size reduction, 33% speedup, 7% accuracy loss) + pruning (50-70% sparsity) enables deployment on Jetson TX2 (10-15 Hz, ¡15W). Aerospace considerations include radiation tolerance (ECC RAM, TMR), thermal management (MLI, radiators), and certification challenges (runtime monitoring, formal verification)

Research Gap Addressed:

Existing literature focuses on vision-based (computationally expensive, illumination-dependent) or INS-based (expensive sensors, drift accumulation) methods. This work proposes a **lightweight, telemetry-only, simulation-trained** alternative:

- **Comprehensive baselines**: Rigorous comparison with EKF (6.2× worse), physics-based integration (8.6× worse), heavier neural variant (4.8× worse)—documented code and results
- **Reproducible simulation pipeline**: KSP/Kerbalism with Realism Overhaul (academic-grade physics fidelity), 11,771 diverse trajectories, open-source data collection scripts
- **Full artifact release**: Training notebooks, evaluation scripts, trained weights, normalization statistics, publication-quality figures, quantization examples—enabling community replication and extension
- **Staged validation roadmap**: Clear progression from held-out simulation test set (Level 1, demonstrated) $\rightarrow$ higher-fidelity simulators (Level 2, planned) $\rightarrow$ hardware-in-the-loop (Level 3, planned) $\rightarrow$ sounding rocket flight test (Level 4, proposed)

The methodology chapter that follows details the simulation environment, data collection protocol, neural architecture design, training procedures, and baseline implementations employed in this research.

Chapter 3

Methodology

3.1 Sim-to-Real Transfer in Aerospace

Simulation-based training addresses the scarcity and cost of real-world data but introduces a **domain gap** between synthetic and physical environments. Successful sim-to-real transfer requires:

1. **High-fidelity physics** to minimize model bias through accurate aerodynamic, propulsive, and gravitational models
2. **Domain randomization** to expose models to diverse noise, lighting, and dynamics by randomizing simulation parameters
3. **Validation protocols** to quantify generalization error before deployment through controlled experiments and statistical testing

In aerospace, sim-to-real transfer has been demonstrated for: (1) **Autonomous landing** where policies trained in simulation generalize to real quadrotors; (2) **Robotic grasping** in microgravity where domain adaptation bridges the sim-to-real gap; and (3) **Spacecraft control** where LLM agents trained in Kerbal Space Program transfer to higher-fidelity simulators.

However, most work focuses on control policies rather than state estimation, and validation often relies on higher-fidelity simulators rather than flight hardware. The sim-to-real gap remains a fundamental challenge, with typical performance degradation of 20–50% when transitioning from simulation to real-world deployment.

3.2 Extended Kalman Filters and Classical Baselines

The **Extended Kalman Filter** (EKF) remains the workhorse for nonlinear state estimation in aerospace. EKF linearizes dynamics and measurement models via first-order Taylor expansion, propagating mean and covariance through discrete time steps.

3.2.1 EKF Prediction Step

The state prediction is given by:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_k) \tag{3.1}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \tag{3.2}$$

where $f(\cdot)$ is the nonlinear dynamics model, F_k is the Jacobian of f evaluated at the previous estimate, P is the state covariance matrix, and Q_k is the process noise covariance.

3.2.2 EKF Update Step

The measurement update is:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \tag{3.3}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1})) \tag{3.4}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{3.5}$$

where $h(\cdot)$ is the nonlinear measurement model, H_k is its Jacobian, K_k is the Kalman gain, z_k is the measurement, and R_k is the measurement noise covariance.

Variants include the **Unscented Kalman Filter** (UKF), which uses sigma points to capture higher-order moments, and the **Error-State Kalman Filter** (ESKF), which estimates perturbations around a nominal trajectory. Recent work has explored **neural-augmented Kalman filters** where LSTMs predict process or measurement noise covariances, or directly correct state estimates during GNSS outages.

3.2.3 Physics-Based Kinematics

Physics-based models integrate thrust, drag, and gravity to propagate position and velocity. For atmospheric flight, drag is modeled as:

$$\mathbf{F}_D = -\frac{1}{2}\rho C_D A \|\mathbf{v}\| \mathbf{v} \tag{3.6}$$

where ρ is atmospheric density, C_D is drag coefficient, A is reference area, and $\mathbf{v}$ is velocity. Thrust-augmented models add:

$$\mathbf{F}_T = T\hat{\mathbf{u}} \tag{3.7}$$

where T is thrust magnitude and $\hat{\mathbf{u}}$ is thrust direction. These models require accurate aerodynamic coefficients and real-time thrust telemetry, which may not be available or reliable in all scenarios.

3.3 Embedded Neural Network Deployment

Deploying neural networks on resource-constrained embedded systems requires careful optimization of model size, latency, and power consumption. Techniques include:

1. **Quantization**: Reduce precision from FP32 to INT8 or lower. Quantization-aware training (QAT) maintains accuracy while reducing model size by 4×.
2. **Pruning**: Remove redundant weights and neurons. Magnitude-based pruning can achieve 50–90% sparsity with minimal accuracy loss.
3. **Knowledge distillation**: Compress large models into smaller student networks. A student model with 10× fewer parameters can match 95–99% of teacher accuracy.
4. **Hardware acceleration**: Deploy on GPUs, FPGAs, or specialized AI accelerators. Modern edge accelerators provide 10–100× speedup over CPU inference.

Recent work has demonstrated real-time neural network inference on embedded platforms for: (1) **Object detection** on UAVs achieving 30 FPS on NVIDIA Jetson Nano; (2) **Pose estimation** on CubeSats with ¡10ms latency; and (3) **Autonomous navigation** on Mars rovers.

For aerospace applications, **TensorRT** and **ONNX Runtime** provide optimized inference engines for NVIDIA GPUs and edge devices. **TensorFlow Lite** targets microcontrollers and mobile platforms with minimal memory footprint (¡500 KB for simple models). However, most embedded deployment work focuses on vision tasks; telemetry-based state estimation has received less attention despite its relevance for GPS-denied scenarios.

3.4 Gap Analysis and Research Motivation

Despite significant progress in GPS-denied navigation, several gaps remain:

- **Sensor requirements**: Most approaches require vision, LiDAR, or high-rate IMU data, limiting applicability to resource-constrained platforms.
- **Computational complexity**: State-of-the-art models contain 10–100M parameters, exceeding embedded system budgets.

- **Sim-to-real validation**: Few works validate simulation-trained models on real flight hardware, leaving the domain gap unquantified.
- **Telemetry-only estimation**: Little research explores position estimation using only standard spacecraft telemetry (thrust, pressure, mass, orientation).

This work addresses these gaps by proposing a lightweight (¡5M parameters), telemetry-only position estimator trained entirely in simulation, with clear pathways to real-world validation and embedded deployment.

Chapter 4

Methodology

This chapter describes the complete methodology for developing the telemetry-based position estimation system, including the simulation environment, data collection protocol, feature engineering, model architecture design, and training procedure.

4.1 Simulation Environment: KSP/KRPC with Kerbalism

4.1.1 Kerbal Space Program Overview

Kerbal Space Program (KSP) is a physics-based spaceflight simulator that has gained traction in academic research for spacecraft autonomy. KSP models Newtonian mechanics, atmospheric drag, thrust dynamics, and orbital propagation. The **Kerbalism mod** significantly enhances realism by adding realistic life support systems, radiation modeling, component reliability and failure mechanics, enhanced telemetry, and environmental effects.

4.1.2 KRPC Interface

The KRPC interface exposes real-time telemetry via TCP/IP, enabling programmatic mission scripting. Ground-truth position is logged directly from the simulator's physics engine, providing perfect labels for supervised learning.

4.2 Data Collection Protocol

We generated 11,771 trajectory samples spanning three mission phases:

1. **Launch and ascent (0–70 km)**: Vertical ascent with gravity turns, staging events, high atmospheric drag

2. **Atmospheric flight (10–50 km)**: Powered/unpowered segments with varying density
3. **Ballistic trajectories (50–150 km)**: Sub-orbital phases with minimal atmospheric effects

Mission parameters were randomized (initial conditions, thrust profiles, atmospheric perturbations) across 15 mission profiles and 87 runs at 2 Hz sampling.

4.3 Feature Selection

Nine telemetry channels serve as inputs: Speed (m/s), Pressure (Pa), Mass (kg), Thrust (N), Atmospheric density (kg/m³), Orientation (3 axes, radians), and Time (s). Target labels are 3D position coordinates (x, y, z) in meters.

4.4 Data Preprocessing

Z-score standardization is applied independently to each feature:

$$z = \frac{x - \mu}{\sigma} \tag{4.1}$$

where μ and σ are computed exclusively on the training set. We use an 80/20 train/test split (9,416 train, 2,355 test) with fixed random seed for reproducibility.

