1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned}
\int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\
&= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\
&= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\
&= 2 \int 1du - \frac{1}{u^2+1} + C \\
&= 2 \int 1du - \tan^{-} u + C \\
&= 2u - \tan^{-} u + C \\
&= \boxed{2\sqrt{x-1} - \tan^{-}\left(\sqrt{x-1}\right) + C}
\end{aligned}$$

2 | $\int \frac{x^2}{x^2+1} dx$

Let $u = x^2 + 1$, $du = 2xdx$

$$\begin{aligned}
\int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\
&= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\
&= \frac{1}{2} \left(u - \ln u \right) + C \qquad &= \frac{1}{2} \left(u - \ln u \right) + C
\end{aligned}$$

3 | **3**

4 | **4**

5 | **5**

6 | $\int \tan^2 x + 1 dx$

$$\begin{aligned}
\int \tan^2 x + 1dx &= \int \sec^2 x - 1 + 1dx \\
&= \int \sec^2 x dx
\end{aligned}$$

Let $u = x, du = 1$

$$\begin{aligned}
&= \int \sec^2 u du \\
&= \tan u + C \\
&= \boxed{\tan x + C}
\end{aligned}$$

7 | **7**

8 | $\int \frac{e^x-1}{e^x} dx$

$$\begin{aligned}\int \frac{e^x-1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C}\end{aligned}$$

9 | $\int \frac{\sec^2 x}{\csc x} sinxdx$

$$\begin{aligned}\int \frac{\sec^2 x}{\csc x} sinxdx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x}\end{aligned}$$

10 | **$∫ sin x cos x dx $**

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned}\int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x}\end{aligned}$$

11 | **TODO** $\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx$

$$\begin{aligned}
\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx &= \int \frac{\sin^2 x+\cos^2 x}{\tan^2 x+1}dx \\
&= \int \frac{\sin^2 x}{\tan^2 x+1}+\frac{\cos^2 x}{\tan^2 x+1}dx \\
&= \int \sin^2 x\cos^2 x+\cos^4 x dx \\
&= \int \frac{1}{\tan^2 x+1}dx \\
&= \int \frac{1}{\sec^2 x}dx \\
&= \int \cos^2 x dx
\end{aligned}$$

12 | **12**

13 | **13**

14 | **14**