Name: ______________ Group Members: ______________

Exploration 3-9a: Introduction to Exponential Function Calculus

Date: ______

Objective: Find a pattern that allows you to differentiate an exponential function.

Compound Interest Problem: When money is left in an account such as an IRA (individual retirement account), it earns interest at a certain *annual percentage rate* (abbreviated APR). Suppose that you invest \$1000 in an IRA that pays 9% per year APR. From previous mathematics courses, you recall that the amount $M(x)$ in the account after x years is given by the exponential function

$$M(x) = 1000(1.09^x)$$

1. Enter $M(x)$ as y_1 in your grapher. Use the LIST feature to make a list containing three columns:

   L_1 with integer values of x from 0 through 5

   L_2 with values of $M(x)$ corresponding to L_1

   L_3 with $\Delta\text{List}(L_2)$

   The third list contains the amount of interest earned for the first through fifth years. How do you explain the fact that these amounts are increasing?

   as you get more money, 9% of that money increases

2. Enter the numerical derivative of $M(x)$ as y_2. Enter the ratio y_2/y_1 as y_3. Use the TABLE feature of your grapher to make a table of values of y_1, y_2, and y_3. What do you notice about the ratio of the derivative to the function value? Make a conjecture about how you could calculate $M'(x)$ from $M(x)$.

   It's all 0.083

   $\frac{M'}{M} = 0.083?$

   w/ smaller Δx

3. By the definition of derivative,

$$M'(x) = \lim_{h\to 0} \frac{M(x+h) - M(x)}{h}$$

   Substitute for M, then use the properties of exponents and limits to show that $M'(x)$ equals $M(x)$ times the limit of a fraction involving h. Show numerically that this fraction approaches the factor you found in Problem 2.

   $\lim_{h\to 0} \frac{a^{(x+h)} - a^x}{h} \Rightarrow \frac{a^x a^h - a^x}{h}$

   $\Rightarrow a^x \frac{(a^h - 1)}{h}$ where $a = 1.09$ and you multiply by 1000

4. Your calculator has a built-in exponential function, e^x, called the **natural exponential function.** Let $f(x) = e^x$. Use the definition of derivative as you did in Problem 3 to write $f'(x)$ as e^x times a limit involving h. What does the limit appear to equal? What is true about the natural exponential function that makes it important enough to deserve its own key on your calculator?

   $\lim_{h\to 0} e^x \underbrace{\frac{(e^h - 1)}{h}}$ → equals to 1 by def of e

   $\therefore = e^x(1) = e^x$

5. Find the value of e on your grapher. Write the answer to as many decimal places as your calculator gives you.

   2.718281828459045

6. The **natural logarithmic function,** $y = \ln x$, also appears on your calculator. This function uses e as the base. Find $\ln 1.09$. Where have you seen this number recently? Based on your answer, write a conjecture for an algebraic formula for $M'(x)$, where ~~M is the compound interest function in Problem 1.~~

   $\boxed{(a^x)' = a^x \ln a}$

   uh plugging in M Just makes it less general

7. Based on your work on this Exploration, make a conjecture for the algebraic derivative of $f(x) = b^x$, where b is a positive constant not equal to 1. Test your conjecture by plotting

   $y_4 = 2^x$

   y_5 = your conjecture for the derivative

   y_6 = numerical derivative of y_4 (thick style) ??

   Does your conjecture agree with the numerical derivative?

   seems like it

8. What did you learn as a result of doing this Exploration that you did not know before?

   that one definition of e