Source: [[KBMATH401ComputingLimits]]

1 | Solving Limits with Elimination

With solving limits via elimination, we are tipically analyzing a rational function that needs factoring of a term out of the polynomials on the top and/or the bottom to get out of the indeterminate form $(\frac{0}{0})$.

- Try factoring both the top and bottom
  - $(x \pm 1)$
  - $(x \pm 2)$
- Rationalize all of the square roots

Tip for picking factors

Tip! If you plug in a value to an expression, and out pops 0, that value is a **zero** of the expression. It is helpful like this

Factor: $(x^6 - 1)$

As you could see, plugging $x = 1$ yields 0, meaning that $(x - 1)$ is a **zero** of $(x^6 - 1)$, and hence would be able to be factored out.

To factor it out, either do synthetic division or long division.

Let's do a problem solve for $\lim_{x \to 2} \frac{(x^2-4)}{(x-2)}$

1. First, notice the fact this function will have a hole at $x = 2$. This is especially important because after we simplify we will loose this hole.
2. Ok, now let's simply. $\frac{(x^2-4)}{(x-2)} = \frac{(x+2)(\cancel{(x-2)})}{\cancel{(x-2)}} = (x + 2)$
3. Great! So, we know that this function behaves linearly with simply a hole at 2.
4. Doing the double-sided limits…
   - Evaluating $\lim_{x \to 2^+}$, the value will be 4 because $2 + 2 = 4$.
   - Evaluating

Here's another one! $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$

1. First, notice that if we are going to solve this problem, we have to divide the top thing ($\sqrt{x+4} - 2$) by x, somehow
2. The only thing we could do here is rationalize the top by multiplying the whole faction by a fancy one $\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$.
3. This results in $\frac{x+4-4}{x \times (\sqrt{x+4}+2)}$, which simplifies to $\frac{\cancel{x}}{\cancel{x} \times (\sqrt{x+4}+2)}$
4. Plugging in $x = 0$, you get $\frac{1}{4}$.

If there is no factors, you got yourself a vertical asymtote. Refer to #missing #disorganized for solution!