1 | $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

$$\begin{aligned}\int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\ &= e - e^{\frac{1}{2}}\end{aligned}$$

2 | $\int_0^1 re^{\frac{r}{2}} dr$

$$\begin{aligned}\int_0^1 re^{\frac{r}{2}} dx &\implies r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4) \\ &= 4 - 2e^{\frac{1}{2}}\end{aligned}$$

3 | **TODO** $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$

4 | **TODO** $\int_0^{\sqrt{\pi}} \cos\sqrt{x} dx$

$$\begin{aligned}\int_0^{\sqrt{\pi}} \cos\sqrt{x} dx &= x\cos\sqrt{x} + \int x\frac{1}{2\sqrt{x}} \sin\sqrt{x} dx \\ &= x\cos\sqrt{x} + \int \frac{\sqrt{x}}{2} \sin\sqrt{x} dx \\ &= x\cos\sqrt{x} + \frac{\sqrt{x}}{2} \int \sin\sqrt{x} dx \\ &= x\cos\sqrt{x} + \frac{\sqrt{x}}{2} \left(x\sin\sqrt{x} - \int \frac{\sqrt{x}}{2} \cos\sqrt{x} dx \right) \\ &= x\cos\sqrt{x} + \frac{\sqrt{x}}{2} \left(x\sin\sqrt{x} - \frac{\sqrt{x}}{2} \int \cos\sqrt{x} dx \right) \\ &= x\cos\sqrt{x} + \frac{\sqrt{x}}{2} x\sin\sqrt{x} - \frac{x}{4} \int \cos\sqrt{x} dx \\ \frac{x+4}{4} \int \cos\sqrt{x} dx &= x\cos\sqrt{x} + \frac{\sqrt{x}}{2} x\sin\sqrt{x}\end{aligned}$$

5 | $\int_1^e \sin \ln x dx$

$$
\begin{aligned}
\int_1^e \sin \ln x dx &= x \sin \ln x - \int x \frac{1}{x} \cos \ln x dx \\
&= x \sin \ln x - \int \cos \ln x dx \\
&= x \sin \ln x - \left(x \cos \ln x + \int \cancel{x} \frac{1}{\cancel{x}} \sin \ln x dx \right) \\
&= x \sin \ln x - x \cos \ln x - \int \sin \ln x dx \\
2 \int \sin \ln x dx &= x \sin \ln x - x \cos \ln x \\
\int \sin \ln x dx &= \frac{1}{2} x (\sin \ln x - \cos \ln x) \\
\implies \quad & \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\
&= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2}
\end{aligned}
$$