Source:

1 | find taylor series

1.1 | $y = \cos(x)$

$$\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= \cos(0) && -\sin(0)x && +\frac{-\cos(0)}{2!}x^2 && +\frac{\sin(0)}{3!}x^3 && +\cdots \\
&= 1 && -0x && -\frac{1}{2!}x^2 && +\frac{0}{3!}x^3 && +\cdots \\
&= 1 && && -\frac{1}{2!}x^2 && && +\frac{1}{4!}x^4+\cdots \\
&= \sum_{k=0} \frac{(-1)^k}{(2k)!}x^{2k}
\end{aligned}$$

1.2 | $y = e^x$

$$\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= e^0 && +e^0x && +\frac{e^0}{2!}x^2 && +\frac{e^0}{3!}x^3 && +\cdots \\
&= 1 && +x && +\frac{1}{2!}x^2 && +\frac{1}{3!}x^3 && +\cdots \\
&= \sum_{k=0} \frac{1}{k!}x^k
\end{aligned}$$

1.3 | **TODO** $y = \sqrt{x}$

2 | prove approximations

2.1 | $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Proof by geometric series

2.2 | $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

Plug $-x$ for x in the previous equation.

2.3 | $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$

Plug x^2 for x in the previous equation.

3 | more finding of polynomial

3.1 | TODO $ y = ln (1+x)$

3.2 | TODO $y = \tan^{-} x$

3.3 | DONE $y = (1+x)^k$

$$\begin{aligned}
P_n(x) &= f(0) && +\frac{d}{dx}f(0)x && +\frac{\frac{d^2}{d^2x}f(0)}{2!}x^2 && +\frac{\frac{d^3}{d^3x}f(0)}{3!}x^3 && +\cdots \\
&= k(1)^k && +k(k-1)(1)^{k-1}x && +\frac{k(k-1)(k-2)(1)^{k-2}}{2!}x^2 && +\frac{k(k-1)(k-2)(k-3)(1)^{k-3}}{3!}x^3 && +\cdots \\
&= k && +k(k-1)x && +\frac{k(k-1)(k-2)}{2!}x^2 && +\frac{k(k-1)(k-2)(k-3)}{3!}x^3 && +\cdots \\
&= k && +\frac{k!}{(k-1)!}x && +\frac{\frac{k!}{(k-2)!}}{2!}x^2 && +\frac{\frac{k!}{(k-3)!}}{3!}x^3 && +\cdots
\end{aligned}$$