1 | **Definitions**

1.1 | **DONE group**

A set and binary operation that satisfies Group Properties

- Closed
- Identity
- Inverse
- Associative

1.2 | **DONE field**

A set and two binary operations: the primary (addition) and secondary (multiplication) that "mostly" satisfies group properties for both operations, and are **commutative and distributive**. It must be a group under the primary operation and a group under the secondary operation except without a secondary inverse for the primary indentity.

1.3 | **DONE non-singular matrices**

singular matrix: has no inverse. non-singular matrix: has an inverse aka determinant non zero

2 | **Connections**

2.1 | **DONE connect direct sum and linear independence**

the sum of two spaces is direct if their basises are linearly independent

2.2 | **DONE matrices to represent complex numbers**

The negative one matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and we want the square root of that. It turns out that $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ satisfies this, and in fact, any complex number $a + bi$ can be represented as $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. These matrices are commutative under multiplication (like complex numbers should be), have their complex conjugates equal to their transposes, and a bunch of other nice properties. Also related to rotation matrices. #source `https://www.nagwa.com/en/explainers/152196980513/`

3 | **Computation**

3.1 | **DONE Find the determinant of matrices**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

3.2 | **DONE compute cross product**

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix} = i \begin{vmatrix} b & c \\ e & f \end{vmatrix} + j \begin{vmatrix} c & a \\ f & d \end{vmatrix} + k \begin{vmatrix} a & b \\ d & e \end{vmatrix} = bf - ce, cd - fa, ae - bd$$

3.3 | **DONE Find equations of lines and planes using cross product and dot product**

Use the cross product to find an orthogonal vector $\vec{p}$. The plane is all vectors that are orthogonal to $\vec{p}$, which is to say that the dot product is zero ($\{\vec{u} : \vec{u} \cdot \vec{p} = 0\}$).

4 | **Derivations**

4.1 | **DONE properties of the determinant**

4.1.1 | **zero when matrix has no inverse (singular)**

4.1.2 | **det = -1 for rotation matrices?**

4.2 | **DONE inverse of a 2x2 matrix**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$ae + bg = 1$$
$$ce + dg = 0$$
$$af + bh = 0$$
$$cf + dh = 1$$

Then you do some algebra to get

$$e = \frac{d}{ad - bc}$$
$$g = \frac{-c}{ad - bc}$$
$$f = \frac{-b}{ad - bc}$$
$$h = \frac{a}{ad - bc}$$

4.3 | **DONE rotation matrices**

Don't try to algebra it. Use polar coordinates and the angle sum trig identities:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

anyways, you get $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$.

5 | **review quizzes**

5.1 | **DONE first quiz**

5.1.1 | **see "find equations of lines and planes using cross product and dot product"**

5.1.2 | **rotation matrices**

5.1.3 | **cross product**

5.2 | **DONE mini take home quiz**

no feedback

5.3 | **DONE linear independence quiz**

teacher gave no problems

5.4 | **DONE quick linear quiz (linear independence and bases)**

no feedback, I think that quiz was pretty solid..