1 | $\int \frac{\sqrt{x-1}}{x}dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}}dx$

$$\begin{aligned}
\int \frac{\sqrt{x-1}}{x}dx &= \int \frac{u}{u^2+1}2udu \\
&= 2\int \frac{(u^2+1)-1}{u^2+1}du \\
&= 2\int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1}du \\
&= 2\int 1du - \frac{1}{u^2+1} + C \\
&= 2\int 1du - \tan^- u + C \\
&= 2u - \tan^- u + C \\
&= \boxed{2\sqrt{x-1} - \tan^-\left(\sqrt{x-1}\right) + C}
\end{aligned}$$

2 | $\int \frac{x^2}{x^2+1}dx$

Let $u = x^2+1$, $du = 2xdx$

$$\begin{aligned}
\int \frac{x^3}{x^2+1}dx &= \frac{1}{2}\int \frac{u-1}{u}du \\
&= \frac{1}{2}\left(u - \int \frac{1}{u}du\right) + C \\
&= \frac{1}{2}\left(u - \ln u\right) + C \\
&= \boxed{\frac{1}{2}\left(x^2+1-\ln(x^2+1)\right) + C}
\end{aligned}$$

3 | **TODO** $\int \frac{x-4}{x^2}dx$

$$\int \frac{x-4}{x^2}dx =$$

4 | $\int (x+1)e^{x^2+2x}dx$

Let $u = x^2+2x$, $du = x+1$

$$\begin{aligned}
\int (x+1)e^{x^2+2x}dx &= \frac{1}{2}\int e^u du \\
&= e^u \\
&= e^{x^2r+2x}
\end{aligned}$$

5 | $\int \tan^2 x + 1 dx$

$$\begin{aligned}\int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx \end{aligned}$$

Let $u = x, du = 1$

$$\begin{aligned}&= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C}\end{aligned}$$

6 | **7**

7 | $\int \frac{e^x - 1}{e^x} dx$

$$\begin{aligned}\int \frac{e^x - 1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C}\end{aligned}$$

8 | $\int \frac{\sec^2 x}{\csc x} sinx dx$

$$\begin{aligned}\int \frac{\sec^2 x}{\csc x} sinx dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x}\end{aligned}$$

9 | **$∫ sin x cos x dx $**

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned}\int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x}\end{aligned}$$

10 | **TODO** $\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx$

$$\begin{aligned}\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx &= \int \frac{\sin^2 x+\cos^2 x}{\tan^2 x+1}dx\\ &= \int \frac{\sin^2 x}{\tan^2 x+1}+\frac{\cos^2 x}{\tan^2 x+1}dx\\ &= \int \sin^2 x\cos^2 x+\cos^4 x dx\end{aligned}$$

$$\begin{aligned}&= \int \frac{1}{\tan^2 x+1}dx\\ &= \int \frac{1}{\sec^2 x}dx\\ &= \int \cos^2 x dx\end{aligned}$$

11 | $\int \frac{\sec x\tan x}{1+\sec^2 x}dx$

Let $$,

12 | **13**

13 | **14**