

Source: [KBhMATH401SubIndex](#)

1 | Series Convergence

1.1 | Geometric Series

In $\sum_{k=0}^{\infty} a(r^k)$, where $|r| < 1$, the series converges to $\sum_{k=0}^{\infty} a(r^k) = \frac{a}{1-r}$

In $\sum_{k=0}^n a(r^k)$, $\sum_{k=0}^n a(r^k) = \frac{a - ar^{n+1}}{1-r}$

1.2 | nth term divergence test

If $\lim_{n \rightarrow \infty} a_n$ is not zero, the series **will** diverge. The inverse is not necessarily true; that is, if this fails, use another test to test convergence.

1.3 | Integral Test

If the integral to infinity is convergent, the sequence is convergent as long as the sequence is continuous, positive, and decreasing. The inverse applies, too.

1.4 | Power Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

If a p-series has a $p > 1$, the p-series will converge

If a p-series has a $p \leq 1$, the p-series will diverge

1.5 | Comparison Test

Both provided that $a_n, b_n \geq 0$ & $a_n \leq b_n$

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Also, if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = C$ ($0 < c < \infty$), the two series will either both converge or both diverge. So you only need to test one.

1.6 | Alternating Series Test

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1.7 | Ratio Test

In a geometric series, the common ratio is simply $r = \frac{r^{n+1}}{r^n}$.

If r is an real value, $|r| < 1$, then series converges. If $|r| \geq 1$, the series diverges.

As limit goes to infinity in the r , if the common ratio approaches <1 , that means that the ratio will get smaller and smaller, just like if r were to be a real value and it was smaller than one. Meaning that the series **converges**.

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And so, formally.

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The inverse is true, too.

However, if the ratio is equal to one, the test is inconclusive.

Absolute Convergence \Rightarrow series who converge and whose absolute value converges

Conditional Convergence \Rightarrow series who converge and whose absolute value does not converge

1.8 | So what is the error of a Taylor series? (Lagrange Error)

The error at point x of a n th degree Taylor polynomial centered at a modeling a function with an absolute maximum value of M in its $n + 1$ th derivative between a bound containing x and a :

$$|E(x)| \leq \frac{M(x-a)^{n+1}}{(n+1)!}$$

1.9 | Power Series

$$f(x) = \sum_{n=0}^{\infty} a_n(x - c)^n$$

For instance, a geometric series is a special power series...

$$g(x) = \sum_{n=0}^{\infty} ax^n$$

This geometric series converges if $|x| < 1$, and so it has an interval of convergence of $-1 < x < 1$. If this converges, this function will converge to $\frac{a}{1-x}$

Interval of Convergence: at what values of x does the series converge?

Radius of Convergence: at what absolute distance from c (the “centering” of the series) will the series converge?

To figure the interval of convergence, simply use the ratio test and solve for x that makes the ratio < 1 . Then, think about the inconclusive cases whereby ratio $= 1$ — then, use the comparison test, or integral test.