

1 | Export

<https://www.overleaf.com/project/606b2fa8be363f9005d8ce03>

2 | Exercise 7

Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely $\dim E(\lambda, T)$ times.

3 | Proof

We will show that for each eigenvalue λ , there are at least $E(\lambda, T)$ occurrences of that eigenvalue and at most $E(\lambda, T)$ occurrences.

Suppose $\dim E(\lambda, T) = m$

Suppose λ is on the diagonal m times. Each of those occurrences corresponds to where the diagonal matrix sends a (linearly independent) basis eigenvector, which implies that the basis of V has at least m eigenvectors corresponding to λ . These m eigenvectors can be extended to a basis of $E(\lambda, T)$, implying that $\dim E(\lambda, T) \geq m$.

3.1 | **TODO** Suppose λ is on the diagonal m times. show that there are at most $\dim E(\lambda, T)$ occurrences on the diagonal