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#flo

## 1 | Polynomials

- See [KBrefPolynomial](#) ## 0 polynomial
- Has degree  $-\infty$
- Degrees are usually positive, except for the 0 degree
- “that’s too hard, and we’re not going to do it here” ## Identically zero
- Like 0 or  $0x^0$
- Most polynomials are sometimes zero, but polynomials that are “identically zero” means that it’s always zero (instead of just sometimes zero)

$\mathcal{P}_m(F)$

- Polynomials with coefficients in  $F$  whose highest degree is  $m$
- It can’t be “whose degree is exactly  $m$ ” because otherwise you won’t have the identity and it won’t be closed under addition (in the case where coefficient sum  $a_m + b_m = 0$ ) ### It’s a finite dimensional vector space

$$a_0z^0 + \dots + a_mz^m + b_0z^0 + \dots + b_mz^m = (a_0 + b_0)z^0 + \dots + (a_m + b_m)z^m$$

### Proof of 2.16

- Structure: proof by contradiction

## 2 | Linear Independence

- “non-trivial” means “simplest possible”, which has usually got the most zeros

### 2.21 Linear Dependence Lemma 2.21

- it’s saying that any linearly independent list has a vector inside that doesn’t “contribute anything”, and that if you remove it you’ll have the same span. Implicitly, maybe through induction?) if you remove a dependent vector enough times then you get a linearly independent list.
- The list  $(1, 1, 1), (2, 2, 2), (3, 3, 3)$  is really dependent, but  $(0), (0), (0)$  is the most dependent (you have to remove all to get independence).

## 3 | Exercise 2.A.1

### Lemma

If vectors  $v_1, v_2, v_3, v_4$  span  $V$ , then the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$