Source: [KBe2020math530refExr0nRetIndex]

#ret

1. Suppose A = $\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$ and B = $\begin{pmatrix}0 & -1\\ 2 & 1\end{pmatrix}$. Multiply AB and BA. What do you notice???

Nothing. Matrix multiplication is not commutative.

2. Use matrices to solve the system: $2x - y = 3$ and $x + 3y = 5$

–

$$\begin{aligned}
\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}3\\ 5\end{bmatrix} && (1)\\
\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix} && (2)\\
\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix} && (3)\\
\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}6 & -3\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}9\\ 5\end{bmatrix} && (4)\\
\begin{bmatrix}7 & 0\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}14\\ 5\end{bmatrix} && (5)\\
\begin{bmatrix}7x\\ x+3y\end{bmatrix} &= \begin{bmatrix}14\\ 5\end{bmatrix} && (6)\\
\begin{bmatrix}\frac{1}{7} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}7x\\ x+3y\end{bmatrix} &= \begin{bmatrix}\frac{1}{7} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}14\\ 5\end{bmatrix} && (7)\\
\begin{bmatrix}x\\ x+3y\end{bmatrix} &= \begin{bmatrix}2\\ 5\end{bmatrix} && (8)\\
x &= 2 && (9)\\
x + 3y &= 5 && (10)\\
& && (11)
\end{aligned}$$

– I'm not sure how to solve the rest of it with matrices, so I'll just do it normally: –

$$\begin{aligned}
x &= 2 && (12)\\
x + 3y &= 5 && (13)\\
2 + 3y &= 5 && (14)\\
3y &= 3 && (15)\\
y &= 1 && (16)\\
& && (17)
\end{aligned}$$

– 3. > Do 2x2 matrices form a group under > a. addition? > b. multiplication?

See [KBe2020math530refGroups] I'll assume that our matrices have real numbers in them.

Operation Property	Closed	Identity	Inverse	Associative?	Final
Addition	Yes	$e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} = e$	"Inherits from addition"	Yes
Multiplication	Yes	$e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	Maybe?	Yes, see below	Undecided

Associativity of 2x2 matrices under multiplication: $-$

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right)\begin{bmatrix} i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}\begin{bmatrix} i & j \\ k & l \end{bmatrix}$$
$$= \begin{bmatrix} aei+bgi+afk+bhk & aej+bgj+afl+bhl \\ cei+dgi+cfk+dhk & cej+dgj+cfl+dhl \end{bmatrix}$$
$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} ei+fk & ej+fl \\ gi+hk & gj+hl \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\begin{bmatrix} i & j \\ k & l \end{bmatrix}\right) \quad (18)$$

$-$

I can't figure out if 2x2 matrices have multiplicative inverses… I tried to work it out using algebra but kept proving identities. Not sure what the right way to go about this is…

I spent far too long on this assignment (1.6h), so I probably won't spend as much time LaTeXing my homework in the future.