

## 1 | Axler 5.B Exercise 13

Suppose  $W$  is a complex vector space and  $T \in \mathcal{L}(W)$  has no eigenvalues. Prove that every subspace of  $W$  invariant under  $T$  is either  $\{0\}$  or infinite-dimensional.

## 2 | Proof

5.21 states

Every operator on a finite-dimensional, nonzero, complex vector space has an eigenvalue.

$W$  is given as a complex vector space, so to have no eigenvalues, it must be zero or infinite-dimensional. If the subspace is zero, then all subspaces must also be zero. Thus, only the infinite-dimensional case remains to be shown.

By definition (5.14), for all subspaces  $U$  of  $W$  invariant under  $T$ ,  $T|_U$  exists in  $\mathcal{L}(U)$ .

Suppose for the sake of contradiction that  $U$  has an eigenvalue. Then, there exists some  $\lambda$