

Source: [\[KBe2020math530refExrOnRetIndex\]](#)

#ret

1. Suppose  $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$ . Multiply  $AB$  and  $BA$ . What do you notice???

Nothing. Matrix multiplication is not commutative.

2. Use matrices to solve the system:  $2x - y = 3$  and  $x + 3y = 5$

$$\begin{aligned}
 & \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} 7 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} 7x \\ x + 3y \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 7x \\ x + 3y \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \end{bmatrix} \\
 & \begin{bmatrix} x \\ x + 3y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\
 & x = 2 \\
 & x + 3y = 5
 \end{aligned}$$

I'm not sure how to solve the rest of it with matrices, so I'll just do it normally:

$$\begin{aligned}
 x &= 2 \\
 x + 3y &= 5 \\
 2 + 3y &= 5 \\
 3y &= 3 \\
 y &= 1
 \end{aligned}$$

3. > Do 2x2 matrices form a group under > a. addition? > b. multiplication?

See [\[KBe2020math530refGroups\]](#) I'll assume that our matrices have real numbers in them.

| Operation      | Property | Closed | Identity   | Inverse   | Associative?             | Final     |
|----------------|----------|--------|--|---|--------------------------|-----------|
| Addition       |          | Yes    | $e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} = e$ | "Inherits from addition" | Yes       |
| Multiplication |          | Yes    | $e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | Maybe?  | Yes, see below           | Undecided |

Associativity of 2x2 matrices under multiplication:

$$\begin{aligned} \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right) \begin{bmatrix} i & j \\ k & l \end{bmatrix} &= \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \\ &= \begin{bmatrix} aei + bgi + afk + bhk & aej + bgj + afl + bhl \\ cei + dgi + cfk + dhk &cej + dgj + cfl + dhl \end{bmatrix} \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} ei + fk & ej + fl \\ gi + hk & gj + hl \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \left( \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \right) \end{aligned}$$

I can't figure out if 2x2 matrices have multiplicative inverses... I tried to work it out using algebra but kept proving identities. Not sure what the right way to go about this is...

I spent far too long on this assignment (1.6h), so I probably won't spend as much time LaTeXing my homework in the future.