

1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 du - \frac{1}{u^2+1} + C \\ &= 2 \int 1 du - \tan^{-1} u + C \\ &= 2u - \tan^{-1} u + C \\ &= \boxed{2\sqrt{x-1} - \tan^{-1}(\sqrt{x-1}) + C} \end{aligned}$$

2 | **2**

3 | **3**

4 | **4**

5 | **5**

6 | $\int \tan^2 x + 1 dx$

$$\begin{aligned} \int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx \end{aligned}$$

Let $u = x$, $du = 1$

$$\begin{aligned} &= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C} \end{aligned}$$

7 | **7**

8 | $\int \frac{e^x-1}{e^x} dx$

$$\int \frac{e^x-1}{e^x} dx = \int 1 - \frac{1}{e^x} dx$$

9 | $\int \frac{\sec^2 x}{\csc x} \sin x dx$

$$\begin{aligned} \int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x} \end{aligned}$$

10 | $\int \sin x \cos x dx$

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned} \int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x} \end{aligned}$$

11 | **TODO** $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx$

$$\begin{aligned} \int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx \\ &= \int \frac{1}{\tan^2 x + 1} dx \\ &= \int \frac{1}{\sec^2 x} dx \\ &= \int \cos^2 x dx \end{aligned}$$

12 | **12**

13 | **13**

14 | **14**