Source:

1 | Definitions

1.1 | Linear Map

A linear map is a function/map from one vector space to another such that it satisfies the properties of additivity and homogeneity. Notationally, a linear map $T \in \mathcal{L}(V, W)$ satisfies $T(a)+T(b) = T(a+b) : a, b \in V$ and $\lambda Ta = T(\lambda a) : \lambda \in \mathbb{F}, a \in V$

1.2 | Null Space

The null space of a linear map is the space of vectors that are sent to 0 by T, aka $\{v : v \in V \wedge Tv = 0\}$

1.3 | Column Space

The column space of a linear map is the subspace of the codomain that is an output to the map, aka $\{w : Tv = w, v \in V, w \in W\}$

1.4 | Homogeneous system of equations

A system of equations where all the right hand sides are 0.

1.5 | Injective

When each element in the column space of a map is mapped to by exactly one element in the domain, aka when $Tu = Tv \implies u = v$.

1.6 | Surjective

When every element in the codomain is mapped to, aka the column space is the codomain, aka $W = \{Tv : v \in V\}$.

2 | Fundamental theorem of linear maps

In a map $T \in \mathcal{L}(U, V)$ where U is finite dimensional, dim $U =$ dim range $T +$ dim null T. Intuitively, the dimension of the input space is the dimension of everything that gets sent to zero plus everything that doesn't get sent to zero.

3 | Why is the range also called the "column space"?

When a linear map is thought of as a matrix, (which Jana promises is always possible,)