1 | $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$

$$\int -e^u du = -e^u + C$$
$$= -e^{\frac{1}{2}} + e^{\frac{1}{1}}$$
$$= e - e^{\frac{1}{2}}$$

2 | $\int_0^1 re^{\frac{r}{2}}dr$

$$\int_0^1 re^{\frac{r}{2}}dx \implies r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}}dr$$
$$= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}}dr$$
$$= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}}$$
$$= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}}$$
$$\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4)$$
$$= 4 - 2e^{\frac{1}{2}}$$

3 | $\int_4^9 \frac{\ln y}{\sqrt{y}}dy$

$$\int \frac{\ln y}{\sqrt{y}}dy = 2\ln y\sqrt{y} - \int 2\frac{1}{y}\sqrt{y}dy$$
$$= 2\ln y\sqrt{y} - \int 2\frac{1}{\sqrt{y}}dy$$
$$= 2\ln y\sqrt{y} - 2\int y^{-\frac{1}{2}}dy$$
$$= 2\ln y\sqrt{y} - 4\sqrt{y} + C$$
$$= 2\sqrt{y}(\ln y - 2) + C$$
$$\implies 6(\ln 9 - 2) - 4(\ln 4 - 2)$$

4 | $\int_0^{\sqrt{\pi}} \cos\sqrt{x}dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$, $dx = 2udu$

$$\int_0^{\sqrt{\pi}} \cos\sqrt{x}dx = 2\int u\cos u du$$
$$= 2u\sin u - 2\int \sin u du$$
$$= 2u\sin u + 2\cos u$$
$$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x}$$
$$\implies 2\pi^{\frac{1}{4}}\sin\pi^{\frac{1}{4}} + 2\cos\pi^{\frac{1}{4}} - 2$$

5 | $\int_1^e \sin \ln x \, dx$

$$\int_1^e \sin \ln x \, dx = x \sin \ln x - \int x \frac{1}{x} \cos \ln x \, dx$$

$$= x \sin \ln x - \int \cos \ln x \, dx$$

$$= x \sin \ln x - \left(x \cos \ln x + \int x \frac{1}{x} \sin \ln x \, dx \right)$$

$$= x \sin \ln x - x \cos \ln x - \int \sin \ln x \, dx$$

$$2 \int \sin \ln x \, dx = x \sin \ln x - x \cos \ln x$$

$$\int \sin \ln x \, dx = \frac{1}{2} x (\sin \ln x - \cos \ln x)$$

$$\implies \quad \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0)$$

$$= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2}$$

6 | $\int_0^1 \frac{x^3}{\sqrt{4+x}} \, dx$

Let $u = 4 + x^2$, $du = 2x\,dx$

$$\int \frac{x^3}{\sqrt{4+x}} \, dx = \frac{1}{2} \int \frac{(u-4)}{\sqrt{u}} \, du$$

$$= \frac{1}{2} \int \sqrt{u} \, dx - 2 \int \frac{1}{\sqrt{u}} \, dx$$

7 | **(additional problems)**

8 | $\int \sin^2 x \, dx$

$$\int \sin^2 x \, dx = - \sin x \cos x + \int \cos^2 x \, dx$$

$$= - \sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx$$

$$2 \int \sin^2 x \, dx = - \sin x \cos x + x$$

$$\int \sin^2 x \, dx = \frac{1}{2} (x - \sin x \cos x)$$

9 | $\int \cos^2 x dx$

$$\int \cos^2 x dx = \cos x \sin x + \int \sin^2 x dx$$

$$= \cos x \sin x + \int 1 dx - \int \cos^2 x dx$$

$$2 \int \cos^2 x dx = \cos x \sin x + x$$

$$\int \cos^2 x dx = \frac{1}{2} \cos x \sin x + \frac{x}{2}$$

10 | $\int \sin^2 x \cos^2 x dx$

First, the integral of $\sin x \cos x$:

$$\int \sin x \cos x dx = \sin^2 x - \int \sin x \cos x dx$$

$$= \frac{1}{2} \sin^2 x + C$$

And the derivative:

$$\frac{d}{dx} \sin x \cos x = -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x$$

Now we can use differentiation by parts on $(\sin x \cos x)(\sin x \cos x)$:

$$\int (\sin x \cos x)(\sin x \cos x) dx = \frac{1}{2}(\sin x \cos x) \sin^2 x - \int (\cos^2 x - \sin^2 x) \left(\frac{1}{2} \sin^2 x \right) dx$$

$$= \frac{1}{2}(\sin x \cos x) \sin^2 x - \frac{1}{2} \int \sin^2 x \cos^2 x dx - \frac{1}{2} \int \sin^4 x dx$$

$$\frac{3}{2} \int \sin^2 x \cos^2 x dx = \frac{1}{2}(\sin x \cos x) \sin^2 x - \frac{1}{2} \int \sin^4 x dx$$

To evaluate the integral of $\sin^4 x$, we can use differentiation by parts after multiplying by one. We will need to use the integral of $\sin x \cos x$ from earlier in this problem, as well as the integral of $\sin^2 x$ from the previous problem.

$$\int 1 \sin^4 x dx = x \sin^4 x - 4 \int x \sin x \cos x dx$$

$$= x \sin^4 x - 4 \left(x \left(\frac{1}{2} \sin^2 x \right) - \int \frac{1}{2} \sin^2 x dx \right)$$

$$= x \sin^4 x - 4 \left(x \left(\frac{1}{2} \sin^2 x \right) - \frac{1}{2} \int \sin^2 x dx \right)$$

$$= x \sin^4 x - 2x \sin^2 x + 4 \frac{1}{2} (x - \sin x \cos x)$$

$$= x \sin^4 x - 2x \sin^2 x + 2x - 2 \sin x \cos x$$

Plugging back into the main problem:

$$\frac{3}{2} \int \sin^2 x \cos^2 x dx = \frac{1}{2}(\sin x \cos x) \sin^2 x - \frac{1}{2} \int \sin^4 x dx$$

$$= \frac{1}{2}(\sin x \cos x) \sin^2 x - \frac{1}{2} x \sin^4 x + x \sin^2 x - x + \sin x \cos x + C$$

$$\int \sin^2 x \cos^2 x dx = \frac{1}{3} \sin^3 x \cos x - \frac{1}{3} x \sin^4 x + \frac{2}{3} x \sin^2 x - \frac{2}{3} x + \frac{3}{2} \sin x \cos x + C$$

I think I did something wrong.

11 | $\int \sin^3 x dx$

$$\int \sin^3 x dx = \int \sin x (1 - \cos^2 x) dx$$

$$= \int \sin x dx - \int \cos^2 x \sin x dx$$

Let $u = \cos x, du = -\sin x dx$

$$=$$

$$\int \sin^3 x dx = \int 1 \sin^3 x dx$$

$$= x \sin^3 x - \int 3x \sin x \cos x dx$$

$$= x \sin^3 x - 3x \sin x \cos x + 3 \int \frac{1}{2} \sin^2 x dx$$

$$= x \sin^3 x - 3x \sin x \cos x + \frac{3}{2} \int \sin^2 x dx$$

$$= x \sin^3 x - 3x \sin x \cos x + \frac{3}{2}\frac{1}{2}(x - \sin x \cos x) + C$$

$$= x \sin^3 x - 3x \sin x \cos x + \frac{3}{4}(x - \sin x \cos x) + C$$