

Source:

## 1 | Definitions

### 1.1 | affine subset

An affine subset of a vector space  $V$  is of the form  $U + v$  where  $U \subseteq V$  and  $v \in V$ .

### 1.2 | product space

The product of some vector spaces  $V_1 \times \cdots \times V_n$  is the set of lists of vectors with one from each respective space:

$$\{(v_1, \dots, v_n) : v_1 \in V_1, \dots, v_n \in V_n\}$$

### 1.3 | quotient space

A quotient space  $V/U$  is the set of affine subsets  $\{U + v : v \in V\}$  (although some of those affine subsets are equivalent).

### 1.4 | equivalence relation

An equivalence relation is a set of elements that are considered equivalent (equal to each other). For example, in a vector space  $U$ ,  $U + 0 = U + u \forall u \in U$ .

## 2 | Why "product" and "quotient" are used to describe these operations

The product of vector spaces is essentially a cartesian product. With real numbers, the product is like stacking copies of an operand in a new direction (product of two numbers for area of a plane, product of three for the volume of a space). Here, we are doing essentially the same thing for vector spaces (each vector is combined within a list with other vector spaces, but they do not interact with each other and are orthogonal, in a sense).

A quotient space is like taking (dividing) out part of a vector space. It's like taking a modulo because a subset ( $U$ ) is collapsed to zero and some things become equivalent. It is like removing ("dividing") a subset of the basis (those that form a basis of  $U$ ), where the basis itself can be represented as the cartesian product  $\mathbb{F}^n$  (where  $n$  is the dimension of  $V$ ).

## 3 | Examples of quotient spaces