Source:

1 | linear approximations

1.1 | cube root

1.1.1 | approximation

$$(1+x)^{\frac{1}{3}} \to \frac{1}{3}(1+x)^{\frac{-2}{3}}$$

at $x=0$ is

$$\frac{1}{3}(1+0)^{\cdots} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

1.1.2 | estimations

value	estimate
0.05	1.016666
-0.25	0.916666

These will be overestimates because the graph is concave down in this reigon.

1.2 | sin(x)

1.2.1 | approximation

$$y \approx \frac{d}{dx}\sin x\Big|_0 (x-0) + \sin 0 = x$$

1.2.2 | estimates

value	estimate
-0.1	-0.1
0.1	0.1

The first estimate will be an underestimate because $\sin x$ is concave up in that reigon. The opposite is true for the second estimate.

1.3 | unknown function (only some points known

1.3.1 | approximation

$$y \approx \frac{d}{dx} f(x)\Big|_c (x-c) + f(c)$$

plugging in $c = 1$,

$$y \approx 5(x-1) - 4$$

1.3.2 | estimations

value	estimate
1.2	-3

This will be an underestimate because the second derivative is positive and the graph is thus concave up.

2 | differentials

For a function $y = f(x)$, dy and dx are diferentials and the relationship is $dy = f'(x)dx = \frac{L(a+\Delta a)-L(a)}{\cancel{dx}}\cancel{dx}$. For a function written $f(x) =$ (something), the differentials are df and dx and the relationship is the same: $df = f'(x)dx$.

2.1 | cube error

2.1.1 | differential

$$\begin{aligned} df &= f'(x)dx \\ &= 3x^2 dx \end{aligned}$$

2.1.2 | volume error

If I understand the use of differentials corretly, then x is the measured value (2) and dx is the uncertainty (delta x), or 0.2ft. Then, the change in the volume (change in fuction or df) would be $3(2)^2(0.2) = 2.4$

2.1.3 | max error for some ϵ

$$\begin{aligned} df &\approx 3x^2 dx \\ dx &\approx \frac{df}{3x^2} \\ &\approx \frac{1}{3(2)^2} \\ &\approx \frac{1}{12}\text{ ft} = 1\text{in} \end{aligned}$$

2.2 | sphere measuring

$$
\begin{aligned}
f(r) &= 4\pi r^2 \\
\frac{d}{dr} f(r) &= 8\pi r \\
df &= 8\pi r(dr) \\
&= 8\pi 21(0.05) = \pm 8.4\pi \text{ cm}^3
\end{aligned}
$$