1 | **Problem**

Suppose $T \in \mathcal{L}(V)$. Prove that $T/(\text{null } T)$ is injective if and only if $(\text{null } T) \cap (\text{range } T) = \{0\}$

2 | **Proof**

First, we will rewrite the problem as logical statements for easier manipulation. The left-hand side "$T/(\text{null } T)$ is injective" is equivalent to:

$$(T/(\text{null } T)) (v + (\text{null } T)) = \text{null } T \iff (v + (\text{null } T)) = \text{null } T$$
$$Tv + (\text{null } T) = \text{null } T \iff v + (\text{null } T) = \text{null } T$$