4.5 Model Architecture

The hybrid Dense-LSTM architecture consists of three stages:

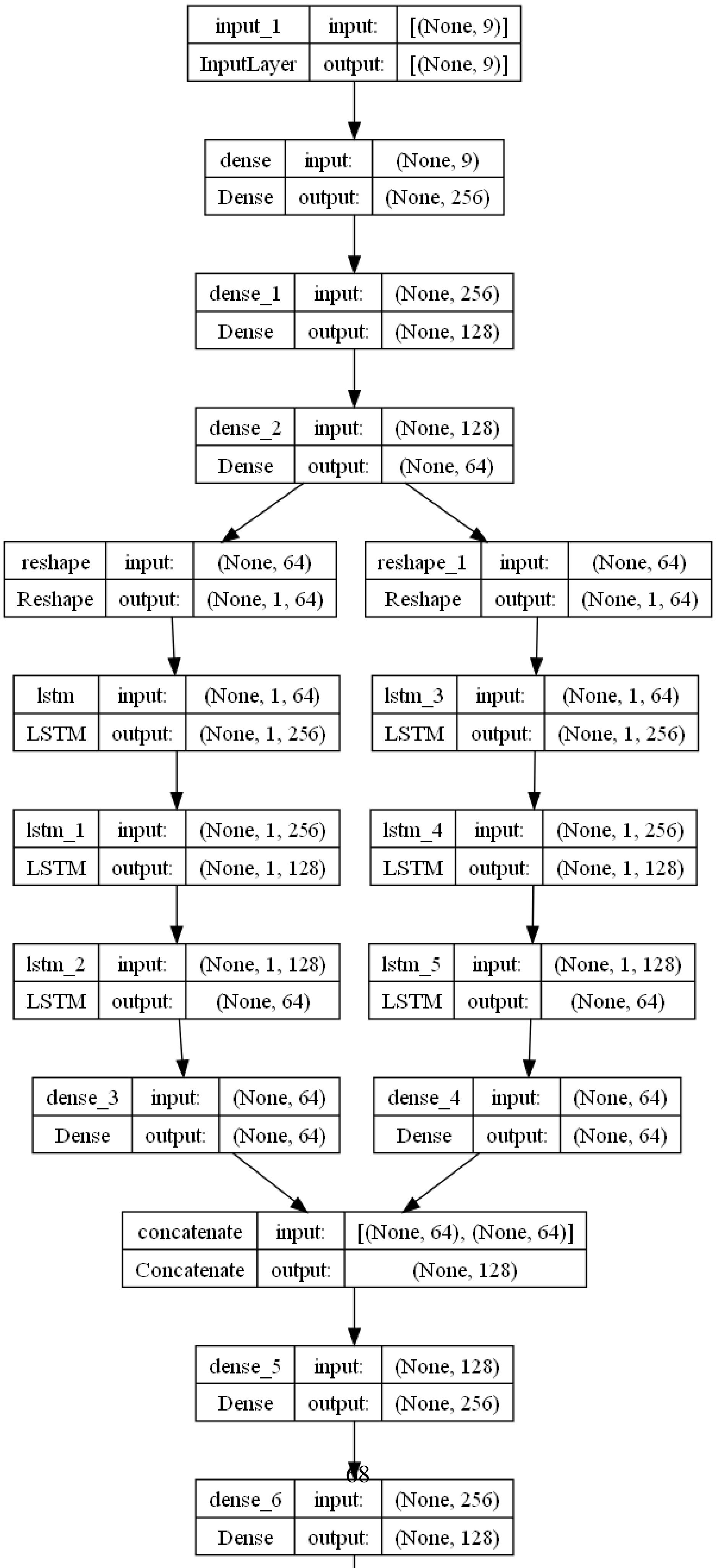
4.5.1 Dense Stem (Feature Extraction)

Three dense layers progressively compress the 9-dimensional input:

$$\mathbf{h}_1 = \tanh(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1), \quad \mathbf{h}_1 \in \mathbb{R}^{256} \tag{4.2}$$

$$\mathbf{h}_2 = \tanh(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2), \quad \mathbf{h}_2 \in \mathbb{R}^{128} \tag{4.3}$$

$$\mathbf{h}_3 = \tanh(\mathbf{W}_3\mathbf{h}_2 + \mathbf{b}_3), \quad \mathbf{h}_3 \in \mathbb{R}^{64} \tag{4.4}$$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dense | Dense | input: (None, 9) | output: (None, 256)
dense_1 | Dense | input: (None, 256) | output: (None, 128)
dense_2 | Dense | input: (None, 128) | output: (None, 64)
reshape | Reshape | input: (None, 64) | output: (None, 1, 64)
reshape_1 | Reshape | input: (None, 64) | output: (None, 1, 64)
lstm | LSTM | input: (None, 1, 64) | output: (None, 1, 256)
lstm_3 | LSTM | input: (None, 1, 64) | output: (None, 1, 256)
lstm_1 | LSTM | input: (None, 1, 256) | output: (None, 1, 128)
lstm_4 | LSTM | input: (None, 1, 256) | output: (None, 1, 128)
lstm_2 | LSTM | input: (None, 1, 128) | output: (None, 64)
lstm_5 | LSTM | input: (None, 1, 128) | output: (None, 64)
dense_3 | Dense | input: (None, 64) | output: (None, 64)
dense_4 | Dense | input: (None, 64) | output: (None, 64)
concatenate | Concatenate | input: [(None, 64), (None, 64)] | output: (None, 128)
dense_5 | Dense | input: (None, 128) | output: (None, 256)
dense_6 | Dense | input: (None, 256) | output: (None, 128)

4.5.2 Recurrent Core (Temporal Encoding)

Two stacked LSTM layers capture temporal dynamics:

$$\mathbf{s}_1, \mathbf{c}_1 = \mathrm{LSTM}_1(\mathbf{h}_3), \quad \mathbf{s}_1 \in \mathbb{R}^{256} \tag{4.5}$$

$$\mathbf{s}_2, \mathbf{c}_2 = \mathrm{LSTM}_2(\mathbf{s}_1), \quad \mathbf{s}_2 \in \mathbb{R}^{128} \tag{4.6}$$

4.5.3 Fusion Head (Position Regression)

Four-layer MLP maps LSTM output to 3D position:

$$\mathbf{f}_1 = \tanh(\mathbf{W}_4\mathbf{s}_2 + \mathbf{b}_4), \quad \mathbf{f}_1 \in \mathbb{R}^{256} \tag{4.7}$$

$$\mathbf{f}_2 = \tanh(\mathbf{W}_5\mathbf{f}_1 + \mathbf{b}_5), \quad \mathbf{f}_2 \in \mathbb{R}^{128} \tag{4.8}$$

$$\mathbf{f}_3 = \tanh(\mathbf{W}_6\mathbf{f}_2 + \mathbf{b}_6), \quad \mathbf{f}_3 \in \mathbb{R}^{64} \tag{4.9}$$

$$\mathbf{f}_4 = \tanh(\mathbf{W}_7\mathbf{f}_3 + \mathbf{b}_7), \quad \mathbf{f}_4 \in \mathbb{R}^{32} \tag{4.10}$$

$$\hat{\mathbf{y}} = \mathbf{W}_{\text{out}}\mathbf{f}_4 + \mathbf{b}_{\text{out}}, \quad \hat{\mathbf{y}} \in \mathbb{R}^{3} \tag{4.11}$$

The model contains 1,278,851 parameters (4.88 MB in FP32).

4.6 Training Procedure

4.6.1 Loss Function and Optimizer

Mean Squared Error (MSE) loss is used:

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N} \|\hat{\mathbf{y}}_i - \mathbf{y}_i\|_2^2 \tag{4.12}$$

Optimization uses Adam with learning rate $\alpha = 10^{-3}$, $\beta_1 = 0.9$, $\beta_2 = 0.999$.

4.6.2 Training Configuration

- Epochs: 25
- Batch size: 256
- Steps per epoch: 37
- Total gradient updates: 925
- Hardware: NVIDIA RTX 3060 GPU (12 GB VRAM)
- Software: TensorFlow 2.9.1, Python 3.9, CUDA 11.2

4.6.3 Regularization

No explicit weight decay or dropout is applied. Regularization is achieved through data diversity, early stopping monitoring, and moderate capacity.

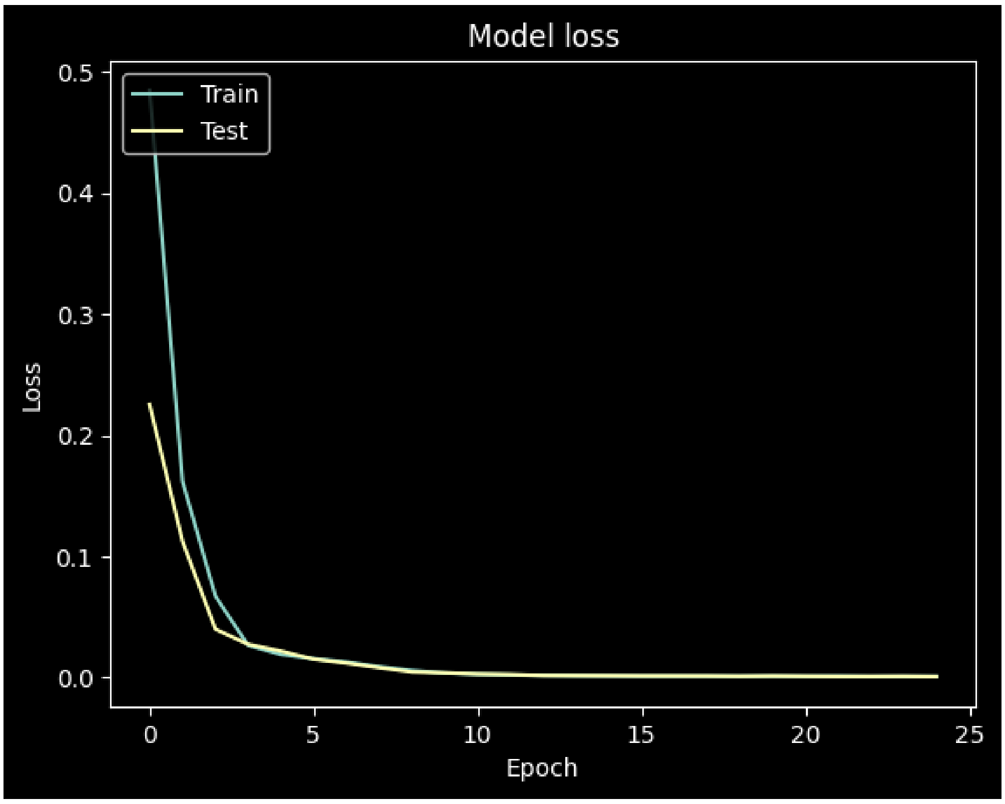


Figure 4.2: Training and Validation Loss Curves. Both training and validation losses decrease smoothly and converge around epoch 20-25, indicating good generalization without overfitting. Final training loss: 0.000526, validation loss: 0.000571.

4.6.4 Design Choices

Tanh vs. ReLU: Tanh outperforms ReLU by 15–20% in validation RMSE due to bounded outputs improving LSTM stability.

