Source:

1 | find taylor series

1.1 | $y = \cos(x)$

$$\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= \cos(0) && -\sin(0)x && +\frac{-\cos(0)}{2!}x^2 && +\frac{\sin(0)}{3!}x^3 && +\cdots \\
&= 1 && -0x && -\frac{1}{2!}x^2 && +\frac{0}{3!}x^3 && +\cdots \\
&= 1 && && -\frac{1}{2!}x^2 && && +\frac{1}{4!}x^4 + \cdots \\
&= \sum_{k=0} \frac{(-1)^k}{(2k)!}x^{2k}
\end{aligned}$$

1.2 | $y = e^x$

$$\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= e^0 && +e^0x && +\frac{e^0}{2!}x^2 && +\frac{e^0}{3!}x^3 && +\cdots \\
&= 1 && +x && +\frac{1}{2!}x^2 && +\frac{1}{3!}x^3 && +\cdots \\
&= \sum_{k=0} \frac{1}{k!}x^k
\end{aligned}$$

1.3 | **TODO** $y = \sqrt{x}$

2 | prove approximations

2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Proof by geometric series