Trapped Ion Quantum Computing

David Skocic

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Basics of Quantum Computing

Qubits

- Analog to the classical bit (Qubit = Quantum bit)
- Any two-state quantum system can theoretically be a qubit
- We use our familiar Dirac notation to represent states

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Qubits can have a superposition unlike a classical bit

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha |0\rangle + \beta |1\rangle$$

Electron Spin

- Spin is a familiar two-state quantum system
- Recall our spin operators for the different directions

$$S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad S_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad S_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Unitary Operators

- The evolution of a quantum system is described by unitary operators

$$|\psi_t\rangle = U(t, t_0)\, |\psi_0\rangle$$

- Unitary Operators are *mathematically reversible*:

$$UU^\dagger = I$$

- You can apply unitary operators to a qubit to alter its state in a desirable manner

Quantum Gates

- In quantum computing, unitary operators are usually called **quantum gates**
- Many of the basic quantum gates are look very familiar!

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- There are also less familiar gates such as the *Haramad* Gate:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Quantum Gates

- The X gate is also called the NOT gate:

$$NOT|0\rangle = |1\rangle \text{ and } NOT|1\rangle = |0\rangle$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

- The Haramad gate creates a superposition:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Quantum Circuits

- A series of quantum gates constitutes a **quantum circuit**
- It is standard to represent these as if they were on a wire

$$\Updownarrow$$

$$MZYH|0\rangle = M\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{M}{\sqrt{2}}\begin{bmatrix}-i\\ -i\end{bmatrix}$$

Where M represents the act of measurement

Trapping an Ion

Classical Ion Trapping