Source:

1 | Broader vector spaces

- · Doesn't have to be physics vectors
- · maybe it's like matrices
- · or linear maps themselves

2 | Axler 3.A ex7

Let w = Tv.

2.1 | If v = 0 then

$$Tv = 0$$

By Axler 3.11 (Maps take 0 to 0). Thus, λ can be anything in \mathbb{F} .

2.2 | Otherwise,

 $\frac{1}{v} \in \mathbb{F}$ because the field has multiplicative inverses for all elements except 0.

$$Tv = w = \left(w\frac{1}{v}\right)v$$

Let $\lambda=w\frac{1}{v}$, then

$$\lambda v = w \frac{1}{v} v = w$$

which is in $\mathbb F$ because $w,\frac{1}{v}\in\mathbb F$ and fields are closed under multiplication.