

Source: [KBe2020math530floIndex](#)

- Vector spaces and fields are like groups
  - With 2 operations
- Vector
  - direction and magnitude
  - numbers with an order
    - list = ordered set
    - $N \times 1$  matrix
  - A vector is not just an  $N \times 1$  matrix. **A vector exists in a vector space**
    - might be full of physics vectors, matrices, or polynomials
- Field
  - Addition and multiplication might be different
    - might be related to normal addition/multiplication
    - might be any binary operation
    - Addition is “primary” operation, multiplication is “secondary”
      - addition is really good (more group like)
      - multiplication needs to exclude the additive identity (because it can't have an inverse)
  - questions
    - multiplication is repeated addition?
      - not necessarily
    - binary expressions?
    - associative?
      - both yes
  - 1.3 demonstrates that the complex numbers are a field
    - commutativity
    - associativity
    - identities
    - additive inverse
    - multiplicative inverse except additive identity
    - distributive
  - usually means Reals or Complex
    - integers mod 3 are a field
      - #bonushw show integers mod 3 are a field
  - higher dimensions
    - $R^2$  is a Cartesian plane,  $R^4$  is a space
  - operations
    - addition is really nice (element wise)
    - scalar multiplication is easy enough
    - vector vector multiplication is hard to find
- two square roots of  $i$ 
  - fundamental theorem of algebra
    - (important)
  - So,  $i$  should have two square roots
  - Powers of  $i$  go in a circle (90 degrees rotation)
    - Complex number rotation gives a preferred direction

- So that's why the quadrants are numbered in that direction
  - One can be found geometrically 20math530srcSquareRootl.png
  - We could also try it algebraically
    - $(a + bi)^2 = i = a^2 - b^2 + 2abi$
    - so  $a^2 - b^2 = 0$  and  $2ab = 1$
  - dot product
    - How much of  $\vec{A}$  is in the direction of  $\vec{B}$  multiplied by the magnitude of  $\vec{B}$
    - $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$ 
      - #bonushw prove that  $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$
    - Identity:  $\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \cos\theta$
  - cross product
    - only works on 3x1 matrices
    - steps
      - determinant
      - $i, j, k$  are the unit vectors
      - $$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{vmatrix} = i \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$
  - dropping zero:  $0a = (0 + 0)a = 0a + 0a \Rightarrow 0a = 0$ , so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
    - 20math530srcFieldsMultiplyCannotBeGroup.png
  - determinant
    - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
  - #todo #exr0n #future prove identities are unique
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