

1 | **Axler6.31 Gram-Schmidt Procedure**

The Gram-Schmidt Procedure is used to turn a list into an orthonormal list with the same span. It's useful for finding orthonormal bases.

Suppose v_1, \dots, v_m is a linearly independent list of vectors in V . Let $e_1 = v_1/\|v_1\|$. For $j = 2, \dots, m$, define e_j inductively by

$$e_j = \frac{v_j - \langle v_j, e_1 \rangle e_1 - \dots - \langle v_j, e_{j-1} \rangle e_{j-1}}{\|\text{<numerator>}\|}$$

Then e_1, \dots, e_m is an orthonormal list of vectors in V s.t. each prefix span is the same as in v_1, \dots, v_m .

1.1 | **intuition**

Basically, for each vector, we divide out the components from the previous vectors and then normalize the size to ensure the norm is one.

It's kind of like the orthogonal decomposition.

2 | **results**

2.1 | **Axler6.34 orthonormal basis exists in finite dim vec spaces**

since every finite dim vec space has a basis that can be Gram-schmidt-ed

2.2 | **Axler6.35 orthonormal lists extend to orthonormal bases**

just extend the orthonormal list into a basis, and then gram-schmidt-ify the vectors you added

2.3 | **Axler6.37 upper-triangular matrix wrt orthonormal basis**

If an upper triangular matrix exists for some operator, then an upper-triangular matrix exists for an orthonormal basis too.

Proof: prefix span invariance is a condition for an upper triangular matrix, so the prefix span equality

An application of this is