

## 1 | Axler6.53 orthogonal projection, $P_U$ def

Suppose  $U$  is a finite-dimensional subspace of  $V$ . The *orthogonal projection* of  $V$  onto  $U$  is the operator  $P_U \in \mathcal{L}(V)$  defined as follows:

For  $v \in V$ , write  $v = u + w$ , where  $u \in U$  and  $w \in U^\perp$ . Then  $P_U v = u$ .

In other words,  $P_U \in \mathcal{L}(V)$  takes  $v$  to the component of  $v$  that is in  $U$ .

### 1.1 | Results

#### 1.1.1 | Axler6.54 calculating $P_U v$

$$P_U v = \frac{\langle v, x \rangle}{\|x\|^2} x$$

Because orthogonal decompositions and stuff

#### 1.1.2 | Axler6.55 properties

Suppose  $U$  is a finite-dimensional subspace of  $V$  and  $v \in V$ . Then,

1.  $P_U \in \mathcal{L}(V)$
2.  $P_U u = u \forall u \in U$
3.  $P_U w = 0 \forall w \in U^\perp$
4.  $\text{range } P_U = U$
5.  $\text{null } P_U = U^\perp$
6.  $P_U^2 = P_U$  (by #2 and #4)
7.  $\|P_U v\| \leq \|v\|$
8. for every orthonormal basis  $e_1, \dots, e_m$  of  $U$ ,

$$P_U v = \langle v, e_1 \rangle e_1 + \dots + \langle v, e_m \rangle e_m$$

(because  $P_U v \in U$ )

#### 1.1.3 | This concept of the orth