1 | Linear independence of rows and columns

1.1 | there will always be the same number of linearly independent rows and columns in the matrix

2 | Functions vs Maps

Technically, maps don't need to be well defined, meaning in a map one input might be allowed to go to multiple outputs. However, the linear maps that we are looking at will always be well defined. Maps give you a notion of "space" where as a function implies that the output depends on the input. Both are true, but each one gives a intuition for part of the picture.

3 | Proof by Contradiction careful

Make sure you actually have a contradiction! You might end up with an equation thats incorrect, but if it might be correct then it is not a valid proof by contradiction.