

True or false (and give a proof of your answer): There exists $T \in \mathcal{L}(\mathbb{R}^3)$ s.t. T is not self-adjoint but there is a basis of \mathbb{R}^3 consisting of eigenvectors of T .

The real spectral theorem suggests that if a map is self-adjoint, it must have an orthogonal basis of eigenvectors. Thus, we can choose a map which does not have an orthogonal basis of eigenvectors. For instance, choose the eigenpairs

$$T(1, 0, 0) = (1, 0, 0)$$

$$T(0, 1, 0) = (0, 2, 0)$$

$$T(1, 1, 1) = (3, 3, 3)$$

In other words, let $T \in \mathcal{L}(\mathbb{R}^3)$ be defined by