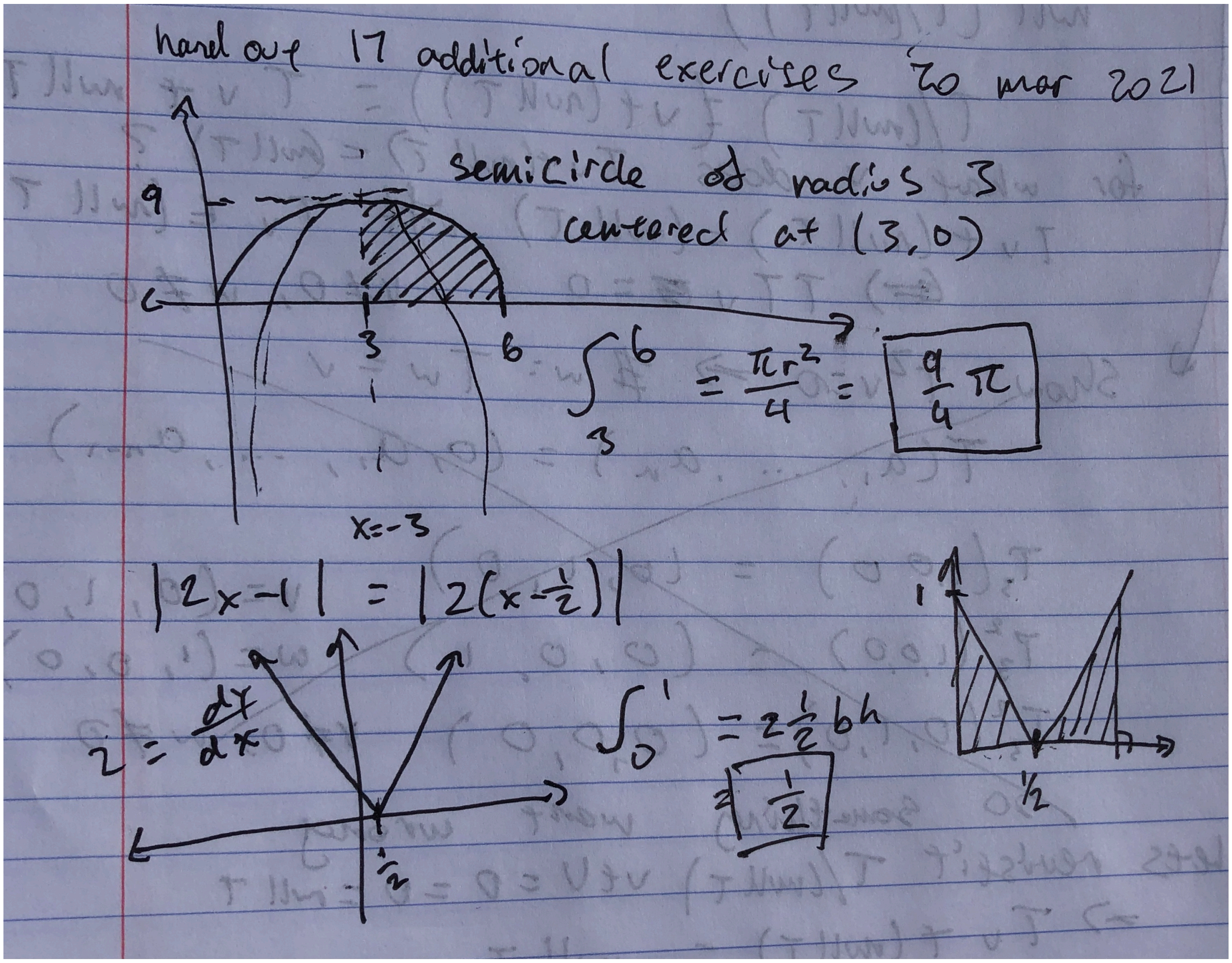
1 | **Exercises**

1.1 | **interpreting in terms of area**



1.3 | **subtracting integrals**

I expect

$$\int_1^2 f(x)dx = \int_1^5 f(x)dx - \int_2^5 f(x)dx = -3 - 4 = -7$$

In fact, I expect

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

1.4 | **show** $\int_a^b x^2 dx = \frac{b^3-a^3}{3}$

(see attached pages)

