Source:

1 | **Problem**

Suppose $T \in \mathcal{L}(V, W)$ and U is a subspace of V. Let π denote the quotient map from V onto V/U. Prove that there exists $S \in \mathcal{L}(V/U, W)$ such that $T = S \circ \pi$ if and only if $U \subseteq$ null T.

Intuitively, if we mod out part of the null T, then we should still be able to have a map that does what T would do.

2 | **Forward Direction by Contrapositive**

If $U \not\subseteq$ null T, then there exists $v \in U$ s.t. $Tv \neq 0$.

3 | **Reverse Direction**