Calculus 2019-2020

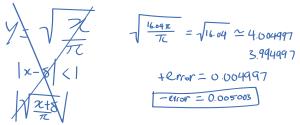
Handout #5: Formal Definition of Limits: An Introduction

Please sketch the graph of the functions in each problem.

- 1. Suppose a company produces a circular plate to repair the skull after brain surgery. The area of such a plate must be  $16\pi$  square millimeters.
  - a. What is the necessary radius for the plate? (recall the area of a circle)



b. Suppose the plate still works with an error in area of  $0.04\pi$  square millimeters. What is the maximum error in radius that will guarantee the area is in error of at most  $0.04\pi$  square millimeters? (Round to nearest 6th decimal place)



2. Suppose we need to manufacture a ball bearing of volume  $36\pi$  cubic inches (recall: the volume of a sphere is  $V=(4/3)\pi r^3$ . Note that the volume of the ball is a function of radius. If the volume of our ball bearing can be in error at most  $0.4\pi$ , what is the maximum permissible error in radius?

$$\sqrt{36.4\pi}$$
 $\sqrt{3}$ 
 $\sqrt{3$ 

3. Suppose a toilet paper company needs to make hollow cylinders (that is, without a top or bottom) with surface area  $12\pi$  square centimeters. Suppose the radius must remain a constant 3 square centimeters but the surface area of the cylinder can be of error at most  $0.2\pi$  square centimeters. What is the maximum permissible error in height?

$$A = c^*L = 2\pi r^*L$$
 $6\pi L = 12.2\pi$ 
 $e = 0.0333$ 

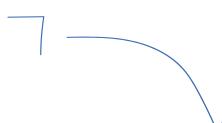
Formal definition of a Limit

$$\lim_{x \to a} f(x) = L$$

**means** for all  $\varepsilon > 0$ , there exists  $\delta > 0$  such that if  $0 < |x - a| < \delta$  then  $|f(x) - L| < \varepsilon$ .

## Let's explore the above definition graphically:

- 4. Consider the function f(x) = 2x + 1.
  - a. Graph the function on Desmos. What do you the limit of the function is at x = 3?



b. Suppose  $\varepsilon=1$ . Find  $\delta>0$  such that it satisfies the limit definition (that is, find the  $\delta>0$  such that if  $|x-3|<\delta$  then  $|(2x+1)-L|<\varepsilon$ ).

$$|2x+1-7| < \varepsilon$$

$$|2x+1-7| < \varepsilon$$

$$|2x-6| < \varepsilon$$

$$|2(x-3)| < \varepsilon$$

$$|2x-3| < \varepsilon$$

$$|x-3| < \varepsilon$$

$$|x-3| < \varepsilon$$

c. Suppose  $\varepsilon = 0.5$ . Find  $\delta > 0$  such that it satisfies the limit definition (up to 4 decimal places).

$$\delta = \frac{6}{2} = \frac{0.05}{2} = 0.2500$$

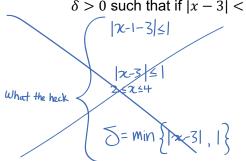
d. Suppose  $\varepsilon=0.25$ . Find  $\delta>0$  such that it satisfies the limit definition (up to 4 decimal places).

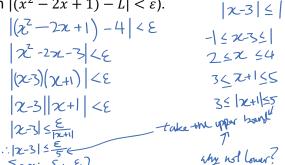
$$\delta = \frac{\epsilon}{2} = \frac{0.25}{2} = 0.1250$$

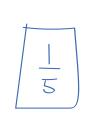
- 5. Consider the function  $f(x) = x^2 2x + 1$ .
  - a. Graph the function on Desmos. What do you the limit of the function is at x = 3?



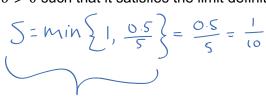
b. Suppose  $\varepsilon = 1$ . Find  $\delta > 0$  such that it satisfies the limit definition (that is, find the  $\delta > 0$  such that if  $|x - 3| < \delta$  then  $|(x^2 - 2x + 1) - L| < \varepsilon$ ).







c. Suppose  $\varepsilon = 0.5$ . Find  $\delta > 0$  such that it satisfies the limit definition (up to 4 decimal places).



d. Suppose  $\varepsilon=0.25$ . Find  $\delta>0$  such that it satisfies the limit definition(up to 4 decimal places).

