Source: [KBe2020math530refExr0nRetIndex]

#ret

1. Suppose A = $\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$. Multiply AB and BA. What do you notice???

Nothing. Matrix multiplication is not commutative.

2. Use matrices to solve the system: $\begin{aligned} 2x - y &= 3 \\ x + 3y &= 5 \end{aligned}$

$$\begin{aligned}
\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} \\
\begin{bmatrix} 7 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
\begin{bmatrix} 7x \\ x + 3y \end{bmatrix} &= \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
\begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 7x \\ x + 3y \end{bmatrix} &= \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
\begin{bmatrix} x \\ x + 3y \end{bmatrix} &= \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\
x &= 2 \\
x + 3y &= 5
\end{aligned}$$

I'm not sure how to solve the rest of it with matrices, so I'll just do it normally:

$$\begin{aligned}
x &= 2 \\
x + 3y &= 5 \\
2 + 3y &= 5 \\
3y &= 3 \\
y &= 1
\end{aligned}$$

3. > Do 2x2 matrices form a group under > a. addition? > b. multiplication?

See [KBe2020math530refGroups] I'll assume that our matrices have real numbers in them.

Operation	Property	Closed	Identity	Inverse	Associative?	Final
Addition		Yes	$e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} = e$	"Inherits from addition"	Yes
Multiplication		Yes	$e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Maybe?	Yes, see below	Undecided

Associativity of 2x2 matrices under multiplication:

$$\begin{aligned}
\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}e & f\\ g & h\end{bmatrix}\right)\begin{bmatrix}i & j\\ k & l\end{bmatrix} &= \begin{bmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{bmatrix}\begin{bmatrix}i & j\\ k & l\end{bmatrix}\\
&= \begin{bmatrix}aei+bgi+afk+bhk & aej+bgj+afl+bhl\\ cei+dgi+cfk+dhk & cej+dgj+cfl+dhl\end{bmatrix}\\
= \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}ei+fk & ej+fl\\ gi+hk & gj+hl\end{bmatrix} &= \begin{bmatrix}a & b\\ c & d\end{bmatrix}\left(\begin{bmatrix}e & f\\ g & h\end{bmatrix}\begin{bmatrix}i & j\\ k & l\end{bmatrix}\right)
\end{aligned}$$

I can't figure out if 2x2 matrices have multiplicative inverses… I tried to work it out using algebra but kept proving identities. Not sure what the right way to go about this is…

I spent far too long on this assignment (1.6h), so I probably won't spend as much time LaTeXing my homework in the future.