1 | an example: semicircle revolved around the x-axis to create a sphere

We can make cuts perpendicular to the axis of rotation. In this case, you end up with a bunch of circular disks, where the height of each slice is your semicircle function.

Thus, the volume of the disk is

$$\pi f^2(x_i)\Delta x = (a^2 - x^2)\pi\Delta x$$

This is kinda like a Riemann Sum, but with more stuff added on. We can take the limit of the sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} \pi(a^2 - x_i^2)\Delta x$$

Where $\Delta x = \frac{1}{n}$ and $x_i = -a + \frac{2ak}{n}$

Expressed as an integral:

$$\int_{-a}^{a} \pi(a^2 - x^2)dx \to \int \pi a^2 dx - \int \pi x^2 dx$$

$$= \pi a^2 x - \pi\frac{1}{3}x^3$$

$$\to -\pi\frac{1}{3}a^3 + \pi\frac{1}{3}(-a)^3$$