1 | **orthogonal decomposition**

An orthogonal decomposition is a way of writing some vector $v \neq 0 \in V$ as the scaled other vector $u \in V$ plus an orthogonal component

> Suppose $u, v \in V$, with $v \neq 0$. Set $c = \frac{\langle u, v \rangle}{\|v\|^2}$ and $w = u - cv$. Then,

$$\langle w, v \rangle = 0 \text{ and } u = cv + w$$

The important algebra is just setting up a system of equations and noticing that orthogonality implies

$$
\begin{aligned}
0 &= \langle u - cv, v \rangle \\
\Longrightarrow 0 = \langle u - cv, v \rangle \quad &= \langle u, v \rangle - \langle cv, v \rangle \\
&= \langle u, v \rangle - c \langle v, v \rangle \\
&= \langle u, v \rangle - c \|v\|^2
\end{aligned}
$$