Integration By Parts April 22, 2021

 $3 \mid \int \ln x dx$

$$\int \ln x dx = \int 1 \ln x dx$$

$$= x \ln x - \int x \frac{1}{x} dx$$

$$= x \ln x - \int 1 dx$$

$$= x \ln x - x$$

 $4 \mid \int \tan^- x dx$

$$\int \tan^{-} x dx = x \tan^{-} x - \int x \frac{1}{x^{2} + 1} dx$$

$$= x \tan^{-} x - \frac{1}{2} \int \frac{du}{u}$$

$$= x \tan^{-} x - \frac{1}{2} \ln u + C$$

$$= x \tan^{-} x - \frac{1}{2} \ln(x^{2} + 1) + C$$

 $5 \mid \int x \sec^2 x dx$

$$\int x \sec^2 x dx = x \tan x - \int \tan x dx$$
$$= x \tan x + \ln|\cos x| + C$$

 $6 \mid \int x^2 e^{5x} dx$

$$\begin{split} \int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\ &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{25} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\ &= \frac{1}{5} e^{5x} (x^2 - \frac{2}{5}x + \frac{2}{25}) + C \end{split}$$

 $7 \mid \int x^2 \cos x dx = f(x) - \int 2x \sin x dx$

Find f(x)

$$f(x) = x^2 \sin x$$

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 $8 \mid \int x \cos x dx$

$$\int x \cos x dx = x \sin x - \int \sin x dx$$
$$= x \sin x + \cos x + C$$

 $9 \mid \int x^2 \sin x dx$

$$\int x^2 \sin x dx = -x \cos x - \int -\cos x dx$$
$$= -x \cos x - \sin x dx + C$$

10 |
$$\int x^3 e^{x^2} dx$$

Let $u=x^2$

$$\int x^3 e^{x^2} dx = \int x^2 x e^{x^2} dx$$

$$= \int u \frac{1}{2} du e^u$$

$$= \frac{1}{2} \int u e^u du$$

$$= \frac{1}{2} u e^u - \frac{1}{2} \int e^u du$$

$$= \frac{1}{2} u e^u - \frac{1}{2} e^u + C$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}$$

11 | $\int x^2 \ln x dx$

$$\int x^2 \ln x dx = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \frac{1}{x} dx$$
$$= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx$$
$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C$$

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12 | $\int \cos \ln x dx$

$$\int 1 \cos \ln x dx = x \cos \ln x + \int \sin \ln x dx$$

$$= x \cos \ln x + x \sin \ln x - \int \cos \ln x dx$$

$$2 \int \cos \ln x dx = x \cos \ln x - x \sin \ln x$$

$$\int \cos \ln x dx = \frac{1}{2} (x \cos \ln x - x \sin \ln x) + C$$

Or you could use $u = \ln x$, apparently.

13 | multiple parts

13.1 | **e**

13.2 $\int e^{2x} \cos 3x dx$

$$\int e^{2x} \cos 3x dx = \cos 3x \frac{1}{2} e^{2x} + \int 3 \sin 3x \frac{1}{2} e^{2x} dx$$

$$= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \int 3 \cos 3x \frac{1}{4} e^{2x} dx$$

$$= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \frac{9}{4} \int e^{2x} \cos 3x dx$$

$$= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \frac{9}{4} \int e^{2x} \cos 3x dx$$

$$\frac{13}{4} \int e^{2x} \cos 3x dx = \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} + C$$

$$\int e^{2x} \cos 3x dx = \frac{4}{13} \left(\cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} \right) + C$$

$$= \frac{2}{13} e^{2x} \left(\cos 3x + \frac{3}{2} \sin 3x \right) + C$$

13.3 | evaluate previous from $[0, \frac{pi}{6}]$

$$\frac{3}{13}e^{\frac{pi}{3}} - \frac{2}{13} =$$

14 |
$$\int \sec^3 x dx$$

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