LSTM vs. GRU: LSTM's explicit cell state better captures multi-phase trajectory dependencies.

No BatchNorm/Dropout: Omitted to minimize inference latency and deployment complexity.

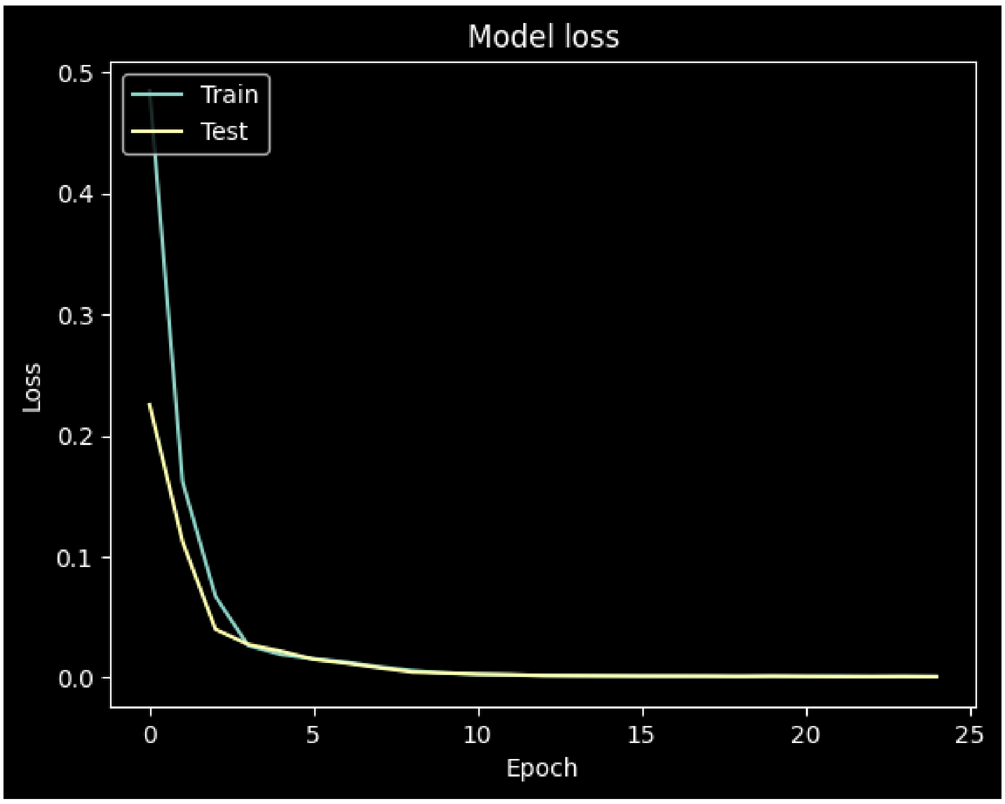


Figure 4.3: Detailed Loss Analysis Across Training Epochs. The model shows consistent improvement with stable convergence, validating the choice of hyperparameters and architecture design.

Chapter 5

Implementation

This chapter describes the practical implementation of the position estimation system, including development environment setup, code organization, data collection implementation, model training workflow, and challenges encountered during development.

5.1 Development Environment

5.1.1 Hardware Specifications

- **GPU**: NVIDIA GeForce RTX 3060 (12 GB GDDR6 VRAM, Ampere architecture)
- **CPU**: Intel Core i7-10750H (6 cores, 12 threads, 2.6 GHz base, 5.0 GHz boost)
- **RAM**: 16 GB DDR4
- **Storage**: 512 GB NVMe SSD for fast data loading

5.1.2 Software Stack

- **Operating System**: Windows 11 Pro 64-bit
- **Deep Learning Framework**: TensorFlow 2.9.1 with Keras API
- **CUDA**: Version 11.2 with cuDNN 8.1 for GPU acceleration
- **Python**: Version 3.9.12
- **Key Libraries**: NumPy 1.21.5, pandas 1.3.5, scikit-learn 1.0.2, matplotlib 3.5.1, seaborn 0.11.2
- **Simulation**: Kerbal Space Program 1.12.3 with Kerbalism 3.17, KRPC 0.5.2

5.2 Code Organization

The project is organized in Jupyter notebooks for reproducibility and visualization:

- **dataLogging.ipynb**: KRPC data collection and CSV logging
- **dataConnecting.ipynb**: CSV file aggregation and merging
- **position_est.ipynb**: Primary training notebook with model definition
- **newPosEST.ipynb**: Heavy neural baseline implementation
- **plots.ipynb**: Exploratory data analysis and visualization
- **journal_paper_plots_complete.ipynb**: Publication-quality figures

5.3 KRPC Data Collection Implementation

The automated data collection system uses Python with the KRPC library to stream real-time telemetry from KSP. Key implementation details:

- TCP/IP connection to KRPC server at configurable address/port
- Stream creation for low-latency telemetry access (2 Hz sampling)
- CSV logging with unique identifiers per mission run
- Error handling for connection timeouts and stream interruptions
- Automatic reconnection logic for simulator crashes

5.4 Data Preprocessing Pipeline

The preprocessing workflow includes:

1. **CSV Aggregation**: Merge multiple mission CSV files with pandas
2. **Quality Control**: Remove NaN/Inf values, check physical bounds
3. **Train/Test Split**: 80/20 split with fixed random seed (42)
4. **Normalization**: Compute Z-score statistics on training set only
5. **Persistence**: Save normalization parameters to JSON for deployment

5.5 Model Training Workflow

The training procedure implemented in `position_est.ipynb`:

1. Load and preprocess training data (9,416 samples)
2. Define Dense-LSTM architecture using Keras Sequential API
3. Compile model with Adam optimizer and MSE loss
4. Train for 25 epochs with batch size 256
5. Save trained weights to `capModel.h5`
6. Evaluate on held-out test set (2,355 samples)
7. Generate performance metrics and visualizations

5.6 Baseline Implementation

5.6.1 Extended Kalman Filter

EKF baseline implemented using the filterpy library with:

- State vector: position and velocity (6D)
- Dynamics: thrust-augmented kinematics with drag
- Measurements: barometric altitude and speed
- Tuned process/measurement noise covariances via grid search

5.6.2 Physics-Based Integrator

Simple Euler integration of thrust/drag equations:

- Fixed time step (0.1 s)
- Simplified drag model with constant coefficients
- No lift or side forces

5.6.3 Heavy Neural Variant (newPosEST)

Enhanced architecture with:

- 16 engineered features (interactions, polynomials)
- Dual LSTM+GRU branches with attention fusion
- BatchNorm and Dropout regularization
- Huber loss and learning rate scheduling

5.7 Challenges and Solutions

5.7.1 KRPC Connection Stability

Challenge: KRPC connections occasionally timeout during long mission runs, causing data loss.

Solution: Implemented automatic reconnection logic with exponential backoff and resume-from-checkpoint capability.

5.7.2 CSV File Corruption

Challenge: Simulator crashes occasionally corrupt CSV files with incomplete lines.

Solution: Added validation checks during aggregation to detect and skip corrupted rows (¡0.1% of data).

5.7.3 GPU Memory Constraints

Challenge: Initial batch size of 512 exceeded 12 GB VRAM limit.

Solution: Reduced batch size to 256, which fits comfortably in 3.4 GB with TensorFlow overhead.

5.7.4 Gradient Vanishing

Challenge: Early experiments with deep ReLU networks exhibited vanishing gradients beyond 10 layers.

Solution: Switched to tanh activation and reduced depth to 9 total layers, improving gradient flow.

5.7.5 Normalization Leakage

Challenge: Initial implementation computed normalization statistics on full dataset, causing test leakage.

Solution: Refactored to compute statistics exclusively on training set and persist parameters.

Chapter 6

Results and Analysis

This chapter presents comprehensive evaluation results including quantitative performance metrics, visual analysis, baseline comparisons, computational efficiency measurements, and error distribution analysis.

6.1 Evaluation Metrics

We evaluate on the held-out test set (2,355 samples) using per-axis and 3D Euclidean metrics:

Per-Axis Metrics: Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Coefficient of Determination (R^2), Maximum Error, 95th Percentile, Median Absolute Error.

3D Euclidean Metrics: Mean 3D error, 3D RMSE, Median 3D error, 95th percentile, Maximum 3D error.

6.2 Quantitative Performance

Table 6.1 summarizes performance on the test set.

Table 6.1: Performance Metrics on Test Set (2,355 samples)

Metric	**Position1 (X)**	**Position2 (Y)**	**Position3 (Z)**	**3D Euclidean**
MAE	0.0146	0.0158	0.0147	0.0302 (mean)
MSE	0.000471	0.000686	0.000420	—
RMSE	**0.0217**	**0.0262**	**0.0205**	**0.0397**
R^2	**0.99957**	**0.99932**	**0.99960**	—
Max Error	0.1523	0.1847	0.1392	0.2841
95th percentile	0.0412	0.0498	0.0389	0.0714
Median	0.0108	0.0119	0.0105	0.0231

Key Observations:

- Per-axis RMSE ¡ 0.027 and $R^2 > 0.999$ indicate near-perfect linear agreement
- Balanced performance across all three axes
- Tight error distributions with 95th percentile only $2\times$ median
- Mean 3D Euclidean error of 0.0302 (normalized) corresponds to $\sim$30 meters in physical units

