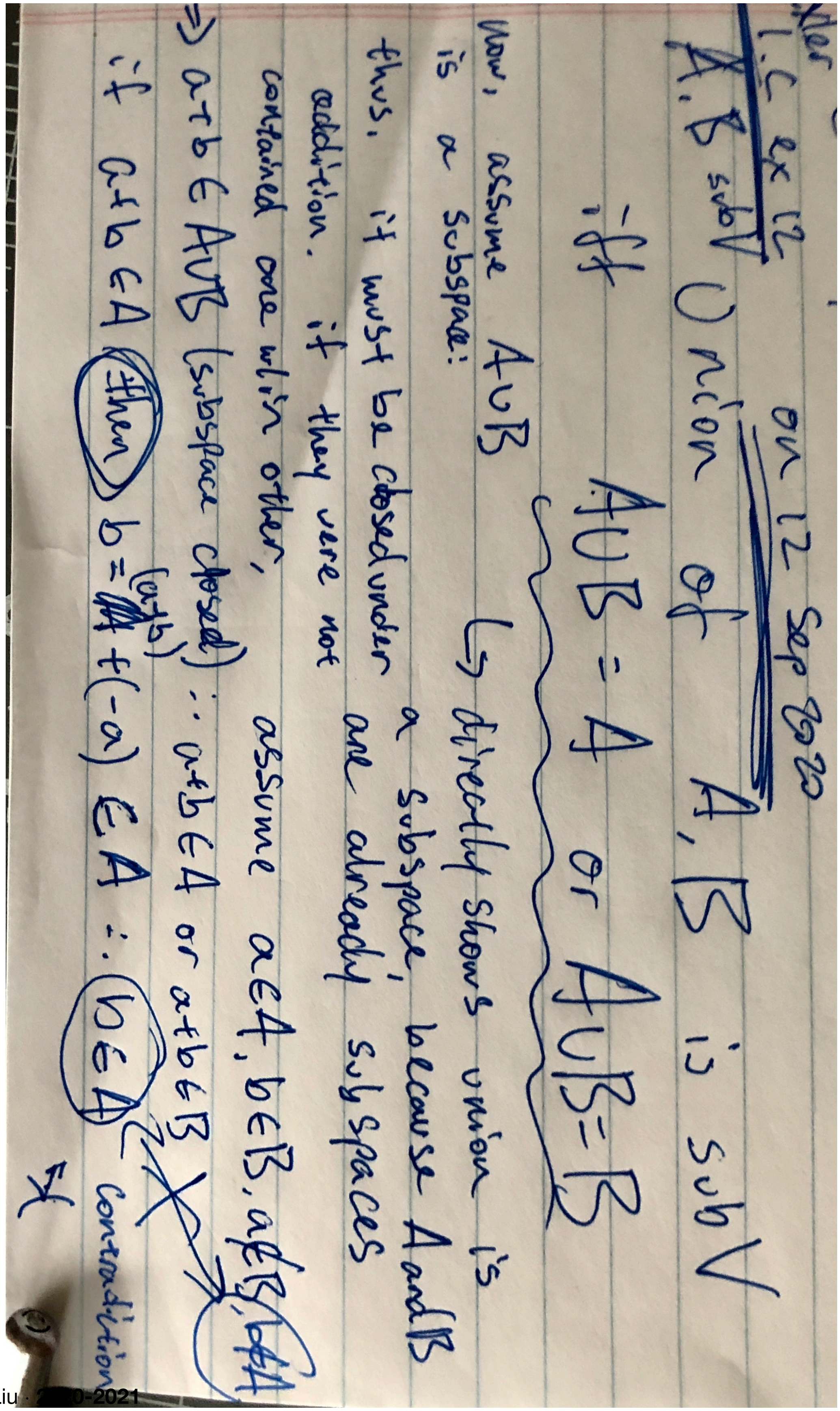
Lemma

Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.

Working it out



1.C ex 12 on 12 Sep 2020

A, B subV Union of A, B is subV iff $A \cup B = A$ or $A \cup B = B$

now, assume $A \cup B$ is a subspace:

thus, it must be closed under addition. if they were not contained one w/in other,

$\Rightarrow a+b \in A \cup B$ (subspace closed) $\therefore a+b \in A$ or $a+b \in B$

if $a+b \in A$ then $b = (a+b) + (-a) \in A \therefore b \in A$ contradiction

↳ directly shows union is a subspace, because A and B are already subspaces

assume $a \in A, b \in B, a \notin B, b \notin A$

Figure 1: Scribbles