1 | cooling pizza

Compute

$$\int_{0}^{5} -110e^{-0.4t}dt$$

to the nearest degree.

$$\int -110e^{-0.4t}dt = \frac{-110}{-0.4}e^{-0.4t} = 275e^{-0.4t}$$

Using the net change theorem,

$$\Delta\beta \int_0^5 -110e^{-0.4t}dt = \int -110e^{-0.4(5)}dt \qquad -\int -110e^{-0.4(0)}dt$$

$$= 275e^{-0.4(5)} \qquad -275e^0$$

$$= 37.21720289 \qquad -275$$

$$= 37.21720289 \qquad -275$$

$$= 350 - 237.78279711 \qquad \approx \boxed{112°F}$$

2 | definite integral as area under a curve

The area in the triangle is 3 square units, so $5+3=\boxed{8}$

3 | minimum value of
$$f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$$

$$\frac{d}{dx}f(x) = e^{(x^2 - 3x)^2}(2x - 3) = 0$$

$$\implies 2x - 3 = 0$$

$$\implies 2x = 3$$

$$\implies x = \frac{3}{2}$$

4 | approximate area under the curve graphically

The function looks symmetric about x = 12, so I will focus on [0, 12].

On the interval [0,6) a little under $6 \cdot 100$ barrels of oil flow through.

On the interval [6,12) a little over $6\cdot 100+\frac{1}{2}6\cdot 100$ barrels flow through, for a total of

$$\approx 2(6 \cdot 100 + 6 \cdot 100 + \frac{1}{2} 6 \cdot 100) = 3000$$

barrels of oil.

D

5 | fundamental theorem of calculus but worded confusingly

F(x) is the antiderivative of f(x), so differences of its values are definite integrals. In this case,

$$F(3) - F(0) = \int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx = 2 + 2.3 = \boxed{4.3}$$

6 | amusement park word problem

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$

valid over the domain [9, 23], and there are zero people in the park at t = 9.

6.1 | number of people who have entered the park by some time

$$\int_{9}^{x} E(t)dt = \int_{9}^{x} \frac{15600}{t^2 - 24t + 160} dx$$

$$= 15600 \ln(t^2 - 24t + 160)(2t - 24)?????$$

I don't know how to integrate this symbolically, and WolframAlpha says it contains an inverse tangent. Thus, I will use a calculator:

$$\int_{9}^{17} \frac{15600}{t^2 - 24t + 160} dt \approx \boxed{6004}$$

6.2 | value of H'(17)

H represents the number of people in the amusement park. $H(17) \approx$

$$\frac{d}{dx} \int_{9}^{t} (E(x) - L(x)) dx = E(x) - L(x)$$

$$\frac{d}{dx} \int_{9}^{t} \left(\frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370} \right) dx E(17) - L(17) = \frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370} \approx -380$$

H' represents the change in the number of people in the park, per hour.