Source:

1 | Isomorphism def

An *isomorphism* is an invertible linear map

2 | Isomorphic def

Two vector spaces are called *isomorphic* if there is an isomorphism from one vector space into the other

2.1 | intuition

Can be thought of as relabeling each element v from one space into an element Tv in the other.

2.2 | results

2.2.1 | equal dimension iff isomorphic Axler3.59

Two vector spaces over some field $\mathbb{F}$ are isomorphic iff they have the same dimension.

2.2.2 | $\mathcal{L}(V, W)$ and $\mathbb{F}^{m,n}$ are isomorphic

Given two bases of V and W, $\mathcal{M}$ is an isomorphism between $\mathcal{L}(V, W)$ and $\mathbb{F}^{m,n}$

2.2.3 | Axler3.61 dim $\mathcal{L}(V, W) = ($dim $V)\,($dim $V)$

2.3 | intuition

Not only do two isomorphic spaces have a one to one correspondence between them, that coresspondence is linear which means that they way the elements interact on one side is the same on the other.