

1 | A real valued matrix

Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

$$AA^T = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{pmatrix}$$

Then, $A^T A$ is the same thing, but with b, c swapped.

2 | For complex matrices

$$\begin{pmatrix} a + bi & c + di \\ f + gi & j + ki \end{pmatrix} \begin{pmatrix} a + bi & f + gi \\ c + di & j + ki \end{pmatrix} = \begin{pmatrix} a^2 - b^2 + 2abi + c^2 - d^2 + 2cdi & af + agi + bfi - bg \\ af + agi + bfi - bg & f^2 - g^2 + 2fgi + j^2 - k^2 + 2jki \end{pmatrix}$$

I'm not sure if I'm noticing anything different from the real ones, although maybe the variables are just too confusing.

3 | Complex conjugate ($A^* A$ vs AA^*)

$$\begin{pmatrix} a + bi & c + di \\ f + gi & j + ki \end{pmatrix} \begin{pmatrix} a - bi & f - gi \\ c - di & j - ki \end{pmatrix} = \begin{pmatrix} a^2 + b^2 + c^2 + d^2 & () \\ () & f^2 + g^2 + j^2 + k^2 \end{pmatrix}$$

$$\begin{pmatrix} a - bi & f - gi \\ c - di & j - ki \end{pmatrix} \begin{pmatrix} a + bi & c + di \\ f + gi & j + ki \end{pmatrix} = \begin{pmatrix} a^2 + b^2 + f^2 + g^2 & () \\ () & c^2 + d^2 + j^2 + k^2 \end{pmatrix}$$

The diagonals are real-valued, and the matrices are symmetric about the diagonal. I wonder if this means the matrices have identical eigenvalues?