

**Source:**

## 1 | Problem

Suppose  $T \in \mathcal{L}(V, W)$  and  $U$  is a subspace of  $V$ . Let  $\pi$  denote the quotient map from  $V$  onto  $V/U$ . Prove that there exists  $S \in \mathcal{L}(V/U, W)$  such that  $T = S \circ \pi$  if and only if  $U \subseteq \text{null } T$ .

## 2 | Forward Direction by Contrapositive

If  $U \not\subseteq \text{null } T$ , then there exists  $v \in U$  s.t.  $Tv \neq 0$ .

## 3 | Reverse Direction