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#ret

## Exercises

### 1.A.2

$$\left(\frac{-1 + \sqrt{3}i}{2}\right)^3 = \left(\frac{-1 + \sqrt{3}i}{2}\right) \left(\frac{-2 - 2\sqrt{3}i}{4}\right) = \frac{2 + \cancel{2\sqrt{3}i} - 2\sqrt{3}i - 2 * 3 * i^2}{8} = \frac{8}{8} = 1$$

### 1.A.10

$$\begin{aligned}(4, -3, 1, 7) + 2(x_1, x_2, x_3, x_4) &= (5, 9, -6, 8) \\ 4 + 2x_1 &= 5, \\ -3 + 2x_2 &= 9, \\ 1 + 2x_3 &= -6, \\ 7 + 2x_4 &= 8 \\ x &= \left(\frac{1}{2}, 6, \frac{-7}{2}, \frac{1}{2}\right)\end{aligned}$$

Not sure how to do this with matrices?

### 1.A.15

$$\begin{aligned}\lambda(x + y) &= \lambda(x_1 + y_1, x_2 + y_2, x_3 + y_3 \dots x_n + y_n) \\ &= (\lambda(x_1 + y_1), \lambda(x_2 + y_2), \lambda(x_3 + y_3) \dots \lambda(x_n + y_n)) \\ &= (\lambda x_1 + \lambda y_1, \lambda x_2 + \lambda y_2, \lambda x_3 + \lambda y_3 \dots \lambda x_n + \lambda y_n) \\ &= (\lambda x_1, \lambda x_2, \lambda x_3 \dots \lambda x_n) + (\lambda y_1, \lambda y_2, \lambda y_3 \dots \lambda y_n) \\ &= \lambda(x) + \lambda(y) = \lambda x + \lambda y\end{aligned}$$

## Matrices for Solving Systems

I'm not sure what I should notice, although it's interesting that they are all 2x2 matrices that are (or can be decomposed into) one number away from the identity. I think we mentioned that they were "essential matrices" or something?

## Geometric Interpretation of Dot Product

We talked about it in class, and learned it in physics, but a dot product  $A \cdot B$  can be interpreted as the magnitude of  $A$ 's projection onto  $B$  multiplied by the magnitude of  $B$ .  $A \cdot B = |A||B|\cos\theta$ .

## Dot Product on Vectors as a Group

No. Dot product returns a scalar, which means that this operation is distinctly not closed.

After class on 3 Sep, Daniel mentioned that it might be a group if you define a modified dot product where you take the normal dot product and put it in the direction of the second vector. However, this doesn't

work because the for any given  $N \times 1$  matrix  $A$  the identity  $e$  has to satisfy  $A \cdot e = e \cdot A = A$ . Thus, the definition that relies on the direction of the second operand will break when the identity is on one of the sides. **Because dot product relies on the angle between the two vectors, I think it would be difficult to find an angle for an identity vector that works with all other angles of vectors. I'm not sure how to formalize this... #todo**

## Inverse of a matrix

I tried this for the previous homework when we were to determine if  $2 \times 2$  matrices were groups under multiplication, but didn't end up getting anywhere. I will try again...

srcIdentityMatrixFormula.png

I got something like  $w = \frac{1 - \frac{bc}{ad}}{a}$ , which I don't think is correct. It's also been an hour and a half, so I think I'll have to leave this here for now. #todo

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