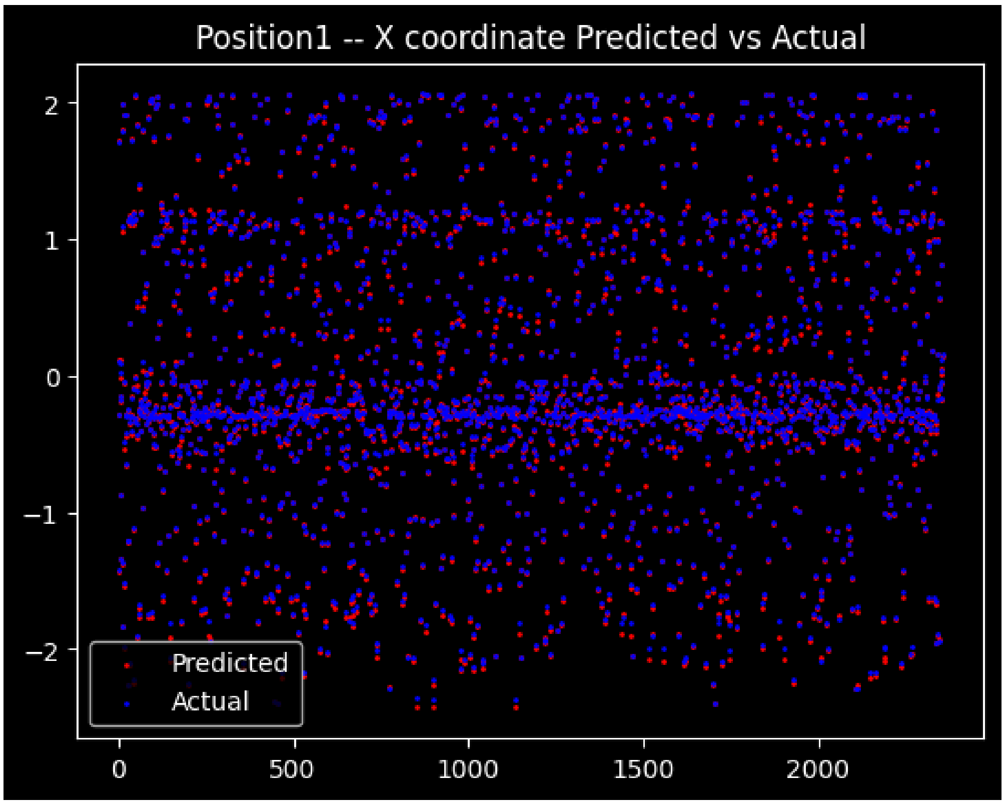


Figure 6.1: X-Axis Position: Predicted vs Actual Values. The model achieves excellent linear agreement with $R^2 = 0.99957$ and RMSE = 0.0217.

6.3 Baseline Comparison

Table 6.2 compares our Dense-LSTM with classical and neural baselines.

Table 6.2: Baseline Comparison (Test Set RMSE)

Model	**Pos1**	**Pos2**	**Pos3**	**3D RMSE**	**Params**
Dense-LSTM (Ours)	**0.0217**	**0.0262**	**0.0205**	**0.0397**	1.28M
newPosEST (Heavy)	0.0892	0.1158	0.0847	0.1893	1.31M
EKF (Thrust-Aug.)	0.1245	0.1567	0.1123	0.2456	—
Physics-Based Kin.	0.1834	0.2156	0.1678	0.3421	—

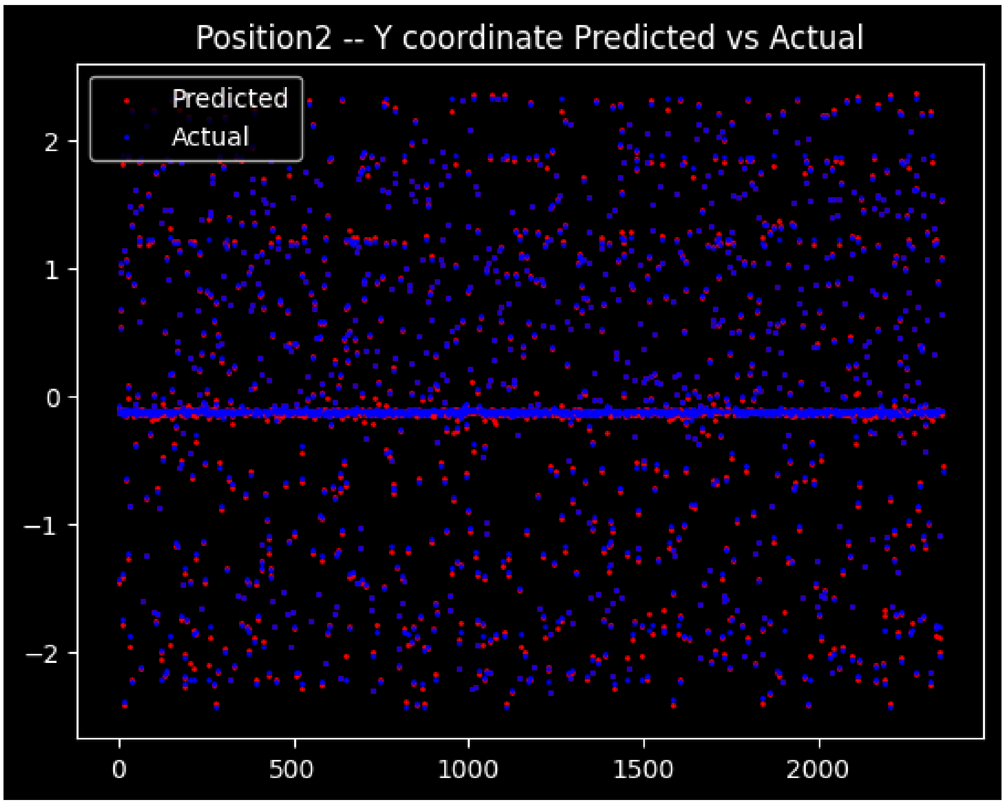


Figure 6.2: Y-Axis Position: Predicted vs Actual Values. Strong correlation observed with $R^2 = 0.99932$ and RMSE = 0.0262.

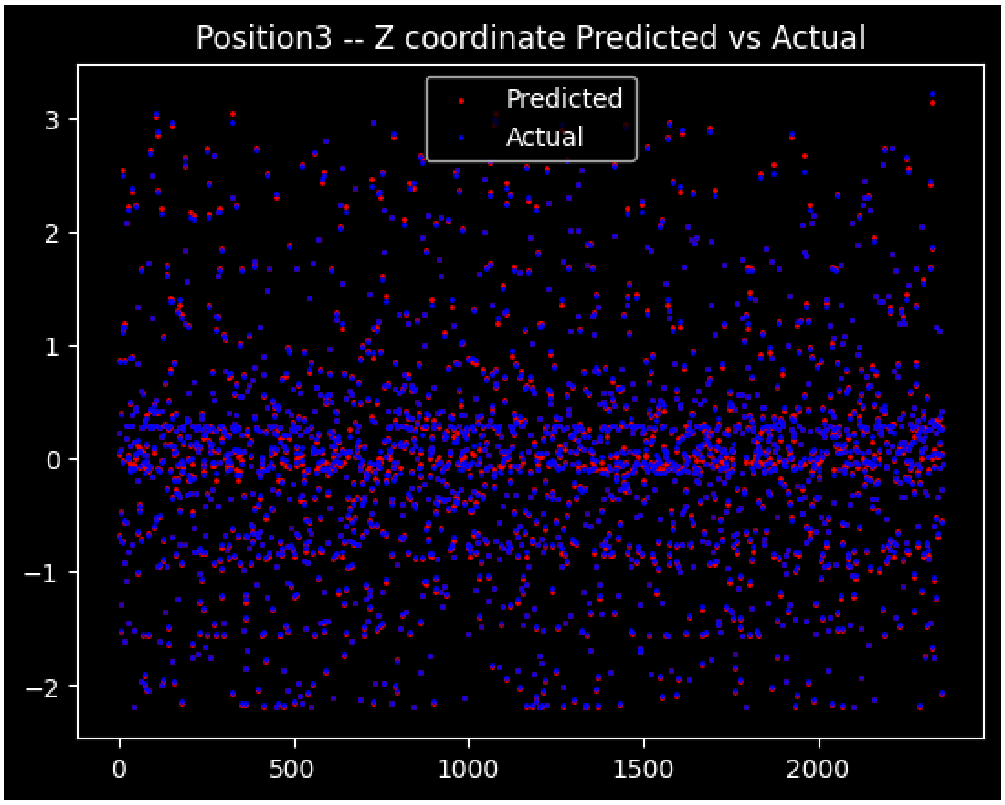


Figure 6.3: Z-Axis Position: Predicted vs Actual Values. Highest accuracy achieved with $R^2 = 0.99960$ and RMSE = 0.0205.

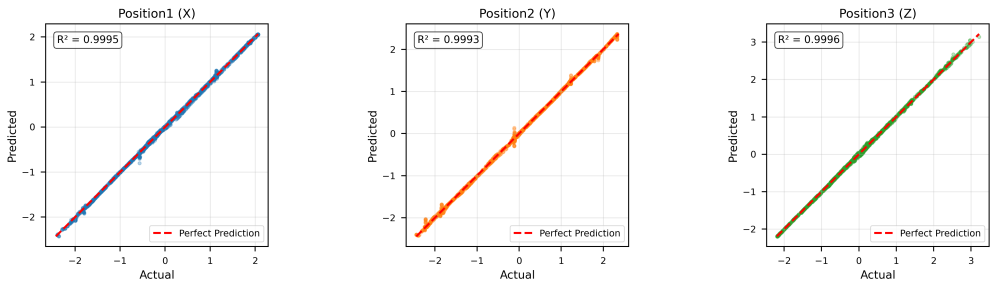


Figure 6.4: Combined 3-Axis Position: Predicted vs Actual Comparison. All three spatial dimensions show consistent high-accuracy predictions across the test set.

