1 | Exercise 3

Axler 6.B ex 3

orthonormality $(1,0,0), (1,1,1), (1,1,2)$

$e_1 = (1,0,0)$, $e_2 = $ normalize$((1,1,1)-(1,0,0))$

$\qquad\qquad\qquad = $ normalize$((0,1,1))$

$\qquad\qquad\qquad = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$e_3 = $ normalize$\left((1,1,2)-(1,0,0) - \frac{3}{\sqrt{2}}\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right)$

$\qquad = $ normalize$\left((0,1,2) - \left(0, \frac{3}{2}, \frac{3}{2}\right)\right)$

$\qquad = $ normalize$\left(\left(0, -\frac{1}{2}, \frac{1}{2}\right)\right)$

$\qquad = \left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$(1,0,0), \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$0 + \frac{-1}{2} + \frac{1}{2} = 0$ ✓   so all pairwise inner
                                                Products are zero

$v = (1, 2, 3)$

$\quad = \left\langle (1,2,3),(1,0,0)\right\rangle e_1 + \left\langle (1,2,3),\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right\rangle e_2 + \left\langle (1,2,3),\left(0,\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right\rangle \times e_3$

$\boxed{\quad = 1e_1 + \frac{5}{\sqrt{2}} e_2 + \frac{1}{\sqrt{2}} e_3 \quad}$

$\quad = (1,0,0) + \left(0, \frac{5}{2}, \frac{5}{2}\right) + \left(0, \frac{-1}{2}, \frac{1}{2}\right)$

$\quad = \left(1, \frac{4}{2}, \frac{6}{2}\right)$ ✓