Source:

1 | **TODO construct and write proof for Axler 2.C ex 17**

2 | **Problem**

Prove or give a counterexample:

$$\begin{aligned}\dim(U_1 + U_2 + U_3)&\\ =&\dim U_1 + \dim U_2 + \dim U_3\\ &- \dim(U_1 \cap U_2) - \dim(U_1 \cap U_3) - \dim(U_2 \cap U_3)\\ &+ \dim(U_1 \cap U_2 \cap U_3)\end{aligned}$$

3 | **Reasoning**

By Axler2.41 we know that

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$$

By applying this formula to itself, we find that

$$

\begin{aligned} dim(U$_1$ + U$_2$ + U$_3$)
&= dim((U$_1$ + U$_2$) + U$_3$)
&= dim(U$_1$ + U$_2$) + dim U$_3$ - dim((U$_1$+U$_2$) ∩ U$_3$) &= dim U$_1$ + dim U$_2$ -dim(U$_1$ ∩ U$_2$) + dim U$_3$