

Source: [\[KBe2020math530index\]](#)

## 1 | Groups

- definition
  - closed
    - if  $a, b \in S$  then  $a + b \in S$
  - has an identity  $e$ 
    - $e + a = a + e = a$
  - each element has an inverse
    - $-a + a = a + -a = e$
  - needs to be associative
    - $(a + b) + c = a + (b + c)$
- commutativity is nice but not required
  - $a + b = b + a$
- Which number systems are groups under addition and multiplication?

Number System	Multiplication	Addition
Natural Numbers	No inverse	No identity
Whole Numbers	No inverse	No inverse
Integers	No inverse	Yes
Rationals	Yes*	Yes
Reals	Yes*	Yes
Complex Numbers	Yes*	Yes

\*Zero doesn't have an inverse, so it usually gets dropped. For example,  $\mathbb{Q}^*$  is  $\mathbb{Q}$  w/o zero #todo-exr0n: rewrite in latex say \$\$\$

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$$\begin{bmatrix} 8 & 2 \\ -2 & 0 \end{bmatrix}$$