Your First Quiz

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HI Jana, sorry there's raw latex syntax. I'm not sure how to make this look pretty without spending another half hour formatting things with pandoc. I spent about 2h 30min on this across two days.

Define:
group
A set of objects and a binary operation that is:

1. closed (two things operated together results in an element of the original set)
2. has an identity (identity operated with any other element results in the other element)
3. inverse (for each element, there is another element that when operated, result in the identity)
4. associative ((a*b)*c = a*(b*c) : a, b, c \in the set and * is the operation)

field
a set of objects and two operations (one primary and one secondary, called addition and multiplication respectively) such that it forms a group under addition and mostly form a group under multiplication (except for the additive identity, for which there cannot be a multiplicative inverse unless the group is boring, as Jana showed after class once). and multiplication distributes over addition.

singular matrix (give an example)

ad-bc = 0 like
0 0
0 0

For vectors in $\mathbb{R}^2$, find matrices that:

do nothing

1 0
0 1

reflect across the y axis

-1 0
0 1

rotate 90º

0&-1\\1&0
I remember going over this in class, but forgot exactly what it was. I ended up rederiving it by drawing the x,y vector and x',y' vector and using a little trig to get x' in terms of x and y, and y' also. I decided to check my work by multiplying

rotate 30º

via the questions from the previous problem, this should be a cube root of the rotate by 90 matrix… how can I find that? Or, i'll have to do it geometrically again… never got to it

the matrix by itself (which should result in a rotation
by 180, aka -1 0 0 -1). Is this a square root of that matrix?
Is this a fourth root of the identity? How many other
square/fourth roots of those matrices are there?
How can we find them? Are there the equivalent of
imaginary matrices?

stretch in the y direction by 3

1 0
0 3

> Use elementary matrices to solve the system:
2x - 3y = 1
x + y = 3

We never learned the definition of an elementary matrix

(2) *= 2: 1 &0 \\0 &2
(1) -= (2): 1 &-1 \\0 &1
(2) /= 2 : 1 &0 \\ 0 &\frac{1}{2}
(1) /= 5 : \frac{1}{5}&0\\0&1
(2) += (1): 1 &0\\1&1
(1) *= -1: -1 &0 \\ 0 &1
swap rows: 0 1 1 0
multiplied together: \frac{1}{5}&\frac{3}{5}\\\frac{-1}{5}&\frac{2}{5}
^^^^ when multiplied with original, we infact get the inverse! (checking work)

now multiply with 1\\3 to get 2\\1

Find the inverse of $\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$, ideally without using a formula.

\frac{1}{5}&\frac{3}{5}\\\frac{-1}{5}&\frac{2}{5} (see above)

Demonstrate that matrix multiplication is not commutative.

\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}w&x\\y&z\end{bmatrix} = \begin{bmatrix}aw+by&ax+bz\\cw+dy&cx+dz\end{bmatrix} \neq \begin{bmatrix} aw+cx&bw+dx\\ay+cz&by+dz\end{bmatrix}\eq\begin{bmatrix}

What set(s) of 2x2 matrices forms a group under multiplication?

The identity? (rational or irrational) Matrices with non-zero determinants (or maybe that's not closed)?
Actually, I just checked it and it turns out that the determinant of two 2x2 matrices multiplied is the product of their determinants. basically I took the determinant of the product [a&b\\c&d] [w&x\\y&z] which simplified to (ad-bc)(wz-xy) which is super cool. So I guess it is closed.

Plus, you could take a number of transformations of the set group of rational matrices with non zero determinants, like maybe you multiply all of them by pi or something.

Or, you could have matrices that are of the form [x&0\\0&y] which is just a more verbose way of writing vector style 2x1 matrices

This is actually kinda fun, but I should get some sleep.

Compute the dot product:

$$\begin{pmatrix}1\\-1\\0\end{pmatrix}\bullet\begin{pmatrix}-2\\1\\3\end{pmatrix}$$

What characteristics does dot product have? Can you give a geometric interpretation? When is it zero?

A\bullet B = |A||B|cos\theta, as I have proved previously… (don't want to latex it again)

So, dot product will be zero when vectors are perpendicular, and generally be affected by the angle between them. Because the vectors are facing away from each other (angle past \frac{\pi}{2}), the dot product is negative.

Compute the cross product:

$$\begin{pmatrix}2\\0\\3\end{pmatrix}\times\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

What characteristics does cross product have? Can you give a geometric interpretation?

i j k

2 0 3

-1 2 1

i(0-6)+j(-3-2)+k(4+0) = -6\\-5\\4

Cross product is perpendicular to the vectors and it’s length is the area of the parallelogram formed by those vectors. Not sure why those are the same geometrically though…

Interesting that it’s just the determinant. Something about the determinant creates perpendicularity, like the same thing happens in two dimensions with the vector $x\\y$ and \left|\begin{matrix}i&j\\x&y\end{matrix}\right|$

Combine the cross product with the dot product to give an equation for the plane containing $\begin{pmatrix}2\\0\\3\end{pmatrix} and \begin{pmatrix}-1\\2\\1\end{pmatrix}$

Seems like a lot of algebra… I’m going to opt to write my history essay.

How are scalar multiplication, cross product, and dot product on a vector space different from multiplication on the real numbers?

Multiplication on real numbers forms a group. Scalar multiplication takes elements of different fields as inputs (by definition, unless your other field is \mathbb{R}, in which case scalar multiplication *is* multiplication on the reals), cross product has no identity (geometrically, a vector is not perpendicular to itself so there can be no other vector such that an output vector is perpendicular to the identity and equal to the first), and dot product is not closed (you get a scalar when multiplying matrices).

Prove that matrix multiplication (of 2x2 matrices) is associative.

I did the first homework by myself, so please see the bottom of my submission for *Solving with matrices*.