

1 | Exercise 7

Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely $\dim E(\lambda, T)$ times.

2 | Proof

There must be $\dim V$ (non-distinct) eigenvalues on the diagonal.

Any eigenvalue λ must have $\dim E(\lambda, T)$ associated eigenvectors, by definition. ("In other words, $E(\lambda, T)$ is the set of all eigenvectors of T corresponding to λ , along with the 0 vector").

λ appears exactly once for each associated eigenvector. The diagonal is comprised of the basis vector coefficients of the eigenspaces, implying that each eigenspace of λ is represented by $\dim E(\lambda, T)$ eigenvectors.