1 | **Exercise 3**

Axler 6.B ex 3

orthonormality $(1,0,0), (1,1,1), (1,1,2)$

$e_1 = (1,0,0), \quad e_2 = \text{normalize}\left((1,1,1)-(1,0,0)\right)$

$= \text{normalize}\left((0,1,1)\right)$

$= (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$e_3 = \text{normalize}\left((1,1,2)-(1,0,0) - \frac{3}{\sqrt{2}}(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\right)$

$= \text{normalize}\left((0,1,2) - (0,\frac{3}{2},\frac{3}{2})\right)$

$= \text{normalize}\left((0,-\frac{1}{2},\frac{1}{2})\right)$

$= (0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$(1,0,0), \quad (0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}), \quad (0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

$0 + -\frac{1}{2} + \frac{1}{2} = 0 \checkmark$ so all pairwise inner products are zero

$v = (1,2,3)$

$= \langle (1,2,3),(1,0,0)\rangle e_1 + \langle (1,2,3),(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\rangle e_2 + \langle (1,2,3),(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\rangle \times e_3$

$= \boxed{1e_1 + \frac{5}{\sqrt{2}}e_2 + \frac{1}{\sqrt{2}}e_3}$

$= (1,0,0) + (0,\frac{5}{2},\frac{5}{2}) + (0,-\frac{1}{2},\frac{1}{2})$

$= (1, \frac{4}{2}, \frac{6}{2}) \checkmark$