1 | Exercise 7

Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely dim $E(\lambda, T)$ times.

2 | Proof

`There must be $\odim V$ (non-distinct) eigenvalues on the diagonal.`

Any eigenvalue λ must have dim $E(\lambda, T)$ associated eigenvectors, by definition. ("In other words, $E(\lambda, T)$ is the set of all eigenvectors of T corresponding to λ, along with the 0 vector").

The diagonal is comprised of the basis vector coefficients of the eigenspaces, implying that each eigenspace of λ is represented by dim $E(\lambda, T)$ eigenvectors. λ appears exactly once for each associated eigenvector, or precisely dim $E(\lambda, T)$ times in total.