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#ret

## Square roots of $i$

20math530retSquareRootsi.pdf Didn't figure it out. How did I get  $a = \pm \frac{\sqrt{2}i}{2}$ ??

## Cross product

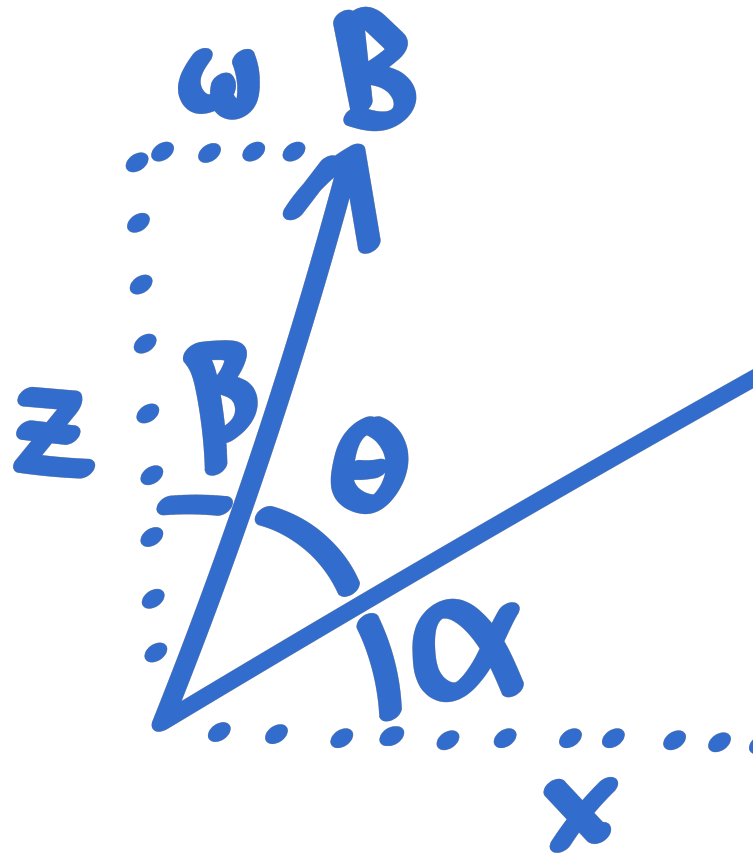
Find the cross product of  $\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$$\begin{aligned} &\Rightarrow \begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ 2 & 2 & -1 \end{vmatrix} \\ &\Rightarrow i \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} + j \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} + k \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} \\ &\Rightarrow -3i + 1j - 4k \\ &\Rightarrow \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix} \end{aligned}$$

## Read Chapter 1.B

### Vector Space Addition/scalarmult

- Every pair of elements in  $V$  can be added together to form another element in  $V$  (closed)
- **A scalar is anything in  $F$ , which means it might be complex!** ### Vector space definition
- commutativity(!):  $u + v = v + u, \forall u, v \in V$
- associativity:  $(u + v) + w = u + (v + w)$  and  $(ab)v = a(bv), \forall u, v, w \in V, \forall a, b \in F$
- additive identity:  $\exists 0 \in V \mid v + 0 = v, \forall v \in V$
- Additive inverse
- Multiplicative identity (denoted 1)
- distributive property (both front and back) A vector space depends on  $F$  so  $V$  is a **vector space over  $F$**  ### Vector spaces with other sets?  $F^S$
- $F^S$  is the set of functions from  $S$  to  $F$ 
  - meaning that it's all functions whose domains are subsets of  $S$  and ranges are subsets of  $F$
- addition  $f, g \in F^S, x \in F : (f + g)(x) = f(x) + g(x)$
- multiplication:  $\lambda \in F$  and  $f \in F^S : \lambda f \in F^S = (\lambda f)(x) = \lambda f(x)$
- functions can be elements in fields or something?
- lists are just functions on a set of numbers..?
- subtraction (additive inverses and identity are unique)
- **When you see  $xy$ , one of them has to be a vector because there is no scalar scalar multiplication defined** ## Show that  $a \bullet b = |a||b|\cos\theta$  Suppose  $a = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $b = \begin{bmatrix} w \\ z \end{bmatrix}$ . We have  $a \bullet b = a^T \cdot b =$



$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix} = xw + yz$ . We need an expression for  $\theta$ :  
 As seen in the diagram,  $\theta = \frac{\pi}{2} - \alpha - \beta$ . Finally:

$$\begin{aligned}
 |A||B|\cos\theta &= |A||B|\cos\left(\frac{\pi}{2} - \alpha - \beta\right) \\
 &= |A||B|\sin(\alpha + \beta) \\
 &= |A||B|(\sin\alpha \cos\beta + \cos\alpha \sin\beta) \\
 &= |A||B|\left(\left(\frac{y}{|A|}\right)\left(\frac{z}{|B|}\right) + \left(\frac{x}{|A|}\right)\left(\frac{w}{|B|}\right)\right) \\
 &= |A||B|\left(\frac{yz}{|A||B|} + \frac{xw}{|A||B|}\right) \\
 &= yz + xw \\
 &= xw + yz
 \end{aligned}$$

## Epilogue

That was two hours... I'll save the proving integers mod 3 are a field for later. #todo-exr0n