Source: [[KBe2020math530floIndex]]

#flo

1 | Polynomials

- See [[KBrefPolynomial]] ## 0 polynomial
- Has degree $-infty$
- Degrees are usually positive, except for the 0 degree
- "that's too hard, and we're not going to do it here" ## Identically zero
- Like 0 or $0x^0$
- Most polynomials are sometimes zero, but polynomials that are "identically zero" means that it's always zero (instead of just sometimes zero)

$\mathcal{P}_m(F)$

- Polynomials with coefficients in F whose highest degree is m
- It can't be "whose degree is exactly m" because otherwise you won't have the identity and it won't be closed under addition (in the case where coefficient sum $a_m + b_m = 0$) ### It's a finite dimensional vector space
- $$a_0 z^0 + ... + a_m z^m + b_0 z^0 + ... + b_m z^m = (a_0 + b_0) z^0 + ... + (a_m + b_m) z^m$$

Proof of 2.16

- Structure: proof by contradiction

2 | Linear Independence

- "non-trivial" means "simplest possible", which has usually got the most zeros

2.21 Linear Dependence Lemma 2.21

- it's saying that any linearly independent list has a vector inside that doesn't "contribute anything", and that if you remove it you'l have the same span. Implicitly, maybe through induction?) if you remove a dependent vector enough times then you get a linearly independent list.
- The list $(1,1,1),(2,2,2),(3,3,3)$ is really dependent, but $(0),(0),(0)$ is the most dependent (you have to remove all to get independence).

Epic

3 | **Exercise 2.A.1**

Lemma

If vectors v_1, v_2, v_3, v_4 span V, then the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

also spans V.

Proof

We prove the lemma by showing that any vector $v \in V$ can be written in the form $a_1v_1 + a_2v_2 + a_3 + v_3 + a_4v_4$ can also be written as a linear combination of the form

$$b_1(v_1 - v_2) + b_2(v_2 - v_3) + b_3(v_3 - v_4) + b_4v_4$$

If we set

$$\begin{aligned} b_1 &= a_1 \\ b_2 &= a_1 + a_2 \\ b_3 &= a_1 + a_2 + a_3 \\ b_4 &= a_1 + a_2 + a_3 + a_4 \end{aligned}$$

then the two combinations will be equivalent:

$$a_1(v_1 - v_2) + (a_1 + a_2)(v_2 - v_3) + (a_1 + a_2 + a_3)(v_3 - v_4) + (a_1 + a_2 + a_3 + a_4)v_4$$

$$= a_1v_1 \cancel{- a_1v_2 + a_1v_2} + a_2v_2 \cancel{- (a_1 + a_2)v_3 + (a_1 + a_2)v_3} + a_3v_3 \cancel{- (a_1 + a_2 + a_3)v_4 + (a_1 + a_2 + a_3)v}$$

$$= a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4$$