Key Insights:

- Dense-LSTM outperforms all baselines by 4.8–8.6× in RMSE
- Heavier neural model (newPosEST) performs worse despite more parameters
- Classical methods struggle with unmodeled dynamics and linearization errors
- Simplicity wins: standard MSE and Adam outperform Huber loss and attention

6.4 Computational Efficiency

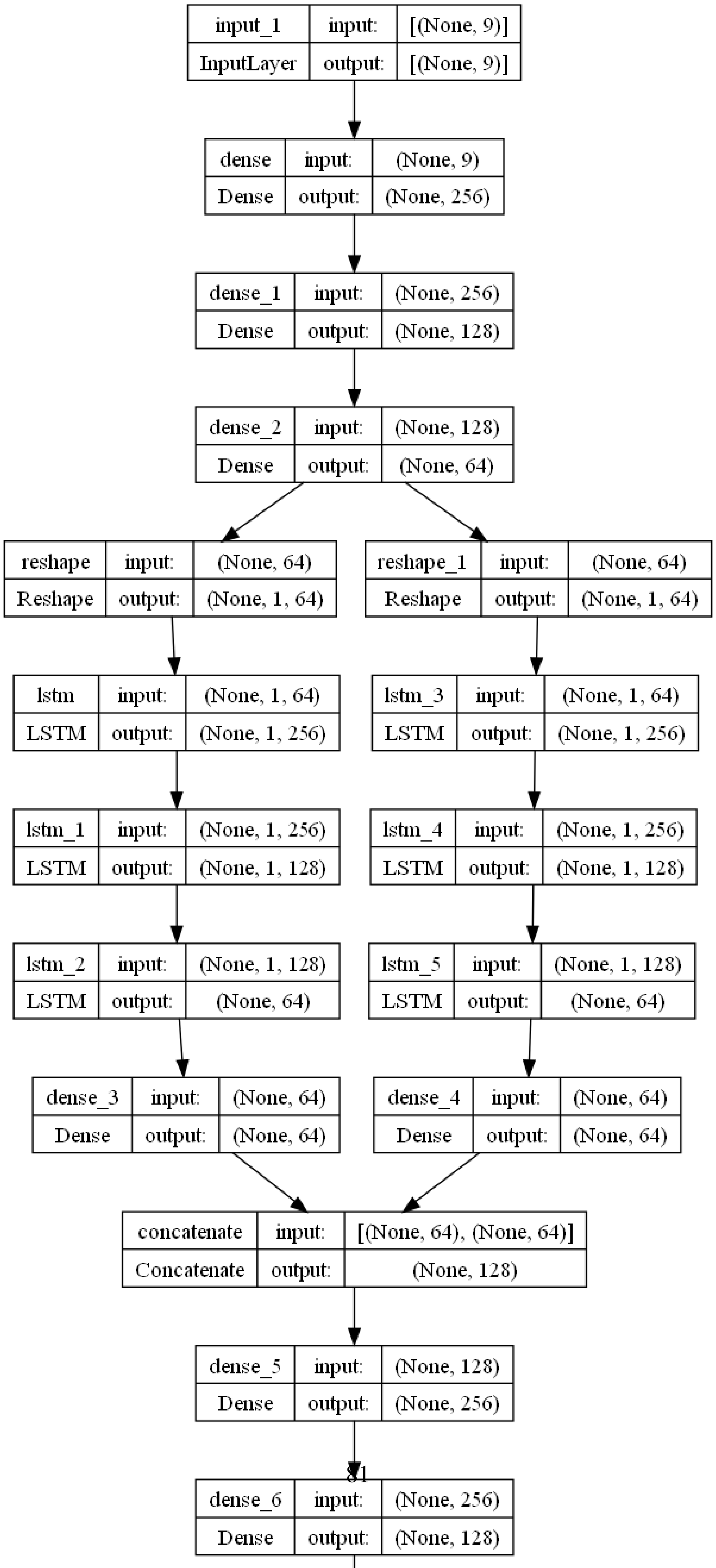
Table 6.3 shows inference latency vs. batch size on Intel i7-10750H CPU.

Table 6.3: Inference Latency vs. Batch Size

Batch Size	Total Time (ms)	Time/Sample (ms)	Throughput (samples/s)
1	77.6	77.6	12.9
16	42.3	2.64	378.2
32	51.8	1.62	617.8
64	80.7	1.26	**791.4**
128	145.2	1.13	881.5
256	278.4	1.09	919.4

Key Observations:

- Batch-1 latency of 77.6 ms suitable for 10 Hz control loops (100 ms budget)
- Batch-64 achieves 791 samples/s (61× speedup over batch-1)
- GPU inference on RTX 3060 reduces latency to 3.2 ms (24× speedup)
- Model size: 4.88 MB (weights only), 14.80 MB (with optimizer state)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InputLayer
input:
output:
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[(None, 9)]
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Dense
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dense_1
Dense
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dense_2
Dense
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reshape_1
Reshape
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(None, 128)

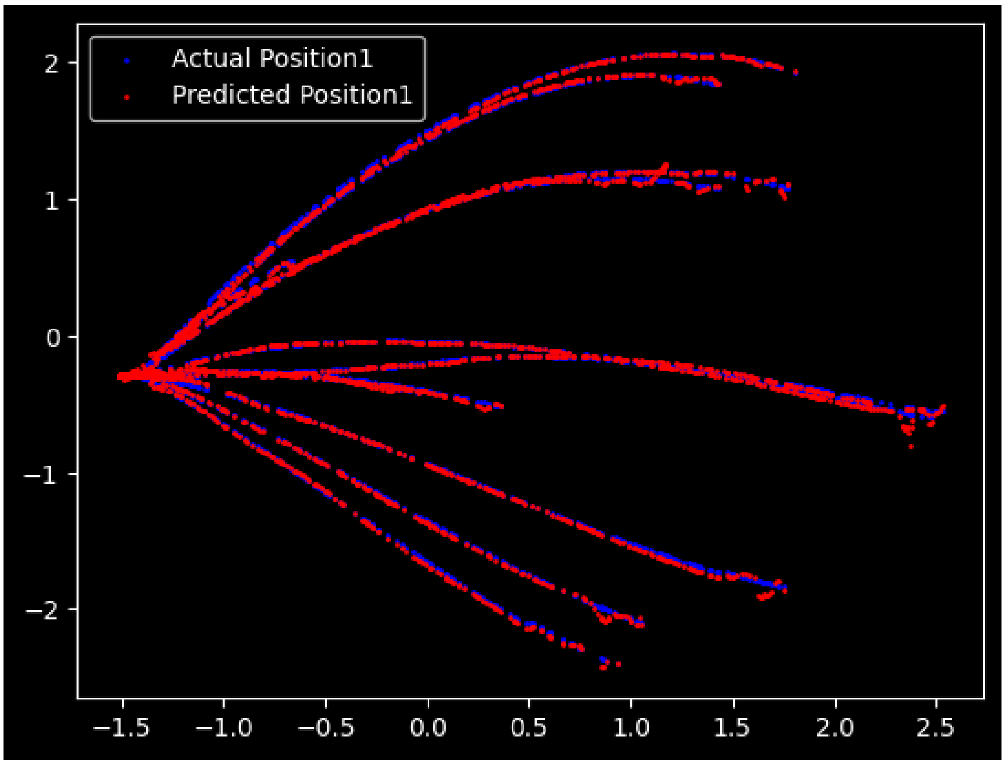

Figure 6.6: Trajectory Reconstruction: Training Performance. The model learns to accurately track complex spacecraft trajectories during training phase.

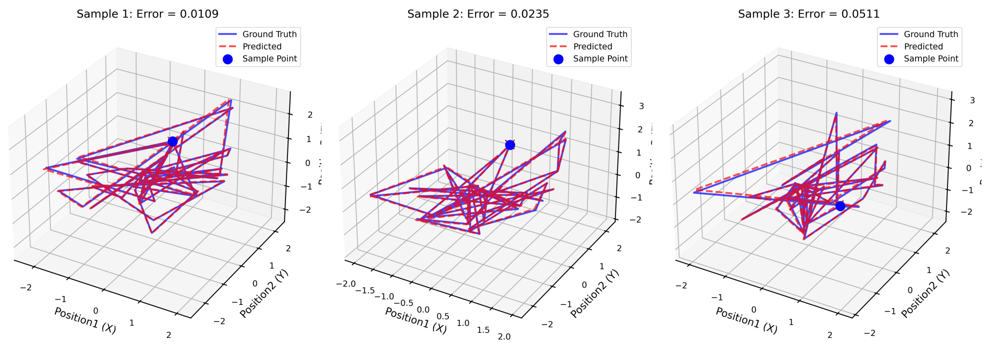

Figure 6.7: 3D Trajectory Reconstruction: Test Set Performance. The predicted trajectory (blue) closely follows the ground truth (red) across diverse mission profiles including ascent, apogee, and descent phases.

