1 | circumfrence

Start with the arc length formula

$$2\int_0^a \sqrt{1+f'^2(x)}dx$$

$f(x)=\sqrt{a^2-x^2}$ so $f'(x)=\frac{1}{2\sqrt{a^2-x^2}}(-2x)=-\frac{x}{\sqrt{a^2-x^2}}$

$$2\int_0^a \sqrt{1+\frac{x^2}{a^2-x^2}}dx = 2\int_0^a \sqrt{\frac{a^2}{a^2-x^2}}dx$$

Now we need to use trig substitution. Lets use $x=a\sin\theta$, with the limit assumptions $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$. Then, $dx=a\cos\theta d\theta$

$$\begin{aligned}
&= 2\int\sqrt{\frac{a^2}{a^2-a^2\sin^2\theta}}d\theta \\
&= 2\int\sqrt{\frac{1}{1-\sin^2\theta}}d\theta \\
&= 2\int\sqrt{\frac{1}{\cos^2\theta}}d\theta \\
&= 2\int\sqrt{\sec^2\theta}d\theta \\
&= 2\int\sec\theta d\theta \\
&= 2\ln|\sec\theta+\tan\theta|
\end{aligned}$$

Now for the triangle part! $\sec\theta=\frac{1}{\cos\theta}=\frac{\sqrt{a^2-x^2}}{a},\tan\theta=\frac{x}{\sqrt{a^2-x^2}}$

$$=2\ln\left|\frac{a}{\sqrt{a^2-x^2}}+\frac{x}{\sqrt{a^2-x^2}}\right|$$

Seems like a lot of dividing by zero

2 | surface area

I don't really understand how we use arclength in the calculation of surface area. I tried just using the circumfrence, but that didn't work with the slicing assumption because the surface curves.