1 | Exercise 7

Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely dim $E(\lambda, T)$ times.

2 | Proof

We will show that for each eigenvalue λ, there are at least $E(\lambda, T)$ occurrences of that eigenvalue and at most $E(\lambda, T)$ occurrences.

Suppose λ is on the diagonal m times. Each of those occurrences corresponds to where the diagonal matrix sends a (linearly independent) basis eigenvector, which implies that the basis of V has at least m eigenvectors corresponding to λ. These m eigenvectors can be extended to a basis of $E(\lambda, T)$, implying that $\dim E(\lambda T) \geq m$.

2.1 | TODO Suppose λ is on the diagonal m times. show that there are at most dim E(;l, T) occurances on the diagonal