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#flo

Talking about the reading (vector spaces)

Vector space

Identity

- It would be the additive identity, because the multiplicative one doesn't count because multiply doesn't take two elements from the same field ##### Operations
- Scalar multiplication
 - Not a multiplication on V
 - We need another field of scalars
 - Fundamental difference: **operates on different objects** (only happens on scalar multiplications)
- addition ##### Linearity
- Something that's linear means "things work for addition and scalar multiplication"
- Take $-2x + 1y = 3$
 - Multiplying by scalars
 - adding them
 - similar to a line in standard form—slope stays constant
- Take $2x - 3y + 1z = 2$
 - a plane in 3d
 - if you pick a direction, the slope stays the same
 - thus, a plane is linear ##### Vector
- Something in a vector space
- infinite lists
 - It's like decimals, except you can choose any number instead of just 0-9
 - base infinity basically
- Most common vector space
 - \mathbb{F}^n , like \mathbb{R}^3 (might also be \mathbb{C}^2 or something, although that's hard to visualize)
 - #definition canonical
 - something "standard", basically everyone should know what you are talking about
 - canonical vector space is \mathbb{R}^2 ##### Distributive property
- Important to tie operations together

Vector Space as a Set of Functions

- like $\mathbb{R}^{[0,1]}$: the functions from $[0, 1]$ that end up as real numbers
 - Identity = $f(x) = 0$ ##### Subspaces
- A subspace of this has to be a group on its own
- Conditions for a subspace
 - See 1.34
 - Just check
 - additive identity
 - closed under addition

- closed under scalar multiplication
- What other subspaces of this vector space are there that also have a domain from $[0, 1]$?
 - Like continuous functions from zero to one
 - functions whose derivatives are continuous or constant or zero
 - even functions are also a subspace [KBe20math530srcEvenFunctionsAreSubspacesOfFlotheS.png](#)
- Subspaces of \mathbb{F}^3
 - Most contain infinite vectors (except $\{0\}$)
 - $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ is a subspace with infinite vectors #### Notation
- #note \mathbb{F}^2 is almost always either \mathbb{R}^2 or \mathbb{C}^2 , mostly \mathbb{R}^2

Direct sums

- Something that isn't a direct sum
 - in \mathbb{R}^3 , $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$
 - Two ways to write 0:
 - $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ ### \mathbb{F}^∞
- Functions from naturals to your field, (assign an element to each natural)
 - that would be the same as ordering the elements in your field?
 - Tons of functions, any one is an infinite vector??

If and Only If proofs (iff)

- You have to take the proof in both directions
- **Assumption:** "now suppose the only way to write 0 as a sum of $u_1 + \dots + u_m$, where each u_j is in U_j , is by taking each u_j equal to 0"
 - Assume the red part, then show the green part. Then, assume the green and show it gets the red.
 - [KBe20math530srcIfOnlyIfProofs.png](#)
- #future geometrical interpretation of determinants