

**11 | cubic and a line****11.1 | show tangency**

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(4x^2 - x^3) = 8x - 3x^2 \Big|_3 \\ &= 24 - 27 = -3 \\ \frac{dy}{dx} &= \frac{d}{dx}(18 - 3x) = -3\end{aligned}$$

**11.2 | area between curves**

$$\begin{aligned}\int_3^6 18 - 3x - 4x^2 + x^3 dx &\rightarrow \frac{1}{4}x^4 - \frac{1}{3}4x^3 - \frac{1}{2}3x^2 + 18x + C \\ &= \frac{1}{4}(6)^4 - \frac{1}{3}4(6)^3 - \frac{1}{2}3(6)^2 + 18(6) - \frac{1}{4}(3)^4 + \frac{1}{3}4(3)^3 + \frac{1}{2}3(3)^2 - 18(3) \\ &= \boxed{\frac{261}{4}}\end{aligned}$$

**12 | estimate area**

Right handed Riemann Sum:

$$0.5 + 4 + 10 + 13 + 10 + 0 = 37.5$$

**13 | estimate area again**

$$4(200 + 2700 + 1100 + 4000 + 200) = 32800$$

**14 | area between curves**

$$\begin{aligned}\int_0^{10} 2200e^{0.024t} dx - \int_0^{10} 1360e^{0.018t} dx &= \frac{1}{0.024} 2200e^{0.024t} - \frac{1}{0.018} 1360e^{0.018t} \\ \Rightarrow \frac{1}{0.024} 2200e^{0.24} - \frac{1}{0.018} 1360e^{0.18} - \frac{1}{0.024} 2200 + \frac{1}{0.018} 1360 &\approx 9964\end{aligned}$$

**15 | meaning of area**

The shaded region represents the profit made between producing 50 units and 100 units.

**16 | TODO slicing pizza into three using parallel cuts**

The problem of placing slices is the same if we only worry about the top half of the pizza. Thus, we can choose some  $x$  for the first slice s.t.

$$\begin{aligned}
2 \int_{-7}^x \sqrt{7^2 - t^2} dt &= \int_x^7 \sqrt{7^2 - t^2} dt \\
2 \int_{-7}^x \sqrt{7^2 - t^2} dt - \int_x^7 \sqrt{7^2 - t^2} dt &= 0 \\
2 \int_{-7}^x \sqrt{7^2 - t^2} dt + \int_7^x \sqrt{7^2 - t^2} dt &= 0 \\
2 \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^{-7} \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\
2 \left( \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\
2 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt + \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + \frac{49\pi}{4} &= 0
\end{aligned}$$

Now, we need to use trigonometric substitution, apparently.

$$x = a \sin \theta, dx = a \cos \theta d\theta$$

Or, you could use David's method, which is just better (cut horizontally instead of vertically)

$$\begin{aligned}
\int_{-7}^7 \sqrt{49 - x^2} - a dx &= \frac{49\pi}{3} \\
\int_{-7}^7 \sqrt{49 - x^2} dx - \int_{-7}^7 a dx &= \frac{49\pi}{3} \\
\frac{49\pi}{2} - \int_{-7}^7 a dx &= \frac{49\pi}{3} \\
\frac{49\pi}{2} - (7a - -7a) &= \frac{49\pi}{3} \\
\frac{49\pi}{6} &= 14a \\
a &= \frac{49\pi}{84} = \frac{7\pi}{12}
\end{aligned}$$

Since  $a$  is the upper half of the center portion, the width of each slice is  $2a = \frac{7\pi}{6}$

## 17 | tractrix

### 17.1 | derivative

At any moment, if the boat is at  $(x, y)$  and the puller is at  $(0, h)$ , then the velocity of the boat is in the direction

$$\frac{\Delta y}{\Delta x}$$

Where  $\Delta x = -x$  and  $\Delta y$  can be found using the Pythagorean Theorem

$$\begin{aligned} L^2 &= \Delta y^2 + x^2 \\ \Rightarrow \Delta y &= \sqrt{L^2 - x^2} \end{aligned}$$

Thus, the boat is moving in the direction

$$\frac{\sqrt{L^2 - x^2}}{-x}$$

## 17.2 | integral

$$\begin{aligned} \int \frac{\sqrt{L^2 - x^2}}{-x} dx &= - \int \frac{1}{x} \sqrt{L^2 - x^2} dx \\ \text{Let } x &= L \sin \theta, dx = L \cos \theta d\theta \\ &= - \int \frac{1}{\sin \theta} \sqrt{L^2 - L^2 \sin^2 \theta} d\theta \end{aligned}$$

## 18 | TODO water displacement

Plan: find a function  $f(r)$  which represents the amount of water displaced for any radius, then take the derivative and find roots.