

## Self-adjoint operators

### Axler7.11 self-adjoint, Hermitian def

An operator  $T \in \mathcal{L}(V)$  is called *self-adjoint* if  $T = T^*$  aka it is adjoint to itself. aka:  $T \in \mathcal{L}(V)$  is self-adjoint iff

$$\langle Tv, w \rangle = \langle v, Tw \rangle$$

Because adjoint-ness is in some ways analogous to complex conjugation, a self-adjoint operator is somewhat analogous to real numbers (kinda like a number who equals its conjugate real, a map that equals its adjoint is "real")

### results

#### Axler7.13 Eigenvalues of self-adjoint operators are real

Every eigenvalue of a self-adjoint operator is real.

#### Axler7.14 Over $\mathbb{C}$ , only the 0 operator has $Tv$ being orthogonal to $v$ for all $v$

For some **complex** vector space  $V$  and  $T \in \mathcal{L}(V)$ , if

$$\langle Tv, v \rangle = 0$$

for all  $v \in V$ , then  $T = 0$ .

#### TODO Axler7.15 and Axler7.16??

Every self-adjoint operator is normal.