

Source: [KBhMATH401Limits](#)

# 1 | Epsilon Delta Proofs

The secrets of the limit

## 1.1 | Formal Definition of a Limit

Definition 1 · **Limit Definition**  $\lim_{x \rightarrow a} f(x) = L$   
for all  $\epsilon > 0$ , there exists a  $\delta$  such that if  $0 < |x - a| < \delta$ , then  $0 < |f(x) - L| < \epsilon$

## 1.2 | An Epsilon Delta Proof

Let's prove  $\lim_{x \rightarrow 2} x^2 = 4$  together!

The crux of the proof is to come up with a value  $\delta$  that is a function of  $\epsilon$  assuming that  $0 < \epsilon$  that meets  $0 < |x - a| < \delta$ .

Oh, here's some symbols

Symbol	Definition
$\forall$	For all
$\exists$	There exists
<i>s.t.</i>	Such that

And so, the formal and pretentious definition of a limit:

$\lim_{x \rightarrow a} f(x) = L$  where  $\forall \epsilon > 0, \exists \delta > 0, s.t. 0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon$ .

This needs to go before **every Epsilon Delta proof**.

- Step 1: Write the Definition Above
- Step 2: Do scratch work to identify delta
- Step 3: Plug it in to verify