

## 1 | upper triangular matrix def

A matrix in which all entries below the diagonal are zero

$$\begin{pmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

### 1.1 | results

#### 1.1.1 | Axler 5.26 Conditions for upper-triangular matrix

Suppose  $T \in \mathcal{L}(V)$  and  $v_1, \dots, v_n$  is a basis of  $V$ . The following are equivalent:

- the matrix of  $T$  with respect to  $v_1, \dots, v_n$  is upper triangular
- $Tv_j \in \text{span}(v_1, \dots, v_j)$  for each  $j = 1, \dots, n$
- The span of each prefix of the basis is invariant under  $T$ .

#### 1.1.2 | Axler 5.27 Over $\mathbb{C}$ , every operator has an upper-triangular matrix

Suppose  $V$  is a finite-dimensional complex vector space and  $T \in \mathcal{L}(V)$ . Then  $T$  has an upper-triangular matrix wrt some basis of  $V$ .

##### 1. intuition

There are  $n$  eigenvalues (fundamental theorem of linear algebra) and each one should have a corresponding eigenvector that can sweep out a column? What happens when an eigenvalue has higher multiplicity?

##### 2. proof

- (a) induction on the dimension of  $V$ . use the fact that the first column can be found, then use the remaining basis vectors as a smaller subspace and do the same thing?

#### 1.1.3 | Axler 5.30 Determination of invertibility from upper-triangular matrix

Suppose  $T \in \mathcal{L}(V)$  has an upper-triangular matrix wrt some basis of  $V$ . Then,  $T$  is invertible iff all the entries on the diagonal of the upper-triangular matrix are nonzero.

##### 1. intuition

- (a) if one of the diagonal vectors is zero, then there is an injectivity/surjectivity problem and the operator is singular
- (b) proof is by assuming all are nonzero and showing surjective, then by contradiction.

#### 1.1.4 | Axler 5.32 Determination of eigenvalues from upper-triangular matrix

Suppose  $T \in \mathcal{L}(V)$  has an upper-triangular matrix wrt some basis of  $V$ . Then the eigenvalues of  $T$  are precisely the entries on the diagonal of that upper-triangular matrix.

1. proof

$$\mathcal{M}(T) = \begin{pmatrix} \lambda_1 & & & * \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$