at everyone

1 | Complete the Representation

Function	First four terms	Generalized
$\frac{1}{1-2x}$	$1+2x+4x^2+8x^3+\cdots$	$\sum_{k=0} 2^k x^k$
$\cos(3x)$	$1-\frac{9x^2}{2!}+\frac{81x^4}{4!}-\frac{729x^6}{6!}+\cdots$	$\sum_{k=0} \frac{(-1)^k(3x)^{2k}}{2k!}$
$\frac{e^x}{e^2}$	$\frac{1}{e^2}+\frac{x}{e^2}+\frac{x^2}{e^2 2!}+\cdots$	$\sum_{k=0} \frac{x^k}{e^2 k!}$
$\sin(x^2)$	$x^2-\frac{x^6}{3!}+\frac{x^10}{5!}+\frac{x^14}{7!}+\cdots$	$\sum_{k=0} \frac{(-1)^k x^{2^{2k+1}}}{(2k+1)!}$
$\frac{1}{1+x^4}$	$1-x^4+x^8-x^{16}+\cdots$	$\sum_{k=0}(-x^4)^k$
e^{x-1}	$1+\frac{(x-1)^2}{2!}+\frac{(x-1)^4}{4!}+\frac{(x-1)^6}{6!}+\cdots$	$\sum_{k=0} \frac{(x-1)^{2k}}{k!}$
$\frac{\cos(x)-1}{x^2}$	$-\frac{1}{2!}+\frac{x^2}{4!}-\frac{x^4}{6!}+\cdots$	$\sum_{k=1} \frac{(-1)^k x^{2(k-1)}}{(2k)!}$
$2x\ln(1+2x)$	$(2x)(2x)-\frac{(2x)(2x)^2}{2}+\frac{(2x)(2x)^3}{3}-\frac{(2x)(2x)^4}{4}+\cdots$	$\sum_{k=1} \frac{2x(-1)^{k-1}(2x)^k}{k}$
$\frac{2x}{1+x^2}$	$2x-2x^3+2x^5-2x^7+\cdots$	$\sum_{k=0} 2x(-1)^k x^{2k}$

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2.1 | a: skipped

2.2 | find maclaurin series for $f'(x)$ where $f(x) = \sum_{k=0} \frac{(2x)^{k+1}}{k+1}$

$$\frac{d}{dx}\frac{(2x)^{n+1}}{n+1} = \frac{\cancel{(n+1)^2}(2x)^n(2)}{\cancel{(n+1)^2}} = 2(2x)^n$$

So, our series is just

$$\sum_{k=0} 2(2x)^k = 2+4x+8x^2+16x^3+\cdots$$

2.3 | estimate $f'\left(-\frac{1}{3}\right)$

When only using the first 4 terms:

$$2+4\frac{-1}{3}+8\left(\frac{-1}{3}\right)^2+16\left(\frac{-1}{3}\right)^2 = \frac{10}{3}$$

For the entire sequence:

$$\sum_{k=0} 2\left(\frac{-2}{3}\right)^k = 2\sum_{k=0}\left(-\frac{2}{3}\right)^k = \frac{2}{1--\frac{2}{3}} = \frac{2}{\frac{5}{3}} = \frac{6}{5}$$

because the series is geometric.

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3.1 | find $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$

That series is just the taylor series for

$$f(x) = \frac{\sin x}{x}$$

So the derivative at zero is zero, and the second derivative:

$$\begin{aligned} \frac{d}{dx}\frac{x\cos x - \sin x}{x^2} &= \frac{x^2\left(-x\sin x \cancel{+ \cos x - \cos x}\right) - (x\cos x - \sin x)(2x)}{x^4} \\ &= \frac{-x^3\sin x - 2x\left(x\cos x - \sin x\right)}{x^4} \end{aligned}$$

is undefined at zero. However, the top of the fraction will be negative ($x^3 \sin x$ is like x^4 when $x \approx 0$ and $x\cos x - \sin x = x(\cos x - \frac{\sin x}{x})$, so the second derivative is zero at x. (Checked with desmos). Thus, the function has a local maximum at $x = 0$.

3.2 | show approximation at $x = 1$ is within $\epsilon < \frac{1}{100}$ with $1 - \frac{1}{3!}$

$$\begin{aligned} f(1) - \left(1 - \frac{1}{3!}\right) &= \frac{1^4}{5!} - \frac{1^6}{7!} + \cdots \\ &= \frac{1}{5!} - \frac{1}{7!} + \cdots \\ &< \frac{1}{5!} = \frac{1}{120} < \frac{1}{100} \end{aligned}$$

3.3 | solution to the differential equation $xy' + y = \cos x$

$$\begin{aligned} xy' + y = \cos x \implies y &= \cos x - xy' \\ &= \cos x - \cancel{x}\frac{x\cos x - \sin x}{x^{\cancel{2}}} \\ &= \cos x - \frac{x\cos x - \sin x}{x} \\ &= \cos x - \frac{\cancel{x}\cos x}{\cancel{x}} + \frac{\sin x}{x} \\ &= \cancel{\cos x - \cos x} + \frac{\sin x}{x} \\ y &= \frac{\sin x}{x} \end{aligned}$$