Source:

1 | Broader vector spaces

- Doesn't have to be physics vectors
- maybe it's like matrices
- or linear maps themselves

2 | The Linear Map 0

A linear map $S = 0$ is a map where $Su = 0 \forall u$.

3 | Axler 3.A ex7 (w/ Vienna + Mason)

Let $w = Tv$.

3.1 | If $v = 0$ then

$$Tv = 0$$

By Axler 3.11 (Maps take 0 to 0). Thus, λ can be anything in $\mathbb{F}$.

3.2 | Otherwise,

$\frac{1}{v} \in \mathbb{F}$ because the field has multiplicative inverses for all elements except 0.

$$Tv = w = \left(w\frac{1}{v}\right)v$$

Let $\lambda = w\frac{1}{v}$, then

$$\lambda v = w\frac{1}{v}v = w$$

which is in $\mathbb{F}$ because $w, \frac{1}{v} \in \mathbb{F}$ and fields are closed under multiplication.

4 | Axler 3.A ex10 (w/ Vienna + Mason)

The additivity of a linear map T requires $T(u+v) = Tu + Tv$. Because $U \subset V, U \neq V$, there must be some element $v \in V$ yet $v \notin U$.

For some element $u \in U$,

$$Tu + Tv = Su + 0 = Su$$

Yet $u + v \notin U$ because if it were, then $(u+v) + (-v) = v$ would be in U. Thus,

$$T(u+v) = 0$$

Because for some u $Su \neq 0$, additivity does not hold over T and thus the map is not linear.