

1 |  $\int \frac{\sqrt{x-1}}{x} dx$

Let  $u = \sqrt{x-1}$ ,  $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned}
 \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\
 &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\
 &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\
 &= 2 \int 1 du - \frac{1}{u^2+1} + C \\
 &= 2 \int 1 du - \tan^{-1} u + C \\
 &= 2u - \tan^{-1} u + C \\
 &= \boxed{2\sqrt{x-1} - \tan^{-1}(\sqrt{x-1}) + C}
 \end{aligned}$$

2 | **2**

3 | **3**

4 | **4**

5 | **5**

6 |  $\int \tan^2 x + 1 dx$

$$\begin{aligned}
 \int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\
 &= \int \sec^2 x dx
 \end{aligned}$$

Let  $u = x$ ,  $du = 1$

$$\begin{aligned}
 &= \int \sec^2 u du \\
 &= \tan u + C \\
 &= \boxed{\tan x + C}
 \end{aligned}$$

7 | **7**8 | **8**9 |  $\int \frac{\sec^2 x}{\csc x} \sin x dx$ 

$$\begin{aligned}
 \int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\
 &= \int \sec^2 x - 1 dx \\
 &= \int \sec^2 x dx - \int 1 dx \\
 &= \boxed{\tan x - x}
 \end{aligned}$$

10 | **10**11 | **TODO**  $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx$ 

$$\begin{aligned}
 \int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx \\
 &= \int \frac{1}{\tan^2 x + 1} dx \\
 &= \int \frac{1}{\sec^2 x} dx \\
 &= \int \cos^2 x dx
 \end{aligned}$$

12 | **12**13 | **13**14 | **14**