

1 | orthogonal def

Two vectors $u, v \in V$ are called *orthogonal* if $\langle u, v \rangle = 0$

2 | results

2.1 | orthogonal \sim perpendicular

2.2 | **Axler 6.12** orthogonality and zero

2.2.1 | **0** is orthogonal to every vector in V

2.2.2 | **0** is the only vector in V that is orthogonal to itself

2.3 | **Axler 6.13** Pythagorean Theorem

Suppose u and v are orthogonal vectors in V . Then

$$\|u + v\|^2 = \|u\|^2 + \|v\|^2$$

2.3.1 | **proof with more algebra written out**

$$\begin{aligned} \|u + v\|^2 &= \langle u + v, u + v \rangle \\ &= \langle u, u + v \rangle + \langle v, u + v \rangle \\ &= \langle u, u \rangle + \cancel{\langle u, v \rangle}^0 + \cancel{\langle v, u \rangle}^0 + \langle v, v \rangle \\ &= \|u\|^2 + \|v\|^2 \end{aligned}$$