Source:

1 | find taylor series

1.1 | $y = \cos(x)$

$$
\begin{aligned}
P_n(x) &= f(0) && + \frac{d}{dx} f(0)x && + \frac{\frac{d^2}{d^2x} f(0)}{2!} x^2 && + \frac{\frac{d^3}{d^3x} f(0)}{3!} x^3 && + \cdots \\
&= \cos(0) && - \sin(0)x && + \frac{-\cos(0)}{2!} x^2 && + \frac{\sin(0)}{3!} x^3 && + \cdots \\
&= 1 && -0x && - \frac{1}{2!} x^2 && + \frac{0}{3!} x^3 && + \cdots \\
&= 1 && && - \frac{1}{2!} x^2 && && + \frac{1}{4!} x^4 + \cdots \\
&= \sum_{k=0} \frac{(-1)^k}{(2k)!} x^{2k}
\end{aligned}
$$

1.2 | $y = e^x$

$$
\begin{aligned}
P_n(x) &= f(0) && + \frac{d}{dx} f(0)x && + \frac{\frac{d^2}{d^2x} f(0)}{2!} x^2 && + \frac{\frac{d^3}{d^3x} f(0)}{3!} x^3 && + \cdots \\
&= e^0 && + e^0 x && + \frac{e^0}{2!} x^2 && + \frac{e^0}{3!} x^3 && + \cdots \\
&= 1 && + x && + \frac{1}{2!} x^2 && + \frac{1}{3!} x^3 && + \cdots \\
&= \sum_{k=0} \frac{1}{k!} x^k
\end{aligned}
$$

1.3 | **TODO** $y = \sqrt{x}$

2 | prove approximations

2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Proof by geometric series

2.2 | $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

Plug $-x$ for x in the previous equation.

2.3 | $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$

Plug x^2 for x in the previous equation.