

## 1 | Problem

What happens if the Gram–Schmidt Procedure is applied to a list of vectors that is not linearly independent?

## 2 | Answer

Suppose the list  $v_1, \dots, v_n$  is linearly dependent. Then, there exists some  $v_j$  s.t.  $v_1, \dots, v_{j-1}$  is linearly independent while  $v_1, \dots, v_j$  is not. Then,  $v_j \in \text{span}(v_1, \dots, v_{j-1})$

Because the Gram-Schmidt procedure preserves prefix spans,

$$v_j \in \text{span}(e_1, \dots, e_{j-1})$$

Thus, the denominator in the  $j$ -th step of the procedure is equivalent to

$$\|v - v\| = \|0\| = 0$$

and a division by zero occurs. Thus, the Gram-Schmidt procedure cannot be used on a linearly dependent list.