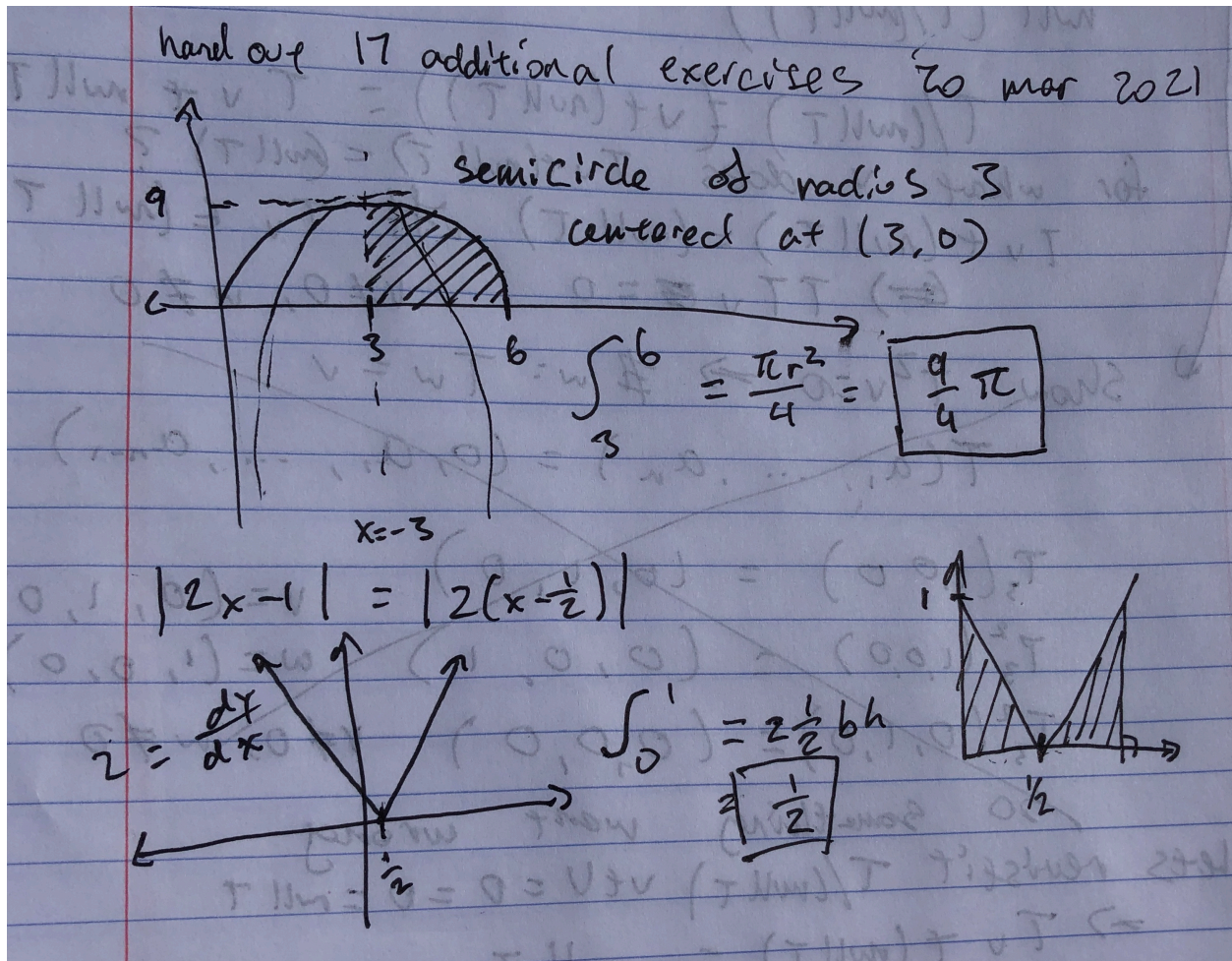


1 | Exercises

1.1 | interpreting in terms of area



1.3 | subtracting integrals

I expect

$$\int_1^2 f(x) dx = \int_1^5 f(x) dx - \int_2^5 f(x) dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

1.4 | **show** $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$

$$\begin{aligned}
 \int_a^b x^2 dx &= \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{b-a}{n} \left(a + k \frac{b-a}{n} \right)^2 \\
 &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=0}^n \left(a + k \frac{b-a}{n} \right)^2 \\
 &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left(na^2 + \sum_{k=0}^n 2ak \frac{b-a}{n} + \left(k \frac{b-a}{n} \right)^2 \right) \\
 &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=0}^n a^2 \\
 &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=0}^n a^2 \\
 &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=0}^n a^2 + \cancel{2ak \frac{b-a}{n}}^0 + \cancel{\left(k \frac{b-a}{n} \right)^2}^0
 \end{aligned}$$