

# Eigenvalues and Eigenvectors

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<b>1.1</b>	<b>linear algebra done right (Axler 5.A)</b>
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## 2 motivation

The simplest non-trivial invariant subspaces are one-dimensional. Let  $U$  be a one-dimensional invariant subspace under  $T$ , then

$$Tu \in U : u \in U$$

Because  $U = \text{span}(u)$ , this implies

$$Tu = \lambda u$$

which defines an eigenvalue ( $\lambda$ ) and eigenvector( $u$ ) pair.

### 3 eigenvalue

DEF

Suppose  $T \in \mathcal{L}(V)$ . A number  $\lambda \in \mathbb{F}$  is called an *eigenvalue* of  $T$  if there exists  $v \in V$  s.t.  $v \neq 0$  and  $Tv = \lambda v$ .

#### 3.1 results

##### 3.1.1 Axler5.6 equivalent conditions

When  $V$  is finite-dimensional,  $T \in \mathcal{L}(V)$  and  $\lambda \in F$ ,

1.  $T - \lambda I$  is not injective
2.  $T - \lambda I$  is not surjective
3.  $T - \lambda I$  is not invertible
4. we don't want  $T - \lambda I$  to be invertible because we want it to be zero (rearranging the prev equation) INTUIT

### 4 eigenvector

DEF

Suppose  $T \in \mathcal{L}(V)$  and  $\lambda \in \mathbb{F}$  is an eigenvalue of  $T$ . A vector  $v \in V$  is called an *eigenvector* of  $T$  corresponding to  $\lambda$  if  $v \neq 0$  and  $Tv = \lambda v$ .

#### 4.1 intuit

INTUIT

$v$  can't be zero because that would be trivial. Otherwise, this is just terminology based on the prev definition: if it gets scaled but stays in the same space, then its called an eigenvector. Note that each eigenvalue  $\lambda$  has a whole span of associated eigenvectors.

#### 4.2 results

##### 4.2.1 equivalent condition

Because  $Tv = \lambda v$  iff  $(T - \lambda I)v = 0$  (algebra), a vector  $v \in V$  with  $v \neq 0$  is an eigenvector of  $T$  corresponding to  $\lambda$  iff  $v \in \text{null}(T - \lambda I)$

### 4.2.2 axler5.10 linearly independent eigenvectors

Let  $T \in L(V)$ . Suppose  $\lambda_1, \dots, \lambda_m$  are distinct eigenvalues of  $T$  and  $v_1, \dots, v_m$  are corresponding eigenvectors. Then  $v_1, \dots, v_m$  is linearly independent.

1. intuit INTUIT If some list of eigenvalues is distinct, then the corresponding eigenvectors will be linearly independent because if any subset linear combination could add to another, then something would be funny about linearity?

### 4.2.3 axler5.11 maximum number of eigenvalues

Suppose  $V$  is finite-dimensional. Then each operator on  $V$  has at most  $\dim V$  distinct eigenvalues.

This follows directly from axler5.10, since all eigenvectors would need to fit into a linearly indep list and a linearly independent list of length more than  $\dim V$  is not possible. ■