

Suppose  $V$  is a complex inner product space and  $T \in \mathcal{L}(V)$  is a normal operator such that  $T^9 = T^8$ . Prove that  $T$  is self-adjoint and  $T^2 = T$ .

If  $T = 0$ , then  $0^2 = 0$  and  $0$  is self-adjoint. Thus, let  $T \neq 0$ .

In 7.1, Axler asserts that  $V$  is finite-dimensional.

$T$  has a diagonal matrix w.r.t. an orthonormal basis of  $V$ .

$$TT^* = T^*T$$

First, we will show that  $T^2 = T$ . Suppose  $T$  is invertible. Then,

$$\begin{aligned} T^9 &= T^8 \\ T^9 T^{-7} &= T^8 T^{-7} \\ T^2 &= T \end{aligned}$$

Suppose  $T$  is not invertible and not equal to zero. Then,  $T$  has some zero entries on its diagonal and some non-zero entries on its diagonal. Let these entries equal  $d_1, \dots, d_n$ .  $T^k$  will have on its diagonal  $d_1^k, \dots, d_n^k$ .