1 | $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

$$
\begin{aligned}
\int -e^u du &= -e^u + C \\
&= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\
&= e - e^{\frac{1}{2}}
\end{aligned}
$$

2 | $\int_0^1 re^{\frac{r}{2}} dr$

$$
\begin{aligned}
\int_0^1 re^{\frac{r}{2}} dx \implies & r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\
&= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\
&= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\
&= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\
&\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4) \\
&= 4 - 2e^{\frac{1}{2}}
\end{aligned}
$$

3 | **TODO** $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$

$$
\int \frac{\ln y}{\sqrt{y}} dx = 2 \ln y \sqrt{y} - \int 2 \frac{1}{y} \sqrt{y} dx 2\sqrt{y}(\ln y - 2)
$$

4 | **TODO** $\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$

$$
\begin{aligned}
\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= x \cos \sqrt{x} + \int x \frac{1}{2\sqrt{x}} \sin \sqrt{x} dx \\
&= x \cos \sqrt{x} + \int \frac{\sqrt{x}}{2} \sin \sqrt{x} dx \\
&= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \int \sin \sqrt{x} dx \\
&= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \left(x \sin \sqrt{x} - \int \frac{\sqrt{x}}{2} \cos \sqrt{x} dx \right) \\
&= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} \left(x \sin \sqrt{x} - \frac{\sqrt{x}}{2} \int \cos \sqrt{x} dx \right) \\
&= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} x \sin \sqrt{x} - \frac{x}{4} \int \cos \sqrt{x} dx \\
\frac{x+4}{4} \int \cos \sqrt{x} dx &= x \cos \sqrt{x} + \frac{\sqrt{x}}{2} x \sin \sqrt{x} \\
\int \cos \sqrt{x} dx &= \frac{4x \cos \sqrt{x} + 2x\sqrt{x} \sin \sqrt{x}}{x+4}
\end{aligned}
$$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$, $dx = 2udu$

$$\begin{aligned}
\int_0^{\sqrt{\pi}} \cos\sqrt{x}dx &= 2\int u\cos udu \\
&= 2u\sin u - 2\int \sin udu \\
&= 2u\sin u + 2\cos u \\
&= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} \implies
\end{aligned}$$

5 | $\int_1^e \sin\ln xdx$

$$\begin{aligned}
\int_1^e \sin\ln xdx &= x\sin\ln x - \int x\frac{1}{x}\cos\ln xdx \\
&= x\sin\ln x - \int \cos\ln xdx \\
&= x\sin\ln x - \left(x\cos\ln x + \int \cancel{x}\frac{\cancel{1}}{\cancel{x}}\sin\ln xdx\right) \\
&= x\sin\ln x - x\cos\ln x - \int \sin\ln xdx \\
2\int \sin\ln xdx &= x\sin\ln x - x\cos\ln x \\
\int \sin\ln xdx &= \frac{1}{2}x(\sin\ln x - \cos\ln x) \\
\implies \quad & \frac{1}{2}e(\sin 1 - \cos 1) - \frac{1}{2}(\sin 0 - \cos 0) \\
&= \frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}
\end{aligned}$$