Formal Languages and Computability 9

Ragnar Björn Ingvarsson, rbi3

28. október 2024

1

We can just see that for every even number we can execute the function

$$f(n) = \frac{n}{2}^2 \tag{1}$$

Which maps each even number to a perfect square. Since this function is computable and provides the needed reduction, we see that B is mapping reducible from A.

2

We can show that A is decidable by giving a description for a TM that decides on A.

Let M be the TM, which takes in a pair of natural numbers (a, b). We first calculate $a^2 + b^2$ and store that. Then we start with $c = 1$ and iterate over all natural numbers, where for each one we calculate c^2 and compare it to our $a^2 + b^2$. If they are equal, we accept, otherwise we continue running. We can then just check if c^2 is larger than $a^2 + b^2$ and if so we reject.

3

We show that EQ_{CFG} is undecidable, we can construct a reduction from ALL_{CFG}. We do so by first assuming a decider M for EQ_{CFG} and then, for each grammar G, we construct a grammar G_1 that generates all possible strings Σ^*. Then we use M to decide if $L(G) = L(G_1)$ and if it accepts, we accept, otherwise we reject.

From here we see that we have reduced ALL_{CFG} to EQ_{CFG} which means that EQ_{CFG} is also undecidable.

4

We will prove this by creating a reduction from A_{TM} to F. Assume that a decider R exists for F and we will create a decider S for A_{TM}.

Let S run on input $\langle M, w \rangle$, and then we construct an encoding for a TM $\langle M_w \rangle$ which, for any input x, runs M on w. If M accepts, M_w accepts, otherwise it rejects. Then we can run R on $\langle M_w \rangle$ and if it accepts, S rejects and otherwise it accepts.

Here we have reduced A_{TM} to F, meaning that since A_{TM} is known to be undecidable, F also has to be undecidable. This works on the basis that if M accepts w, $\langle M_w \rangle$ accepts all possible strings, which is an infinite set, so R rejects it, meaning we can accept in turn. However, if M doesnt accept w or loops, $\langle M_w \rangle$ rejects every possible string, meaning $L(M_w) = \varnothing$ which is a finite set, meaning R accepts it so S can reject.