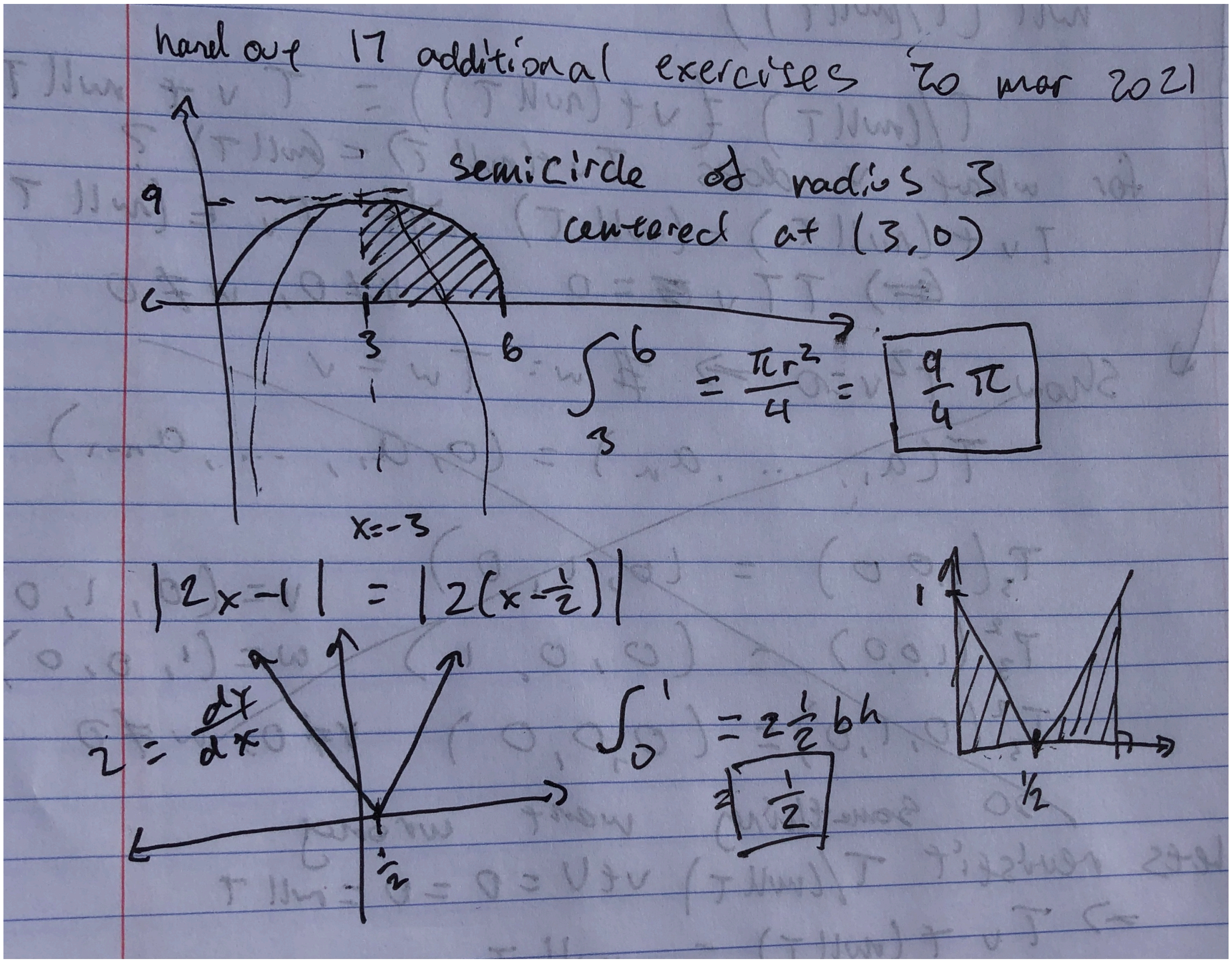
1 | Exercises

1.1 | interpreting in terms of area



1.3 | subtracting integrals

I expect

$$\int_1^2 f(x)dx = \int_1^5 f(x)dx - \int_2^5 f(x)dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

1.4 | show $\int_a^b x^2 dx = \frac{b^3-a^3}{3}$

(see attached pages)

Keep in mind

$$\sum_{k=1}^{n} af(x) = a\sum_{k=1}^{n} f(x)$$
$$\sum_{k=1}^{n} (a + f(x)) = an + \sum_{k=1}^{n} f(x)$$

$$\begin{aligned}
\int_a^b x^2dx &= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{b-a}{n}\left(a + k\frac{b-a}{n}\right)^2\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} \left(a + k\frac{b-a}{n}\right)^2 \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} \left(a^2 + \left(k\frac{b-a}{n}\right)^2 + 2ak\frac{b-a}{n}\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} a^2 + \sum_{k=1}^{n} \left(k\frac{b-a}{n}\right)^2 + \sum_{k=1}^{n} 2ak\frac{b-a}{n} \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(a^2n + \sum_{k=1}^{n} k^2 \left(\frac{b-a}{n}\right)^2 + 2a\frac{b-a}{n}\sum_{k=1}^{n} k\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(a^2n + \left(\frac{b-a}{n}\right)^2 \sum_{k=1}^{n} k^2 + 2a\frac{b-a}{n}\sum_{k=1}^{n} k\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(a^2n + \left(\frac{b-a}{n}\right)^2 \frac{n(n+1)(2n+1)}{6} + 2a\frac{b-a}{n}\frac{n(n+1)}{2}\right) \\
&= \lim_{n\to\infty} (b-a) \left(a^2 + \left(\frac{b-a}{n}\right)^2 \frac{(n+1)(2n+1)}{6} + \cancel{2}a\frac{b-a}{n}\frac{(n+1)}{\cancel{2}}\right) \\
&= \lim_{n\to\infty} (b-a) \left(a^2 + \frac{(b-a)^2}{n^2} \left(n\frac{(2n+1)}{6} + \frac{(2n+1)}{6}\right) + a\frac{b-a}{n}(n+1)\right) \\
&= \lim_{n\to\infty} (b-a) \left(a^2 + \frac{(b-a)^2}{n^2} \left(n\frac{(2n+1)}{6} + \frac{(2n+1)}{6}\right) + an\frac{b-a}{n} + a\frac{b-a}{n}\right)
\end{aligned}$$