

Source:

1 | Problem

Suppose $T \in \mathcal{L}(V, W)$ and U is a subspace of V . Let π denote the quotient map from V onto V/U . Prove that there exists $S \in \mathcal{L}(V/U, W)$ such that $T = S \circ \pi$ if and only if $U \subseteq \text{null } T$.

Intuitively, if we mod out part of the null T , then we should still be able to have a map that does what T would do, and if we are able to do what T would do, then we only removed part of null T and lost no information.

2 | Forward Direction by Contrapositive

Intuitively, if we lost information, then we can't reconstruct what T would do.

If $U \not\subseteq \text{null } T$, then there exists $v \in U$ s.t. $Tv \neq 0$. This is some of the "information" that was "lost". Because $v \in U$,

$$\pi v = U + v = U$$

3 | Reverse Direction