

$$1 \mid \int \frac{\sqrt{x-1}}{x} dx$$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 du - \frac{1}{u^2+1} + C \\ &= 2 \int 1 du - \tan^{-1} u + C \\ &= 2u - \tan^{-1} u + C \\ &= \boxed{2\sqrt{x-1} - \tan^{-1}(\sqrt{x-1}) + C} \end{aligned}$$

$$2 \mid \int \frac{x^2}{x^2+1} dx$$

Let $u = x^2 + 1$, $du = 2x dx$

$$\begin{aligned} \int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\ &= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\ &= \frac{1}{2} (u - \ln u) + C \\ &= \boxed{\frac{1}{2} (x^2 + 1 - \ln(x^2 + 1)) + C} \end{aligned}$$

3 | **3**4 | **4**5 | **5**6 | $\int \tan^2 x + 1 dx$

$$\begin{aligned}\int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx\end{aligned}$$

Let $u = x, du = 1$

$$\begin{aligned}&= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C}\end{aligned}$$

7 | **7**8 | $\int \frac{e^x - 1}{e^x} dx$

$$\begin{aligned}\int \frac{e^x - 1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C}\end{aligned}$$

9 | $\int \frac{\sec^2 x}{\csc x} \sin x dx$

$$\begin{aligned}\int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x}\end{aligned}$$

10 | **\$\int \sin x \cos x dx\$**Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned}
 \int \sin x \cos x dx &= \int u du \\
 &= \frac{1}{2} u^2 \\
 &= \boxed{\frac{1}{2} \sin^2 x}
 \end{aligned}$$

11 | **TODO** $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx$

$$\begin{aligned}
 \int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx \\
 &= \int \frac{\sin^2 x}{\tan^2 x + 1} + \frac{\cos^2 x}{\tan^2 x + 1} dx \\
 &= \int \sin^2 x \cos^2 x + \cos^4 x dx
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{1}{\tan^2 x + 1} dx \\
 &= \int \frac{1}{\sec^2 x} dx \\
 &= \int \cos^2 x dx
 \end{aligned}$$

12 | $\int \frac{\sec x \tan x}{1 + \sec^2 x} dx$

Let $u = 1 + \sec^2 x$,

13 | **13**

14 | **14**