

1 | Axler6.31 Gram-Schmidt Procedure

The Gram-Schmidt Procedure is used to turn a list into an orthonormal list with the same span. It's useful for finding orthonormal bases.

Suppose v_1, \dots, v_m is a linearly independent list of vectors in V . Let $e_1 = v_1/\|v_1\|$. For $j = 2, \dots, m$, define e_j inductively by

$$e_j = \frac{v_j - \langle v_j, e_1 \rangle e_1 - \dots - \langle v_j, e_{j-1} \rangle e_{j-1}}{\|\text{<numerator>}\|}$$

Then e_1, \dots, e_m is an orthonormal list of vectors in V s.t. each prefix span is the same as in v_1, \dots, v_m .

1.1 | intuition

Basically, for each vector, we divide out the components from the previous vectors and then normalize the size to ensure the norm is one.

It's kind of like the orthogonal decomposition.