Line Assignment

P Pavan Kumar

October 2022

***Problem Statement* - Two sides of a rhombus ABCD are parallel to the ines y=x+2 and y=7x+3.1f the diagonals of the rhombus intersect at the point (1,2) and the vertex A is on the y-axis,find possible co-ordinate of A.**

Figure

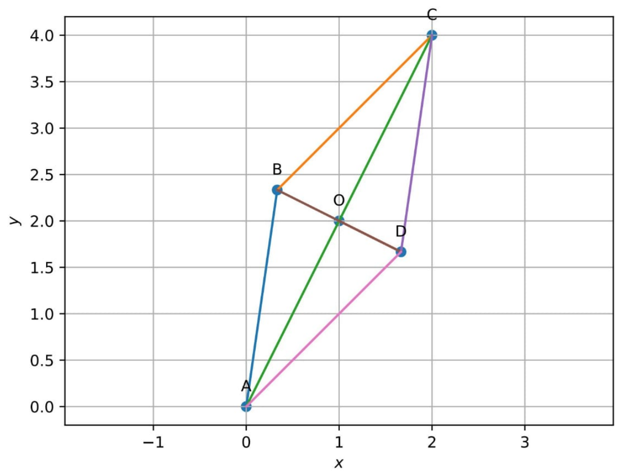

Figure 1: Diagonals intersect at point O(1,2)

Solution

The equation of the line1 y=x+2 and line2 y=7x+3.
we know that vector equation of the line is

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = c \tag{1}$$

The vector equation of the line1 and line2 is

$$\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} = 2 \tag{2}$$
$$\begin{pmatrix}-7 & 1\end{pmatrix}\mathbf{x} = 3 \tag{3}$$

from above equations the direction vectors of AB and AD are

$$\mathbf{m_1} = \begin{pmatrix}1\\7\end{pmatrix} \tag{4}$$

$$\mathbf{m_2} = \begin{pmatrix}1\\1\end{pmatrix} \tag{5}$$

To find the coordinate of A we use the parametric equation of the line

$$\mathbf{X} = \mathbf{P} + \lambda\left(\frac{m_1}{\|m_1\|} + \frac{m_2}{\|m_2\|}\right) \tag{6}$$

where

$$\mathbf{x} = \mathbf{A} = \alpha\mathbf{e_2}, \mathbf{P} = \begin{pmatrix}1\\2\end{pmatrix} \tag{7}$$

$$\alpha\mathbf{e_2} = \mathbf{P} + \lambda\left(\frac{m_1}{\|m_1\|} + \frac{m_2}{\|m_2\|}\right) \tag{8}$$

multlipliying the above equation with e1 transpose

$$\lambda = -1.178 \tag{9}$$

after finding the lambda substitute in the eq(12) slove for the equation for vertex A

$$\alpha\mathbf{e_2} = \mathbf{P} + \lambda\left(\frac{m_1}{\|m_1\|} + \frac{m_2}{\|m_2\|}\right) \tag{10}$$

the vertex A is

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix} \tag{11}$$

The midpoint gives the vertex c

$$\mathbf{P} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{12}$$

$$\mathbf{C} = \begin{pmatrix}2\\4\end{pmatrix} \tag{13}$$

the normal vectors are

$$\mathbf{n_1} = omat * \mathbf{m_1}, \mathbf{n_2} = omat * \mathbf{m_2} \tag{14}$$

on sloving,we get

$$\mathbf{n_1} = \begin{pmatrix}7\\-1\end{pmatrix}, \mathbf{n_2} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{15}$$

the vertex B is the intersection of the lines

$$\mathbf{n_1^T}(\mathbf{x}-\mathbf{A}) = 0, \quad \mathbf{n_2^T}(\mathbf{x}-\mathbf{C}) = 0 \tag{16}$$

on solving, we get

$$\mathbf{B} = \begin{pmatrix}1/3\\7/3\end{pmatrix} \tag{17}$$

the vertex D is the intersection of the lines

$$\mathbf{n_1^T}(\mathbf{x}-\mathbf{C}) = 0, \quad \mathbf{n_2^T}(\mathbf{x}-\mathbf{A}) = 0 \tag{18}$$

on solving, we get

$$\mathbf{D} = \begin{pmatrix}5/3\\5/3\end{pmatrix} \tag{19}$$

Hence the vertices of the rhombus are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{20}$$

$$\mathbf{B} = \begin{pmatrix} 1/3 \\ 7/3 \end{pmatrix} \tag{21}$$

$$\mathbf{C} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{22}$$

$$\mathbf{D} = \begin{pmatrix} 5/3 \\ 5/3 \end{pmatrix} \tag{23}$$