1 | Axler 5.B Exercise 13

> Suppose W is a complex vector space and $T \in \mathcal{L}(W)$ has no eigenvalues. Prove that every subspace of W invariant under T is either $\{0\}$ or infinite-dimensional.

2 | Proof

5.21 states

> Every operator on a finite-dimensional, nonzero, complex vector space has an eigenvalue.

W is given as a complex vector space, so to have no eigenvalues, it must be zero or infinite-dimensional. If the subspace is zero, then all subspaces must also be zero. Thus, only the infinite-dimensional case remains to be shown.

By definition (5.14), for all subspaces U of W invariant under T, $T\big|_U$ exists in $\mathcal{L}(U)$.

Suppose for the sake of contradiction that U has an eigenvalue. Then, there exists some $\lambda \in \mathbb{C}$ and some $v \neq 0 \in U$