Formal Languages and Computability 1

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6. september 2024

1 Let $t_{i+1} = (2t_i - 1)^2$. Show by induction that $0 \leq t_i \leq 1$ for all i if $0 \leq t_0 \leq 1$.

We notice that the base case $0 \leq t_0 \leq 1$ has already been provided, therefore we continue with $k \in \mathbb{N}$ where we assume $0 \leq t_k \leq 1$.

We notice that $2t_k$ results in

$$0 \leq 2t_k \leq 2$$

and then

$$-1 \leq 2t_k - 1 \leq 1$$

Now squaring $2t_k - 1$ will give a range between 0 and 1 since it will not be negative and the lowest number it can give is 0 where $t_k = 1/2$. Therefore:

$$0 \leq (2t_k - 1)^2 \leq 1$$

And since we have the precondition $t_{i+1} = (2t_i - 1)^2$ we get

$$0 \leq t_{k+1} \leq 1$$

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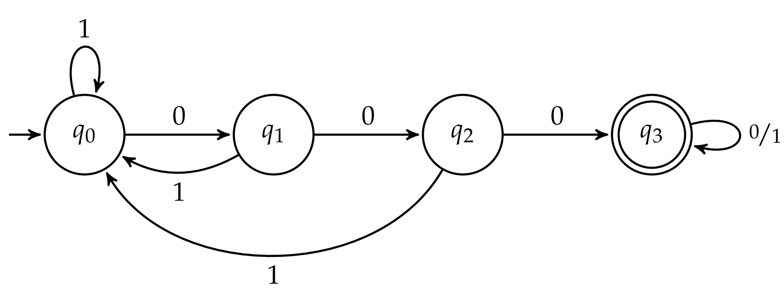
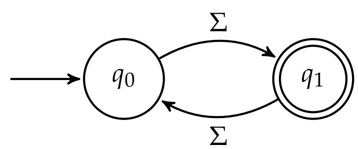
2 Draw a state diagram for a DFA that *accepts* any binary string that has three or more consecutive zeroes.

Here we simply need to have three ladder-like states where each consecutive 0 brings you to the next one, but each 1 knocks you all the way down. If then, we get three consecutive zeroes we get locked to the fourth state where the string can continue with any number of zeroes or ones and then end at any time.



3 Suppose A is a regular language with alphabet Σ. Let B denote the language where we have omitted strings with an even length. Show that B is regular.

We can show that a language is regular by showing, among other methods, a DFA that represents the language:



So that for each word in the language Σ there exists both a transition from q_0 to q_1 and from q_1 to q_0.

4 Let A denote the language that consists of strings with a correct checksum

a) Proof that $w^R \in A$ for all $w \in A$.

We see that since $w \in A$ and

$$w = a_1 a_2 ... a_n c$$

so that

$$c = a_1 \oplus a_2 \oplus ... \oplus a_n$$

In the case with $w^R = c a_n a_{n-1} ... a_2 a_1$ we see that for w^R to be in A, a_1 has to be a correct checksum, that is,

$$a_1 = c \oplus a_n \oplus ... \oplus a_2$$

And according to our definition of c we get

$$c \oplus a_n \oplus a_{n-1} \oplus ... \oplus a_2$$
$$= a_1 \oplus a_2 \oplus ... \oplus a_n \oplus a_n \oplus a_{n-1} \oplus ... \oplus a_2$$

We observe here that since $a \oplus a = 0$ we see that each duplicate a_i cancels each other out of the equation and we are left with:

$$= a_1 \oplus a_n \oplus a_n = a_1 \oplus 0$$
$$= a_1$$

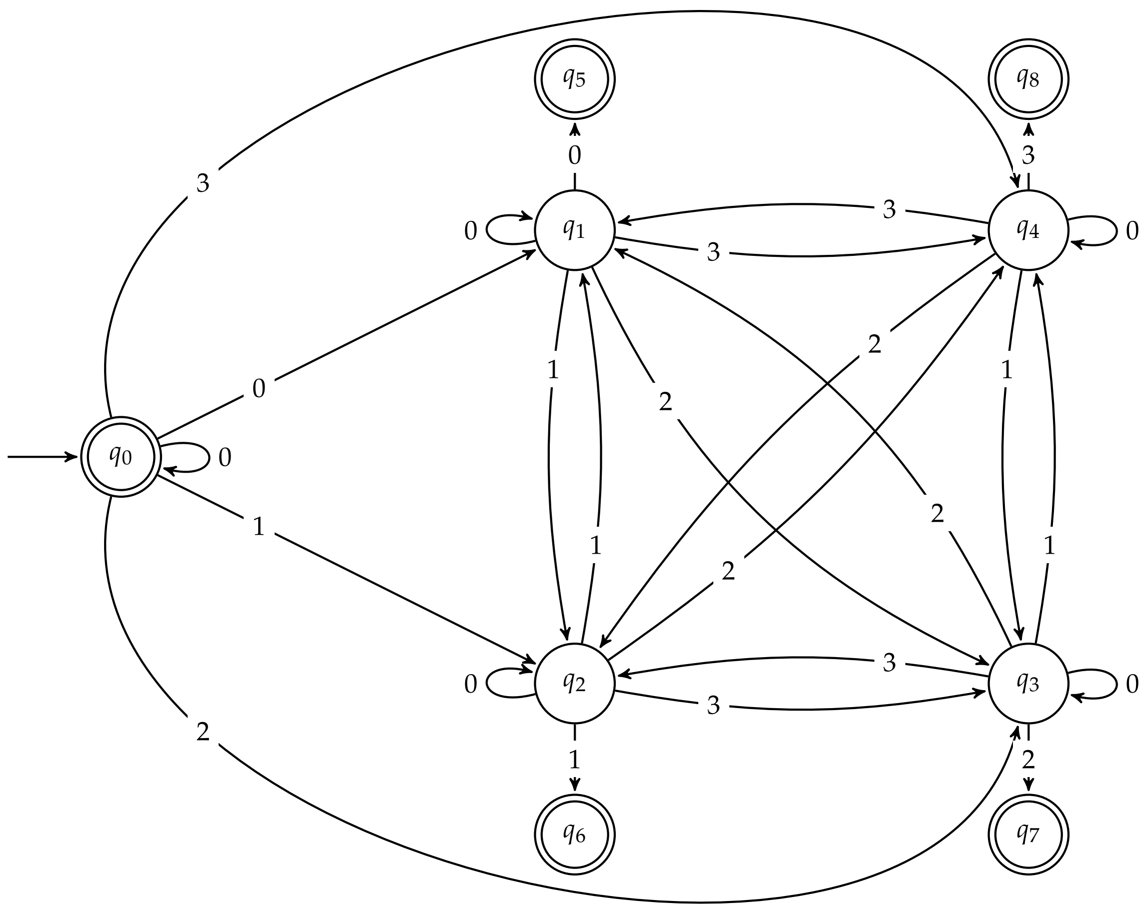
And therefore, $w^R \in A$.

$\blacksquare$

b) Show that A is regular.

Here we will show a NFA that represents the language A.

Since we need the checksum, or the last symbol, to be dependent on what we have so far, we will alternate between four states, where each one has the possibility to end on the corresponding c, or continue further.



We have then shown that a NFA can be constructed, and therefore A is regular.