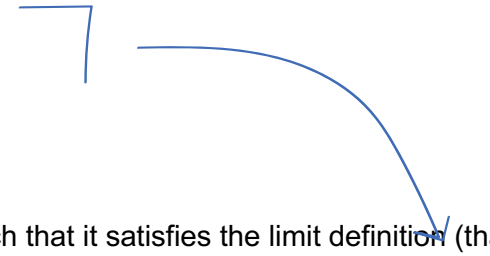
Formal definition of a Limit

$$\lim_{x \to a} f(x) = L$$

means for all $\varepsilon > 0$, there exists $\delta > 0$ such that if $0<|x - a| < \delta$ then $|f(x) - L| < \varepsilon$.

Let's explore the above definition graphically:

4. Consider the function $f(x) = 2x + 1$.
   a. Graph the function on Desmos. What do you the limit of the function is at $x = 3$?



   b. Suppose $\varepsilon = 1$. Find $\delta > 0$ such that it satisfies the limit definition (that is, find the $\delta > 0$ such that if $|x - 3| < \delta$ then $|(2x + 1) - L| < \varepsilon$).

$L = 7$

$|2x+1-7| < \varepsilon$
$|2x-6| < \varepsilon$
$|2(x-3)| < \varepsilon$
$2|x-3| < \varepsilon$
$|x-3| < \frac{\varepsilon}{2}$ $\quad \delta = \frac{\varepsilon}{2}$
$|x-3| < \delta$

   c. Suppose $\varepsilon = 0.5$. Find $\delta > 0$ such that it satisfies the limit definition (up to 4 decimal places).

$\delta = \frac{\varepsilon}{2} = \frac{0.05}{2} = 0.2500$

   d. Suppose $\varepsilon = 0.25$. Find $\delta > 0$ such that it satisfies the limit definition (up to 4 decimal places).

$\delta = \frac{\varepsilon}{2} = \frac{0.25}{2} = 0.1250$