

Source:

1 | sum of a vector and a subspace def

1.1 | for $v \in V$ and $U \subset V$, $v + U = \{v + u : u \in U\}$ (aka shift everything by v)

2 | affine subset, parallel def

2.1 | an affine subset of V is a subset of V that is "shifted" by a vector in V

2.2 | all affine subsets from a subspace are said to be parallel to that subspace

3 | quotient space def

3.1 | A quotient space V/U where $U \subset V$ is the set of affine subsets parallel to U (all shifts)

3.2 | result

3.2.1 | two affine subsets parallel to U are equal or disjoint (Axler 3.85)

1. intuition

(a) if they are 'parallel', then they must be equal (inf intersection) or disjoint (zero intersection)

3.2.2 | the quotient space is a vector space

3.2.3 | quotient map, π def

1. The quotient map $\pi : V \rightarrow V/U$ is defined by $\pi(v) = v + U \forall v$
2. basically it gives a quotient space from a vector (syntactic sugar)

3.2.4 | dimension of a quotient space

1. $\dim V/U = \dim V - \dim U$

4 | squiggle T (the condensed map)

4.1 | for $T \in \mathcal{L}(V, W)$, $T_{\text{squiggle}} : V/(\text{null } T) \rightarrow W$ s.t. $T_{\text{squiggle}}(v + \text{null } T) = Tv$

4.2 | basically it takes an affine subset that encodes the important part of the input (takes v from $\pi(v)$) and maps it to W

4.3 | makes an isomorphism to $\text{range } T$