Source:

1 | linear approximations

1.1 | cube root

1.1.1 | approximation

$$(1+x)^{\frac{1}{3}} \to \frac{1}{3}(1+x)^{\frac{-2}{3}}$$

at $x = 0$ is

$$\frac{1}{3}(1+0)^{\cdots} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

1.1.2 | estimations

value	estimate
0.05	1.016666
-0.25	0.916666

These will be overestimates because the graph is concave down in this reigon.

1.2 | sin(x)

1.2.1 | approximation

$$y \approx \frac{d}{dx} \sin x \Big|_0 (x-0) + \sin 0 = x$$

1.2.2 | estimates

value	estimate
-0.1	-0.1
0.1	0.1

The first estimate will be an underestimate because $\sin x$ is concave up in that reigon. The opposite is true for the second estimate.

1.3 | unknown function (only some points known

1.3.1 | approximation

$$y \approx \frac{d}{dx} f(x) \Big|_c (x - c) + f(c)$$

plugging in $c = 1$,

$$y \approx 5(x - 1) - 4$$

1.3.2 | estimations

value	estimate
1.2	-3

This will be an underestimate because the second derivative is positive and the graph is thus concave up.

2 | differentials

For a function $y = f(x)$, dy and dx are diferentials and the relationship is $dy = f'(x)dx$.
For a function written $f(x) =$ (something), the differentials are df and dx and the relationship is the same: $df = f'(x)dx$.

2.1 | cube error

2.1.1 | differential

$$df = f'(x)dx = 3x^2 dx$$