

Source:

1 | **Lemma**

The size of a linearly independent set is less than or equal to the size of a spanning set over some vector space V .

2 | **Intermediate Result: Span of a linearly independent extension of a linearly independent list has more elements than the span of the original list.**

2.1 | **Lemma**

Given a linearly independent list $v = v_1, \dots, v_k$ where each vector $v_1, \dots, v_k \in V$ and another vector v_{k+1} which is linearly independent with v , show that

$$\text{span}(v_1, \dots, v_k, v_{k+1})$$

contains elements that are not in

$$\text{span}(v_1, \dots, v_k)$$

2.2 | **Proof**

Because v_{k+1} is linearly independent with v , it cannot be written as a linear combination of elements in v .