Source:

1 | Definitions

1.1 | DONE group

A set and binary operation that satisfies Group Properties

- Closed
- Identity
- Inverse
- Associative

1.2 | DONE field

A set and two binary operations: the primary (addition) and secondary (multiplication) that ”mostly” satisfies group properties for both operations, and are **commutative and distributive**. It must be a group under the primary operation and a group under the secondary operation except without a secondary inverse for the primary indentity.

1.3 | DONE non-singular matrices

singular matrix: has no inverse. non-singular matrix: has an inverse aka determinant non zero

2 | Connections

2.1 | DONE connect direct sum and linear independence

the sum of two spaces is direct if their basises are linearly independent

2.2 | DONE matrices to represent complex numbers

The negative one matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and we want the square root of that. It turns out that $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ satisfies this, and in fact, any complex number $a + bi$ can be represented as $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. These matrices are commutative under multiplication (like complex numbers should be), have their complex conjugates equal to their transposes, and a bunch of other nice properties. Also related to rotation matrices. #source `https://www.nagwa.com/en/explainers/152196980513/`

3 | Computation

3.1 | DONE Find the determinant of matrices

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

3.2 | DONE compute cross product

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix} = i \begin{vmatrix} b & c \\ e & f \end{vmatrix} + j \begin{vmatrix} c & a \\ f & d \end{vmatrix} + k \begin{vmatrix} a & b \\ d & e \end{vmatrix} = bf - ce, cd - fa, ae - bd$$

3.3 | TODO Find equations of lines and planes using cross product and dot product

Use the cross product to find an orthagonal vector $\vec{p}$. The plane is all vectors that are orthagonal to $\vec{p}$, which is to say that the dot product is zero ($\{\vec{u} : \vec{u} \cdot \vec{p} = 0\}$)

4 | Derivations

4.1 | DONE properties of the determinant

4.1.1 | zero when matrix has no inverse (singular)

4.1.2 | det = -1 for rotation matrices?

4.2 | DONE inverse of a 2x2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$ae + bg = 1$$
$$ce + dg = 0$$
$$af + bh = 0$$
$$cf + dh = 1$$

Then you do some algebra to get

$$e = \frac{d}{ad - bc}$$
$$g = \frac{-c}{ad - bc}$$
$$f = \frac{-b}{ad - bc}$$
$$h = \frac{a}{ad - bc}$$

4.3 | DONE rotation matrices

Don't try to algebra it. Use polar coordinates and the angle sum trig identities:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

anyways, you get $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

5 | review quizzes

5.1 | DONE first quiz

5.1.1 | see ”find equations of lines and planes using cross product and dot product”

5.1.2 | rotation matrices

5.1.3 | cross product

5.2 | DONE mini take home quiz

no feedback

5.3 | DONE linear independence quiz

teacher gave no problems

5.4 | DONE quick linear quiz (linear independence and bases)

no feedback, I think that quiz was pretty solid..