Source:

1 | **Openstax Calc vol 1 chap 2.4 ex 134**

$$g(t) = \frac{1}{t} + 1$$

which is basically $\frac{1}{x}$ shied up by one, so there is an $\boxed{\text{infinite discontinuity at} x = 0}$

2 | **136**

There is a jump discontinuity at $x = 2$, because normally $y = \frac{x}{x}$ simplifies to $y = 1$, but the sign flips at $x = 2$.

3 | **142**

$$f(y) = \frac{\sin(\pi y)}{\tan(\pi y)} = \frac{\cancel{\sin(\pi y)}\cos(\pi y)}{\cancel{\sin(\pi y)}}$$

So there is a removable discontinuity at $y = 1$, because there is a discontinuity but it can be removed with algebra.

4 | **148**

$$\begin{aligned} e^{4k} &= 4 + 3 \\ e^{4k} &= 7 \\ 4k &= \ln(7) \\ k &= \frac{\ln(7)}{4} \end{aligned}$$

5 | **TODO 174**

Prove $f(x)$ is continues everywhere, meaning show that $\forall c \in \mathbb{R}$

$$\lim_{x \to c} f(x) = f(c)$$

Because we can always evaluate $f(x)$, the limit always exists.

6 | **Paul's online math notes Section 2-9: 23**

The IVT states that when a function is continuous over a closed interval $[a, b]$, then for all $\min\{f(a), f(b)\} \leq y \leq \max\{f(a), f(b)\}$ there exists some $a \leq c \leq b$ s.t. $f(c) = y$. In this case, we have $f(4) = 193$ and $f(8) = -511$. $f(x)$ is a polynomial, so it is continuous over the range. Because our values stradle zero, there must be some value $4 \leq c \leq 8$ s.t. $f(c) = 0$.

7 | Boundedness theorem

Given a function $f(x)$ that is continuous on a closed interval $[a, b]$, there exists some $M \in \mathbb{R}$ s.t. $f(c) \leq M$ for all $a \leq c \leq b$ aka M is an upper bound on $f(x)$ over the interval $[a, b]$. There's also one that's less than all c. Doesn't work for open intervals.

7.1 | $(0, 1]$: not continuous, not a closed interval

7.2 | $[0, 1)$: not a closed interval

7.3 | $(0, 1]$: not a closed interval

7.4 | $(0, 1]$: not continuous, not a closed interval

7.5 | $f(x) = \frac{1}{x}$: not continuous

8 | Epilouge

Other than Problem 5, this took roughly 40 minutes. I still don't know how to do problem 5..