1 | **general trends**

1.1 | **split fractions**

Always consider splitting sums of fractions

$$\int \frac{a+b}{c}dx = \int \frac{a}{c}dx + \int \frac{b}{c}dx$$

1.2 | **pull out constant factors**

$$\int af(x)dx = a\int f(x)dx$$

2 | **additive**

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

3 | **change of lower bound**

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$
$$\implies \int_a^x f(t)dt = \int_b^x f(t)dt - \int_a^b f(t)dt$$

4 | **fundamental theorem of calculus**

$$\int f'(x)dx = f(x) + C$$
$$\frac{d}{dx}\int f(x)dx = f(x)$$
$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

(second one doesn't have a $+C$ because the derivative sends that to zero)

5 | **net change theorem**

$$\int_a^b f'(x)dx = f(b) - f(a)$$
$$\frac{d}{dx}\int_a^x f(x)dx = f(x)$$
$$\int_a^x f'(x)dx = f(x) - f(a)$$

6 | **variable bounds**

$$\frac{d}{dx}\int_a^{p(x)} f(x)dx = f(p(x))p'(x)$$

$$\int_{p(x)}^{q(x)} f(t)dt = \int_0^{q(x)} f(t)dt - \int_0^{p(x)} f(t)dt$$

An example

$$k(x) = \int_{x^2}^{e^x} \sqrt{\sin t}\,dt = \int_0^{e^x} \sqrt{\sin t}\,dt - \int_0^{x^2} \sqrt{\sin t}\,dt$$

$$k'(x) = \sqrt{\sin e^x}\,e^x - 2x\sqrt{\sin x^2}$$

7 | **mean value theorem (for integrals)**

There exists some point c over an integrable interval $[a, b]$ s.t.

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

8 | **integration rules**

8.1 | **power rule for integrals**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

8.2 | **k-angle formulas for sinusoids**

$$\int \sin kx\,dx = -\frac{\cos kx}{k}$$

$$\int \cos kx\,dx = \frac{\sin kx}{k}$$

8.2.1 | **for $\sin^2 x$ or $\cos^2 x$,**

Try to get a double angle using the $\cos 2x$ identities.

8.2.2 | **for a product of $\sin x \cos x$ or similar,**

Use the double angle identity for $\sin 2x = 2\sin x \cos x$.

8.2.3 | **for $\sec x$ and $\tan x$,**

We know their derivatives contain themselves, so we can look for something cyclic.
For example,

$$\int \sec x dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$$
$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

Let $u = \sec x + \tan x$, $du = \sec x \tan x + \sec^2 x$

$$\int \sec x dx = \int \frac{du}{u} dx$$
$$= \int \frac{1}{u} du$$
$$= \ln |u|$$
$$= \ln |\sec x + \tan x|$$

8.3 | u-substitution (on products, chain rule)

if it happens to work:

$$\int f'(g(x))g'(x)dx = f(g(x)) + C$$

an example:

$$\int 2x \sin\left(x^2\right) dx$$

Let $u = x^2$ and $du = fracdudxdx = 2xdx$

$$\int \sin x^2 2x dx = \int \sin u du$$
$$= -\cos(u) + C$$
$$= -\cos\left(x^2\right) + C$$

8.4 | integration by parts (on products, product rule)

$$\frac{d}{dx} f(x)g(x) \qquad = f'(x)g(x) + g'(x)f(x)$$
$$\int \frac{d}{dx} f(x)g(x)dx = \int f'(x)g(x) + g'(x)f(x)dx$$
$$f(x)g(x) + C \qquad = \int f'(x)g(x)dx + \int g'(x)f(x)dx$$
$$\implies \int f'(x)g(x)dx \qquad = f(x)g(x) - \int g'(x)f(x)dx + C$$

8.4.1 | tips

1. can use 1 as the other part of the product