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Exploration 2-1a: Int roduction to Limits

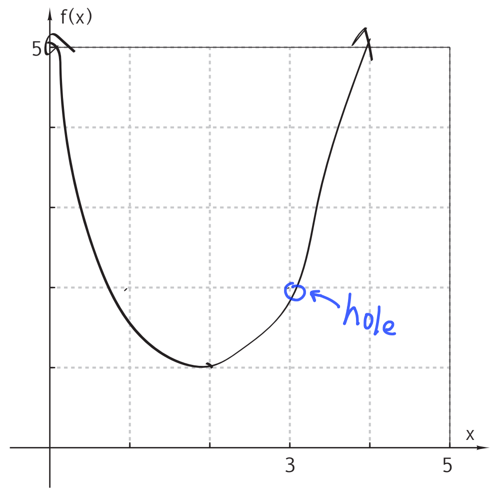
Date: for 1 Sep 2020

Objective: Find the limit of a function that approaches an indeterminate form at a particular value of x and relate it to the definition.

1. Plot on your grapher the graph of this function.

   f(x) H $\frac{x^3 \text{ D } 7x^2 \text{ C } 17x \text{ D } 15}{x \text{ D } 3}$

   Use a friendly window with x H 3 as a grid point, but with the grid turned off. Sketch the result shere. Show the behavior of the function in a neighborhood of x H 3.



2. Substitute 3 for x in the equation for f(x). What form does the answer take? What name is given to an expression of this form?
   indeterminate form

3. The graph of f has a removable discontinuity at x H 3. The y-value at this discontinuity is the limit of f(x) as x approaches 3. What number does this limit equal?
   2

4. Make a table of values of f(x) for each 0.1 unit change in x-value from 2.5 through 3.5.

x	f(x)
2.5	1.25
2.6	1.36
2.7	1.49
2.8	1.64
2.9	1.81
3.0	undefined
3.1	2.21
3.2	2.44
3.3	2.69
3.4	2.96
3.5	3.25

5. Between what two numbers does f(x) stay when x is kept in the open interval (2.5, 3.5)?
   1.25, 3.25

6. Simplify the fraction for f(x). Solve numerically to find the two numbers close to 3 between which x must be kept if f(x) is to stay between 1.99 and 2.01.

   $\frac{\cancel{(x-3)}(x(x-5)+(x+5))}{\cancel{x-3}}$

   $x = \pm\sqrt{y-1} + 2$

   2.995 ... 3.005

7. How far from x H 3 (to the left and to the right) are the two x-values in Problem 6?
   0.005

8. For the statement "If x is within 0.005 units of 3 (but not equal to 3), then f(x) is within 0.01 unit of 2," write the largest number that can go in the blank.

9. The formal definition of limit is

   L H $\lim_{x\to c}$ f(x) if and only if
   - for any positive number ε (no matter how small)
   - there is a positive number δ such that
   - if x is within δ units of c, but not equal to c,
   - then f(x) is within ε units of L.

   The four numbers L, c, ε, and δ all appear in Problem 8. Which is which?

   ε = 0.005
   δ = 0.01
   c = 3
   L = 2

10. What did you learn as a result of doing this Exploration that you did not know before?
    Simplify more complex rational functions, formal definition of "really small range" for limits.