# 1 | Axler6.53 orthogonal projection, $P_U$ def

Suppose $U$ is a finite-dimensional subspace of $V$. The *orthogonal projection* of $V$ onto $U$ is the operator $P_U \in \mathcal{L}(V)$ defined as follows:
For $v \in V$, write $v = u + w$, where $u \in U$ and $w \in U^{\perp}$. Then $P_U v = u$.

In other words, $P_U \in \mathcal{L}(V)$ takes $v$ to the component of $v$ that is in $U$.

## 1.1 | Results

### 1.1.1 | Axler6.54 calculating $P_U v$

$$P_U v = \frac{\langle v, x\rangle}{\|x\|^2} x$$

Because orthogonal decompositions and stuff

### 1.1.2 | Axler6.55 properties

Suppose $U$ is a finite-dimensional subspace of $V$ and $v \in V$. Then,

1. $P_U \in \mathcal{L}(V)$
2. $P_U u = u \forall u \in U$
3. $P_U w = 0 \forall w \in U^{\perp}$
4. range $P_U = U$
5. null $P_U = U^{\perp}$
6. $P_U^2 = P_U$ (by #2)