1 | Reading

1.1 | Definition of a Definite Integral

For each interval $[x_i, x_{i+1}]$, we choose x_i^* in the interval to be the position of the minimum (for lower bound) or maximum (upper bound) value.

2 | Problems

2.1 | exr1.3

Using the left edge: -8.4375

Summation notation for left edge approximation:

$$\sum_{i=0}^{n} \underbrace{\frac{b-a}{n}}_{\text{width}} \underbrace{f\left(a + \frac{b-a}{n} i\right)}_{\text{height}}$$

2.2 | exr1.4 (in class)

0.21875 using the left estimate

2.3 | exr1.5

2.3.1 | left estimate

34.7 feet (add all except last number and divide by two, because we are stopping at 3.0 seconds in.)

2.3.2 | right estimate

44.8 feet (add the last number and drop the zero from the beginning)

2.3.3 | middle estimate

Not enough information to do it for $\Delta x = 0.5$, so I will use $n = 3$ aka $\Delta x = 1$

$$6.2 + 14.9 + 19.4 = 40.5 \text{ feet}$$

2.4 | exr1.6

2.4.1 | $\int_0^1 \sqrt{x^2+1}dx$

$\sqrt{x^2+1}$ is the length the hypotenuse of a triangle with leg-lengths 1 and x. Because x is continuous, this is like the area of a right triangle with leg-lengths 1 and 1, which is $\boxed{\frac{1}{2}}$.

1. TODO Wolfram Alpha doesn't agree though

   Probably because as you take approximations, there will be overlap, so the actual value is bigger than I think it is.

   I also don't know how to take the anti-chain-rule, so I don't know how to integrate the function symbolically.

2.4.2 | $\int_0^3 (x-1)dx$

Not sure area wise, but the anti-derivative is guess-able:

$$\frac{d}{dx}\left(\frac{x^2}{2} - x\right) = x - 1$$

$$\frac{3^2}{2} - 3 = 1.5$$

2.5 | **exr1.7**

2.5.1 | **right endpoint approx for** $y = x^2$

$$\sum_{i=1}^{n+1} \Delta x f(i\Delta x) = \sum_{i=1}^{n+1} \frac{1}{n}\left(\frac{i}{n}\right)^2$$

where $\Delta x = \frac{1}{n}$

2.5.2 | **general form for left-side riemann sum**

See exr1.3

2.6 | **exr1.11**

$$\int_{\pi}^{2\pi} \cos(x)dx = \lim_{n\to\infty} \sum_{i=0}^{n} \frac{\pi}{n} \cos\left(\pi + \frac{i\pi}{n}\right)$$

2.7 | **exr1.12**