

Source:**1 | find taylor series**

1.1 | $y = \cos(x)$

$$\begin{aligned}
P_n(x) &= f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 + \frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 + \dots \\
&= \cos(0) - \sin(0)x + \frac{-\cos(0)}{2!}x^2 + \frac{\sin(0)}{3!}x^3 + \dots \\
&= 1 - 0x - \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \dots \\
&= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots \\
&= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}
\end{aligned}$$

1.2 | $y = e^x$

$$\begin{aligned}
P_n(x) &= f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 + \frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 + \dots \\
&= e^0 + e^0x + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \dots \\
&= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots \\
&= \sum_{k=0}^{\infty} \frac{1}{k!} x^k
\end{aligned}$$

1.3 | **TODO** $y = \sqrt{x}$

2 | prove approximations

2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

Proof by geometric series

2.2 | $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$

Plug $-x$ for x in the previous equation.

2.3 | $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \dots$

Plug x^2 for x in the previous equation.

3 | more finding of polynomial

3.1 | **TODO** \$ y = \ln(1+x)\$

3.2 | **TODO** \$ y = \tan^{-1} x\$

3.3 | **DONE** \$ y = (1+x)^k\$

$$\begin{aligned}
 P_n(x) &= f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^2}{dx^2}f(0)}{2!}x^2 + \frac{\frac{d^3}{dx^3}f(0)}{3!}x^3 + \dots \\
 &= k(1)^k + k(k-1)(1)^{k-1}x + \frac{k(k-1)(k-2)(1)^{k-2}}{2!}x^2 + \frac{k(k-1)(k-2)(k-3)(1)^{k-3}}{3!}x^3 + \dots \\
 &= k + k(k-1)x + \frac{k(k-1)(k-2)}{2!}x^2 + \frac{k(k-1)(k-2)(k-3)}{3!}x^3 + \dots \\
 &= k + \frac{k!}{(k-1)!}x + \frac{\frac{k!}{(k-2)!}}{2!}x^2 + \frac{\frac{k!}{(k-3)!}}{3!}x^3 + \dots \\
 &= k + \frac{k!x}{(k-1)!} + \frac{k!}{(k-2)!2!}x^2 + \frac{k!}{(k-3)!3!}x^3 + \dots \\
 &= \binom{k}{1}x + \binom{k}{2}x^2 + \binom{k}{3}x^3 + \dots
 \end{aligned}$$