Source: [KBe2020math530refExr0nRetIndex]

#ret

1. Suppose $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$. Multiply AB and BA. What do you notice???

Nothing. Matrix multiplication is not commutative.

$$\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 7x \\ x+3y \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ x+3y \end{bmatrix} = \begin{bmatrix} x \\ x+3y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$x = 2$$

$$x + 3y = 5$$

I'm not sure how to solve the rest of it with matrices, so I'll just do it normally:

$$x = 2$$

$$x + 3y = 5$$

$$2 + 3y = 5$$

$$3y = 3$$

$$y = 1$$

3. > Do 2x2 matrices form a group under > a. addition? > b. multiplication? See [KBe2020math530refGroups] I'll assume that our matrices have real numbers in them.

Operation Property	Closed	Identity	Inverse	Associative?	Final
Addition	Yes	$e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} :$	"Inherits from addition" =	Yes
Multiplication	Yes	$e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Maybe?	Yes, see below	Undecided

Associativity of 2x2 matrices under multiplication:

$$\begin{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \end{pmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix}$$

$$= \begin{bmatrix} aei + bgi + afk + bhk & aej + bgj + afl + bhl \\ cei + dgi + cfk + dhk & cej + dgj + cfl + dhl \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} ei + fk & ej + fl \\ gi + hk & gj + hl \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix}$$

I can't figure out if 2x2 matrices have multiplicative inverses... I tried to work it out using algebra but kept proving identities. Not sure what the right way to go about this is...

I spent far too long on this assignment (1.6h), so I probably won't spend as much time LaTeXing my homework in the future.