1 | Slicing into Rectangles

The general idea of Riemann sums is to slice a curve into vertical non-overlapping rectangles to approximate the area between the curve and the x-axis. This can be expressed mathematically as a summation given the function $f(x)$, the range $[a, b]$, and the number of rectangles n:

$$\sum_{k=1}^{n} \frac{b-a}{n} f(a + k\frac{b-a}{n})$$

This can be written more concisely by defining $\Delta x = \frac{b-a}{n}$ and $x_i = a + k\Delta x$:

$$\sum_{k=1}^{n} \Delta x f(x_i)$$

These estimates all have the right endpoint of the rectangle touching the curve. You could also use the left endpoint, or use the minimum value one and add a triangle to form a trapezoid.

2 | Area Interpretation

Areas under curves can be estimated if you recognize the function. For example:

$$\int_0^1 \sqrt{x^2 - 1} dx$$

3 | Upper and Lower Bound

4 | the Definite Integral