1 | $\int \frac{\sqrt{x-1}}{x}dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}}dx$

$$
\begin{aligned}
\int \frac{\sqrt{x-1}}{x}dx &= \int \frac{u}{u^2+1}2u\,du \\
&= 2\int \frac{(u^2+1)-1}{u^2+1}du \\
&= 2\int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1}du \\
&= 2\int 1\,du - \frac{1}{u^2+1} + C \\
&= 2\int 1\,du - \tan^- u + C \\
&= 2u - \tan^- u + C \\
&= \boxed{2\sqrt{x-1} - \tan^-\left(\sqrt{x-1}\right) + C}
\end{aligned}
$$

2 | $\int \frac{x^2}{x^2+1}dx$

Let $u = x^2 + 1$, $du = 2x\,dx$

$$
\begin{aligned}
\int \frac{x^3}{x^2+1}dx &= \frac{1}{2}\int \frac{u-1}{u}du \\
&= \frac{1}{2}\left(u - \int \frac{1}{u}du\right) + C \\
&= \frac{1}{2}\left(u - \ln u\right) + C \\
&= \boxed{\frac{1}{2}\left(x^2+1 - \ln(x^2+1)\right) + C}
\end{aligned}
$$

3 | **3**

4 | **4**

5 | **5**

6 | $\int \tan^2 x + 1 dx$

$$\int \tan^2 x + 1 dx = \int \sec^2 x - 1 + 1 dx$$
$$= \int \sec^2 x dx$$

Let $u = x, du = 1$

$$= \int \sec^2 u du$$
$$= \tan u + C$$
$$= \boxed{\tan x + C}$$

7 | **7**

8 | $\int \frac{e^x - 1}{e^x} dx$

$$\int \frac{e^x - 1}{e^x} dx = \int 1 - \frac{1}{e^x} dx$$
$$= \int 1 - e^{-x} dx$$
$$= x + e^{-x} + C$$
$$= \boxed{e^{-x} + x + C}$$

9 | $\int \frac{\sec^2 x}{\csc x} sinx dx$

$$\int \frac{\sec^2 x}{\csc x} sinx dx = \int \tan^2 x dx$$
$$= \int \sec^2 x - 1 dx$$
$$= \int \sec^2 x dx - \int 1 dx$$
$$= \boxed{\tan x - x}$$

10 | $\int \sin x \cos x \, dx$

Let $u = \sin x$, then $du = \cos x dx$

$$\int \sin x \cos x \, dx = \int u \, du$$
$$= \frac{1}{2} u^2$$
$$= \boxed{\frac{1}{2} \sin^2 x}$$

11 | **TODO** $\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx$

$$\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx = \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx$$

$$= \int \frac{\sin^2 x}{\tan^2 x + 1} + \frac{\cos^2 x}{\tan^2 x + 1} dx$$
$$= \int \sin^2 x \cos^2 x + \cos^4 x \, dx$$

$$= \int \frac{1}{\tan^2 x + 1} dx$$
$$= \int \frac{1}{\sec^2 x} dx$$
$$= \int \cos^2 x \, dx$$

12 | $\int \frac{\sec x \tan x}{1 + \sec^2 x} dx$

Let $$,

13 | **13**

14 | **14**