

I chose to use Region B (bounded by $x=0$, $y=\sqrt{x}$, and $x=9$) and the semicircle as my cross section. The integral to calculate the volume is the integral of the area of each slice.

$$\begin{aligned}
 & \int_0^9 \pi r_x^2 dx \\
 &= \int_0^9 \pi \left(\frac{\sqrt{x}}{2} \right)^2 dx \\
 &= \frac{\pi}{4} \int_0^9 x dx \\
 &= \frac{\pi}{8} 9^2 \\
 &= \boxed{\frac{81\pi}{8}}
 \end{aligned}$$

This value is corroborated to four decimal points using the slice generator, seen here:

