1 | Axler6.53 orthogonal projection, P_U def

Suppose U is a finite-dimensional subspace of V. The *orthogonal projection* of V onto U is the operator $P_U \in \mathcal{L}(V)$ defined as follows:

For $v \in V$, write $v = u + w$, where $u \in U$ and $w \in U^\perp$. Then $P_U v = u$.

In other words, $P_U \in \mathcal{L}(V)$ takes v to the component of v that is in U.

This concept is closely related to the Orthogonal Decomposition

1.1 | Results

1.1.1 | Axler6.54 calculating $P_U v$

$$P_U v = \frac{\langle v, x \rangle}{\|x\|^2} x$$

Because orthogonal decompositions and stuff

1.1.2 | Axler6.55 properties

Suppose U is a finite-dimensional subspace of V and $v \in V$. Then,

1. $P_U \in \mathcal{L}(V)$
2. $P_U u = u \forall u \in U$
3. $P_U w = 0 \forall w \in U^\perp$
4. range $P_U = U$
5. null $P_U = U^\perp$
6. $P_U^2 = P_U$ (by \#2 and \#4)
7. $\|P_U v\| \leq \|v\|$
8. for every orthonormal basis $e_1, \ldots, e_m$ of U,

   $$P_U v = \langle v, e_1 \rangle e_1, + \cdots + \langle v, e_m \rangle e_m$$

   (because $P_U v \in U$)

1.1.3 | Axler6.56 Minimizing the distance to a subspace