> Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V. Prove that U is invariant under T iff $U^\perp$ is invariant under T^*.

For all $u \in U$, $Tu = u' \in U$.

This implies that both directions, since $U = U^{\perp\perp}$ and $T = (T^*)^*$.