Formal Languages and Computability 4

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24. september 2024

1 Give an example where A, B are nonregular, $A \neq B$, but $A \cap B$ is regular.

We can take for example the nonregular languages $A = \{0^n 1^n | n \geq 0\}$ and $B = \{2^n 3^n | n \geq 0\}$ where $A \neq B$ and the intersection $A \cap B = \{\epsilon\}$ which is clearly regular.

2 Let us define language A accordingly, $A = \{i^n d^m | 0 \leq n < m\}$. Show that A is nonregular.

We will use pumping lemma:

Let us assume that there exists a DFA that describes A, so that pumping lemma holds for some p. Let $s = i^p d^{p+1}$ so that $|s| = p + p + 1 = 2p + 1 > p$

Then we should be able to split s into three pieces, $s = xyz$ so that $xy^i z \in A$ for all $i \geq 0$. Since $|xy| \leq p$, x and y both have to consist solely of i's. Since that is the case, and $|y| > 0$, we can always find an i so that $xy^i z$ gives us a case where $|xy^i| \geq |z|$, causing a contradiction.

Therefore, the language is nonregular.

3 Let us consider ternary digits and a language A which consists of strings w that start with a prefix defining the most frequent symbol in the rest of the string. Show that no NFA exists that could recognize A

We will use pumping lemma to prove that the language is nonregular. Let us assume that there exists a NFA that recognizes A so that pumping lemma holds for some p. Let $s = 02^p 1^p 0^p 0$ so that $|s| = 3p + 2$

We can then split any string s into three parts, $s = xyz$ where $xy^i z \in A$ for all $i \geq 0$. We see that since $|xy| \leq p$, there are a few cases for y:

- y consists of only 0. Here we can simply say that $i = 0$ and get the string $2^p 1^p 0^p 0$ where the string states that there are mostly 2's in the string which is incorrect and we get a contradiction.
- y consists of 0 and then any number of 2's. Here we can repeat the same trick and get $xy^0 z$, stating that there are mostly 2's in the string which is incorrect.
- y consists only of 2's. Here we can pump until 2 is the most common symbol, overtaking the number of 0's, leading to a contradiction.

Here, all three cases can lead to a contradiction and therefore pumping lemma deduces that the language is nonregular and therefore there exists no NFA that could recognize it.