> Suppose $T \in \mathcal{L}(V)$ and $\lambda \in \mathbb{F}$. Prove that λ is an eigenvalue of T iff $\overline{\lambda}$ is an eigenvalue of T^*.

Given λ is an eigenvalue of T, show that $\overline{\lambda}$ is an eigenvalue of T^*. This will imply both directions, since $\lambda = \overline{\overline{\lambda}}$ and $T = T^{*^*}$

Suppose $\mathcal{M}(T)$ is the matrix of T wrt some orthonormal basis. Then, the matrix $\mathcal{M}(T^*)$ of T^* wrt the same orthonormal basis will equal the conjugate transpose of $\mathcal{M}(T)$.

Eigenvalues lie on the diagonal of a matrix, so the conjugate transpose will have the effect of conjugating each eigenvalue. Thus, the eigenvalues of $\mathcal{M}(T)$ are conjugates of the eigenvalues of $\mathcal{M}(T^*)$.

There exists some v s.t.

$$Tv = \lambda v$$

$$\langle \lambda v, w\rangle = \langle Tv, w\rangle = \langle v, T^* w\rangle$$