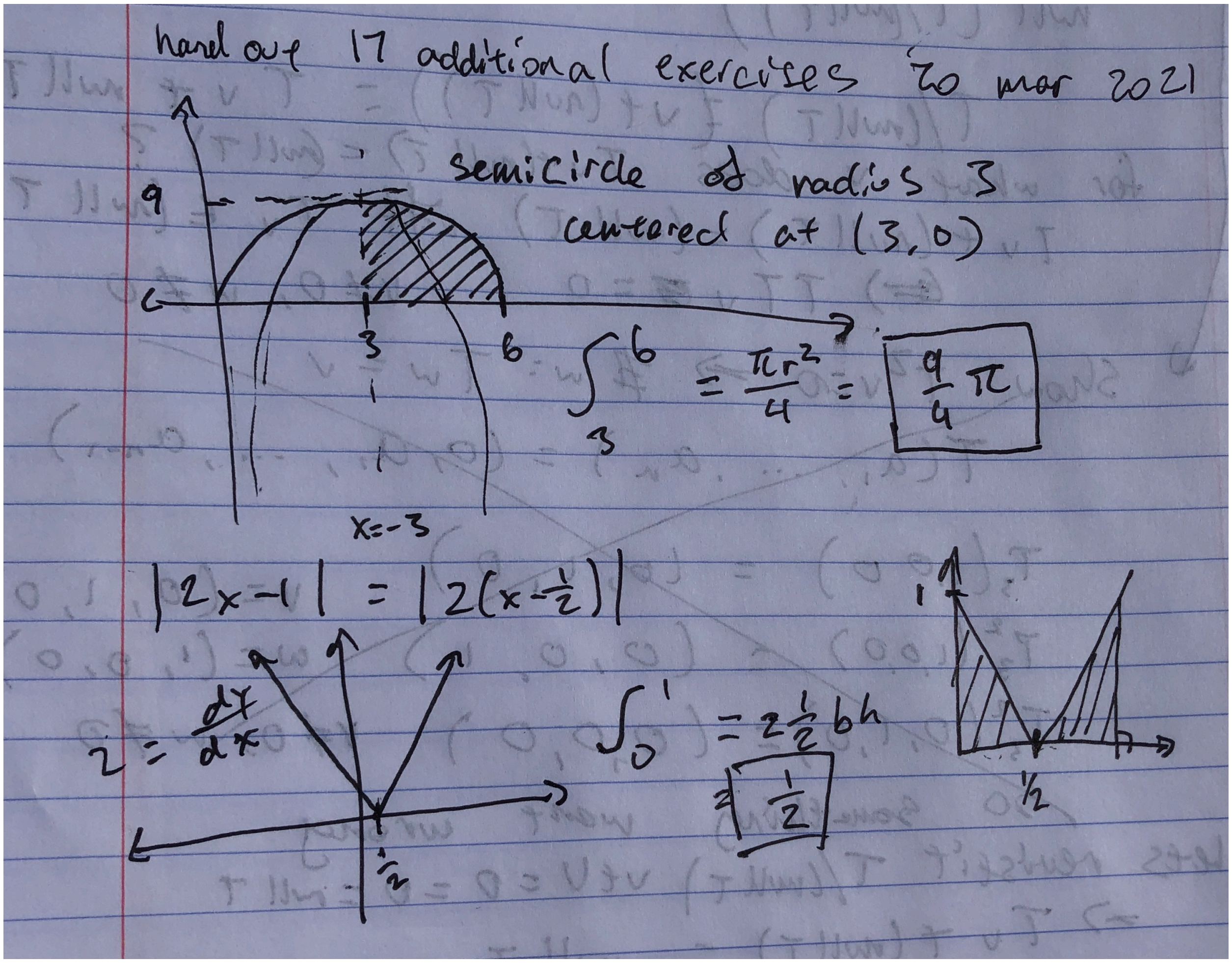
1 | Exercises

1.1 | interpreting in terms of area



1.3 | subtracting integrals

I expect

$$\int_1^2 f(x)dx = \int_1^5 f(x)dx - \int_2^5 f(x)dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

1.4 | show $\int_a^b x^2 dx = \frac{b^3-a^3}{3}$

17a.4 Show $\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$

$$\int_a^b x^2\,dx = \lim_{n\to\infty} \sum_{k=0}^{n}\left((a+k\Delta x)^2 \Delta x\right) \text{ where } \Delta x = \frac{b-a}{n}$$

$$= \lim_{n\to\infty} \Delta x \sum_{k=0}^{n}\left(a^2 + k^2\Delta x^2 + 2ak\Delta x\right)$$

$$= \lim_{n\to\infty} \Delta x\left(\sum_{k=0}^{n} a^2 + \sum_{k=0}^{n} k^2\Delta x^2 + \sum_{k=0}^{n} 2ak\Delta x\right)$$

$$= " \qquad " + \Delta x^2 \sum_{k=0}^{n} k^2 + 2a\Delta x \sum_{k=0}^{n} k$$

$$= " \qquad " + \Delta x^2 \frac{(n)(n+1)(2n+1)}{6} + 2a\Delta x \frac{n(n+1)}{2}$$

$$\overset{\lim}{=} \Delta x\left(a^2 n + \cancel{2}a(b-a)\frac{(n+1)}{\cancel{2}} + (b-a)\Delta x \frac{(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} a^2(b-a) + (b-a)\frac{\cancel{2}a(b-a)}{\cancel{2}} + \cancel{\Delta x\, a(b-a)}^{\,0}$$

$$+ \begin{cases} + \Delta x(b-a)\left((b-a)\frac{(2n+1)}{6} + \Delta x \frac{2n+1}{6}\right) \\ = (b-a)\,\Delta x(2n+1)\left(\frac{b-a}{6} + \frac{\Delta x}{6}\right) \\ = (b-a)\left(\frac{b-a}{6} + \cancel{\frac{\Delta x}{6}}^{\,0}\right)\left(2(b-a) + \cancel{\Delta x}^{\,0}\right) \end{cases}$$

$$= \cancel{\lim_{n\to\infty}}\; a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{3}$$

cont

$$a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{3}$$

$$= a^2(b-a) + a(b^2+a^2-2ab) + \frac{(b-a)(b-a)^2}{3}$$

$$= \frac{1}{3}\left(3a^2(b-a) + 3a(b-a)^2 + (b-a)(b-a)^2\right)$$

$$+\cancel{(b-a)^2\left(3a+(b-a)\right)}$$

$$= \frac{1}{3}\left(3a^2b - \cancel{3a^3} + 3ab^2 + \cancel{3a^3} - 6a^2b + b(b-a)^2 - a(b-a)^2\right.$$

$$= \frac{1}{3}\left(3a^3b + \cancel{3ab^2} - \cancel{6a^2b} + b^3 + a^2b - 2ab^2 - ab^2 - a^3 + 2a^2b\right.$$

$$-3a^2b \quad + b^3 - a^3 - \cancel{3ab^2} + \cancel{3a^2b}$$

$$= \frac{1}{3}\left(\cancel{3a^2b - 3a^2b} + b^3 - a^3\right)$$

$$= \boxed{\frac{b^3-a^3}{3}}$$

a number of written mistakes slowed me down