Keep in mind

$$\sum_{k=1}^{n} af(x) = a\sum_{k=1}^{n} f(x)$$
$$\sum_{k=1}^{n} (a + f(x)) = an + \sum_{k=1}^{n} f(x)$$

$$\begin{aligned}
\int_a^b x^2dx &= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{b-a}{n}\left(a+k\frac{b-a}{n}\right)^2\right)\\
&= \lim_{n\to\infty}\frac{b-a}{n}\sum_{k=1}^{n}\left(a+k\frac{b-a}{n}\right)^2\\
&= \lim_{n\to\infty}\frac{b-a}{n}\sum_{k=1}^{n}\left(a^2+\left(k\frac{b-a}{n}\right)^2+2ak\frac{b-a}{n}\right)\\
&= \lim_{n\to\infty}\frac{b-a}{n}\sum_{k=1}^{n}a^2+\sum_{k=1}^{n}\left(k\frac{b-a}{n}\right)^2+\sum_{k=1}^{n}2ak\frac{b-a}{n}\\
&= \lim_{n\to\infty}\frac{b-a}{n}\left(a^2n+\sum_{k=1}^{n}k^2\left(\frac{b-a}{n}\right)^2+2a\frac{b-a}{n}\sum_{k=1}^{n}k\right)\\
&= \lim_{n\to\infty}\frac{b-a}{n}\left(a^2n+\left(\frac{b-a}{n}\right)^2\sum_{k=1}^{n}k^2+2a\frac{b-a}{n}\sum_{k=1}^{n}k\right)\\
&= \lim_{n\to\infty}\frac{b-a}{n}\left(a^2n+\left(\frac{b-a}{n}\right)^2\frac{n(n+1)(2n+1)}{6}+2a\frac{b-a}{n}\frac{n(n+1)}{2}\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\left(\frac{b-a}{n}\right)^2\frac{(n+1)(2n+1)}{6}+\cancel{2}a\frac{b-a}{n}\frac{(n+1)}{\cancel{2}}\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{n^2}\left(n\frac{(2n+1)}{6}+\frac{(2n+1)}{6}\right)+a\frac{b-a}{n}(n+1)\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{n^2}\left(n\frac{(2n+1)}{6}+\frac{(2n+1)}{6}\right)+a\cancel{n}\frac{b-a}{\cancel{n}}+a\frac{b-a}{n}\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{n^2}\left(n\frac{(2n+1)}{6}+\frac{(2n+1)}{6}\right)+a(b-a)+a\cancelto{0}{\frac{b-a}{n}}\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{n^{\cancel{2}}}\cancel{n}\frac{(2n+1)}{6}+\cancelto{0}{\frac{(b-a)^2}{n^2}\frac{(2n+1)}{6}}+a(b-a)\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{n}\frac{(2n+1)}{6}+a(b-a)\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{\cancel{n}}{3}\frac{(b-a)^2}{\cancel{n}}+\cancelto{0}{\frac{1}{6}\frac{(b-a)^2}{n}}+a(b-a)\right)\\
&= \lim_{n\to\infty}(b-a)\left(a^2+\frac{(b-a)^2}{3}+a(b-a)\right)\\
&= \lim_{n\to\infty}a^2(b-a)+\frac{b^2+a^2-2ab}{3}(b-a)+a(b-a)^2\\
&= \lim_{n\to\infty}\frac{1}{3}\left(3a^2(b-a)+b^2(b-a)+a^2(b-a)-2ab(b-a)+3a(b-a)^2\right)\\
&= \lim_{n\to\infty}\frac{1}{3}\left(3a^2b-3a^3+b^3-ab^2+a^2b-a^3-2ab^2+2a^2b+3a(b^2+a^2-2ab)\right)\\
&= \lim_{n\to\infty}\frac{1}{3}\left(3a^2b-3a^3+b^3-ab^2+a^2b-a^3-2ab^2+2a^2b+3ab^2+3a^3-6a^2b\right)\\
&= \lim_{n\to\infty}\frac{1}{3}\left(3a^2b-3a^3+b^3-ab^2+a^2b-a^3-2ab^2+2a^2b+3ab^2+3a^3-6a^2b\right)
\end{aligned}$$

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