Formal Languages and Computability 8

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1

a) We see that putting 1011 into the DFA leads us into the reject state q_r so the string is not in the language that M describes.

b) Here, since the string 1011 is not a string accepted by M, $\langle M, 1011\rangle$ is not in A_{DFA}, since A_{DFA} requires that M accepts the string.

c) Here we see that 1010 is accepted by M so $\langle M, 1010\rangle \in A_{DFA}$.

d) This is not true since EQ_{DFA} requires two DFA's, $\langle M, N\rangle$ and so with only $\langle M\rangle$ we cannot compare two DFA's since only one is given.

2

To prove that D is countable, we can make do with the fact that since $\mathbb{N}$ is countable, we only need to show that there exists an injection $D \to \mathbb{N}$ since if that is the case, the the cardinality of D, $|D| \leq |\mathbb{N}|$, meaning that since $\mathbb{N}$ is countable, D also has to be countable.

We then let $f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be defined as $f(n, m, o) = 2^n \cdot 3^m \cdot 5^o$, $n, m, o \in \mathbb{N}$. To prove that it is injective, we assume that for some $a, b, c, m, n, o \in \mathbb{N}$,

$$f(a, b, c) = f(n, m, o)$$

$$\iff 2^a \cdot 3^b \cdot 5^c = 2^n \cdot 3^m \cdot 5^o$$

which clearly shows that $a = n$, $b = m$, $c = o$. Therefore, $f : D \to \mathbb{N}$ is injective so D is countable.

3

a) Since we are working with a set of binary strings, the set A is countably infinite as A is a subset of the set of all possible binary strings, according to the Note on Lemma 53 in lecture 8. If we would be working with binary sequences however, the set A would be uncountable.

b) Since $w = 1.010101010\ldots$ is an infinite binary sequence, it is not in A since A only accepts finite binary strings.

c) A is decidable since for any string w which is finite according to our definition in a), we can calculate $\sqrt{8}$ to the precision of w and then compare them, all in finite time, then reject if $w > \sqrt{8}$ and accept otherwise.

4

Here we will construct a new DFA, C, that accepts only strings that are accepted by S and not by R. We then see that if $L(S) \subseteq L(R)$ then $L(C) = \varnothing$.

Lets assume a TM $F = F(\langle R, S\rangle)$ which constructs a DFA C as we described. Then we can run a TM T that decides on E_{DFA} on C. If T accepts, F accepts, if not, it rejects.

TM F then accepts if $L(C) = \varnothing$ and rejects otherwise so it decides on the language A.