Source:

1 | Definitions

1.1 | affine subset

An affine subset of a vector space V is of the form $U + v$ where $U \subseteq V$ and $v \in V$.

1.2 | product space

The product of some vector spaces $V_1 \times \cdots \times V_n$ is the set of lists of vectors with one from each respective space:

$$\{(v_1, \ldots, v_n) : v_1 \in V_1, \ldots, v_n \in V_n\}$$

1.3 | quotient space

A quotient space V/U is the set of affine subsets $\{U + v : v \in V\}$ (although some of those affine subsets are equivalent).

1.4 | equivalence relation

An equivalence relation is a set of elements that are considered equivalent (equal to eachother). For example, in a vector space U, $U + 0 = U + u \forall u \in U$.

2 | Why ”product” and ”quotient” are used to describe these operations

The product of vector spaces is essentially a cartesian product. With real numbers, the product is like stacking copies of an operand in a new direction (product of two numbers for area of a plane, product of three for the volume of a space). Here, we are doing essentially the same thing for vector spaces (each vector is combined within a list with other vector spaces, but they do not interact with eachother and are orthagonal, in a sense).

A quotient space is like taking (dividing) out part of a vector space. It's like taking a modulo because a subset (U) is collapsed to zero and some things become equivalent. It is like removing (”dividing”) a subset of the basis (those that form a basis of U), where the basis itself can be represented as the cartesian product $\mathbb{F}^n$ (where n is the dimension of V).

3 | Examples of quotient spaces

Let $U = \{(x, y) : y = 2x; x, y \in \mathbb{R}\}$. In $\mathbb{R}^2/U$, vectors in U are equivalent to $U + 0$ and