1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned}
\int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\
&= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\
&= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\
&= 2 \int 1du - \frac{1}{u^2+1} + C \\
&= 2 \int 1du - \tan^{-} u + C \\
&= 2u - \tan^{-} u + C \\
&= \boxed{2\sqrt{x-1} - \tan^{-} \left(\sqrt{x-1}\right) + C}
\end{aligned}$$

2 | $\int \frac{x^2}{x^2+1} dx$

Let $u = x^2+1$, $du = 2xdx$

$$\begin{aligned}
\int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\
&= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\
&= \frac{1}{2} \left(u - \ln u \right) + C \\
&= \boxed{\frac{1}{2} \left(x^2 + 1 - \ln(x^2+1) \right) + C}
\end{aligned}$$

3 | **TODO** $\int \frac{x-4}{x^2} dx$

Let $u = x-4$, $du = dx$

$$\begin{aligned}
\int \frac{x-4}{x^2} dx &= \int \frac{u}{(u+4)^2} dx \\
&= \int \frac{u}{u^2+8u+16} du
\end{aligned}$$

4 | $\int (x+1)e^{x^2+2x}dx$

Let $u = x^2 + 2x$, $du = x + 1$

$$\begin{aligned} \int (x+1)e^{x^2+2x}dx &= \frac{1}{2}\int e^u du \\ &= \frac{1}{2}e^u \\ &= \boxed{\frac{1}{2}e^{x^2+2x}} \end{aligned}$$

5 | $\int \tan^2 x + 1dx$

$$\begin{aligned} \int \tan^2 x + 1dx &= \int \sec^2 x - 1 + 1dx \\ &= \int \sec^2 xdx \end{aligned}$$

Let $u = x, du = 1$

$$\begin{aligned} &= \int \sec^2 udu \\ &= \tan u + C \\ &= \boxed{\tan x + C} \end{aligned}$$

6 | **TODO** $\int \frac{6x^2-4}{x}dx$

7 | $\int \frac{e^x-1}{e^x}dx$

$$\begin{aligned} \int \frac{e^x - 1}{e^x}dx &= \int 1 - \frac{1}{e^x}dx \\ &= \int 1 - e^{-x}dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C} \end{aligned}$$

8 | $\int \frac{\sec^2 x}{\csc x} sinxdx$

$$\begin{aligned} \int \frac{\sec^2 x}{\csc x} sinxdx &= \int \tan^2 xdx \\ &= \int \sec^2 x - 1dx \\ &= \int \sec^2 xdx - \int 1dx \\ &= \boxed{\tan x - x} \end{aligned}$$

9 | **$∫ sin x cos x dx $**

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned}\int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2}u^2 \\ &= \boxed{\frac{1}{2}\sin^2 x}\end{aligned}$$

10 | **TODO** $\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx$

$$\begin{aligned}\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx &= \int \frac{\sin^2 x+\cos^2 x}{\tan^2 x+1}dx \\ &= \int \frac{\sin^2 x}{\tan^2 x+1}+\frac{\cos^2 x}{\tan^2 x+1}dx \\ &= \int \sin^2 x\cos^2 x+\cos^4 x dx \\ &= \int \frac{1}{\tan^2 x+1}dx \\ &= \int \frac{1}{\sec^2 x}dx \\ &= \int \cos^2 x dx\end{aligned}$$

11 | $\int \frac{\sec x\tan x}{1+\sec^2 x}dx$

Let $$,

12 | **13**

13 | **14**