

1 | Axler6.31 Gram-Schmidt Procedure

The Gram-Schmidt Procedure is used to turn a list into an orthonormal list with the same span. It's useful for finding orthonormal bases.

Suppose v_1, \dots, v_m is a linearly independent list of vectors in V . Let $e_1 = v_1/\|v_1\|$. For $j = 2, \dots, m$, define e_j inductively by

$$e_j = \frac{v_j - \langle v_j, e_1 \rangle e_1 - \dots - \langle v_j, e_{j-1} \rangle e_{j-1}}{\|\text{<numerator>}\|}$$

Then e_1, \dots, e_m is an orthonormal list of vectors in V s.t. each prefix span is the same as in v_1, \dots, v_m .