

**11 | area between curves****12 | estimate area**

Right handed Riemann Sum:

$$0.5 + 4 + 10 + 13 + 10 + 0 = 37.5$$

**13 | estimate area again**

$$4(200 + 2700 + 1100 + 4000 + 200) = 32800$$

**14 | area between curves**

$$\begin{aligned} \int_0^{10} 2200e^{0.024t} dx - \int_0^{10} 1360e^{0.018t} dx &= \frac{1}{0.024} 2200e^{0.024t} - \frac{1}{0.018} 1360e^{0.018t} \\ \Rightarrow \frac{1}{0.024} 2200e^{0.24} - \frac{1}{0.018} 1360e^{0.18} - \frac{1}{0.024} 2200 + \frac{1}{0.018} 1360 &\approx 9964 \end{aligned}$$

**15 | meaning of area**

The shaded region represents the profit made between producing 50 units and 100 units.

**16 | TODO slicing pizza into three using parallel cuts**

The problem of placing slices is the same if we only worry about the top half of the pizza. Thus, we can choose some  $x$  for the first slice s.t.

$$\begin{aligned}
2 \int_{-7}^x \sqrt{7^2 - t^2} dt &= \int_x^7 \sqrt{7^2 - t^2} dt \\
2 \int_{-7}^x \sqrt{7^2 - t^2} dt - \int_x^7 \sqrt{7^2 - t^2} dt &= 0 \\
2 \int_{-7}^x \sqrt{7^2 - t^2} dt + \int_7^x \sqrt{7^2 - t^2} dt &= 0 \\
2 \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^{-7} \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\
2 \left( \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt \right) + \left( \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt \right) &= 0 \\
2 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt + \int_0^x \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + 2 \int_{-7}^0 \sqrt{7^2 - t^2} dt - \int_0^7 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + \int_{-7}^0 \sqrt{7^2 - t^2} dt &= 0 \\
3 \int_0^x \sqrt{7^2 - t^2} dt + \frac{49\pi}{4} &= 0
\end{aligned}$$

Now, we need to use trigonometric substitution, apparently.

$$x = a \sin \theta, dx = a \cos \theta d\theta$$

Or, you could use David's method, which is just better (cut horizontally instead of vertically)

$$\begin{aligned}
\int_{-7}^7 \sqrt{49 - x^2} - a dx &= \frac{49\pi}{3} \\
\int_{-7}^7 \sqrt{49 - x^2} dx - \int_{-7}^7 a dx &= \frac{49\pi}{3} \\
\frac{49\pi}{2} - \int_{-7}^7 a dx &= \frac{49\pi}{3} \\
\frac{49\pi}{2} - (7a - -7a) &= \frac{49\pi}{3} \\
\frac{49\pi}{6} &= 14a \\
a &= \frac{49\pi}{84} = \frac{7\pi}{12}
\end{aligned}$$

## 17 | TODO tractrix

At any moment, if the boat is at  $(x, y)$  and the puller is at  $(0, h)$ , then the velocity of the boat is in the direction

$$\frac{y - h}{x}$$

## 18 | TODO water displacement

Plan: find a function  $f(r)$  which represents the amount of water displaced for any radius, then take the derivative and find roots.