Orthonormal List May 2, 2021

1 | Axler6.23 orthonormal def

- A list of vectors is called orthonormal if each vector in the list has norm 1 and is orthogonal
 to all other vectors in the list
- in other words, a list e_1, \ldots, e_m of vectors in V is orthonormal if

$$\langle e_j, e_k \rangle = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$$

2 | results

2.1 | Axler6.25 norm of an orthonormal linear combination Pythagorean theorem

If e_1, \ldots, e_m is an orthonormal list of vectors in V then

$$||a_1e_1 + \dots + a_me_m||^2 = |a_1|^2 + \dots + |a_m|^2$$

Pythagorean theorem

2.2 | Axler6.26 orthonormal lists are linearly independent

Any orthogonal list is linearly independent, so an orthonormal list must also be linearly independent. Or you can use 6.25, set it to zero, and see that all the coefficients must be zero.

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