

0 | **meta**

This homework took ~3h whilst discussing with peers... I really need to practice this type of algebra.

1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 du - \frac{1}{u^2+1} + C \\ &= 2 \int 1 du - \tan^{-1} u + C \\ &= 2u - \tan^{-1} u + C \\ &= \boxed{2\sqrt{x-1} - \tan^{-1}(\sqrt{x-1}) + C} \end{aligned}$$

Polynomial long division?

When you have a square root with a sum/difference inside, there's not much you can do. So, your best bet is to substitute either the stuff inside the root as u or the entire radical as u .

2 | $\int \frac{x^2}{x^2+1} dx$

Let $u = x^2 + 1$, $du = 2x dx$

$$\begin{aligned} \int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\ &= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\ &= \frac{1}{2} (u - \ln u) + C \\ &= \boxed{\frac{1}{2} (x^2 + 1 - \ln(x^2 + 1)) + C} \end{aligned}$$

**JUST SPLIT THE FRACTION
AND LOOK FOR TANsr X**

$$3 \mid \int \frac{x-4}{x^2} dx$$

$$\begin{aligned} \int \frac{x-4}{x^2} dx &= \int \frac{x}{x^2} \frac{4}{x^2} dx \\ &= \int \frac{1}{x} dx + 4 \int \frac{1}{x^2} dx \\ &= \boxed{\ln x - \frac{4}{x} + C} \end{aligned}$$

$$4 \mid \int (x+1)e^{x^2+2x} dx$$

Let $u = x^2 + 2x$, $du = x + 1 dx$

$$\begin{aligned} \int (x+1)e^{x^2+2x} dx &= \frac{1}{2} \int e^u du \\ &= \frac{1}{2} e^u \\ &= \boxed{\frac{1}{2} e^{x^2+2x} + C} \end{aligned}$$

$$5 \mid \int \tan^2 x + 1 dx$$

$$\begin{aligned} \int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx \end{aligned}$$

Let $u = x$, $du = dx$

$$\begin{aligned} &= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C} \end{aligned}$$

$$6 \mid \int \frac{6x^2-4}{x} dx$$

$$\begin{aligned} \int \frac{6x^2-4}{x} dx &= \int \frac{6x^2}{x} dx - 4 \int \frac{1}{x} dx \\ &= \int 3x dx - 4 \ln |x| + C \\ &= \boxed{3x^2 - 4 \ln |x| + C} \end{aligned}$$

$$7 \mid \int \frac{e^x - 1}{e^x} dx$$

$$\begin{aligned} \int \frac{e^x - 1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C} \end{aligned}$$

$$8 \mid \int \frac{\sec^2 x}{\csc x} \sin x dx$$

$$\begin{aligned} \int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x + C} \end{aligned}$$

$$9 \mid \int \sin x \cos x dx$$

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned} \int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x + C} \end{aligned}$$

Product of \sin and \cos , so we can use a double angle formula

$$\begin{aligned} \int \sin x \cos x dx &= \int \frac{1}{2} \sin 2x dx \\ &= \frac{1}{2} \int \sin 2x dx \\ &= -\frac{1}{4} \cos 2x \end{aligned}$$

10 | $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx$

$$\begin{aligned} \int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx \\ &= \int \frac{1}{\tan^2 x + 1} dx \\ &= \int \frac{1}{\sec^2 x} dx \\ &= \int \cos^2 x dx \\ &= \int \frac{1}{2} (\cos 2x + 1) dx \\ &= \frac{1}{2} \int \cos 2x + 1 dx \\ &= \frac{1}{2} \left(\int \cos 2x dx + \int 1 dx \right) \end{aligned}$$

Let $u = 2x$, $du = 2dx$

$$\begin{aligned} \frac{1}{2} \int \cos 2x dx + \frac{x}{2} + C &= \frac{1}{4} \int \cos u du + \frac{x}{2} + C \\ &= \frac{1}{4} \sin u + \frac{x}{2} + C \\ &= \boxed{\frac{1}{4} \sin 2x + \frac{x}{2} + C} \end{aligned}$$

11 | $\int \frac{\sec x \tan x}{1 + \sec^2 x} dx$

Let $u = \sec x$, $du = \sec x \tan x dx$

$$\begin{aligned} \int \frac{\sec x \tan x}{1 + \sec^2 x} dx &= \int \frac{du}{1 + u^2} dx \\ &= \int \frac{1}{1 + u^2} du \\ &= \tan^{-1} u + C \\ &= \boxed{\tan^{-1} \sec x + C} \end{aligned}$$

12 | $\int x^2 e^{x^3} dx$

Let $u = x^3$, $du = 3x^2 dx$

$$\begin{aligned} \int x^2 e^{x^3} dx &= \frac{1}{3} \int e^u du \\ &= \frac{1}{3} e^u + C \\ &= \boxed{\frac{1}{3} e^{x^3} + C} \end{aligned}$$

$$13 \mid \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$\text{Let } u = \sqrt{x}, du = \frac{1}{2\sqrt{x}} dx$$

$$\begin{aligned} \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2 \int e^u du \\ &= 2e^u + C \\ &= \boxed{2e^{\sqrt{x}} + C} \end{aligned}$$