

Suppose V is a complex inner product space and $T \in \mathcal{L}(V)$ is a normal operator such that $T^9 = T^8$. Prove that T is self-adjoint and $T^2 = T$.

If $T = 0$, then $0^2 = 0$ and 0 is self-adjoint. Thus, let $T \neq 0$.

In 7.1, Axler asserts that V is finite-dimensional.

T has a diagonal matrix w.r.t. an orthonormal basis of V .

$$TT^* = T^*T$$

First, we will show that $T^2 = T$. Suppose T is invertible. Then,

$$\begin{aligned} T^9 &= T^8 \\ T^9 T^{-7} &= T^8 T^{-7} \\ T^2 &= T \end{aligned}$$

Suppose T is not invertible and not equal to zero. Then, T has