Source:

What sizes of matrix can you add? When can't you add matrices?

Matrices of the same dimensions (because we do it element wise). Maybe you can add a vector to a matrix if the number of rows is equal to the dimensionality of the vector.

What sizes of matrix can you multiply? When can't you multiply matrices?

Multiply: $N \times M * M \times K => N \times K$.

Multiply
$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
by vectors in $\mathbb{R}^2$ (for example, you could multiply by $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$).

Can you characterize the transformations you get by multiplying (lots of vectors) by each of these matrices?

Action	Matrix
Identity	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
Select left column	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Select right column	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Treat as expression (linear combination/transformation?)*	$\begin{bmatrix} a \\ b \end{bmatrix}$

*I'm not sure what linear combinations/transformations are, but I think this is somehow related? Anyways, it takes each row i and returns $\sigma A_{i,j} * B_j$

Which of the number systems we discussed today form a group under addition? Under multiplication?

Source: [KBe2020math530refGroups]

Number System	Multiplication	Addition
Natural Numbers	No inverse	No identity
Whole Numbers	No inverse	No inverse
Integers	No inverse	Yes
Rationals	Yes	Yes
Reals	Yes	Yes
Complex Numbers	Yes	Yes