> Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V. Prove that U is invariant under T iff $U^\perp$ is invariant under T^*.

For all $u \in U$, $Tu = u' \in U$. Let $w = U^\perp$. Then, $T^*w = $

$$\langle u, T^*w \rangle = \langle Tu, w \rangle = \langle u', w \rangle$$

Let $u \in U$ and $w \in U^\perp$. Then,

$$\langle Tu, w \rangle = 0$$
$$\langle u, T^*w \rangle = 0$$

This implies both directions, since $U = U^{\perp^\perp}$ and $T = (T^*)^*$.