

Source:

1 | **cube root**

1.1 | **approximation**

$$(1+x)^{\frac{1}{3}} \rightarrow \frac{1}{3}(1+x)^{-\frac{2}{3}}$$

at $x = 0$ is

$$\frac{1}{3}(1+0)^{-\frac{2}{3}} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

1.2 | **estimations**

value	estimate
0.05	1.016666
-0.25	0.916666

These will be overestimates because the graph is concave down in this region.

2 | **sin(x)**

2.1 | **approximation**

$$y \approx \frac{d}{dx} \sin x \Big|_0 (x-0) + \sin 0 = x$$

2.2 | **estimates**

3 | **unknown function (only some points known)**

3.1 | **approximation**

$$y \approx \frac{d}{dx} f(x) \Big|_c (x-c) + f(c)$$

plugging in $c = 1$,

$$y \approx 5(x-1) - 4$$

3.2 | **estimations**

value	estimate
1.2	-3

This will be an underestimate because the second derivative is positive and the graph is thus concave up.