

**Source:**

## 1 | eigenvalues

eigenvalue: multiplied by a scalar? a subspace that, when put through a linear map, only gets scaled.

$$Tv = \lambda v$$

Where  $v \neq 0$ . (we ignore it because its no fun to send zero to zero, and bc the span is empty).

**T must be an operator!** Otherwise the matrix sizes don't work out when subtracting  $\lambda I$ .

where  $v$  is the eigenvector and  $\lambda$  is the eigenvalue. The equation is often rewritten as:

$$Tv - \lambda v = 0Tv - \lambda Iv = 0(T - \lambda I)v = 0$$

We want  $T - \lambda I$  to be singular, because we want the null space to include  $v$ . So we subtract  $\lambda$  from the  $I$  diagonal of  $T$  and then find values of  $v$  which are equal to zero?

now this can be factored and roots can be found. also it's an operator.

### 1.1 | Axler 5.6 equivalent conditions

Only when  $V$  is finite dimensional!

1.1.1  $|T - \lambda I$  is not injective, because both  $v, 0$  are in the null space.

1.1.2  $|T - \lambda I$  is also not surjective or invertible bc finite dim operator.

## 2 | an example

Given the matrix  $\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ , find the eigenvalues and eigenvectors.

Now that we have that other fomulation, we can just subtract  $\lambda I$  from  $T$  to get

$$\begin{pmatrix} 3 - \lambda & 1 \\ 0 & 2 - \lambda \end{pmatrix}$$

Then, we just need to find whether it is non-invertible aka singular aka determinant.

$$(3 - \lambda)(2 - \lambda) = 0$$

The solutions are  $\lambda = 2$  or  $3$ . These are the eigenvalues.

Now just plug in  $\lambda$  and find the null space using RREF. The null space for  $\lambda = 3$  has null space  $\text{span}(x, 0)$ , so we just pick one of those vectors (ex.  $(1, 0)$ ) to be the eigenvector.

## 2.1 | **review of terms**

2.1.1 |  $\text{span}(1, 0)$  is an invariant subspace. (also whatever you get for  $\lambda = 2$ )

2.1.2 | **any vector in an invariant subspace is an eigenvector**

2.1.3 | **the eigenvalues are 2, 3**

## 2.2 | **general idea**

the point of eigenvectors is to figure out where other vectors go by looking at pieces that only get stretched or shrunk.

## 3 | **depends on**

3.1 | **finding roots is helpful**