Source:

1 | product of vector spaces def

Suppose $V_1, \dots, V_m$ are vector spaces over $\mathbb{F}$

- The *product* $V_1 \times \cdots \times V_m = \{(v_1, \dots, v_m) : v_1 \in V_1, \dots, v_m \in V_m\}$
- Addition on $V_1 \times \cdots \times V_m$ is defined as

$$(u_1, \dots, u_m) + (v_1, \dots, v_m) = (u_1 + v_1, \dots, u_m + v_m)$$

- Scalar multiplication on $V_1 \times \cdots \times V_m$ is defined by

$$\lambda(v_1, \dots, v_m) = (\lambda v_1, \dots, \lambda v_m)$$

1.1 | careful

1.1.1 | product of multiple vector spaces (not just two)

1. similar to how sums/direct sums are not just sums of a pair but rather sums of a list

1.2 | properties

1.2.1 | addition has to be over applicable products

$v_i \in V_i + u_i \in U_i$ must exist for each $1 \leq i \leq m$ for the sum $(V_i \times \cdots \times V_m) + (U_i \times \cdots \times U_m)$

1.3 | results

1.3.1 | Axler3.73 product of vector spaces is a vector space

If vector spaces in a product are over $\mathbb{F}$, then their product is a vector space over $\mathbb{F}$.

1. Proof proof
   (a) commutativity, associativity inherited from $\mathbb{F}$
   (b) additive identity, additive inverse, multiplicative identity inherited separataely from each space (they don't interact)