# 1 | Cauchy-Schwarz Inequality important

'One of the most important inequalities in mathematics'

Suppose $u, v \in V$ (where $V$ is an inner product space). Then

$$|\langle u, v\rangle| \leq \|u\|\|v\|$$

The inequality is an equality iff one of $u, v$ is a scalar multiple of the other.

## 1.1 | proof is by the orthogonal decomposition

## 1.2 | results

### 1.2.1 | triangle inequality

Suppose $u, v \in V$. Then

$$\|u + v\| \leq \|u\| + \|v\|$$

The inequality is an equality if and only if one of $u, v$ is a non-negative multiple of the other (degenerate triangle)

### 1.2.2 | Parallelogram Equality

Suppose $u, v \in V$. Then

$$\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$$