1 | Axler6.31 Gram-Schmidt Procedure

The Gram-Schmidt Procedure is used to turn a list into an orthonormal list with the same span. It's useful for finding orthonormal bases.

> Suppose $v_1, \dots, v_m$ is a linearly independent list of vectors in V. Let $e_1 = v_1/\|v_1\|$. For $j = 2, \dots, m$, define e_j inductively by
>
> $$e_j = \frac{v_j - \langle v_j, e_1 \rangle e_1 - \dots - \langle v_j, e_{j-1} \rangle e_{j-1}}{\|\text{<numerator>}\|}$$
>
> Then $e_1, \dots, e_m$ is an orthonormal list of vectors in V s.t. each prefix span is the same as in $v_1, \dots, v_m$.

1.1 | intuition

Basically, for each vector, we divide out the components from the previous vectors and then normalize the size to ensure the norm is one.

It's kind of like the orthogonal decomposition.

2 | results

2.1 | Axler6.34 orthonormal basis exists in finite dim vec spaces

since every finite dim vec space has a basis that can be Gram-schmidt-ed

2.2 | Axler6.35 orthonormal lists extend to orthonormal bases

just extend the orthonormal list into a basis, and then gram-schmidt-ify the vectors you added

2.3 | Axler6.37 upper-triangular matrix wrt orthonormal basis

If an upper triangular matrix exists for some operator, then an upper-triangular matrix exists for an orthonormal basis too.

Proof: the prefix span-equality implies subspace invariance or something.

An application of this is