1 | **Axler 6.A exercise 9**

Suppose $u, v \in V$ and $\|u\| \leq 1$ and $\|v\| \leq 1$. Prove that

$$\sqrt{1 - \|u\|^2}\sqrt{1 - \|u\|^2} \leq 1 - |\langle u, v \rangle|$$

2 | **Proof**

We will prove this by showing that the left hand side is less than or equal to an intermediate term, which is less than or equal to the right hand side.

$$|\langle u, v \rangle| \leq \|u\|\|v\|$$
$$1 - \|u\|\|v\| \leq 1 - |\langle u, v \rangle|$$

This intermediate value is obtained using the Cauchy-Schwarz inequality.

Now, to show that the left hand side is less than or equal to the right hand side,

$$\sqrt{1 - \|u\|^2}\sqrt{1 - \|v\|^2}$$
$$= \sqrt{(1 - \|u\|^2)(1 - \|v\|^2)}$$
$$= \sqrt{1 - \|u\|^2 - \|v\|^2 + \|u\|^2\|v\|^2}$$