

$$1 \mid \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$\begin{aligned} \int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^1 \\ &= e - e^{\frac{1}{2}} \end{aligned}$$

$$2 \mid \int_0^1 r e^{\frac{r}{2}} dr$$

$$\begin{aligned} \int_0^1 r e^{\frac{r}{2}} dx &\Rightarrow r 2 e^{\frac{r}{2}} - \int 2 e^{\frac{r}{2}} dr \\ &= 2 r e^{\frac{r}{2}} - \int 2 e^{\frac{r}{2}} dr \\ &= 2 r e^{\frac{r}{2}} - 4 e^{\frac{r}{2}} \\ &= 2 r e^{\frac{r}{2}} - 4 e^{\frac{r}{2}} \\ &\Rightarrow 2 e^{\frac{1}{2}} - 4 e^{\frac{1}{2}} - (-4) \\ &= 4 - 2 e^{\frac{1}{2}} \end{aligned}$$

$$3 \mid \textbf{TODO} \int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

$$\int \frac{\ln y}{\sqrt{y}} dx = 2 \ln y \sqrt{y} - \int 2 \frac{1}{y} \sqrt{y} dx 2 \sqrt{y} (\ln y - 2)$$

$$4 \mid \textbf{TODO} \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$$

$$\begin{aligned} \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= x \cos \sqrt{x} + \int x \frac{1}{2\sqrt{x}} \sin \sqrt{x} dx \\ &= x \cos \sqrt{x} + \int \frac{\sqrt{x}}{2} \sin \sqrt{x} dx \\ &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \int \sin \sqrt{x} dx \\ &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \left(x \sin \sqrt{x} - \int \frac{\sqrt{x}}{2} \cos \sqrt{x} dx \right) \\ &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \left(x \sin \sqrt{x} - \frac{\sqrt{x}}{2} \int \cos \sqrt{x} dx \right) \\ &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} x \sin \sqrt{x} - \frac{x}{4} \int \cos \sqrt{x} dx \\ \frac{x+4}{4} \int \cos \sqrt{x} dx &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} x \sin \sqrt{x} \\ \int \cos \sqrt{x} dx &= \frac{4x \cos \sqrt{x} + 2x\sqrt{x} \sin \sqrt{x}}{x+4} \end{aligned}$$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$, $dx = 2udu$

$$\begin{aligned}
 \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= 2 \int u \cos u du \\
 &= 2u \sin u - 2 \int \sin u du \\
 &= 2u \sin u + 2 \cos u \\
 &= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} \\
 \implies &-2
 \end{aligned}$$

5 | $\int_1^e \sin \ln x dx$

$$\begin{aligned}
 \int_1^e \sin \ln x dx &= x \sin \ln x - \int x \frac{1}{x} \cos \ln x dx \\
 &= x \sin \ln x - \int \cos \ln x dx \\
 &= x \sin \ln x - \left(x \cos \ln x + \int x \cancel{\frac{1}{x}} \sin \ln x dx \right) \\
 &= x \sin \ln x - x \cos \ln x - \int \sin \ln x dx \\
 2 \int \sin \ln x dx &= x \sin \ln x - x \cos \ln x \\
 \int \sin \ln x dx &= \frac{1}{2} x (\sin \ln x - \cos \ln x) \\
 \implies \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\
 &= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2}
 \end{aligned}$$