

1 | Problem

Suppose $T \in \mathcal{L}(V)$. Prove that $T/(\text{null } T)$ is injective if and only if $(\text{null } T) \cap (\text{range } T) = \{0\}$

2 | Proof

First, we will rewrite the problem as logical statements for easier manipulation. The left-hand side " $T/(\text{null } T)$ is injective" is equivalent to:

$$T/(\text{null } T)(v + (\text{null } T)) = (\text{null } T) \iff v + (\text{null } T) = (\text{null } T)$$