

Source: [KBe2020math530floIndex](#)

- Vector spaces and fields are like groups
 - With 2 operations
- Vector
 - direction and magnitude
 - numbers with an order
 - list = ordered set
 - $N \times 1$ matrix
 - A vector is not just an $N \times 1$ matrix. **A vector exists in a vector space**
 - might be full of physics vectors, matrices, or polynomials
- Field
 - Addition and multiplication might be different
 - might be related to normal addition/multiplication
 - might be any binary operation
 - Addition is “primary” operation, multiplication is “secondary”
 - addition is really good (more group like)
 - multiplication needs to exclude the additive identity (because it can't have an inverse)
 - questions
 - multiplication is repeated addition?
 - not necessarily
 - binary expressions?
 - associative?
 - both yes
 - 1.3 demonstrates that the complex numbers are a field
 - commutativity
 - associativity
 - identities
 - additive inverse
 - multiplicative inverse except additive identity
 - distributive
 - usually means Reals or Complex
 - integers mod 3 are a field
 - #bonushw show integers mod 3 are a field
 - higher dimensions
 - R^2 is a Cartesian plane, R^4 is a space
 - operations
 - addition is really nice (element wise)
 - scalar multiplication is easy enough
 - vector vector multiplication is hard to find
- two square roots of i
 - fundamental theorem of algebra
 - (important)
 - So, i should have two square roots
 - Powers of i go in a circle (90 degrees rotation)
 - Complex number rotation gives a preferred direction

- So that's why the quadrants are numbered in that direction
 - One can be found geometrically 20math530srcSquareRootl.png
 - We could also try it algebraically
 - $(a + bi)^2 = i = a^2 - b^2 + 2abi$
 - so $a^2 - b^2 = 0$ and $2ab = 1$
 - dot product
 - How much of \vec{A} is in the direction of \vec{B} multiplied by the magnitude of \vec{B}
 - $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$
 - #bonushw prove that $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$
 - Identity: $\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \cos\theta$
 - cross product
 - only works on 3x1 matrices
 - steps
 - determinant
 - i, j, k are the unit vectors
 - $$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{vmatrix} = i \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$
 - dropping zero: $0a = (0 + 0)a = 0a + 0a \Rightarrow 0a = 0$, so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
 - 20math530srcFieldsMultiplyCannotBeGroup.png
 - determinant
 - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
 - #todo #exr0n #future prove identities are unique
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