**Source**:

# 1 | Definition

#definition Axler3.2 Linear Map #aka linear transformation A *linear map* from $V$ to $W$ is a function $T : V \to W$ with the following properties:

## 1.1 | Additivity

$$T(u+v) = Tu + Tv \forall u, v \in V$$

## 1.2 | Homogenity

$$T(\lambda v) = \lambda(Tv) \forall \lambda \in \mathbb{F}, v \in V$$

# 2 | Other Notation

## 2.1 | Set of Maps

#definition Axler3.3 $\mathcal{L}(V, W)$

> The set of all linear maps from $V$ to $W$ is denoted $\mathcal{L}(V, W)$.

# 3 | Examples

## 3.1 | zero ($0$)

Zero is a function $0 : V \to W$ s.t. $0v = 0 \forall v \in V$. (It takes all vectors in $V$ and maps them to the additive identity of $W$)

## 3.2 | identity ($I$)

The identity maps each from one vector space to itself (in the same vector space):

$$I \in \mathcal{L}(V, V), v \in V : Iv = v$$

## 3.3 | differentiation ($D$)

$$D \in \mathcal{L}\left(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R})\right) : Dp = p'$$

Basically stating that for two polynomials $a, b \in \mathcal{P}(\mathbb{R})$, $a' + b' = (a+b)'$ and with a constant $\lambda \in \mathcal{R}$ $(\lambda a)' = \lambda a'$.

## 3.4 | integration

## 3.5 | multiplication by $x^2$

$$T \in \mathcal{L}\left(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R})\right) : (Tp)(x) = x^2 p(x)$$

is a linear map

## 3.6 | **backward shift**

$F^{\infty}$ is the vector space of all sequences of elements in $\mathbb{F}$.

$$T \in \mathcal{L}\left(\mathcal{F}^{\infty}, \mathcal{F}^{\infty}\right) : T(x_1, x_2, x_3, \ldots) = T(x_2, x_3, \ldots)$$