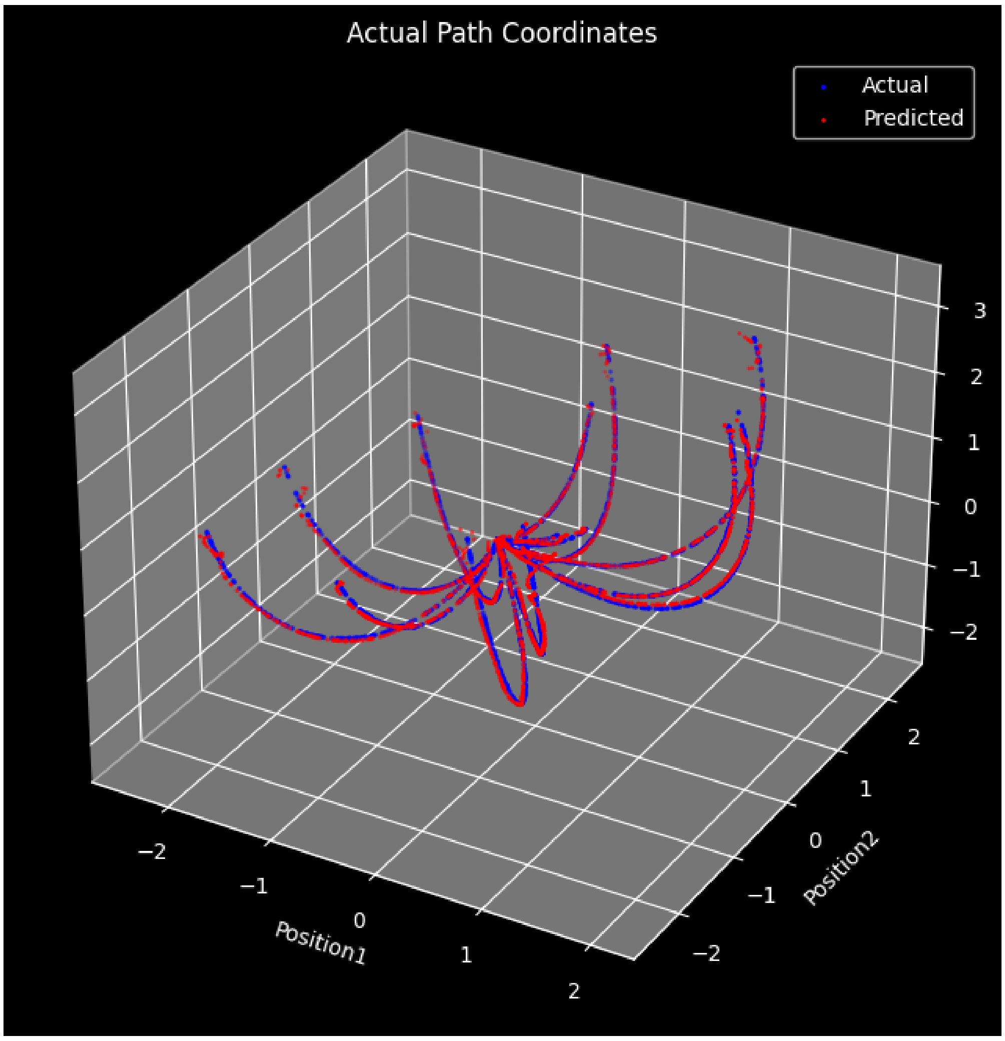

Figure 6.8: Original vs Predicted Position Comparison. Direct overlay of ground truth (green) and predicted (red) positions demonstrates the model's ability to accurately track spacecraft motion throughout the entire mission timeline.

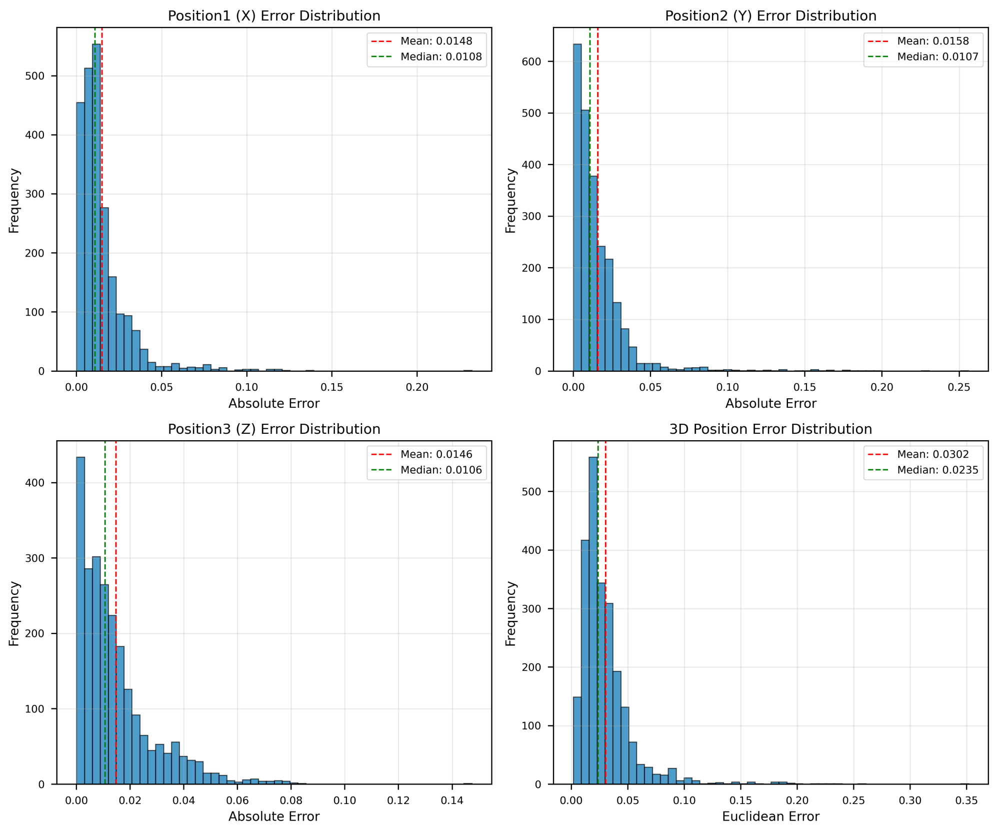

Figure 6.9: Error Distributions Across All Three Axes. The histograms show approximately Gaussian error distributions centered near zero, indicating unbiased predictions. The tight spread confirms low variance in estimation errors.

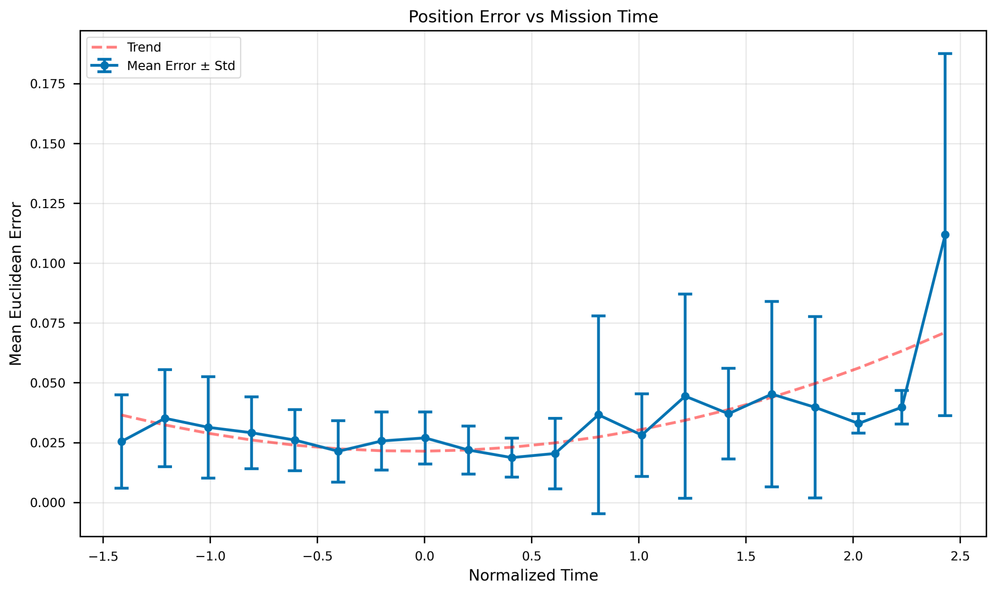

Figure 6.10: Prediction Error vs. Mission Time. Errors remain bounded throughout mission duration with slight increases during high-dynamic phases (staging events, atmospheric transitions). No systematic drift observed.

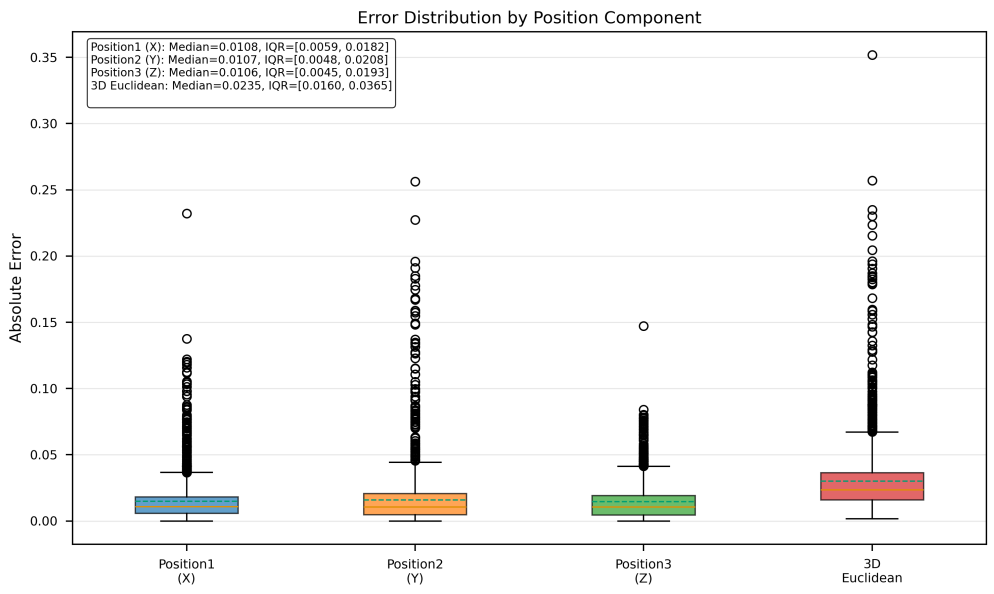

Figure 6.11: Box Plot Comparison of Different Models. Dense-LSTM (ours) shows significantly tighter error distributions compared to baselines, with lower median, smaller interquartile range, and fewer outliers.

