

**Source:**

1 | **new schedule today :/**

2 | **Systems of equations, matrix equations, and vectors**

2.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

2.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

2.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

3 | **in class work! See `./KBe20math530srcNull_space_and_column_space_intro.pdf`**

3.1 |  $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3.1.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

The only solution is  $x=0$ , because  $Ax = x$ .

3.1.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

N/A

3.1.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

There can be infinitely many values of  $b$ ..?

3.2 |  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

3.2.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

Infinitely many (anything of the form  $\begin{pmatrix} 0 \\ 0 \\ x \end{pmatrix}$ )

3.2.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

A column in the matrix is zero? Maybe the columns are linearly dependent

### 3.2.3 | How many possible outcomes $b$ are there for the equation $Ax = b$ for any $x$ .

Infinite with dim 2?

$$3.3 \mid A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

#### 3.3.1 | How many solutions $x$ satisfy $Ax = 0$ ?

Only one value of  $x$  makes the product zero.

#### 3.3.2 | When the answer is "infinitely many" what tools might we have to describe the size of that set?

n/a

### 3.3.3 | How many possible outcomes $b$ are there for the equation $Ax = b$ for any $x$ .

column vector has dimension 3, but the vector space has dim 2

$$3.4 \mid A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

#### 3.4.1 | How many solutions $x$ satisfy $Ax = 0$ ?

infinite, same vectors as subproblem 2

#### 3.4.2 | When the answer is "infinitely many" what tools might we have to describe the size of that set?

dimension 2? column vectors in the matrix are linearly dependent.

### 3.4.3 | How many possible outcomes $b$ are there for the equation $Ax = b$ for any $x$ .

infinite, dim 2 (but each vector is dim 3)

$$3.5 \mid A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

#### 3.5.1 | How many solutions $x$ satisfy $Ax = 0$ ?

infinite, vectors of the form  $\begin{pmatrix} 0 \\ a \\ -a \end{pmatrix}$

3.5.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

dimension 2? column vectors in the matrix are linearly dependent.

3.5.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

infinite, dim 2 (but each vector is dim 3)