**Source**:

# 1 | eigenvalues

eigenvalue: multiplied by a scalar? a subspace that, when put through a linear map, only gets scaled.

$$Tv = \lambda v$$

Where $v \neq 0$. (we ignore it because its no fun to send zero to zero, and bc the span is empty).
**T must be an operator!** Otherwise the matrix sizes don't work out when subtracting $\lambda I$.
where $v$ is the eigenvector and $\lambda$ is the eigenvalue. The equation is often rewritten as:

$$Tv - \lambda v = 0Tv - \lambda Iv \quad = 0(T - \lambda I)v = 0$$

now this can be factored and roots can be found. also it's an operator.

## 1.1 | Axler 5.6 equivalent conditions

Only when $V$ is finite dimensional!

### 1.1.1 | $T - \lambda I$ is not injective, because both $v, 0$ are in the null space.

### 1.1.2 | $T - \lambda I$ is also not surjective or invertible bc finite dim operator.

# 2 | an example

Given the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$, find the eigenvalues and eigenvectors.

Now that we have that other fomulation, we can just subtract $\lambda I$ from $T$ to get

$$\begin{pmatrix} 3 - \lambda & 1 \\ 0 & 2 - lambda \end{pmatrix}$$

Then, we just need to find whether it is non-invertible aka singular aka determinant.

$$(3 - \lambda)(2 - \lambda) = 0$$

The solutions are $\lambda = 2$ or $3$. These are the eigenvalues.

Now just plug in $\lambda$ and find the null space using RREF. The null space for $\lambda = 3$ has null space span$(x, 0)$, so we just pick one of those vectors (ex. $(1, 0)$) to be the eigenvector.

### 2.1 | review of terms

#### 2.1.1 | $span(1, 0)$ is an invariant subspace. (also whatever you get for $\lambda = 2$

#### 2.1.2 | any vector in an invariant subspace is an eigenvector

#### 2.1.3 | the eigenvalues are $2, 3$

### 2.2 | general idea

the point of eigenvectors is to figure out where other vectors go by looking at pieces that only get streched or shrunk.

## 3 | depends on

### 3.1 | finding roots is helpful