Source:

1 | Axler3.6 sum ($S + T$)

If $S, T \in \mathcal{L}(V, W)$ then the *sum* $S + T$ is defined by

$$(S + T)(v) = Sv + Tv$$

$(S + T)$ is a linear map.

2 | Axler3.6 scalar product λT

If $T \in \mathcal{L}(V, W)$ and $\lambda \in \mathbb{F}$ then the *product* $(\lambda T)v = \lambda Tv$. λT is a linear map.

3 | Axler3.8 Product of Linear Maps

It's basically the composition of linear maps. Let U, V, W be vector spaces over $\mathbb{F}$ and T, S be linear maps s.t. $T \in \mathcal{L}(U, V)$ and $S \in \mathcal{L}(V, W)$. Then the *product*

$$ST \in \mathcal{L}(U, W) : (ST)(u) = S(Tu)$$

#aka $ST = S \circ T$

3.1 | careful

3.1.1 | Evaluate backwards

Like the composition of functions, remember to evaluate these guys backwards. $(ST)(u) = S(Tu)$ meaning you evaluate Tu first, then S of that.

3.1.2 | T maps into the domain of S

Otherwise it's not defined.

4 | Results

4.1 | Axler3.7 $\mathcal{L}(V, W)$ is a vector space over $\mathbb{F}$

4.2 | Axler3.9 Algebraic properties

4.2.1 | associativity

$$(T_1 T_2) T_3 = T_1 (T_2 T_3)$$

when it makes sense to multiply them.

1. TODO #question what about $(T_1 + T_2) + T_3 \stackrel{?}{=} T_1 + (T_2 + T_3)$?

4.2.2 | **identity**

$$TI = IT = T$$

where $T \in \mathcal{L}(U, V)$ and I is the identity of U or V respectively.

4.2.3 | **distributive properties**

$$(S_1 + S_2)T = S_1TS_2T\text{and}T(S_1 + S_2) = TS_1 + TS_2$$