Source: [[KBhMATH401SubIndex]]

#disorganized

1 | Limits

Some Vocab

Here's a function

$y = \frac{1}{x}$.

We know that it has

- Domain $D(-\infty, 0)(0, \infty)$
- Range $R(-\infty, 0)(0, \infty)$
- $As\ x \to \infty,\ y \to 0$
- Function is *odd*, that is, $f(-x) = -f(x)$

The Limit Notation

Definition 1 · **Right Single-Sided Limit** $\lim_{x \to a^+} f(x)$
"What is y approaching when x approaches a from the right $(+)$?"

Definition 2 · **Left Single-Sided Limit** $\lim_{x \to a^-} f(x)$
"What is y approaching when x approaches a from the left $(-)$?"

Watch! If both the left and right single-sided limit exists and is the same, the Double-Sided Limit exists.

Definition 3 · **Left Single-Sided Limit** $\lim_{x \to a} f(x)$
"What is y approaching when x approaches a?" This exists only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

Vocab! When the Double-Sided Limit does not exist, it is called *DOES NOT EXIST!*. It is not! $\cancel{undefined}$

Computing Limits Algebraically

Let's do a problem solve for $\lim_{x \to 2} \frac{(x^2-4)}{(x-2)}$

1. First, notice the fact this function will have a hole at $x = 2$. This is especially important because after we simplify we will loose this hole.
2. Ok, now let's simply. $\frac{(x^2-4)}{(x-2)} = \frac{(x+2)(\cancel{(x-2)})}{\cancel{(x-2)}} = (x+2)$
3. Great! So, we know that this function behaves linearly with simply a hole at 2.
4. Doing the double-sided limits…
   - Evaluating $\lim_{x \to 2^+}$, the value will be 4 because $2 + 2 = 4$.
   - Evaluating
   - Infinite Discountinuity (verticle asymtote)
     - Double Sided Limit does not exist
   - Jump Discontinuity

    - Double Sided Limit does exist
    - Function defined
- Point Discountinuity
    - Double Side Limit exists
    - Function is not defined