

## 1 | Exercises

### 1.1 | interpreting in terms of area



### 1.3 | subtracting integrals

I expect

$$\int_1^2 f(x) dx = \int_1^5 f(x) dx - \int_2^5 f(x) dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

### 1.4 | show $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$

(see attached pages)

Keep in mind

$$\sum_{k=1}^n af(x) = a \sum_{k=1}^n f(x)$$
$$\sum_{k=1}^n (a + f(x)) = an + \sum_{k=1}^n f(x)$$

$$\begin{aligned}
\int_a^b x^2 dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{b-a}{n} \left( a + k \frac{b-a}{n} \right)^2 \right) \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n \left( a + k \frac{b-a}{n} \right)^2 \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n \left( a^2 + \left( k \frac{b-a}{n} \right)^2 + 2ak \frac{b-a}{n} \right) \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n a^2 + \sum_{k=1}^n \left( k \frac{b-a}{n} \right)^2 + \sum_{k=1}^n 2ak \frac{b-a}{n} \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left( a^2 n + \sum_{k=1}^n k^2 \left( \frac{b-a}{n} \right)^2 + 2a \frac{b-a}{n} \sum_{k=1}^n k \right) \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left( a^2 n + \left( \frac{b-a}{n} \right)^2 \sum_{k=1}^n k^2 + 2a \frac{b-a}{n} \sum_{k=1}^n k \right) \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \left( a^2 n + \left( \frac{b-a}{n} \right)^2 \frac{n(n+1)(2n+1)}{6} + 2a \frac{b-a}{n} \frac{n(n+1)}{2} \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \left( \frac{b-a}{n} \right)^2 \frac{(n+1)(2n+1)}{6} + 2a \frac{b-a}{n} \frac{(n+1)}{2} \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{n^2} \left( n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a \frac{b-a}{n} (n+1) \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{n^2} \left( n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a \cancel{\frac{b-a}{n}} + a \frac{b-a}{n} \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{n^2} \left( n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a(b-a) + a \cancel{\frac{b-a}{n}} \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{n^2} \left( n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a(b-a) \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{n} \frac{(2n+1)}{6} + a(b-a) \right) \\
&= \lim_{n \rightarrow \infty} (b-a) \left( a^2 + \frac{(b-a)^2}{3} + a(b-a) \right) \\
&= \lim_{n \rightarrow \infty} a^2(b-a) + \frac{b^2 + a^2 - 2ab}{3}(b-a) + a(b-a)^2
\end{aligned}$$

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