1 | Export

`https://www.overleaf.com/project/606b2fa8be363f9005d8ce03`

2 | Exercise 7

> Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely dim $E(\lambda, T)$ times.

3 | Proof

We will show that for each eigenvalue λ, there are at least $E(\lambda, T)$ occurrences of that eigenvalue and at most $E(\lambda, T)$ occurrences.

Suppose first that $\dim E(\lambda, T) = m$ and $v_1, \ldots, v_m$ is a basis of $E(\lambda, T)$. In the diagonal matrix, the column corresponding to each of the m eigenvectors is comprised of the coefficients of

$$Tv_j = \lambda v_j$$

Thus, λ appears on the diagonal at least m times.

Suppose then that λ is on the diagonal m times. Each of those occurrences corresponds to where the diagonal matrix sends a (linearly independent) basis eigenvector, which implies that the basis of V has at least m eigenvectors corresponding to λ. These m eigenvectors can be extended to a basis of $E(\lambda, T)$, implying that $\dim E(\lambda, T) \geq m$.