

## 1 | cooling pizza

Compute

$$\int_0^5 -110e^{-0.4t} dt$$

to the nearest degree.

$$\int -110e^{-0.4t} dt = \frac{-110}{-0.4} e^{-0.4t} = 275e^{-0.4t}$$

Using the net change theorem,

$$\begin{aligned} \Delta\beta \int_0^5 -110e^{-0.4t} dt &= \int -110e^{-0.4(5)} dt - \int -110e^{-0.4(0)} dt \\ &= 275e^{-0.4(5)} - 275e^0 \\ &= 37.21720289 - 275 \\ &= 37.21720289 - 275 \\ &= 350 - 237.78279711 \approx \boxed{112^\circ F} \end{aligned}$$

## 2 | definite integral as area under a curve

The area in the triangle is 3 square units, so  $5 + 3 = \boxed{8}$

## 3 | minimum value of $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$

$$\begin{aligned} \frac{d}{dx} f(x) &= e^{(x^2-3x)^2} (2x-3) = 0 \\ \implies 2x-3 &= 0 \\ \implies 2x &= 3 \\ \implies x &= \boxed{\frac{3}{2}} \end{aligned}$$

## 4 | approximate area under the curve graphically

The function looks symmetric about  $x = 12$ , so I will focus on  $[0, 12]$ .

On the interval  $[0, 6)$  a little under  $6 \cdot 100$  barrels of oil flow through.

On the interval  $[6, 12)$  a little over  $6 \cdot 100 + \frac{1}{2} \cdot 6 \cdot 100$  barrels flow through, for a total of

$$\approx 2(6 \cdot 100 + 6 \cdot 100 + \frac{1}{2} \cdot 6 \cdot 100) = 3000$$

barrels of oil.

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## 5 | fundamental theorem of calculus but worded confusingly

$F(x)$  is the antiderivative of  $f(x)$ , so differences of its values are definite integrals. In this case,

$$F(3) - F(0) = \int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx = 2 + 2.3 = \boxed{4.3}$$

## 6 | amusement park word problem

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$

valid over the domain  $[9, 23]$ , and there are zero people in the park at  $t = 9$ .

### 6.1 | number of people who have entered the park by some time

$$\begin{aligned} \int_9^x E(t)dt &= \int_9^x \frac{15600}{t^2 - 24t + 160} dx \\ &= 15600 \ln(t^2 - 24t + 160)(2t - 24)???? \end{aligned}$$

I don't know how to integrate this symbolically, and WolframAlpha says it contains an inverse tangent. Thus, I will use a calculator:

$$\int_9^{17} \frac{15600}{t^2 - 24t + 160} dt \approx \boxed{6004}$$

### 6.2 | value of $H'(17)$

$H$  represents the number of people in the amusement park.  $H(17) \approx 3725$

$$\begin{aligned} \frac{d}{dx} \int_9^t (E(x) - L(x)) dx &= E(x) - L(x) \\ E(17) - L(17) &= \frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370} \approx -380 \end{aligned}$$

$H'$  represents the change in the number of people in the park, per hour.