

3 | Exercise 2.A.1

Lemma

If vectors v_1, v_2, v_3, v_4 span V , then the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

also spans V .

Proof

We prove the lemma by showing that any vector $v \in V$ can be written in the form $a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4$ can also be written as a linear combination of the form

$$b_1(v_1 - v_2) + b_2(v_2 - v_3) + b_3(v_3 - v_4) + b_4 v_4$$

If we set

$$b_1 = a_1$$

$$b_2 = a_1 + a_2$$

$$b_3 = a_1 + a_2 + a_3$$

$$b_4 = a_1 + a_2 + a_3 + a_4$$

then the two combinations will be equivalent:

$$\begin{aligned} & a_1(v_1 - v_2) + (a_1 + a_2)(v_2 - v_3) + (a_1 + a_2 + a_3)(v_3 - v_4) + (a_1 + a_2 + a_3 + a_4)v_4 \\ &= \cancel{a_1 v_1 - a_1 v_2 + a_1 v_2} + \cancel{a_2 v_2 - (a_1 + a_2)v_3 + (a_1 + a_2)v_3} + \cancel{a_3 v_3 - (a_1 + a_2 + a_3)v_4 + (a_1 + a_2 + a_3)v_4} \\ &= a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 \end{aligned}$$
