Source: [KBe2020math530floIndex]

- Vector spaces and fields are like groups
  - With 2 operations
- Vector
  - direction and magnitude
  - numbers with an order
    - list = ordered set
    - Nx1 matrix
  - A vector is not just an Nx1 matrix. **A vector exists in a vector space**
    - might be full of physics vectors, matrices, or polynomials
- Field
  - Addition and multiplication might be different
    - might be related to normal addition/multiplication
    - might by any binary operation
    - Addition is "primary" operation, multiplication is "secondary"
      - addition is really good (more group like)
      - multiplication needs to exclude the additive identity (because it can't have an inverse)
    - questions
      - multiplication is repeated addition?
        - not nessacarily
      - binary expressions?
      - associative?
        - both yes
    - 1.3 demonstrates that the complex numbers are a field
      - communativity
      - associativity
      - identities
      - additive inverse
      - multiplicitive inverse except additive identity
      - distributive
  - usually means Reals or Complex
    - ints mod 3 are a field
      - #bonushw show ints mod 3 are a field
  - higher dimensions
    - R^2 is a cartesian plane, R^4 is a space
  - operations
    - addition is really nice (element wise)
    - scalar multiplication is easy enough
    - vector vector multiplication is hard to find
- two square roots of i
  - fundamental theorem of algebra
    - (important)
  - So, i should have two square roots
  - Powers of i go in a circle (90 degrees rotation)
    - Complex number rotation gives a preferred direction

        - So that's why the quadrants are numbered in that direction
    - One can be found geometrically 20math530srcSquareRootI.png
    - We could also try it algebraically
        - $(a+bi)^2 = i = a^2 - b^2 + 2abi$
        - so $a^2 - b^2 = 0$ and $2ab = 1$
- dot product
    - How much of $\vec{A}$ is in the direction of $\vec{B}$ multiplied by the magnitude of $\vec{B}$
    - $\vec{A} \cdot \vec{B} = |A||B|cos\theta$
        - #bonushw prove that ^^
    - Identity: $\frac{\vec{A}\cdot\vec{B}}{|A||B|} = cos\theta$
- cross product
    - only works on 3x1 matrices
    - steps
        - determinant
        - i, j, k are the unit vectors
        - $$\begin{bmatrix}2\\1\\0\end{bmatrix}\begin{bmatrix}1\\2\\-1\end{bmatrix} = \left|\begin{bmatrix}i & j & k\\2 & 1 & 0\\1 & 2 & -1\end{bmatrix}\right| = i\left|\begin{bmatrix}1 & 0\\2 & -1\end{bmatrix}\right| - j\left|\begin{bmatrix}2 & 0\\1 & -1\end{bmatrix}\right| + k\left|\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\right| = \begin{bmatrix}-1\\2\\3\end{bmatrix}$$
- dropping zero: $0a = (0+0)a = 0a + 0a \Rightarrow 0a = 0$, so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
    - 20math530srcFieldsMultiplyCannotBeGroup.png
- determinant
    - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
- #todo #exr0n #future prove identities are unique
