

Source:

## 1 | linear approximations

### 1.1 | cube root

#### 1.1.1 | approximation

$$(1+x)^{\frac{1}{3}} \rightarrow \frac{1}{3}(1+x)^{-\frac{2}{3}}$$

at  $x = 0$  is

$$\frac{1}{3}(1+0)^{-\frac{2}{3}} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

#### 1.1.2 | estimations

value	estimate
0.05	1.016666
-0.25	0.916666

These will be overestimates because the graph is concave down in this region.

### 1.2 | sin(x)

#### 1.2.1 | approximation

$$y \approx \left. \frac{d}{dx} \sin x \right|_0 (x-0) + \sin 0 = x$$

#### 1.2.2 | estimates

value	estimate
-0.1	-0.1
0.1	0.1

The first estimate will be an underestimate because  $\sin x$  is concave up in that region. The opposite is true for the second estimate.

### 1.3 | unknown function (only some points known)

#### 1.3.1 | approximation

$$y \approx \left. \frac{d}{dx} f(x) \right|_c (x - c) + f(c)$$

plugging in  $c = 1$ ,

$$y \approx 5(x - 1) - 4$$

#### 1.3.2 | estimations

value	estimate
1.2	-3

This will be an underestimate because the second derivative is positive and the graph is thus concave up.

## 2 | differentials

For a function  $y = f(x)$ ,  $dy$  and  $dx$  are differentials and the relationship is  $dy = f'(x)dx = \frac{L(a+\Delta a) - L(a)}{\Delta x} dx$ .

For a function written  $f(x) = (\text{something})$ , the differentials are  $df$  and  $dx$  and the relationship is the same:  $df = f'(x)dx$ .

### 2.1 | cube error

#### 2.1.1 | differential

$$\begin{aligned} df &= f'(x)dx \\ &= 3x^2 dx \end{aligned}$$

#### 2.1.2 | volume error

If I understand the use of differentials correctly, then  $x$  is the measured value (2) and  $dx$  is the uncertainty (delta  $x$ ), or 0.2ft. Then, the change in the volume (change in function or  $df$ ) would be  $3(2)^2(0.2) = 2.4$

#### 2.1.3 | max error for some $\epsilon$

$$\begin{aligned} df &\approx 3x^2 dx \\ dx &\approx \frac{df}{3x^2} \\ &\approx \frac{1}{3(2)^2} \\ &\approx \frac{1}{12} \text{ ft} = 1 \text{ in} \end{aligned}$$

## 2.2 | sphere measuring

$$f(r) = \frac{4}{3}\pi r^3$$

$$\frac{df}{dr} = 4\pi r^2$$

$$df = 8\pi r(dr)$$

$$= 8\pi 21(0.05) = \pm 8.4\pi \text{ cm}^3$$