

at everyone

## 1 | Complete the Representation

| Function                | First four terms  | Generalized                                  |
|-------------------------|---|--|
| $\frac{1}{1-2x}$        | $1 + 2x + 4x^2 + 8x^3 + \dots$  | $\sum_{k=0} 2^k x^k$                         |
| $\cos(3x)$              | $1 - \frac{9x^2}{2!} + \frac{81x^4}{4!} - \frac{729x^6}{6!} + \dots$                    | $\sum_{k=0} \frac{(-1)^k (3x)^{2k}}{2k!}$    |
| $\frac{e^x}{e^2}$       | $\frac{1}{e^2} + \frac{x}{e^2} + \frac{x^2}{e^2 2!} + \dots$                            | $\sum_{k=0} \frac{x^k}{e^2 k!}$              |
| $\sin(x^2)$             | $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$                  | $\sum_{k=0} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$ |
| $\frac{1}{1+x^4}$       | $1 - x^4 + x^8 - x^{12} + \dots$  | $\sum_{k=0} (-x^4)^k$                        |
| $e^{x-1}$               | $1 + \frac{(x-1)^2}{2!} + \frac{(x-1)^4}{4!} + \frac{(x-1)^6}{6!} + \dots$              | $\sum_{k=0} \frac{(x-1)^{2k}}{k!}$           |
| $\frac{\cos(x)-1}{x^2}$ | $-\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \dots$                               | $\sum_{k=1} \frac{(-1)^k x^{2(k-1)}}{(2k)!}$ |
| $2x \ln(1+2x)$          | $(2x)(2x) - \frac{(2x)(2x)^2}{2} + \frac{(2x)(2x)^3}{3} - \frac{(2x)(2x)^4}{4} + \dots$ | $\sum_{k=1} \frac{2x(-1)^{k-1}(2x)^k}{k}$    |
| $\frac{x}{1+x^2}$       | $2x - 2x^3 + 2x^5 - 2x^7 + \dots$   | $\sum_{k=0} 2x(-1)^k x^{2k}$                 |

## 2 | page 3

### 2.1 | a: skipped

### 2.2 | find maclaurin series for $f'(x)$ where $f(x) = \sum_{k=0} \frac{(2x)^{k+1}}{k+1}$

$$\frac{d}{dx} \frac{(2x)^{n+1}}{n+1} = \frac{\cancel{(n+1)}^2 (2x)^n (2)}{\cancel{(n+1)}^2} = 2(2x)^n$$

So, our series is just

$$\sum_{k=0} 2(2x)^k = 2 + 4x + 8x^2 + 16x^3 + \dots$$

### 2.3 | estimate $f'(-\frac{1}{3})$

$$2 + 4\left(-\frac{1}{3}\right) + 8\left(-\frac{1}{3}\right)^2 + 16\left(-\frac{1}{3}\right)^3 = \frac{10}{3}$$

## 3 | page 4

### 3.1 | find $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$

That series is just the Taylor series for

$$f(x) = \frac{\sin x}{x}$$

So the derivative at zero is zero, and the second derivative:

$$\begin{aligned} \frac{d}{dx} \frac{x \cos x - \sin x}{x^2} &= \frac{x^2 (-x \sin x + \cos x - \cos x) - (x \cos x - \sin x) (2x)}{x^4} \\ &= \frac{-x^3 \sin x - 2x (x \cos x - \sin x)}{x^4} \end{aligned}$$

is undefined at zero. However, the top of the fraction will be negative ( $x^3 \sin x$  is even and  $x \cos x - \sin x = x(\cos x - \frac{\sin x}{x})$ ), so the second derivative is zero at  $x$ . (Checked with desmos).