

Source:

1 | Algebraic and Geometric Multiplicities

I missed the last ten minutes of class and had to look up what the algebraic and geometric multiplicities are. I used this source.

Also it says something about

It is a fact that summing up the algebraic multiplicities of all the eigenvalues of an $n \times n$ matrix A gives exactly n .

Which reminds me of the fundamental theorem of algebra...

$$1.1 \mid \begin{pmatrix} 4 & -12 \\ 2 & 0 \end{pmatrix}$$

1.1.1 | Geometric multiplicity

The null space is span $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ which is dimension 1.

1.1.2 | Algebraic multiplicity

The determinant of $\begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix}$ is

$$-2(4 - \lambda) - 2(-\lambda) = 4(\lambda - 1)$$

So the algebraic multiplicity is 1?

$$1.2 \mid \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

1.2.1 | Geometric

Null space of 1 $((x, 0, 0))$ has dim 1. Null space of 3 $((x, \frac{-2x}{3}, \frac{4x}{3}))$ has dim 1 as well.

1.2.2 | Algebraic

The determinant simplifies to one factored term:

$$(1 - \lambda)^2(3 - \lambda)$$

So 1 has a multiplicity 2 and 3 has multiplicity 1?

$$1.3 \mid \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

1.3.1 | **Geometric**

For $\lambda = 1$, null space is $(x, y, 0)$ so dim 2. For $\lambda = 3$, null space is $(x, \frac{-x}{2}, x)$ so dim 1.

1.3.2 | **Algebraic**