

$$1 \mid \int \ln x dx$$

$$\begin{aligned}\int \ln x dx &= \int 1 \ln x dx \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int 1 dx \\ &= \boxed{x \ln x - x}\end{aligned}$$

$$2 \mid \int \tan^{-1} x dx$$

$$\begin{aligned}\int \tan^{-1} x dx &= x \tan^{-1} x - \int x \frac{1}{x^2 + 1} dx \\ &= x \tan^{-1} x - \frac{1}{2} \int \frac{du}{u} \\ &= x \tan^{-1} x - \frac{1}{2} \ln u + C &= x \tan^{-1} x - \frac{1}{2} \ln(x^2 + 1) + C\end{aligned}$$

$$3 \mid \int x \sec^2 x dx$$

$$\begin{aligned}\int x \sec^2 x dx &= x \tan x - \int \tan x dx \\ &= x \tan x + \ln |\cos x| + C\end{aligned}$$

$$4 \mid \int x^2 e^{5x} dx$$

$$\begin{aligned}\int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\ &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{5} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\ &= \frac{1}{5} e^{5x} \left(x^2 - 2x + \frac{2}{25} \right) + C\end{aligned}$$