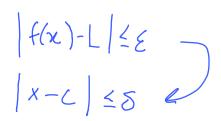
Exploration 2-3a: Limit of a Sum, Epsilon and Delta Proof

Date: _____

Objective: Prove by ε - δ techniques that the limit of a sum of two functions equals the sum of the two limits.

1. Write the definition of limit using ε and δ .



For Problems 2-5,

Let
$$y_1 = 5x + 1$$
.

Let
$$y_2 = x^2$$
.

Let
$$y_3 = y_1 + y_2$$
.

2. Find

$$L_1 = \lim_{x \to 2} y_1 = \frac{1}{1 + 1}$$
 and $L_2 = \lim_{x \to 2} y_2 = \frac{1}{1 + 1}$

3. Let $\varepsilon = 0.1$. Find δ_1 such that y_1 is within $\varepsilon/2$ unit of L_1 on the positive side by substituting $(L_1 + 0.05)$ for y_1 and $(2 + \delta_1)$ for x.

$$L_{1} + \frac{\varepsilon}{2} = 5(C+\delta) + 1$$

$$11 + \frac{\varepsilon}{2} = 5(2+\delta) + 1$$

$$\frac{10+\frac{E}{2}}{5} = 2+6$$

$$5 = \frac{E}{10} = 0.001$$

4. Using $\varepsilon = 0.1$ as in Problem 3, find δ_2 such that y_2 is within $\varepsilon/2$ units of L_2 on the positive side by substituting ($L_2 + 0.05$) for y_2 and ($2 + \delta_2$) for x.

$$\begin{vmatrix} \chi^{2} - 4 \end{vmatrix} \leq \underbrace{\varepsilon}_{2} + \underbrace{(2+\delta)^{2}}_{2} + \underbrace{(2+\delta)^{2}}_{3} + \underbrace{(2+\delta)^{2}}_{4} + \underbrace{($$

5. Add $L_1 + L_2$ to get $L = \lim_{y \to 2} y_3$.

Let $\delta = \min \{\delta_1, \delta_2\}$, which means that δ is the smaller of δ_1 and δ_2 . Make a table of values of y_3 for several values of x between 2 and $(2 + \delta)$. Does the table show that y_3 is within $\varepsilon = 0.1$ unit of L whenever x is within δ units of 2 on the positive side?

5 = 0.001

and 4 works

For Problems 6 and 7,

Let
$$f(x) = g(x) + h(x)$$
.

Let
$$L_1 = \lim_{x \to a} g(x)$$
.

Let
$$L_2 = \lim_{x \to c} h(x)$$
.

6. Suppose that someone has chosen a number $\varepsilon > 0$. Because L_1 is the limit of g(x) as x approaches c, you can keep g(x) as close as you like to L_1 just by keeping x close enough to c. Thus, there is a number δ_1 such that g(x) can be kept within $\varepsilon/2$ units of L_1 . That is,

$$L_1 + e/2 < g(x) < L_1 + \varepsilon/2$$

Similarly, you can keep h(x) within $\varepsilon/2$ units of L_2 just by keeping x within δ_2 units of c. Write an inequality for h(x) if x is within δ_2 units of c.

$$L_z - \frac{\varepsilon}{2} < h(x) < L_z + \frac{\varepsilon}{2}$$

7. Let $\delta = \min \{\delta_1, \delta_2\}$. Thus, both inequalities in Problem 6 will be true. By appropriate operations on these inequalities, show that f(x) is within ε units of $(L_1 + L_2)$, and thus that $(L_1 + L_2)$ is the limit of f(x) as x approaches c.

 $L_{1} - \frac{\varepsilon}{2} + L_{2} - \frac{\varepsilon}{2} < g(x) + h(x) < L_{1} + \frac{\varepsilon}{2} + L_{2} + \frac{\varepsilon}{2}$ $L_{1} + L_{2} - \varepsilon < g + h(x) < L_{1} + L_{2} + \varepsilon$

8. State the property of the limit of a sum verbally.

limit of a sum of functions is the sum of their limits.

9. What did you learn as a result of doing this Exploration that you did not know before?

You can directly add inequalities