

## 1 | Cauchy-Schwarz Inequality important

'One of the most important inequalities in mathematics'

Suppose  $u, v \in V$  (where  $V$  is an inner product space). Then

$$|\langle u, v \rangle| \leq \|u\| \|v\|$$

The inequality is an equality iff one of  $u, v$  is a scalar multiple of the other.

### 1.1 | intuition

For the Euclidean inner product, this is true because  $\langle u, v \rangle = \|u\| \|v\| \cos \theta$

### 1.2 | proof is by the orthogonal decomposition

### 1.3 | results

#### 1.3.1 | triangle inequality

Suppose  $u, v \in V$ . Then

$$\|u + v\| \leq \|u\| + \|v\|$$

The inequality is an equality if and only if one of  $u, v$  is a non-negative multiple of the other (degenerate triangle)

#### 1.3.2 | Parallelogram Equality

Suppose  $u, v \in V$ . Then

$$\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$$