

$$1 \mid \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$\begin{aligned} \int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\ &= e - e^{\frac{1}{2}} \end{aligned}$$

$$2 \mid \int_0^1 r e^{\frac{r}{2}} dr$$

$$\begin{aligned} \int_0^1 r e^{\frac{r}{2}} dx &\Rightarrow r 2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2r e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2r e^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &= 2r e^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &\Rightarrow 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4) \\ &= 4 - 2e^{\frac{1}{2}} \end{aligned}$$

$$3 \mid \int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

$$\begin{aligned} \int \frac{\ln y}{\sqrt{y}} dy &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{y} \sqrt{y} dy \\ &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{\sqrt{y}} dy \\ &= 2 \ln y \sqrt{y} - 2 \int y^{-\frac{1}{2}} dy \\ &= 2 \ln y \sqrt{y} - 4\sqrt{y} + C \\ &= 2\sqrt{y}(\ln y - 2) + C \\ \Rightarrow &6(\ln 9 - 2) - 4(\ln 4 - 2) \end{aligned}$$

$$4 \mid \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$$

$$\text{Let } u = \sqrt{x}, du = \frac{1}{2\sqrt{x}} dx, dx = 2u du$$

$$\begin{aligned} \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= 2 \int u \cos u du \\ &= 2u \sin u - 2 \int \sin u du \\ &= 2u \sin u + 2 \cos u \\ &= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} \\ \Rightarrow &2\pi^{\frac{1}{4}} \sin \pi^{\frac{1}{4}} + 2 \cos \pi^{\frac{1}{4}} - 2 \end{aligned}$$

5 | $\int_1^e \sin \ln x dx$

$$\begin{aligned}
 \int_1^e \sin \ln x dx &= x \sin \ln x - \int x \frac{1}{x} \cos \ln x dx \\
 &= x \sin \ln x - \int \cos \ln x dx \\
 &= x \sin \ln x - \left(x \cos \ln x + \int x \cancel{\frac{1}{x}} \sin \ln x dx \right) \\
 &= x \sin \ln x - x \cos \ln x - \int \sin \ln x dx \\
 2 \int \sin \ln x dx &= x \sin \ln x - x \cos \ln x \\
 \int \sin \ln x dx &= \frac{1}{2} x (\sin \ln x - \cos \ln x) \\
 \implies \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\
 &= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2}
 \end{aligned}$$

6 | $\int_0^1 \frac{x^3}{\sqrt{4+x}} dx$

Let $u = 4 + x^2$, $du = 2x dx$

$$\begin{aligned}
 \int \frac{x^3}{\sqrt{4+x}} dx &= \frac{1}{2} \int \frac{(u-4)}{\sqrt{u}} du \\
 &= \frac{1}{2} \int \sqrt{u} dx - 2 \int \frac{1}{\sqrt{u}} dx
 \end{aligned}$$

7 | (additional problems)

7.1 | $\int \sin^2 x dx$

$$\begin{aligned}
 \int \sin^2 x dx &= -\sin x \cos x + \int \cos^2 x dx \\
 &= -\sin x \cos x + \int 1 dx - \int \sin^2 x dx \\
 2 \int \sin^2 x dx &= -\sin x \cos x + x \\
 \int \sin^2 x dx &= \frac{1}{2} (x - \sin x \cos x)
 \end{aligned}$$

$$7.2 \mid \int \cos^2 x dx$$

$$\begin{aligned}\int \cos^2 x dx &= \cos x \sin x + \int \sin^2 x dx \\ &= \cos x \sin x + \int 1 dx - \int \cos^2 x dx \\ 2 \int \cos^2 x dx &= \cos x \sin x + x \\ \int \cos^2 x dx &= \frac{1}{2} \cos x \sin x + \frac{x}{2} + C\end{aligned}$$

$$7.3 \mid \int \sin^2 x \cos^2 x dx$$

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \int \sin^2 x \cos^2 x dx &= \frac{1}{4} \int \sin^2 2x dx \\ \text{Let } u &= 2x, du = 2dx \\ &= \frac{1}{8} \int \sin^2 u du \\ &= \frac{1}{8} \frac{1}{2} (u - \sin u \cos u) \\ &= \frac{1}{16} (2x - \sin 2x \cos 2x) + C\end{aligned}$$

$$7.4 \mid \int \sin^3 x dx$$

$$\begin{aligned}\int \sin^3 x dx &= \int \sin x (1 - \cos^2 x) dx \\ &= \int \sin x dx - \int \cos^2 x \sin x dx \\ \text{Let } u &= \cos x, du = -\sin x dx \\ &= \int \sin x dx + \int u^2 du \\ &= -\cos x + \frac{1}{3} u^3 + C \\ &= \frac{1}{3} \cos^3 x - \cos x + C\end{aligned}$$

$$7.5 \mid \int \cos^3 x dx$$

$$\begin{aligned}\int \cos^3 x dx &= \int \cos x \cos^2 x dx \\ &= \int \cos x (1 - \sin^2 x) dx \\ &= \int \cos x - \sin^2 x \cos x dx \\ &= \sin x - \int \sin^2 x \cos x dx\end{aligned}$$