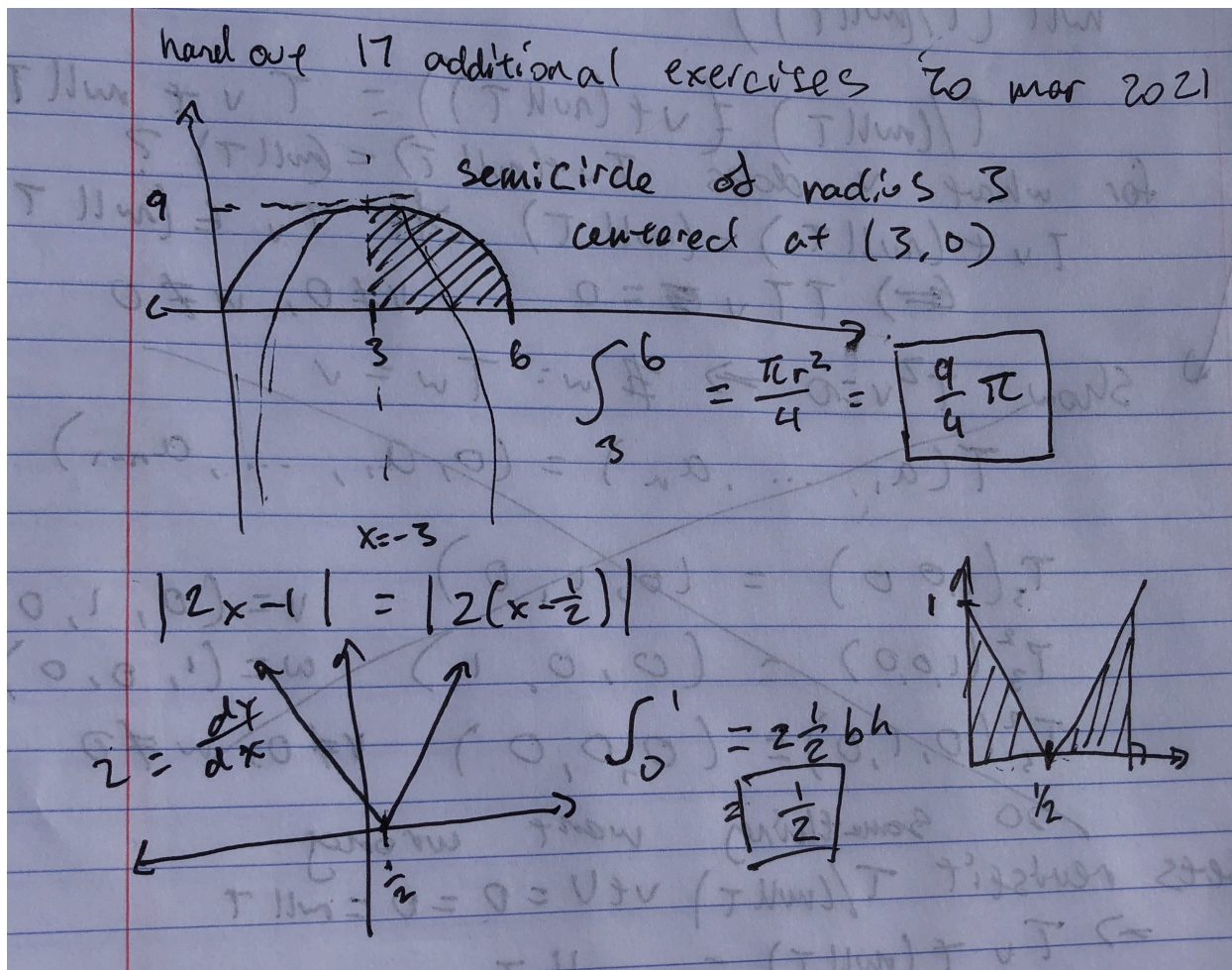


## 1 | Exercises

### 1.1 | interpreting in terms of area



### 1.3 | subtracting integrals

I expect

$$\int_1^2 f(x) dx = \int_1^5 f(x) dx - \int_2^5 f(x) dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

1.4 | **show**  $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$

17a.4 Show  $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$

$$\int_a^b x^2 dx = \lim_{n \rightarrow \infty} \sum_{k=0}^n \left( (a + k \Delta x)^2 \Delta x \right) \text{ where } \Delta x = \frac{b-a}{n}$$

$$= \lim_{n \rightarrow \infty} \Delta x \sum_{k=0}^n (a^2 + k^2 \Delta x^2 + 2ak \Delta x)$$

$$= \lim_{n \rightarrow \infty} \Delta x \left( \sum_{k=0}^n a^2 + \sum_{k=0}^n k^2 \Delta x^2 + \sum_{k=0}^n 2ak \Delta x \right)$$

$$= \Delta x \left( a^2 \sum_{k=0}^n 1 + \Delta x^2 \sum_{k=0}^n k^2 + 2a \Delta x \sum_{k=0}^n k \right)$$

$$= \Delta x \left( a^2 (n+1) + \Delta x^2 \frac{n(n+1)(2n+1)}{6} + 2a \Delta x \frac{n(n+1)}{2} \right)$$

$$\lim_{n \rightarrow \infty} \Delta x \left( a^2 n + 2a(b-a) \frac{n+1}{2} + (b-a) \Delta x \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \lim_{n \rightarrow \infty} a^2(b-a) + (b-a) \frac{2a(b-a)}{2} + \Delta x a(b-a) \rightarrow 0$$

$$+ \Delta x (b-a) \left( (b-a) \frac{(2n+1)}{6} + \Delta x \frac{2n+1}{6} \right)$$

$$= (b-a) \Delta x (2n+1) \left( \frac{b-a}{6} + \frac{\Delta x}{6} \right)$$

$$= (b-a) \left( \frac{b-a}{6} + \frac{\Delta x}{6} \right) (2(b-a) + \Delta x)$$

$$= \lim_{n \rightarrow \infty} a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{3}$$



cont

$$\begin{aligned}
 & a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{3} \\
 &= a^2(b-a) + a(b^2 + a^2 - 2ab) + \frac{(b-a)(b-a)^2}{3} \\
 &= \frac{1}{3} \left( 3a^2(b-a) + 3a(b-a)^2 + (b-a)(b-a)^2 \right) \\
 &= \frac{1}{3} \left( 3a^2b - 3a^3 + 3ab^2 + 3a^3 - 6a^2b + b(b-a)^2 - a(b-a)^2 \right) \\
 &= \frac{1}{3} \left( 3a^3b + 3ab^2 - 6a^2b + b^3 + a^2b - 2ab^2 - ab^2 - a^3 + 2a^2b \right) \\
 &= \frac{1}{3} \left( 3a^3b - 3a^2b + b^3 - a^3 \right) \\
 &= \boxed{\frac{b^3 - a^3}{3}}
 \end{aligned}$$

a number of written mistakes slowed me down