

## 1 | Axler6.53 orthogonal projection, $P_U$ def

Suppose  $U$  is a finite-dimensional subspace of  $V$ . The *orthogonal projection* of  $V$  onto  $U$  is the operator  $P_U \in \mathcal{L}(V)$  defined as follows:

For  $v \in V$ , write  $v = u + w$ , where  $u \in U$  and  $w \in U^\perp$ . Then  $P_U v = u$ .

In other words,  $P_U \in \mathcal{L}(V)$  takes  $v$  to the component of  $v$  that is in  $U$ .

### 1.1 | Results

#### 1.1.1 | Axler6.54 calculating $P_U v$

$$P_U v = \frac{\langle v, x \rangle}{\|x\|^2} x$$

Because orthogonal decompositions and stuff

#### 1.1.2 | Axler6.55 properties

Suppose  $U$  is a finite-dimensional subspace of  $V$  and  $v \in V$ . Then,

1.  $P_U \in \mathcal{L}(V)$