Source: [KBe2020math530refExr0nRetIndex]

#ret

Square roots of i

20math530retSquareRootsi.pdf Didn't figure it out. How did I get $a = \pm\frac{\sqrt{2}i}{2}$??

Cross product

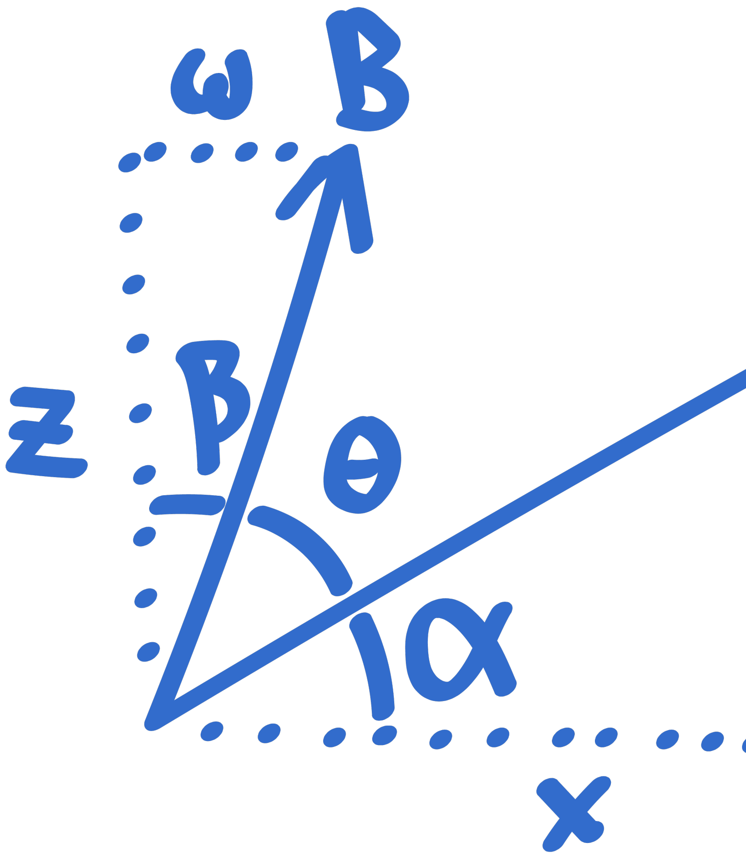
Find the cross product of $\begin{pmatrix}1\\3\\0\end{pmatrix} \times \begin{pmatrix}2\\2\\-1\end{pmatrix}$

$$\begin{aligned}
&\Rightarrow \begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ 2 & 2 & -1 \end{vmatrix} \\
&\Rightarrow i \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} + j \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} + k \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} \\
&\Rightarrow -3i + 1j - 4k \\
&\Rightarrow \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}
\end{aligned}$$

Read Chapter 1.B

Vector Space Addition/scalarmult

- Every pair of elements is in V can be added together to form another element in V (closed)
- **A scalar is anything in F, which means it might be complex!** ### Vector space definition
- communativity(!): $u + v = v + u.\forall u, v \in V$
- associativity: $(u + v) + w = u + (v + w)and(ab)v = a(bv).\forall u, v, w \in V.\forall a, b \in F$
- additive identity: $\exists 0 \in V \mid v + 0 = v.\forall v \in V$
- Additive inverse
- Multiplicative identity (denoted 1)
- distributive property (both front and back) A vector space depneds on F so V is a **vector space over** F ### Vector spaces with other sets? F^S
- F^S is the set of functions from S to F
  - meaning that it's all functions whose domains are subsets of S and ranges are subsets of F?
- addition $f, g \in F^S, x \in F : (f + g)(x) = f(x) + g(x)$
- multiplication: $\lambda \in F$ and $f \in F^S : \lambda F \in F^S = (\lambda f)(x) = \lambda f(x)$
- functions can be elements in fields or something?
- lists are just functions on a set of numbers..?
- subtraction (additive inverses and identity are unique)
- **When you see xy, one of them has to be a vector because there is no scalar scalar multiplication defined** ## Show that $a \bullet b = |a||b|cos\theta$ Suppose $a = \begin{bmatrix}x\\y\end{bmatrix}$ and $b = \begin{bmatrix}w\\z\end{bmatrix}$. We have $a \bullet b = a^T \cdot b =$



$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix} = xw + yz$. We need an expresion for θ:
As seen in the diagram, $\theta = \frac{\pi}{2} - \alpha - \beta$. Finally:

$$\begin{aligned}
|A||B|cos\theta &= |A||B|cos\left(\frac{\pi}{2} - \alpha - \beta\right) \\
&= |A||B|sin(\alpha + \beta) \\
&= |A||B|\left(sin\alpha\ cos\beta + cos\alpha\ sin\beta\right) \\
&= |A||B|\left(\left(\frac{y}{|A|}\right)\left(\frac{z}{|B|}\right) + \left(\frac{x}{|A|}\right)\left(\frac{w}{|B|}\right)\right) \\
&= |A||B|\left(\frac{yz}{|A||B|} + \frac{xw}{|A||B|}\right) \\
&= yz + wz \\
&= xw + yz
\end{aligned}$$

Epilogue

That was two hours… I'll save the proving integers mod 3 are a field for later. #todo-exr0n