12 | **estimate area**

Right handed Riemann Sum:

$$0.5 + 4 + 10 + 13 + 10 + 0 = 37.5$$

13 | **estimate area again**

$$4(200 + 2700 + 1100 + 4000 + 200) = 32800$$

14 | **area between curves**

$$\int_0^{10} 2200e^{0.024t}dx - \int_0^{10} 1360e^{0.018t}dx = \frac{1}{0.024}2200e^{0.024t} - \frac{1}{0.018}1360e^{0.018t}$$

$$\implies \quad \frac{1}{0.024}2200e^{0.24} - \frac{1}{0.018}1360e^{0.18} - \frac{1}{0.024}2200 + \frac{1}{0.018}1360 \quad \approx 9964$$

15 | **meaning of area**

The shaded region represents the profit made between producing 50 units and 100 units.

16 | **TODO slicing pizza into three using parallel cuts**

The problem of placing slices is the same if we only worry about the top half of the pizza. Thus, we can choose some x for the first slice s.t.

$$2\int_{-7}^{x}\sqrt{7^2-t^2}dt=\int_{x}^{7}\sqrt{7^2-t^2}dt$$
$$2\int_{-7}^{x}\sqrt{7^2-t^2}dt-\int_{x}^{7}\sqrt{7^2-t^2}dt=0$$
$$2\int_{-7}^{x}\sqrt{7^2-t^2}dt+\int_{7}^{x}\sqrt{7^2-t^2}dt=0$$
$$2\left(\int_{0}^{x}\sqrt{7^2-t^2}dt-\int_{0}^{-7}\sqrt{7^2-t^2}dt\right)+\left(\int_{0}^{x}\sqrt{7^2-t^2}dt-\int_{0}^{7}\sqrt{7^2-t^2}dt\right)=0$$
$$2\left(\int_{0}^{x}\sqrt{7^2-t^2}dt+\int_{-7}^{0}\sqrt{7^2-t^2}dt\right)+\left(\int_{0}^{x}\sqrt{7^2-t^2}dt-\int_{0}^{7}\sqrt{7^2-t^2}dt\right)=0$$
$$2\int_{0}^{x}\sqrt{7^2-t^2}dt+2\int_{-7}^{0}\sqrt{7^2-t^2}dt+\int_{0}^{x}\sqrt{7^2-t^2}dt-\int_{0}^{7}\sqrt{7^2-t^2}dt=0$$
$$3\int_{0}^{x}\sqrt{7^2-t^2}dt+2\int_{-7}^{0}\sqrt{7^2-t^2}dt-\int_{0}^{7}\sqrt{7^2-t^2}dt=0$$
$$3\int_{0}^{x}\sqrt{7^2-t^2}dt+\int_{-7}^{0}\sqrt{7^2-t^2}dt=0$$
$$3\int_{0}^{x}\sqrt{7^2-t^2}dt+\frac{49\pi}{4}=0$$

Now, we need to use trigonometric substitution, apparently.$x=$

$$\int_{-7}^{7}\sqrt{49-x^2}-adx=\frac{49\pi}{3}$$

17 | **TODO tractrix**

At any moment, if the boat is at (x,y) and the puller is at $(0,h)$, then the velocity of the boat is in the direction

$$\frac{y-h}{x}$$

18 | **TODO water displacement**

Plan: find a function $f(r)$ which represents the amount of water displaced for any radius, then take the derivative and find roots.