Source:

1 | **Broader vector spaces**

- Doesn't have to be physics vectors
- maybe it's like matrices
- or linear maps themselves

2 | **Axler 3.A ex7**

Let $w = Tv$.

2.1 | **If $v = 0$ then**

$$Tv = 0$$

By Axler 3.11 (Maps take 0 to 0). Thus, λ can be anything in $\mathbb{F}$.

2.2 | **Otherwise,**

$\frac{1}{v} \in \mathbb{F}$ because the field has multiplicative inverses for all elements except 0.

$$Tv = w = \left(w\frac{1}{v} \right) v$$

Let $\lambda = w\frac{1}{v}$, then

$$\lambda v = w\frac{1}{v}v = w$$

which is in $\mathbb{F}$ because $w, \frac{1}{v} \in \mathbb{F}$ and fields are closed under multiplication.