1 | **Axler 6.A exercise 9**

Suppose $u, v \in V$ and $\|u\| \leq 1$ and $\|v\| \leq 1$. Prove that

$$\sqrt{1-\|u\|^2}\sqrt{1-\|u\|^2} \leq 1 - |\langle u, v\rangle|$$

2 | **Proof**

We will prove this by showing that the left hand side is less than or equal to an intermediate term, which is less than or equal to the right hand side.

$$|\langle u, v\rangle| \leq \|u\|\|v\|$$
$$1 - \|u\|\|v\| \leq 1 - |\langle u, v\rangle|$$

This intermediate value is obtained using the Cauchy-Schwarz inequality.

Now, to show that the square of the left hand side is less than or equal to the square of the right hand side,

$$\begin{aligned}
&=(1-\|u\|^2)(1-\|v\|^2)\\
&=1-\|u\|^2-\|v\|^2+\|u\|^2\|v\|^2\\
&=1-\|u\|^2-\|v\|^2+\|u\|^2\|v\|^2\\
&=1-2\|u\|\|v\|+\|u\|^2\|v\|^2
\end{aligned}$$