1 | $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

$$\begin{aligned} \int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\ &= e - e^{\frac{1}{2}} \end{aligned}$$

2 | $\int_0^1 re^{\frac{r}{2}} dr$

$$\begin{aligned} \int_0^1 re^{\frac{r}{2}} dx &\implies r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4) \\ &= 4 - 2e^{\frac{1}{2}} \end{aligned}$$

3 | **TODO** $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$

4 | **TODO** $\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$

$$\begin{aligned} \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= x \cos \sqrt{x} - \int x \frac{1}{2\sqrt{x}} \sin \sqrt{x} dx \\ &= x \cos \sqrt{x} - \int \sqrt{x} \frac{1}{2} \sin \sqrt{x} dx \end{aligned}$$

5 | $\int_1^e \sin\ln x dx$

$$
\begin{aligned}
\int_1^e \sin\ln x dx &= x\sin\ln x - \int x\frac{1}{x}\cos\ln x dx \\
&= x\sin\ln x - \int \cos\ln x dx \\
&= x\sin\ln x - \left(x\cos\ln x + \int \cancel{x}\frac{\cancel{1}}{\cancel{x}}\sin\ln x dx\right) \\
&= x\sin\ln x - x\cos\ln x - \int \sin\ln x dx \\
2\int \sin\ln x dx &= x\sin\ln x - x\cos\ln x \\
\int \sin\ln x dx &= \frac{1}{2}x(\sin\ln x - \cos\ln x) \\
\implies \quad & \frac{1}{2}e(\sin 1 - \cos 1) - \frac{1}{2}(\sin 0 - \cos 0) \\
&= \frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}
\end{aligned}
$$