

12 | **estimate area**

Right handed Riemann Sum:

$$0.5 + 4 + 10 + 13 + 10 + 0 = 37.5$$

13 | **estimate area again**

$$4(200 + 2700 + 1100 + 4000 + 200) = 32800$$

14 | **area between curves**

$$\begin{aligned} \int_0^{10} 2200e^{0.024t} dx - \int_0^{10} 1360e^{0.018t} dx &= \frac{1}{0.024} 2200e^{0.024t} - \frac{1}{0.018} 1360e^{0.018t} \\ \Rightarrow \frac{1}{0.024} 2200e^{0.24} - \frac{1}{0.018} 1360e^{0.18} - \frac{1}{0.024} 2200 + \frac{1}{0.018} 1360 &\approx 9964 \end{aligned}$$

15 | **meaning of area**

The shaded region represents the profit made between producing 50 units and 100 units.

16 | **TODO slicing pizza into three using parallel cuts**

The problem of placing slices is the same if we only worry about the top half of the pizza. Thus, we can choose some  $x$  for the first slice s.t.

$$\begin{aligned} 2 \int_{-7}^x \sqrt{7^2 - t^2} dt &= \int_x^7 \sqrt{7^2 - t^2} dt \\ 2 \int_{-7}^x \sqrt{7^2 - t^2} dt - \int_x^7 \sqrt{7^2 - t^2} dt &= 0 \\ 2 \int_{-7}^x \sqrt{7^2 - t^2} dt + \int_7^x \sqrt{7^2 - t^2} dt &= 0 \\ 2 \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^{-7} \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\ 2 \left( \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\ 2 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt + \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\ 3 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\ 3 \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt &= 0 \\ 3 \int_0^x \sqrt{7^2 - t^2} dt + \frac{49\pi}{4} &= 0 \end{aligned}$$

17 | **tractrix**

At any moment, if the boat is at  $(x, y)$  and the puller is at  $(0, h)$ , then the velocity of the boat is in the direction

$$\frac{y - h}{x}$$