

1 | Exercise 3

Axler 6.B ex 3

orthonormality $(1, 0, 0), (1, 1, 1), (1, 1, 2)$

$$e_1 = (1, 0, 0), \quad e_2 = \text{normalize}((1, 1, 1) - (1, 0, 0)) \\ = \text{normalize}(0, 1, 1) \\ = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$e_3 = \text{normalize}\left((1, 1, 2) - (1, 0, 0) - \frac{3}{\sqrt{2}}(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\right)$$

$$= \text{normalize}\left((0, 1, 2) - (0, \frac{3}{2}, \frac{3}{2})\right)$$

$$= \text{normalize}\left(0, \frac{1}{2}, \frac{1}{2}\right)$$

$$= (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$(1, 0, 0), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$0 + \frac{1}{2} + \frac{1}{2} = 0 \quad \checkmark$$

so all pairwise inner products are zero

$$v = (1, 2, 3)$$

$$= \langle (1, 2, 3), (1, 0, 0) \rangle e_1 + \langle (1, 2, 3), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rangle e_2 + \langle (1, 2, 3), (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rangle e_3$$

$$= \boxed{1e_1 + \frac{5}{\sqrt{2}}e_2 + \frac{1}{\sqrt{2}}e_3}$$

$$= (1, 0, 0) + (0, \frac{5}{2}, \frac{5}{2}) + (0, \frac{1}{2}, \frac{1}{2})$$

$$= (1, \frac{4}{2}, \frac{6}{2}) \quad \checkmark$$