Source: [KBe2020math530floIndex]

- Vector spaces and fields are like groups
    - With 2 operations
- Vector
    - direction and magnitude
    - numbers with an order
        - list = ordered set
        - Nx1 matrix
    - A vector is not just an Nx1 matrix. **A vector exists in a vector space**
        - might be full of physics vectors, matrices, or polynomials
- Field
    - Addition and multiplication might be different
        - might be related to normal addition/multiplication
        - might by any binary operation
        - Addition is “primary” operation, multiplication is “secondary”
            - addition is really good (more group like)
            - multiplication needs to exclude the additive identity (because it can’t have an inverse)
        - questions
            - multiplication is repeated addition?
                - not necessarily
            - binary expressions?
            - associative?
                - both yes
        - 1.3 demonstrates that the complex numbers are a field
            - commutativity
            - associativity
            - identities
            - additive inverse
            - multiplicative inverse except additive identity
            - distributive
    - usually means Reals or Complex
        - integers mod 3 are a field
            - #bonushw show integers mod 3 are a field
    - higher dimensions
        - R^2 is a Cartesian plane, R^4 is a space
    - operations
        - addition is really nice (element wise)
        - scalar multiplication is easy enough
        - vector vector multiplication is hard to find
- two square roots of i
    - fundamental theorem of algebra
        - (important)
    - So, i should have two square roots
    - Powers of i go in a circle (90 degrees rotation)
        - Complex number rotation gives a preferred direction

    - So that's why the quadrants are numbered in that direction
  - One can be found geometrically 20math530srcSquareRootI.png
  - We could also try it algebraically
    - $(a+bi)^2 = i = a^2 - b^2 + 2abi$
    - so $a^2 - b^2 = 0$ and $2ab = 1$
- dot product
  - How much of $\vec{A}$ is in the direction of $\vec{B}$ multiplied by the magnitude of $\vec{B}$
  - $\vec{A} \cdot \vec{B} = |A||B|cos\theta$
    - #bonushw prove that ^^
  - Identity: $\frac{\vec{A} \cdot \vec{B}}{|A||B|} = cos\theta$
- cross product
  - only works on 3x1 matrices
  - steps
    - determinant
    - i, j, k are the unit vectors
    - $$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \left| \begin{bmatrix} i & j & k \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \right| = i \left| \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \right| - j \left| \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \right| + k \left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right| = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$
- dropping zero: $0a = (0+0)a = 0a + 0a \Rightarrow 0a = 0$, so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
  - 20math530srcFieldsMultiplyCannotBeGroup.png
- determinant
  - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
- #todo #exr0n #future prove identities are unique
