Source:

1 | well defined def

A function is well defined if each element of the domain has exactly one image. Formally,

> A function $f : X \to Y$ is a relation f from X to Y satisfying:
>
> 1. $\forall x \in X$, $\exists y \in Y$ s.t. $(x, y) \in f$ (every element of the domain has an image)
> 2. $\forall x \in X$, $\forall y_1, y_2 \in Y$, $(x, y_1), (x, y_2) \in f$ implies $y_1 = y_2$ (each element of the domain has at most one image)

2 | counterexample

2.1 | $f(\frac{a}{b}) = a + b$

This is actually not a well defined function, because $f(\frac{1}{2}) = 3$ while $f(\frac{2}{4}) = 6$, yet $\frac{1}{2} = \frac{2}{4}$. This is actuall a really beautiful counter example.

3 | sources source

3.1 | **Math Stack Exchange Answer quoting definition**

3.2 | **Math Stack Exchange Answer with counterexample**