Source:

1 | find taylor series

1.1 $|y = \cos(x)|$

$$P_n(x) = f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 + \frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 + \cdots$$

$$= \cos(0) - \sin(0)x + \frac{-\cos(0)}{2!}x^2 + \frac{\sin(0)}{3!}x^3 + \cdots$$

$$= 1 -0x - \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \cdots$$

$$= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k}$$

1.2 | $y = e^x$

$$P_n(x) = f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 + \frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 + \cdots$$

$$= e^0 + e^0x + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \cdots$$

$$= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

1.3 | $y = \sqrt{x}$ centered at x = 1

$$P_n(x) = f(1) + \frac{d}{dx}f(1)(x-1) + \frac{\frac{d^2}{d^2x}f(1)}{2!}(x-1)^2 + \frac{\frac{d^3}{d^3x}f(1)}{3!}(x-1)^3 + \cdots$$

$$= 1 + \frac{1}{2}(x-1) + \frac{\frac{1}{2}\frac{-1}{2}}{2!}(x-1)^2 + \frac{\frac{1}{2}\frac{-1}{2}\frac{-3}{2}}{3!}(x-1)^3 + \cdots$$

I don't know how to write it using summation notation though...

2 | prove approximations

2.1
$$\left| \frac{1}{1-x} \right| = 1 + x + x^2 + x^3 + \cdots$$

Proof by geometric series

$$2.2 \mid \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

Plug -x for x in the previous equation.

$$2.3 \mid \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$$

Plug x^2 for x in the previous equation.

3 | more finding of polynomial

$$3.1 \mid TODO \$ y = In (1+x)\$$$

3.2 | **TODO**
$$y = \tan^- x$$

$$3.3 \mid y = (1+x)^k$$

$$P_{n}(x) = f(0) + \frac{d}{dx}f(0)x + \frac{\frac{d^{2}}{d^{2}x}f(0)}{2!}x^{2} + \frac{\frac{d^{3}}{d^{3}x}f(0)}{3!}x^{3} + \cdots$$

$$= 1 + k(1)^{k} + k(k-1)(1)^{k-1}x + \frac{k(k-1)(k-2)(1)^{k-2}}{2!}x^{2} + \frac{k(k-1)(k-2)(k-3)(1)^{k-3}}{3!}x^{3} + \cdots$$

$$= 1 + k + k(k-1)x + \frac{k(k-1)(k-2)}{2!}x^{2} + \frac{k(k-1)(k-2)(k-3)}{3!}x^{3} + \cdots$$

$$= 1 + k + \frac{k!}{(k-1)!}x + \frac{\frac{k!}{(k-2)!}}{2!}x^{2} + \frac{\frac{k!}{(k-3)!}}{3!}x^{3} + \cdots$$

$$= 1 + k + \frac{k!x}{(k-1)!} + \frac{k!}{(k-2)!2!}x^{2} + \frac{k!}{(k-3)!3!}x^{3} + \cdots$$

$$= \binom{k}{0} + \binom{k}{1}x + \binom{k}{2}x^{2} + \binom{k}{3}x^{3} + \cdots$$

$$= \sum_{k=0}^{\infty} \binom{k}{i}x^{i}$$

4 | find sum of series by recognizing Taylor Series approximations of some functions

$$4.1 \mid 3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$$

$$3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} = e^3 - 1$$

$$4.2 \mid 1 - \ln 2 + \frac{\ln^2 2}{2!} + \frac{\ln^3 2}{3!} + \cdots$$

$$1 - \ln 2 + \frac{\ln^2 2}{2!} + \frac{\ln^3 2}{3!} + \dots = e^{-\ln 2} = \frac{1}{2}$$

4.3 |
$$\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k+1}}{4^{2k+1}(2k+1)!}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k+1}}{4^{2k+1} (2k+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{4}^{2k+1}\right)}{(2k+1)!} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

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5 | evaluate limits using taylor series approx

$$5.1 \mid \lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

$$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

$$= \lim_{x \to 0} \frac{\cancel{x} - \frac{x^3}{6} + \frac{x^5}{5!} + \cdots + \frac{x^3}{6}}{x^5}$$

$$= \lim_{x \to 0} \frac{\cancel{x} + \frac{x^5}{5!} + \cdots + \cancel{x}}{x^5}$$

$$= \lim_{x \to 0} \frac{\cancel{x} + \frac{x^5}{5!} + \cdots}{x^5}$$

$$= \lim_{x \to 0} \frac{1}{5!} + \frac{x^7}{x^57!} + \frac{x^9}{x^59!} \cdots$$

$$= \frac{1}{5!}$$

5.2 | **TODO**
$$\lim_{x\to 0} \frac{x-\tan^- x}{x^3}$$

6 | find taylor series approximations

6.1
$$|y = e^x + e^{-x}$$

$$e^{x} + e^{-x}$$

$$= 1 + x + \frac{x^{2}}{2!} \frac{x^{3}}{3!} + \dots + 1 - x + \frac{x^{2}}{2!} - \frac{x^{3}}{3!}$$

$$= 1 + 1 + x - x + \frac{x^{2}}{2!} + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} - \frac{x^{3}}{3^{1}} + \dots$$

$$= 2\left(1 + \frac{x^{2}}{2!} + \frac{x^{4}}{4!} + \dots\right)$$

$$= 2\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

$$6.2 \mid y = \sin(\pi x)$$

$$\sin(\pi x) = \sum_{k=0}^{\infty} \frac{(-1)^k (\pi x)^{2k+1}}{(2k+1)!}$$

(just plug it in)

6.3 | **TODO**
$$y = \sin^2 x$$