

$$1 \mid \int \frac{\sqrt{x-1}}{x} dx$$

$$\text{Let } u = \sqrt{x-1}, du = \frac{1}{2\sqrt{x-1}} dx$$

$$\begin{aligned} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 - \frac{1}{u^2+1} du \\ &= 2 \int 1 - \tan^{-1} u du \end{aligned}$$

$$2 \mid \mathbf{2}$$

$$3 \mid \mathbf{3}$$

$$4 \mid \mathbf{4}$$

$$5 \mid \mathbf{5}$$

$$6 \mid \int \tan^2 x + 1 dx$$

$$\begin{aligned} \int \tan^2 x + 1 dx &= \int \sec^2 x - 1 + 1 dx \\ &= \int \sec^2 x dx \end{aligned}$$

$$\text{Let } u = x, du = 1$$

$$\begin{aligned} &= \int \sec^2 u du \\ &= \tan u + C \\ &= \boxed{\tan x + C} \end{aligned}$$

7 | **7**8 | **8**9 | $\int \frac{\sec^2 x}{\csc x} \sin x dx$

$$\begin{aligned}
 \int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\
 &= \int \sec^2 x - 1 dx \\
 &= \int \sec^2 x dx - \int 1 dx \\
 &= \tan x - x
 \end{aligned}$$

10 | **10**11 | **TODO** $\int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx$

$$\begin{aligned}
 \int \frac{e^{2 \ln \sin x} + e^{2 \ln \cos x}}{e^{2 \ln \tan x} + e^{2 \ln 1}} dx &= \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx \\
 &= \int \frac{1}{\tan^2 x + 1} dx \\
 &= \int \frac{1}{\sec^2 x} dx \\
 &= \int \cos^2 x dx
 \end{aligned}$$

12 | **12**13 | **13**14 | **14**