Axler 6.B ex 3

orthonormality $(1,0,0), (1,1,1), (1,1,2)$

$e_1 = (1,0,0)$, $e_2 = \text{normalize}((1,1,1)-(1,0,0))$

$= \text{normalize}((0,1,1))$

$= (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$e_3 = \text{normalize}\left((1,1,2) - (1,0,0) - \frac{3}{\sqrt{2}}(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\right)$

$= \text{normalize}\left((0,1,2) - (0, \frac{3}{2}, \frac{3}{2})\right)$

$= \text{normalize}\left((0, \frac{1}{2}, \frac{1}{2})\right)$

$= (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$(1,0,0)$, $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$0 + \frac{-1}{2} + \frac{1}{2} = 0$ ✓ so all pairwise inner products are zero

$v = (1,2,3)$

$= \langle (1,2,3), (1,0,0) \rangle e_1 + \langle (1,2,3), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rangle e_2 + \langle (1,2,3), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rangle e_3$

$= \boxed{1e_1 + \frac{5}{\sqrt{2}} e_2 + \frac{1}{\sqrt{2}} e_3}$

$= (1,0,0) + (0, \frac{5}{2}, \frac{5}{2}) + (0, \frac{1}{2}, \frac{1}{2})$

$= (1, \frac{4}{2}, \frac{5}{2})$ ✓