

Source: [KBe2020math530fioIndex](#)

## 1 | #exercise 2.A.17

- All polynomials have  $(x + 2)$  as a factor, and therefore can be written in the form  $(x + 2)f_j(x)$  where  $f_j(x)$  has degree at most  $m - 1$ .
  - Because the  $z^0, z^1, \dots, z^{m-1}$  is a spanning list of  $P_m - 1(F)$ , the spanning list of  $P_{m-1}(F)$  is of length  $m$ .
  - The original list had  $m + 1$  elements, so by Axler 2.23 the list cannot be linearly independent.
  - We can therefore find a non-trivial combination that equals zero, and can thus find a non-trivial combination of the original list by multiplying each vector by  $(x - 2)$ .
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