

12 | estimate area

Right handed Riemann Sum:

$$0.5 + 4 + 10 + 13 + 10 + 0 = 37.5$$

13 | estimate area again

$$4(200 + 2700 + 1100 + 4000 + 200) = 32800$$

14 | area between curves

$$\begin{aligned} \int_0^{10} 2200e^{0.024t} dx - \int_0^{10} 1360e^{0.018t} dx &= \frac{1}{0.024} 2200e^{0.024t} - \frac{1}{0.018} 1360e^{0.018t} \\ \Rightarrow \frac{1}{0.024} 2200e^{0.24} - \frac{1}{0.018} 1360e^{0.18} - \frac{1}{0.024} 2200 + \frac{1}{0.018} 1360 &\approx 9964 \end{aligned}$$

15 | meaning of area

The shaded region represents the profit made between producing 50 units and 100 units.

16 | slicing pizza into three using parallel cuts

The problem of placing slices is the same if we only worry about the top half of the pizza. Thus, we can choose some x for the first slice s.t.

$$\begin{aligned} 2 \int_{-7}^x \sqrt{7^2 - t^2} dt &= \int_x^7 \sqrt{7^2 - t^2} dt \\ 2 \int_{-7}^x \sqrt{7^2 - t^2} dt - \int_x^7 \sqrt{7^2 - t^2} dt &= 0 \\ 2 \int_{-7}^x \sqrt{7^2 - t^2} dt + \int_7^x \sqrt{7^2 - t^2} dt &= 0 \\ 2 \left(\int_0^x \sqrt{7^2 - t^2} dt - \int_0^{-7} \sqrt{7^2 - t^2} dt \right) + \left(\int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\ 2 \left(\int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt \right) + \left(\int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\ 2 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt + \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\ 3 \int_0^x \sqrt{7^2 - t^2} dt + 3 \int_{-7}^0 \sqrt{7^2 - t^2} dt + \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \end{aligned}$$