

Suppose  $V$  is a complex inner product space and  $T \in \mathcal{L}(V)$  is a normal operator such that  $T^9 = T^8$ . Prove that  $T$  is self-adjoint and  $T^2 = T$ .

If  $T = 0$ , then  $0^2 = 0$  and  $0$  is self-adjoint. Thus, let  $T \neq 0$ .

In 7.1, Axler asserts that  $V$  is finite-dimensional.

By the complex spectral theorem,  $T$  has a diagonal matrix w.r.t. an orthonormal basis of  $V$ .

Let these entries equal  $d_1, \dots, d_n$ .  $T^k$  will have on its diagonal  $d_1^k, \dots, d_n^k$ . For each  $d_i$ ,  $d_i^8 = d_i^9$ . The only values in  $\mathbb{F}$  that satisfy this are zero and one; thus every  $d_i$  must be a zero or a one.