Source:

1 | Definition

#definition Axler3.2 Linear Map #aka linear transformation A *linear map* from V to W is a function $T : V \to W$ with the following properties:

1.1 | Additivity

$$T(u+v) = Tu + Tv \forall u, v \in V$$

1.2 | Homogenity

$$T(\lambda v) = \lambda(Tv) \forall \lambda \in \mathbb{F}, v \in V$$

2 | Other Notation

2.1 | Set of Maps

#definition Axler3.3 $\mathcal{L}(V, W)$

> The set of all linear maps from V to W is denoted $\mathcal{L}(V, W)$.

3 | Examples

3.1 | zero (0)

Zero is a function $0 : V \to W$ s.t. $0v = 0 \forall v \in V$. (It takes all vectors in V and maps them to the additive identity of W)

3.2 | identity (I)

The identity maps each from one vector space to itself (in the same vector space):

$$I \in \mathcal{L}(V, V), v \in V : Iv = v$$

3.3 | differentiation (D)

$$D \in \mathcal{L}\left(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R})\right) : Dp = p'$$

Basically stating that for two polynomials $a, b \in \mathcal{P}(\mathbb{R})$, $a' + b' = (a+b)'$ and with a constant $\lambda \in \mathcal{R}$ $(\lambda a)' = \lambda a'$.

3.4 | integration

3.5 | multiplication by x^2

$$T \in \mathcal{L}\left(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R})\right) : (Tp)(x) = x^2 p(x)$$

is a linear map

3.6 | **backward shift**

F^∞ is the vector space of all sequences of elements in $\mathbb{F}$.

$$T \in \mathcal{L}(\mathbb{F}^\infty, \mathbb{F}^\infty) : T(x_1, x_2, x_3, \ldots) = (x_2, x_3, \ldots)$$

3.7 | $\mathbb{F}^n \to \mathbb{F}^m$

Given a ”coefficent matrix” $A : A_{j,k} \in \mathbb{F} \forall j = 1, \ldots, m; \forall k = 1, \ldots, n$