Source:

1 | find taylor series

1.1 | $y = \cos(x)$

$$
\begin{aligned}
P_n(x) &= f(0) & +\frac{d}{dx}f(0)x & +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 & +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 & +\cdots \\
&= \cos(0) & -\sin(0)x & +\frac{-\cos(0)}{2!}x^2 & +\frac{\sin(0)}{3!}x^3 & +\cdots \\
&= 1 & -0x & -\frac{1}{2!}x^2 & +\frac{0}{3!}x^3 & +\cdots \\
&= 1 & & -\frac{1}{2!}x^2 & & +\frac{1}{4!}x^4+\cdots \\
&= \sum_{k=0} \frac{(-1)^k}{(2k)!}x^{2k}
\end{aligned}
$$

1.2 | $y = e^x$

$$
\begin{aligned}
P_n(x) &= f(0) & +\frac{d}{dx}f(0)x & +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 & +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 & +\cdots \\
&= e^0 & +e^0x & +\frac{e^0}{2!}x^2 & +\frac{e^0}{3!}x^3 & +\cdots \\
&= 1 & +x & +\frac{1}{2!}x^2 & +\frac{1}{3!}x^3 & +\cdots \\
&= \sum_{k=0} \frac{x^k}{k!}
\end{aligned}
$$

1.3 | $y = \sqrt{x}$ **centered at** $x = 1$

$$
\begin{aligned}
P_n(x) &= f(1) & +\frac{d}{dx}f(1)(x-1) & +\frac{\frac{d^2}{d^2x}f(1)}{2!}(x-1)^2 & +\frac{\frac{d^3}{d^3x}f(1)}{3!}(x-1)^3 & +\cdots \\
&= 1 & +\frac{1}{2}(x-1) & +\frac{\frac{1}{2}\frac{-1}{2}}{2!}(x-1)^2 & +\frac{\frac{1}{2}\frac{-1}{2}\frac{-3}{2}}{3!}(x-1)^3 & +\cdots
\end{aligned}
$$

I don't know how to write it using summation notation though..

2 | prove approximations

2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Proof by geometric series

2.2 | $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

Plug $-x$ for x in the previous equation.

2.3 | $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$

Plug x^2 for x in the previous equation.

3 | more finding of polynomial

3.1 | TODO $ y = ln (1+x)$

3.2 | TODO $y = \tan^{-} x$

3.3 | $y = (1+x)^k$

$$
\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= 1 + k(1)^k && +k(k-1)(1)^{k-1}x && +\frac{k(k-1)(k-2)(1)^{k-2}}{2!}x^2 && +\frac{k(k-1)(k-2)(k-3)(1)^{k-3}}{3!}x^3 && +\cdots \\
&= 1 + k && +k(k-1)x && +\frac{k(k-1)(k-2)}{2!}x^2 && +\frac{k(k-1)(k-2)(k-3)}{3!}x^3 && +\cdots \\
&= 1 + k && +\frac{k!}{(k-1)!}x && +\frac{\frac{k!}{(k-2)!}}{2!}x^2 && +\frac{\frac{k!}{(k-3)!}}{3!}x^3 && +\cdots \\
&= 1 + k && +\frac{k!x}{(k-1)!} && +\frac{k!}{(k-2)!2!}x^2 && +\frac{k!}{(k-3)!3!}x^3 && +\cdots \\
&= \binom{k}{0} + \binom{k}{1}x && +\binom{k}{2}x^2 && +\binom{k}{3}x^3 && +\cdots \\
&= \sum_{i=0} \binom{k}{i}x^i
\end{aligned}
$$

4 | find sum of series by recognizing Taylor Series approximations of some functions

4.1 | $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$

$$3 + \quad \frac{9}{2!} + \quad \frac{27}{3!} + \quad \frac{81}{4!} = e^3 - 1$$

4.2 | $1 - \ln 2 + \frac{\ln^2 2}{2!} + \frac{\ln^3 2}{3!} + \cdots$

$$1 - \ln 2 + \frac{\ln^2 2}{2!} + \frac{\ln^3 2}{3!} + \cdots = e^{-\ln 2}$$

4.3 | $\sum_{k=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$

$$\sum_{k=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^n \frac{\pi}{4}^{2n+1}}{(2n+1)!} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$