

## 1 | Exercise 7

Suppose  $T \in \mathcal{L}(V)$  has a diagonal matrix  $A$  with respect to some basis of  $V$  and that  $\lambda \in \mathbb{F}$ . Prove that  $\lambda$  appears on the diagonal of  $A$  precisely  $\dim E(\lambda, T)$  times.

## 2 | Proof

Any eigenvalue  $\lambda$  must have  $\dim E(\lambda, T)$  linearly independent associated eigenvectors, by definition. ("In other words,  $E(\lambda, T)$  is the set of all eigenvectors of  $T$  corresponding to  $\lambda$ , along with the 0 vector").

The diagonal is comprised of the basis vector coefficients of the eigenspaces, implying that each eigenspace of  $\lambda$  is represented by  $\dim E(\lambda, T)$  eigenvectors.  $\lambda$  appears exactly once for each associated eigenvector, or precisely  $\dim E(\lambda, T)$  times in total.