

1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 du - \frac{1}{u^2+1} + C \\ &= 2 \int 1 du - \tan^{-1} u + C \\ &= 2u - \tan^{-1} u + C \\ &= \boxed{2\sqrt{x-1} - \tan^{-1}(\sqrt{x-1}) + C} \end{aligned}$$

2 | $\int \frac{x^2}{x^2+1} dx$

Let $u = x^2 + 1$, $du = 2x dx$

$$\begin{aligned} \int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\ &= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\ &= \frac{1}{2} (u - \ln u) + C \\ &= \boxed{\frac{1}{2} (x^2 + 1 - \ln(x^2 + 1)) + C} \end{aligned}$$

3 | **TODO** $\int \frac{x-4}{x^2} dx$

Let $u = x - 4$, $du = dx$

$$\begin{aligned} \int \frac{x-4}{x^2} dx &= \int \frac{u}{(u+4)^2} dx \\ &= \int \frac{u}{u^2+8u+16} du \end{aligned}$$

Let $v = u + 4$

$$4 \mid \int (x+1)e^{x^2+2x} dx$$

$$\text{Let } u = x^2 + 2x, du = x + 1$$

$$\begin{aligned} \int (x+1)e^{x^2+2x} dx &= \frac{1}{2} \int e^u du \\ &= \frac{1}{2} e^u \\ &= \boxed{\frac{1}{2} e^{x^2+2x}} \end{aligned}$$

$$5 \mid \int \tan^2 x + 1 dx$$

$$\begin{aligned} \int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx \end{aligned}$$

$$\text{Let } u = x, du = 1$$

$$\begin{aligned} &= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C} \end{aligned}$$

$$6 \mid \textbf{TODO} \int \frac{6x^2-4}{x} dx$$

$$7 \mid \int \frac{e^x-1}{e^x} dx$$

$$\begin{aligned} \int \frac{e^x-1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C} \end{aligned}$$

$$8 \mid \int \frac{\sec^2 x}{\csc x} \sin x dx$$

$$\begin{aligned} \int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x} \end{aligned}$$

9 | **9** $\int \sin x \cos x \, dx$

Let $u = \sin x$, then $du = \cos x \, dx$

$$\begin{aligned}\int \sin x \cos x \, dx &= \int u \, du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x}\end{aligned}$$

10 | **10** $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} \, dx$

$$\begin{aligned}\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} \, dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} \, dx \\ &= \int \frac{\sin^2 x}{\tan^2 x + 1} + \frac{\cos^2 x}{\tan^2 x + 1} \, dx \\ &= \int \sin^2 x \cos^2 x + \cos^4 x \, dx\end{aligned}$$

$$\begin{aligned}&= \int \frac{1}{\tan^2 x + 1} \, dx \\ &= \int \frac{1}{\sec^2 x} \, dx \\ &= \int \cos^2 x \, dx\end{aligned}$$

11 | $\int \frac{\sec x \tan x}{1 + \sec^2 x} \, dx$

Let $u = \sec x$,

12 | **13**13 | **14**