

Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V . Prove that U is invariant under T iff U^\perp is invariant under T^* .

For all $u \in U$, $Tu = u' \in U$. Let $w \in U^\perp$. Then, $\langle T^*w, u \rangle = \langle w, Tu \rangle = \langle w, u' \rangle = 0$

$$\langle u, T^*w \rangle = \langle Tu, w \rangle = \langle u', w \rangle$$

Let $u \in U$ and $w \in U^\perp$. Then,

$$\langle Tu, w \rangle = 0$$

$$\langle u, T^*w \rangle = 0$$

This implies that both directions, since $U = U^{\perp\perp}$ and $T = (T^*)^*$.