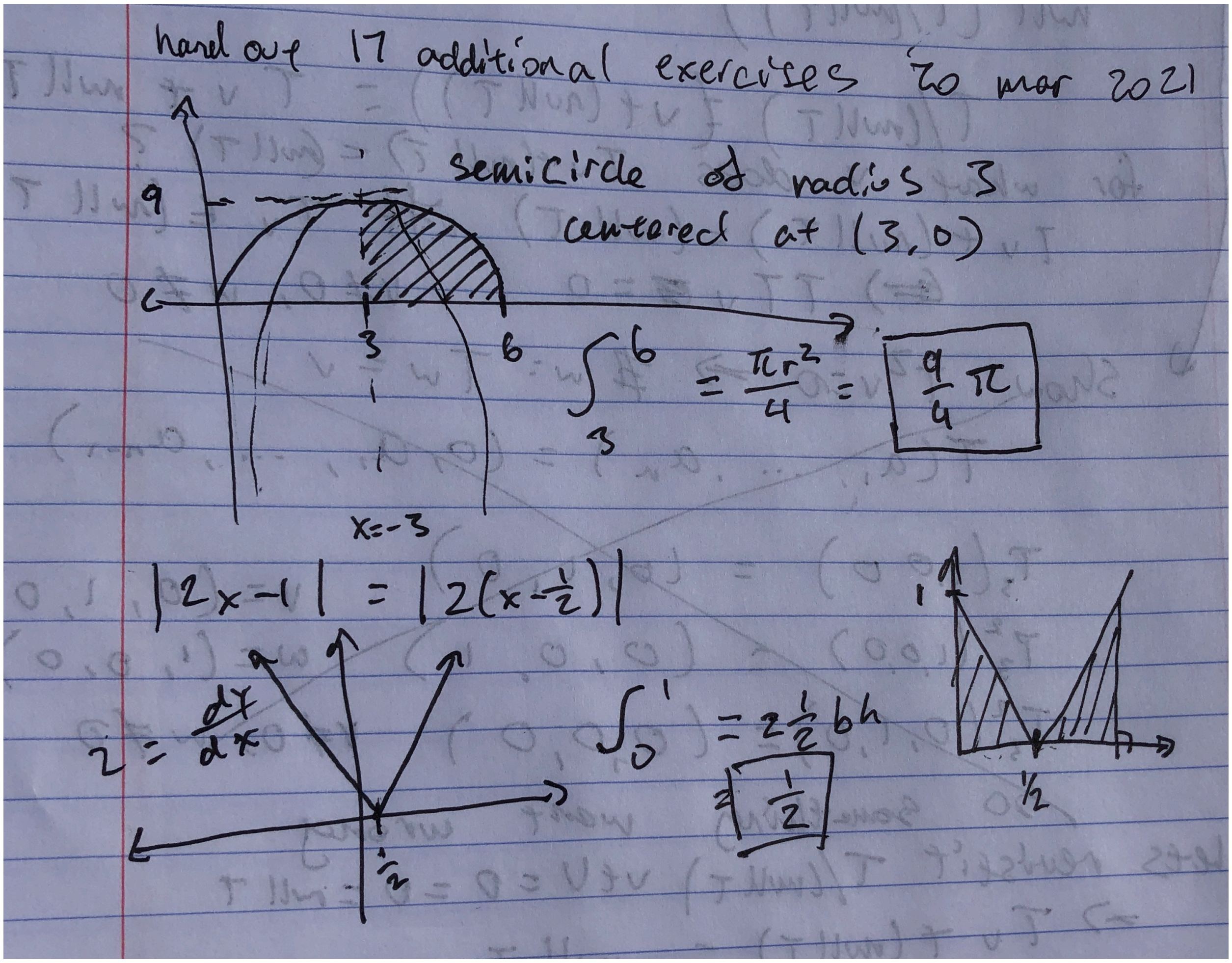
1 | Exercises

1.1 | interpreting in terms of area



1.3 | subtracting integrals

I expect

$$\int_1^2 f(x)dx = \int_1^5 f(x)dx - \int_2^5 f(x)dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

1.4 | **show** $\int_a^b x^2 dx = \frac{b^3-a^3}{3}$

$$\begin{aligned}
\int_a^b x^2 dx &= \lim_{n\to\infty} \sum_{k=0}^{n} \frac{b-a}{n} \left(a + k\frac{b-a}{n}\right)^2 \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} \left(a + k\frac{b-a}{n}\right)^2 \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} \left(a^2 + 2ak\frac{b-a}{n} + \left(k\frac{b-a}{n}\right)^2\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(na^2 + \sum_{k=0}^{n} \left(2ak\frac{b-a}{n} + \left(k\frac{b-a}{n}\right)^2\right)\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} a^2 \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} a^2 \\
&= \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} a^2 + \cancelto{0}{2ak\frac{b-a}{n}} + \cancelto{0}{\left(k\frac{b-a}{n}\right)^2}
\end{aligned}$$