Source: [[KBe2020math530refExr0nRetIndex]]

1 | **Problem**

Do subspaces form a group under subspace addition? - Properties for a group: [[KBe2020math530refGroups]]
- Closed - Identity - Inverses - Associative

2 | **Working it out**

I don't actually remember the exact definition of subspace addition. If I remember correctly from the proof of Axler exercise 1.C.14, the sum of two subspaces is the subspace where each vector is the sum of two vectors in the original two subspaces?

Closed

I don't remember if it was guaranteed to be a subspace, but it must have the identity (because the constituents both had the identity), it is closed under scalar multiplication (because you could take the sum apart, multiply the bits from each smaller subspace which are closed under scalar multiplication, and then put them back together again). I think it is closed under addition because both parts are closed under addition. This is by no means a rigorous proof, but it is as close as I can get without knowing the actual definition of a subspace sum, and I think its reasonably convincing.

Identity

If the above is true, then subspaces are closed under subspace addition. The identity subspace would be the one with only the field additive identity, because there is only one element so the resulting subspace sum has the same number of elements as the other original subspace, and because the identity vector plus any vector of the other subspace will be that other vector by definition.

Inverse

Because the subspace sum is all of the possible outputs when adding each vector in the two subspaces, if a subspace has two or more unique elements then it's not possible to have an inverse subspace: it would not be possible to create a subspace and force the parings such that

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