1 | Axler6.27 orthonormal basis def

An *orthonormal basis* of V is an orthonormal list of vectors in V that is also a basis of V.

Pretty self explanatory.

2 | results

2.1 | Axler6.28 orthonormal list of the right length is a basis

Because it's linearly independent, and linearly independent lists of the right length are bases (Axler2.39).

2.2 | Axler6.30 vector as a linear combo of orthonormal basis

Suppose $e_1, \ldots, e_m$ is an orthonormal basis of V and $v \in V$. Then,

$$v = \langle v, e_1 \rangle e_1 + \cdots \langle v, e_n \rangle e_n$$

and

$$\|v\|^2 = |\langle v, e_1 \rangle|^2 + \cdots + |\langle v, e_n \rangle|^2$$

3 | see also

3.1 | orthonormal