1 | Axler5.22 matrix of an operator, $\mathcal{M}(T)$ def

Suppose $T \in \mathcal{L}(V)$ and $v_1, \ldots, v_n$ is a basis of V. The *matrix of T* wrt this basis is the *n*-by-*n* matrix

$$\mathcal{M}(T) = \begin{pmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & \ddots & \vdots \\ A_{n,1} & \cdots & A_{n,n} \end{pmatrix}$$

whose entries $A_{j,k}$ are defined by

$$Tv_k = A_{1,k}v_1 + \cdots + A_{n,k}v_n$$

Specify a basis with $\mathcal{M}\left(T, (v_1, \ldots, v_n)\right)$

1.1 | intuition

1.1.1 | each column is where the map takes a basis vector

2 | Simplifying The Matrix Representation

2.1 | 'A central goal of linear algebra is to show that given an operator $T \in \mathcal{L}(V)$, there exists a basis of V wrt which T has a reasonably simple matrix'

2.2 | If by simple we mean "has many zeros" or RREF, then we know enough to ensure that there exists a basis s.t. the first column has zeros everywhere except the first row.

$$\begin{pmatrix} \lambda & & \\ 0 & * & \\ \vdots & & \\ 0 & & \end{pmatrix}$$

Where $*$ denotes all the other entries. Find λ by taking the lone eigenvalue and letting it's eigenvector be the first basis vector.