$$1 \mid \int \frac{\sqrt{x-1}}{x} dx$$

Let
$$u = \sqrt{x-1}$$
, $du = \frac{1}{2\sqrt{x-1}}dx$

$$\int \frac{\sqrt{x-1}}{x} dx = \int \frac{u}{u^2+1} 2u du$$

$$= 2 \int \frac{(u^2+1)-1}{u^2+1} du$$

$$= 2 \int \frac{u^2+1}{u^2+1} + \frac{-1}{u^2+1} du$$

$$= 2 \int 1 du - \frac{1}{u^2+1} + C$$

$$= 2 \int 1 du - \tan^- u + C$$

$$= 2u - \tan^- u + C$$

$$= \left[2\sqrt{x-1} - \tan^- \left(\sqrt{x-1}\right) + C \right]$$

- 2 | 2
- 3 | 3
- 4 | 4
- 5 **| 5**

$$6 \mid \int \tan^2 x + 1 dx$$

$$\int \tan^2 x + 1 dx = \int \sec^2 x - 1 + 1 dx$$

$$= \int \sec^2 x dx$$
Let $u = x, du = 1$

$$= \int \sec^2 u du$$

$$= \tan u + C$$

$$= \boxed{\tan x + C}$$

7 | 7

$$8 \mid \int \frac{e^x - 1}{e^x} dx$$

$$\int \frac{e^x - 1}{e^x} dx = \int 1 - \frac{1}{e^x} dx$$
$$= \int 1 - e^{-x} dx$$
$$= x + e^{-x} + C$$
$$= \boxed{e^{-x} + x + C}$$

 $9 \mid \int \frac{\sec^2 x}{\csc x} sinx dx$

$$\int \frac{\sec^2 x}{\csc x} \sin x dx = \int \tan^2 x dx$$
$$= \int \sec^2 x - 1 dx$$
$$= \int \sec^2 x dx - \int 1 dx$$
$$= [\tan x - x]$$

10 | **\$∫ sin x cos x dx \$**

Let $u = \sin x$, then $du = \cos x dx$

$$\int \sin x \cos x dx = \int u du$$
$$= \frac{1}{2}u^2$$
$$= \left[\frac{1}{2}\sin^2 x\right]$$

11 | TODO $\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx$

$$\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx = \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx$$
$$= \int \frac{1}{\tan^2 x + 1} dx$$
$$= \int \frac{1}{\sec^2 x} dx$$
$$= \int \cos^2 x dx$$

- 12 | **12**
- 13 | **13**
- 14 | **14**

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