

## 1 | **Axler6.31 Gram-Schmidt Procedure**

The Gram-Schmidt Procedure is used to turn a list into an orthonormal list with the same span. It's useful for finding orthonormal bases.

Suppose  $v_1, \dots, v_m$  is a linearly independent list of vectors in  $V$ . Let  $e_1 = v_1/\|v_1\|$ . For  $j = 2, \dots, m$ , define  $e_j$  inductively by

$$e_j = \frac{v_j - \langle v_j, e_1 \rangle e_1 - \dots - \langle v_j, e_{j-1} \rangle e_{j-1}}{\|\text{<numerator>}\|}$$

Then  $e_1, \dots, e_m$  is an orthonormal list of vectors in  $V$  s.t. each prefix span is the same as in  $v_1, \dots, v_m$ .

### 1.1 | **intuition**

Basically, for each vector, we divide out the components from the previous vectors and then normalize the size to ensure the norm is one.

It's kind of like the orthogonal decomposition.

## 2 | **results**

### 2.1 | **Axler6.34 orthonormal basis exists in finite dim vec spaces**

since every finite dim vec space has a basis that can be Gram-schmidt-ed

### 2.2 | **Axler6.35 orthonormal lists extend to orthonormal bases**

since every list can be extended to a basis and an orthonormal basis exists

### 2.3 | **Axler6.37 upper-triangular matrix wrt orthonormal basis**