

$$3 \mid \int \ln x dx$$

$$\begin{aligned}\int \ln x dx &= \int 1 \ln x dx \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int 1 dx \\ &= \boxed{x \ln x - x}\end{aligned}$$

$$4 \mid \int \tan^{-1} x dx$$

$$\begin{aligned}\int \tan^{-1} x dx &= x \tan^{-1} x - \int x \frac{1}{x^2 + 1} dx \\ &= x \tan^{-1} x - \frac{1}{2} \int \frac{du}{u} \\ &= x \tan^{-1} x - \frac{1}{2} \ln u + C \\ &= x \tan^{-1} x - \frac{1}{2} \ln(x^2 + 1) + C\end{aligned}$$

$$5 \mid \int x \sec^2 x dx$$

$$\begin{aligned}\int x \sec^2 x dx &= x \tan x - \int \tan x dx \\ &= x \tan x + \ln |\cos x| + C\end{aligned}$$

$$6 \mid \int x^2 e^{5x} dx$$

$$\begin{aligned}\int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\ &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{25} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\ &= \frac{1}{5} e^{5x} \left(x^2 - \frac{2}{5} x + \frac{2}{25} \right) + C\end{aligned}$$

$$7 \mid \int x^2 \cos x dx = f(x) - \int 2x \sin x dx$$

Find $f(x)$

$$f(x) = x^2 \sin x$$

$$8 \mid \int x \cos x dx$$

$$\begin{aligned}\int x \cos x dx &= x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C\end{aligned}$$

$$9 \mid \int x^2 \sin x dx$$

$$\begin{aligned}\int x^2 \sin x dx &= -x \cos x - \int -\cos x dx \\ &= -x \cos x - \sin x dx\end{aligned}$$

$$10 \mid \int x^3 e^{x^2} dx$$

Let $u = x^2$

$$\begin{aligned}\int x^3 e^{x^2} dx &= \int x^2 x e^{x^2} dx \\ &= \int u \frac{1}{2} du e^u \\ &= \frac{1}{2} \int u e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} \int e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} e^u + C \\ &= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}\end{aligned}$$

$$11 \mid \int x^2 \ln x dx$$

$$\begin{aligned}\int x^2 \ln x dx &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \frac{1}{x} dx \\ &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx \\ &= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C\end{aligned}$$