

1 | **sum of a vector and a subspace def**

1.1 | **for $v \in V$ and $U \subset V$, $v + U = \{v + u : u \in U\}$ (aka shift everything by v)**

2 | **affine subset, parallel def**

2.1 | **an affine subset of V is a subset of V that is "shifted" by a vector in V**

2.2 | **all affine subsets from a subspace are said to be parallel to that subspace**

3 | **quotient space def**

3.1 | **A quotient space V/U where $U \subset V$ is the set of affine subsets parallel to U (all shifts)**

3.2 | **result**

3.2.1 | **two affine subsets parallel to U are equal or disjoint (Axler 3.85)**

1. intuition

1. if they are 'parallel', then they must be equal (inf intersection) or disjoint (zero intersection)

3.2.2 | **the quotient space is a vector space**

3.2.3 | **quotient map, π def**

1. The quotient map $\pi : V \rightarrow V/U$ is defined by $\pi(v) = v + U \forall v$

2. basically it gives a quotient space from a vector (syntactic sugar)

3.2.4 | **dimension of a quotient space**

1. $\dim V/U = \dim V - \dim U$

4 | **squiggle T (the condensed map)**

4.1 | **for $T \in \mathcal{L}(V, W)$, $T_{\text{squiggle}} : V/(\text{null } T) \rightarrow W$ s.t. $T_{\text{squiggle}}(v + \text{null } T) = Tv$**

4.2 | **basically it takes an affine subset that encodes the important part of the input (takes v from $\pi(v)$) and maps it to W**

4.3 | **makes an isomorphism to $\text{range } T$**