

1 | **Axler5.21 Complex Vector Spaces have atleast one eigenvalue**

Every operator on a finite-dimensional, nonzero, complex vector space has an eigenvalue.

2 | **intuition**

- 2.1 | **by the fundamental theorem of algebra, the characteristic polynomial will have roots and thus there will be eigenvalues.**

3 | **proof**

- 3.1 | **by factoring, we turn the polynomial of maps into a composition of linear maps of the form $(T - \lambda I)$ and the input vector has to go to all of them. We choose a v s.t. it should be equal to zero, which means that one of the maps needs to send the v to zero (and that map will be injective and that lambda will be an eigenvalue).**
- 3.2 | **to formalize the "one of the maps sends the input to zero," you can just use a prev proof "if a chain of maps is not injective, then one of the maps is not injective" or induct because there is a finite number of maps.**