**Source**:

# 1 | find taylor series

## 1.1 | $y = \cos(x)$

$$\begin{aligned}
P_n(x) &= f(0) & &+\frac{d}{dx}f(0)x & &+\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 & &+\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 & &+\cdots \\
&= \cos(0) & &-\sin(0)x & &+\frac{-\cos(0)}{2!}x^2 & &+\frac{\sin(0)}{3!}x^3 & &+\cdots \\
&= 1 & &-0x & &-\frac{1}{2!}x^2 & &+\frac{0}{3!}x^3 & &+\cdots \\
&= 1 & & & &-\frac{1}{2!}x^2 & & & &+\frac{1}{4!}x^4+\cdots \\
&= \sum_{k=0}\frac{(-1)^k}{(2k)!}x^{2k}
\end{aligned}$$

## 1.2 | $y = e^x$

$$\begin{aligned}
P_n(x) &= f(0) & &+\frac{d}{dx}f(0)x & &+\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 & &+\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 & &+\cdots \\
&= e^0 & &+e^0x & &+\frac{e^0}{2!}x^2 & &+\frac{e^0}{3!}x^3 & &+\cdots \\
&= 1 & &+x & &+\frac{1}{2!}x^2 & &+\frac{1}{3!}x^3 & &+\cdots \\
&= \sum_{k=0}\frac{1}{k!}x^k
\end{aligned}$$

## 1.3 | **TODO** $y = \sqrt{x}$

# 2 | prove approximations

## 2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Proof by geometric series

## 2.2 | $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

Plug $-x$ for $x$ in the previous equation.

## 2.3 | $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$

Plug $x^2$ for $x$ in the previous equation.

# 3 | more finding of polynomial

## 3.1 | TODO $ y = ln (1+x)$

## 3.2 | TODO $y = \tan^{-} x$

## 3.3 | DONE $y = (1+x)^k$

$$\begin{aligned}
P_n(x) &= f(0) &&+\frac{d}{dx}f(0)x &&+\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 &&+\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 &&+\cdots \\
&= k(1)^k &&+k(k-1)(1)^{k-1}x &&+\frac{k(k-1)(k-2)(1)^{k-2}}{2!}x^2 &&+\frac{k(k-1)(k-2)(k-3)(1)^{k-3}}{3!}x^3 &&+\cdots \\
&= k &&+k(k-1)x &&+\frac{k(k-1)(k-2)}{2!}x^2 &&+\frac{k(k-1)(k-2)(k-3)}{3!}x^3 &&+\cdots \\
&= k &&+\frac{k!}{(k-1)!}x &&+\frac{\frac{k!}{(k-2)!}}{2!}x^2 &&+\frac{\frac{k!}{(k-3)!}}{3!}x^3 &&+\cdots \\
&= k &&+\frac{k!x}{(k-1)!} &&+\frac{k!}{(k-2)!2!}x^2 &&+\frac{\frac{k!}{(k-3)!}}{3!}x^3 &&+\cdots
\end{aligned}$$