Source:

1 | eigenvalues

eigenvalue: multiplied by a scalar? a subspace that, when put through a linear map, only gets scaled.

$$Tv = \lambda v$$

Where $v \neq 0$. (we ignore it because its no fun to send zero to zero, and bc the span is empty).
T must be an operator! Otherwise the matrix sizes don't work out when subtracting λI.
where v is the eigenvector and λ is the eigenvalue. The equation is often rewritten as:

$$Tv - \lambda v = 0 Tv - \lambda I v \quad = 0 (T - \lambda I)v = 0$$

now this can be factored and roots can be found. also it's an operator.

1.1 | Axler 5.6 equivalent conditions

Only when V is finite dimensional!

1.1.1 | $T - \lambda I$ is not injective, because both $v, 0$ are in the null space.

1.1.2 | $T - \lambda I$ is also not surjective or invertible bc finite dim operator.

2 | an example

Given the matrix $\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$, find the eigenvalues and eigenvectors.
Now that we have that other fomulation, we can just subtract λI from T to get

$$\begin{pmatrix} 3 - \lambda & 1 \\ 0 & 2 - lambda \end{pmatrix}$$

Then, we just need to find whether it is non-invertible aka singular aka determinant.

$$(3 - \lambda)(2 - \lambda) = 0$$

The solutions are $\lambda = 2$ or 3. These are the eigenvalues.
Now just plug in λ and find the null space using RREF. The null space for $\lambda = 3$ has null space span$(x, 0)$, so we just pick one of those vectors (ex. $(1, 0)$) to be the eigenvector.

2.1 | **review of terms**

2.1.1 | $span(1, 0)$ **is an invariant subspace. (also whatever you get for** $\lambda = 2$

2.1.2 | **any vector in an invariant subspace is an eigenvector**

2.1.3 | **the eigenvalues are** $2, 3$

2.2 | **general idea**

the point of eigenvectors is to figure out where other vectors go by looking at pieces that only get streched or shrunk.

3 | **depends on**

3.1 | **finding roots is helpful**