Source:

1) Evaluate the following limit using Squeeze theorem (Think about the range of sin(□) □ □ cos(□)) to find the enveloping functions

   a)

$$\lim_{\theta\to\infty} -\frac{1}{\theta} \le \lim_{\theta\to\infty} \frac{\sin\theta}{\theta} \le \lim_{\theta\to\infty} \frac{1}{\theta}$$

$$0 \le \lim_{\theta\to\infty} \frac{\sin\theta}{\theta} \le 0$$

$$\lim_{\theta\to\infty} \frac{\sin\theta}{\theta} = 0 \text{ by the squeeze theorem}$$

   b)

$$\lim_{\theta\to\infty} \frac{1-cos\theta}{\theta} = \lim_{\theta\to\infty} \frac{1}{\theta} - \lim_{\theta\to\infty} \frac{\cos\theta}{\theta}$$

$$= 0 - \lim_{\theta\to\infty} \frac{\cos\theta}{\theta} = -\lim_{\theta\to\infty} \frac{\cos\theta}{\theta}$$

$$\lim_{\theta\to\infty} -\frac{1}{\theta} \le -\lim_{\theta\to\infty} \frac{\cos\theta}{\theta} \le \lim_{\theta\to\infty} \frac{1}{\theta}$$

$$0 \le -\lim_{\theta\to\infty} \frac{\cos\theta}{\theta} \le 0$$

$$-\lim_{\theta\to\infty} \frac{\cos\theta}{\theta} = 0 \text{ by squeeze theorem}$$

$$\lim_{\theta\to\infty} \frac{\cos\theta}{\theta} = 0$$

   c)

$$\lim_{\theta\to\infty} \theta^2 \cos\frac{1}{\theta^2} = \infty$$

There are no functions that can serve as enveloping functions.

2) Prove that

$$\lim_{x\to 0} \frac{\sin\theta}{\theta} = 1$$

   using steps below and using the sketch of a unit circle where the angle □ is in radians. K is a point on the unit circle.

   a)

$$K = (\cos\theta, \sin\theta)$$

   b)

$$\text{Slope of } OK = \frac{\sin\theta}{\cos\theta}$$

   c)

$$OL : y - \sin\theta = \frac{\sin\theta}{\cos\theta}(x - \cos\theta)$$

   d)

$$A = (1, 0)$$

   e)

$$L = (1, \frac{\sin\theta}{\cos\theta})$$

   f)

$$\triangle OAK = \frac{\sin\theta}{2}$$

g)

$$\square\ OAK = \frac{\theta}{2}$$

h)

$$\triangle OAL = \frac{\sin\theta}{2\cos\theta}$$

i)

$$\frac{\sin\theta}{2} \leq \frac{\theta}{2} \leq \frac{\sin\theta}{2\cos\theta}$$

j)

$$\lim_{\theta\to 0} 1 \leq \lim_{\theta\to 0} \frac{\theta}{\sin\theta} \leq \lim_{\theta\to 0} \frac{1}{\cos\theta}$$

$$\lim_{\theta\to 0} \frac{1}{1} \leq \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} \cos\theta$$

$$1 \leq \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \leq 1$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \text{ by the squeeze theorem}$$