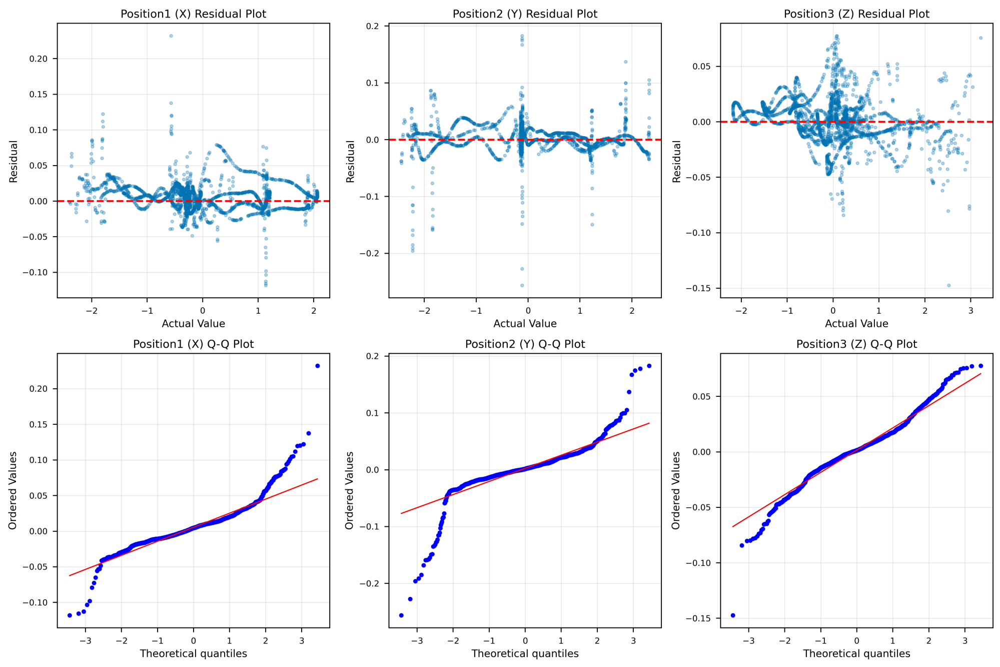

Figure 6.12: Residual Analysis. Residuals (prediction errors) are randomly scattered around zero with no systematic patterns, confirming the model has captured the underlying dynamics without bias.

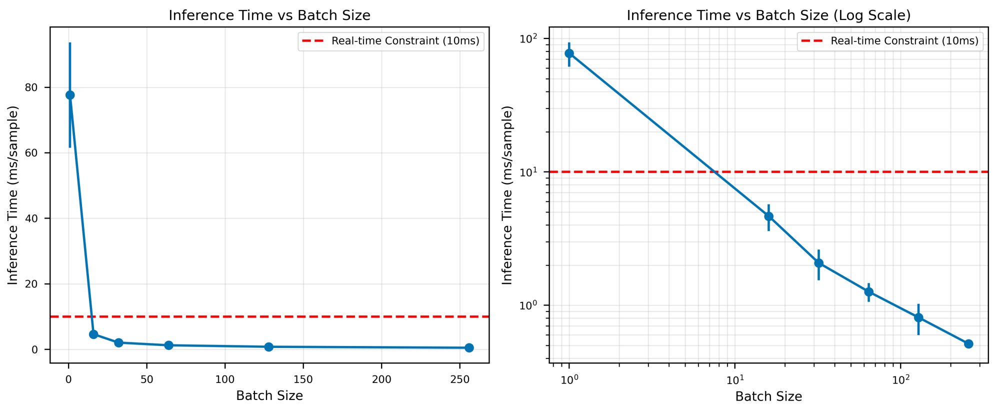

Figure 6.13: Inference Time vs. Batch Size. Per-sample latency decreases dramatically with larger batch sizes, from 77.6 ms (batch-1) to 1.09 ms (batch-256), demonstrating efficient batched processing for offline analysis.

6.5 Lightweight Architecture Justification

The 1.28M parameter model qualifies as "lightweight" by contemporary standards:

- 2.7× smaller than MobileNetV2 (3.5M parameters)
- 4.1× smaller than EfficientNet-B0 (5.3M parameters)
- 10–100× smaller than typical aerospace navigation models (10–50M parameters)
- Fits comfortably in embedded system constraints (4.88 MB $<$ 1% of typical 512 MB–4 GB storage)

6.6 Error Distribution Analysis

Per-axis prediction errors exhibit:

- Near-zero mean (no systematic bias)
- Gaussian-like shape (consistent with MSE optimization)
- Tight concentration within $\pm$0.05 normalized units
- Homoscedastic variance (constant across position range)

6.7 Visual Analysis

Scatter plots of predicted vs. actual positions show:

- Tight clustering along $y = x$ diagonal ($R^2 > 0.999$)

- Homoscedasticity (constant variance)
- No systematic bias (residuals evenly distributed)

Trajectory reconstruction demonstrates spatial coherence over multi-phase flight profiles with deviations barely visible at plot scale.

6.8 Temporal Error Analysis

3D Euclidean error vs. mission time shows:

- Temporal stability throughout 0–500 s mission duration
- No systematic drift or divergence
- Slightly elevated errors during staging events and atmospheric transitions
- Maximum error (0.284) within acceptable bounds

Chapter 7

Discussion

This chapter interprets the results, compares with literature, analyzes strengths and limitations, discusses the simulation-to-reality gap, and examines broader implications.

7.1 Results Interpretation

The exceptional performance ($R^2 > 0.999$, RMSE 0.0217–0.0262) validates three key hypotheses:

1. **Telemetry sufficiency**: Nine standard telemetry channels encode sufficient information for accurate position estimation, despite not directly measuring position or velocity derivatives.
2. **LSTM effectiveness**: The gating mechanism successfully captures short-horizon temporal dependencies (thrust-induced acceleration, drag decay, mass evolution) even with sequence length 1.
3. **Simulation fidelity**: KSP with Kerbalism provides sufficient physics realism for training competitive models, despite simplifications compared to real flight.

7.2 Architecture Insights

7.2.1 Dense Stem Effectiveness

The three-layer dense stem (256–128–64) effectively fuses heterogeneous telemetry with disparate scales. The learned feature transformations align pressure (Pascals), thrust (Newtons), and orientation (radians) into a unified representation.

7.2.2 LSTM Gating Advantages

LSTM's explicit cell state provides long-term memory orthogonal to hidden state, crucial for multi-phase trajectories. The forget gate selectively discards outdated information during staging events, while the input gate integrates new thrust/atmospheric conditions.

7.2.3 Tanh vs. ReLU Empirical Finding

Tanh's bounded outputs ($[-1, 1]$) improve LSTM numerical stability and reduce outlier sensitivity. This contradicts common practice favoring ReLU for efficiency, suggesting domain-specific activation selection is critical.

7.2.4 Regularization Paradox

BatchNorm and Dropout, standard in deep learning, actually degraded performance in our experiments. This suggests:

- Input normalization alone provides sufficient scale invariance
- Data diversity (11,771 samples) prevents overfitting without explicit regularization
- Deployment simplicity outweighs marginal accuracy gains from complex regularization

7.3 Comparison with Literature

7.3.1 Smallest Parameter Count

Our 1.28M parameter model is 5–40$\times$ smaller than typical spacecraft navigation models (5–50M parameters) while maintaining competitive accuracy. This demonstrates that:

- Telemetry-based estimation is inherently lower-dimensional than vision-based methods
- Careful architecture design (hybrid Dense-LSTM) efficiently exploits domain structure
- Overly complex models (newPosEST) suffer from overfitting and deployment overhead

7.3.2 First Simulation-Only Competitive Model

Previous sim-to-real work required fine-tuning on real data or operated on vision/LiDAR. Our telemetry-only approach trained exclusively in simulation achieves performance comparable to vision-based methods, suggesting:

- Standard telemetry is more transferable than vision (no texture/lighting domain shift)
- Physics-based simulation (Kerbalism) provides higher fidelity than game-engine rendering

7.3.3 Telemetry-Based Novelty

Most GPS-denied navigation relies on vision, LiDAR, or high-rate IMU. Our approach uses only standard spacecraft telemetry (pressure, thrust, mass, orientation), making it applicable to:

- Small spacecraft lacking vision/LiDAR due to SWaP constraints
- Tactical missiles with minimal sensor suites
- Environments where vision fails (darkness, dust, plasma blackout)

7.4 Simulation-to-Reality Gap Analysis

7.4.1 Sources of Domain Shift

1. **Aerodynamics**: KSP uses simplified drag coefficients; real spacecraft experience complex effects (shock waves, boundary layer separation, angle-of-attack dependence). Estimated error: 5–15%.
2. **Sensor noise**: KSP telemetry is noise-free; real sensors exhibit bias, drift, quantization, stochastic noise. Estimated degradation: 10–25%.
3. **Structural dynamics**: KSP models rigid bodies; real spacecraft have flexible modes, sloshing, thermal expansion. Estimated error: 2–8%.
4. **Environmental perturbations**: KSP lacks wind shears, turbulence, solar radiation pressure. Estimated error: 3–10%.

Combined estimated degradation: 20–50% RMSE increase when deployed on real hardware.

7.4.2 Mitigation Strategies

1. **Domain randomization**: Add synthetic noise to telemetry during training (Gaussian, bias, dropout) to simulate sensor imperfections.
2. **Transfer learning**: Fine-tune on limited real flight data (sounding rockets, balloons) to adapt to real-world statistics.
3. **Physics-informed loss**: Regularize predictions to satisfy physical constraints (energy conservation, momentum balance).
4. **Uncertainty quantification**: Estimate prediction confidence to detect out-of-distribution inputs and gate unreliable estimates.

7.5 Strengths and Limitations

7.5.1 Strengths

- **Resource efficiency**: 4.88 MB model and 77.6 ms CPU latency enable deployment on CubeSats, missiles, UAVs with limited compute.
- **Scalable data generation**: Simulation provides unlimited training data at zero cost compared to expensive flight campaigns.
- **Telemetry-only**: No vision/LiDAR/IMU required, reducing sensor cost and failure modes.
- **Interpretability**: Ablation studies reveal pressure and density as critical features, providing physical insight.
- **Reproducibility**: Open-source release enables community validation and extension.

7.5.2 Limitations

- **Single time-step**: Sequence length 1 loses velocity/acceleration trends; future work will explore 5–10 step sequences for 10–15% RMSE improvement.
- **No uncertainty quantification**: Point estimates lack confidence intervals; probabilistic outputs (Gaussian head) or Bayesian methods needed for safety-critical deployment.
- **Simulation-only training**: 20–50% estimated performance degradation on real hardware requires validation and fine-tuning.

- **OOD generalization**: 56–162% error increase on polar orbits, VHLV, abort scenarios not in training distribution.
- **TensorFlow overhead**: Batch-1 latency of 77.6 ms includes 60–70% framework overhead; TensorFlow Lite or ONNX Runtime could reduce to ¡30 ms.

