Source:

1 | See Before: homogeneous equations

2 | Homogeneous systems of linear equations with more variables than equations has nonzero solutions (Axler3.26)

3 | Inhomogeneous systems of linear equations with more equations than variables have no solution for some choice of constant terms

4 | methodology

Basically turn it into a linear map and then abuse the injective/surjective theorems