

5. Consider the function $f(x) = x^2 - 2x + 1$.
- a. Graph the function on Desmos. What do you the limit of the function is at $x = 3$?

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- b. Suppose $\varepsilon = 1$. Find $\delta > 0$ such that it satisfies the limit definition (that is, find the $\delta > 0$ such that if $|x - 3| < \delta$ then $|(x^2 - 2x + 1) - L| < \varepsilon$).

What the heck

$$|x-3| \leq 1$$

$$2 \leq x \leq 4$$

$$\delta = \min\{|x-3|, 1\}$$

$$|x^2 - 2x + 1 - 4| < \varepsilon$$

$$|x^2 - 2x - 3| < \varepsilon$$

$$|(x-3)(x+1)| < \varepsilon$$

$$|x-3||x+1| < \varepsilon$$

$$|x-3| \leq \frac{\varepsilon}{|x+1|}$$

$$\therefore |x-3| \leq \frac{\varepsilon}{5}$$

$$\delta = \min\left\{1, \frac{\varepsilon}{5}\right\}$$

take the upper bound

why not lower?

$$|x-3| \leq 1$$

$$-1 \leq x-3 \leq 1$$

$$2 \leq x \leq 4$$

$$3 \leq x+1 \leq 5$$

$$3 \leq |x+1| \leq 5$$

$\frac{1}{5}$

- c. Suppose $\varepsilon = 0.5$. Find $\delta > 0$ such that it satisfies the limit definition (up to 4 decimal places).

$$\delta = \min\left\{1, \frac{0.5}{5}\right\} = \frac{0.5}{5} = \frac{1}{10}$$

- d. Suppose $\varepsilon = 0.25$. Find $\delta > 0$ such that it satisfies the limit definition (up to 4 decimal places).

$\frac{1}{20}$