

Source:

1 | cube root

1.1 | approximation

$$(1+x)^{\frac{1}{3}} \rightarrow \frac{1}{3}(1+x)^{-\frac{2}{3}}$$

at $x = 0$ is

$$\frac{1}{3}(1+0)^{-\frac{2}{3}} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

1.2 | estimations

value	estimate
0.05	1.016666
-0.25	0.916666

These will be overestimates because the graph is concave down in this region.

2 | sin(x)

2.1 | approximation

$$y \approx \left. \frac{d}{dx} \sin x \right|_0 (x-0) + \sin 0 = x$$

2.2 | estimates

value	estimate
-0.1	-0.1
0.1	0.1

The first estimate will be an underestimate because $\sin x$ is concave up in that region. The opposite is true for the second estimate.

3 | unknown function (only some points known)

3.1 | approximation

$$y \approx \left. \frac{d}{dx} f(x) \right|_c (x - c) + f(c)$$

plugging in $c = 1$,

$$y \approx 5(x - 1) - 4$$

3.2 | estimations

value	estimate
1.2	-3

This will be an underestimate because the second derivative is positive and the graph is thus concave up.