

at everyone

## 1 | Complete the Representation

Function	First four terms	Generalized
$\frac{1}{1-2x}$	$1 + 2x + 4x^2 + 8x^3 + \dots$	$\sum_{k=0}^{\infty} 2^k x^k$
$\cos(3x)$	$1 - \frac{9x^2}{2!} + \frac{81x^4}{4!} - \frac{729x^6}{6!} + \dots$	$\sum_{k=0}^{\infty} \frac{(-1)^k (3x)^{2k}}{2k!}$
$\frac{e^x}{e^2}$	$\frac{1}{e^2} + \frac{x}{e^2} + \frac{x^2}{e^2 2!} + \dots$	$\sum_{k=0}^{\infty} \frac{x^k}{e^2 k!}$
$\sin(x^2)$	$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$	$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$
$\frac{1}{1+x^4}$	$1 - x^4 + x^8 - x^{12} + \dots$	$\sum_{k=0}^{\infty} (-x^4)^k$
$e^{x-1}$	$1 + \frac{(x-1)^2}{2!} + \frac{(x-1)^4}{4!} + \frac{(x-1)^6}{6!} + \dots$	$\sum_{k=0}^{\infty} \frac{(x-1)^{2k}}{k!}$
$\frac{\cos(x)-1}{x^2}$	$-\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \dots$	$\sum_{k=1}^{\infty} \frac{(-1)^k x^{2(k-1)}}{(2k)!}$
$2x \ln(1+2x)$	$(2x)(2x) - \frac{(2x)(2x)^2}{2} + \frac{(2x)(2x)^3}{3} - \frac{(2x)(2x)^4}{4} + \dots$	$\sum_{k=1}^{\infty} \frac{2x(-1)^{k-1} (2x)^k}{k}$
$\frac{x}{1+x^2}$	$2x - 2x^3 + 2x^5 - 2x^7 + \dots$	$\sum_{k=0}^{\infty} 2x(-1)^k x^{2k}$

## 2 | page 3

### 2.1 | a: skipped

### 2.2 | find maclaurin series for $f'(x)$ where $f(x) = \sum_{k=0}^{\infty} \frac{(2x)^{k+1}}{k+1}$

$$\frac{d}{dx} \frac{(2x)^{n+1}}{n+1} = \frac{(n+1)^2 (2x)^n (2)}{(n+1)^2} = 2(2x)^n$$

So, our series is just

$$\sum_{k=0}^{\infty} 2(2x)^k = 2 + 4x + 8x^2 + 16x^3 + \dots$$

### 2.3 | estimate $f'(-\frac{1}{3})$

$$2 + 4\left(-\frac{1}{3}\right) + 8\left(-\frac{1}{3}\right)^2 + 16\left(-\frac{1}{3}\right)^3 = \frac{10}{3}$$

for the entire sequence...

$$\sum_{k=0}^{\infty} 2\left(-\frac{2}{3}\right)^k = 2 \sum_{k=0}^{\infty} \left(-\frac{2}{3}\right)^k$$

## 3 | page 4

3.1 | **find**  $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$

That series is just the Taylor series for

$$f(x) = \frac{\sin x}{x}$$

So the derivative at zero is zero, and the second derivative:

$$\begin{aligned} \frac{d}{dx} \frac{x \cos x - \sin x}{x^2} &= \frac{x^2 (-x \sin x + \cancel{\cos x} - \cancel{\cos x}) - (x \cos x - \sin x)(2x)}{x^4} \\ &= \frac{-x^3 \sin x - 2x(x \cos x - \sin x)}{x^4} \end{aligned}$$

is undefined at zero. However, the top of the fraction will be negative ( $x^3 \sin x$  is like  $x^4$  when  $x \approx 0$  and  $x \cos x - \sin x = x(\cos x - \frac{\sin x}{x})$ ), so the second derivative is zero at  $x$ . (Checked with desmos). Thus, the function has a local maximum at  $x = 0$ .

3.2 | **show approximation at  $x = 1$  is within  $\epsilon < \frac{1}{100}$  with  $1 - \frac{1}{3!}$**

$$\begin{aligned} f(1) - \left(1 - \frac{1}{3!}\right) &= \frac{1^4}{5!} - \frac{1^6}{7!} + \dots \\ &= \frac{1}{5!} - \frac{1}{7!} + \dots \\ &< \frac{1}{5!} = \frac{1}{120} < \frac{1}{100} \end{aligned}$$

3.3 | **solution to the differential equation  $xy' + y = \cos x$**

$$\begin{aligned} xy' + y &= \cos x \implies y' = \frac{\cos x - y}{x} \\ &= \cos x - \cancel{\frac{x \cos x - \sin x}{x}} \\ &= \cos x - \frac{x \cos x - \sin x}{x} \\ &= \cos x - \cancel{\frac{x \cos x}{x}} + \frac{\sin x}{x} \\ &= \cancel{\cos x} - \cancel{\cos x} + \frac{\sin x}{x} \\ y &= \frac{\sin x}{x} \end{aligned}$$