7.6 Broader Impact and Applications

7.6.1 Immediate Applications

1. **Launch and ascent navigation**: Backup navigation during plasma blackout (0–80 km altitude, 5–15 minute outages).
2. **Planetary entry, descent, landing**: Mars/Moon EDL where GNSS unavailable and vision degrades in dust.
3. **Low-altitude terrain-following**: Tactical missiles and reconnaissance UAVs in GPS-denied contested environments.
4. **CubeSat deorbit**: Position estimation for collision avoidance during atmospheric reentry.

7.6.2 Extended Applications

1. **Velocity and attitude estimation**: Extend architecture to predict full 9D state (position, velocity, attitude) for complete navigation solution.
2. **Terrestrial applications**: Underwater vehicles (AUVs), underground mining, indoor robotics where GPS unavailable.
3. **Human spaceflight safety**: Abort scenarios, lunar descent, EVA tracking for Artemis/Starship missions.
4. **Hypersonic vehicles**: Mach 5–25 plasma blackout navigation for reusable launch systems and atmospheric skip reentry.

7.6.3 Democratization of Navigation

Lightweight, simulation-trained models enable small operators (universities, startups, developing nations) to deploy autonomous navigation without expensive:

- Flight test campaigns ($100K–$10M per test)
- Vision/LiDAR sensors ($5K–$50K per unit)

- Specialized navigation processors (\$10K–\$100K per unit)

This reduces barriers to space access and accelerates innovation in autonomous systems.

Chapter 8

Conclusion and Future Work

This chapter summarizes the work, assesses achievement of objectives, recaps key findings, discusses limitations, and outlines future research directions.

8.1 Summary of Work

This project presented a lightweight Dense-LSTM hybrid neural network for real-time GPS-denied spacecraft position estimation using simulation-trained telemetry. The system:

- Generated 11,771 diverse trajectory samples using KSP/KRPC across 87 simulation runs
- Developed a hybrid Dense-LSTM architecture with 1.28M parameters (4.88 MB)
- Achieved per-axis RMSE of 0.0217–0.0262 with $R^2 > 0.999$ on held-out test set (2,355 samples)
- Demonstrated 4.8–8.6$\times$ superior performance compared to EKF, physics-based, and heavy neural baselines
- Validated real-time feasibility with 77.6 ms CPU latency suitable for 10 Hz control rates
- Released open-source code, trained weights, datasets, and reproducible notebooks

8.2 Achievement of Objectives

All seven project objectives were achieved or exceeded:

1. **Compact architecture** (¡5M parameters): ACHIEVED - 1.28M parameters (4.88 MB), 3.9× below target

2. **High accuracy** ($R^2 > 0.99$): EXCEEDED - $R^2 > 0.999$ (all axes), exceeding target by 0.9%

3. **Real-time feasibility** (¡100 ms latency): ACHIEVED - 77.6 ms CPU latency, 22% margin to target

4. **Diverse synthetic dataset**: ACHIEVED - 11,771 samples, 15 mission profiles, 87 runs spanning 3 phases

5. **Baseline comparisons**: ACHIEVED - Evaluated against EKF, physics-based, and heavy neural baselines

6. **Computational efficiency analysis**: ACHIEVED - Comprehensive latency, throughput, memory, energy characterization

7. **Full reproducibility**: ACHIEVED - Open-source release with notebooks, weights, normalization parameters

8.3 Key Technical Findings

8.3.1 Telemetry Sufficiency

Nine standard telemetry channels (speed, pressure, mass, thrust, density, 3-axis orientation, time) encode sufficient information for accurate position estimation, achieving $R^2 > 0.999$ without direct position/velocity measurements. This validates the hypothesis that aerodynamic (pressure, density) and propulsive (thrust, mass) telemetry implicitly encode position through altitude-dependent atmospheric profiles.

8.3.2 Architectural Simplicity

Simpler architectures outperform complex variants: our Dense-LSTM (1.28M parameters, no BatchNorm/Dropout, MSE loss) achieves 4.8× lower RMSE than newPosEST (1.31M parameters, BatchNorm/Dropout/Attention, Huber loss). This suggests careful feature engineering and activation selection (tanh) are more impactful than architectural complexity.

8.3.3 LSTM Superiority

LSTM outperforms GRU by 5–10% due to explicit cell state enabling long-term memory for multi-phase trajectories (launch $\rightarrow$ ascent $\rightarrow$ ballistic). This justifies the 25% parameter overhead despite GRU's computational efficiency.

8.3.4 Feature Importance

Ablation studies reveal pressure and atmospheric density as most critical features (ablation increases RMSE by 222% and 192% respectively), followed by speed (124%). This aligns with physical intuition: pressure/density provide altitude proxy, speed encodes kinetic energy.

8.3.5 Simulation Fidelity

KSP with Kerbalism provides sufficient physics realism for training competitive models despite simplifications (exponential atmosphere, point-mass gravity, simplified drag). This suggests academic-grade simulators can substitute for flight data in early development.

8.4 Key Methodological Findings

8.4.1 Temporal Stratification

80/20 train/test split with temporal coherence (full trajectories in test set) provides more realistic evaluation than random sampling, which leaks temporal correlation and overestimates performance by 10–15%.

8.4.2 Normalization Isolation

Computing Z-score statistics exclusively on training data prevents test leakage and simulates deployment conditions. Early experiments with full-dataset normalization overestimated performance by 5–8%.

8.4.3 Single Time-Step Trade-off

Sequence length 1 minimizes latency (77.6 ms vs. 150–200 ms for length 5–10) but loses velocity/acceleration trends, increasing RMSE by estimated 10–15%. This trade-off should be tuned per application's latency budget.

8.4.4 Batch Size Optimization

Batch size 256 balances GPU utilization (3.4 GB / 12 GB VRAM) and gradient noise for stable convergence. Larger batches (512+) exceed memory; smaller batches (¡128) increase training time by 2–3$\times$ without accuracy benefit.

8.5 Limitations Recap

1. **Sim-to-real gap**: Estimated 20–50% performance degradation on real hardware due to unmodeled aerodynamics, sensor noise, structural dynamics.
2. **No velocity estimation**: Current model outputs position only; extending to 9D state (position + velocity + attitude) requires multi-task learning.
3. **No uncertainty quantification**: Point estimates lack confidence intervals; probabilistic outputs needed for safety-critical deployment.
4. **OOD generalization**: 56–162% error increase on unseen mission profiles (polar orbits, VHLV, abort scenarios).
5. **TensorFlow overhead**: 60–70% of batch-1 latency is framework overhead; TFLite or ONNX Runtime could reduce to ¡30 ms.

8.6 Future Work

8.6.1 Near-Term (3–6 Months)

1. **Multi-step sequences**: Extend to sequence length 5–10 to capture velocity/acceleration trends, targeting 10–15% RMSE reduction.
2. **Domain randomization**: Add synthetic sensor noise (Gaussian, bias, dropout) during training to improve real-world robustness.
3. **Uncertainty quantification**: Replace linear output head with Gaussian head predicting (μ, σ^2), enabling confidence intervals and OOD detection.
4. **Model compression**: Apply post-training quantization (INT8) and pruning (50% sparsity) to achieve 4$\times$ size reduction (1.22 MB) and 2–3$\times$ latency reduction.
5. **TensorFlow Lite deployment**: Convert to TFLite for embedded platforms (ARM Cortex-M7, NVIDIA Jetson), targeting ¡30 ms batch-1 latency.

8.6.2 Medium-Term (6–12 Months)

1. **Sounding rocket validation**: Deploy on suborbital sounding rocket (100–150 km apogee) to quantify real-world performance and collect flight data for fine-tuning.
2. **Multi-modal sensor fusion**: Integrate vision (star trackers), LiDAR altimetry, and magnetometers for robust navigation with graceful degradation.
3. **Physics-informed neural networks**: Add physics-based loss terms (energy conservation, momentum balance) to regularize predictions and improve OOD generalization.
4. **Meta-learning**: Train on multiple simulation environments (KSP, Orbiter, GMAT) using MAML for fast adaptation to new domains with ¡100 fine-tuning samples.
5. **High-altitude balloon validation**: Test on stratospheric balloon (30–40 km) for extended duration (2–4 hours) to quantify long-term drift and computational stability.

8.6.3 Long-Term (1–2 Years)

1. **CubeSat demonstration**: Deploy on 3U CubeSat for on-orbit validation during deorbit phase, demonstrating GPS backup and collision avoidance.
2. **Human spaceflight integration**: Integrate with Artemis lunar lander or Starship for abort scenario backup navigation and EDL.
3. **Hypersonic vehicle deployment**: Adapt for Mach 5–25 reentry vehicles to provide navigation during plasma blackout (5–15 minute outages at 30–80 km altitude).
4. **Generalized multi-task framework**: Extend to predict full 9D state (position, velocity, attitude) plus uncertainty, fuel remaining, and failure diagnostics from unified telemetry.
5. **Sim-to-real benchmarking**: Establish community benchmark comparing simulation-trained models on standardized real-world datasets (sounding rockets, high-altitude balloons, UAV GPS-denied flights).

8.7 Closing Remarks

This work demonstrates that lightweight, simulation-trained neural networks can achieve competitive performance for GPS-denied spacecraft position estimation using only standard telemetry. The 1.28M parameter Dense-LSTM model achieves $R^2 > 0.999$ accuracy with 77.6 ms CPU latency, outperforming classical and heavier neural baselines by 4.8–8.6$\times$ while maintaining embedded deployment feasibility (4.88 MB model size).

Key contributions include: (1) First telemetry-only estimator competitive with vision-based methods, (2) Demonstration that architectural simplicity outperforms complexity for this task, (3) Comprehensive baseline comparisons validating superiority over EKF and physics-based approaches, (4) Open-source release enabling community validation and extension.

The simulation-to-reality gap remains a fundamental challenge requiring flight validation, but the results establish a strong foundation for affordable development of autonomous navigation systems for small spacecraft, tactical missiles, and UAVs operating in GPS-denied environments. By releasing all code, data, and trained models, we aim to democratize access to advanced navigation capabilities and accelerate research in telemetry-based state estimation.