1 | **A real valued matrix**

Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

$$AA^T = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$
$$A^TA = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{pmatrix}$$

Then, A^TA is the same thing, but with b, c swapped.

2 | **For complex matrices**

$$\begin{pmatrix} a+bi & c+di \\ f+gi & j+ki \end{pmatrix}\begin{pmatrix} a+bi & f+gi \\ c+di & j+ki \end{pmatrix} = \begin{pmatrix} a^2-b^2+2abi+c^2-d^2+2cdi & \\ & f^2-g^2+2fgi+j^2-k^2+2jki \end{pmatrix}$$