1 | Problem

Suppose $T \in \mathcal{L}(V)$. Prove that $T/(\text{null}\, T)$ is injective if and only if $(\text{null}\, T) \cap (\text{range}\, T) = \{0\}$

2 | Proof

First, we will rewrite the problem as logical statements for easier manipulation. The left-hand side "$T/(\text{null}\, T)$ is injective" is equivalent to:

$$\begin{aligned} T/(\text{null}\, T)\,(v + (\text{null}\, T)) = (\text{null}\, T) &\iff v + (\text{null}\, T) = (\text{null}\, T) \\ Tv + (\text{null}\, T) = \text{null}\, T &\iff \end{aligned}$$