

**Source:**

1 | **new schedule today :/**

2 | **Systems of equations, matrix equations, and vectors**

3 | **in class work! See ./KBe20math530srcNull\_space\_and\_column\_space\_intro.pdf**

$$3.1 \mid A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3.1.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

The only solution is  $x=0$ , because  $Ax = x$ .

3.1.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

N/A

3.1.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

There can be infinitely many values of  $b$ ..? The vector space is dim 2

$$3.2 \mid A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

3.2.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

Infinitely many (anything of the form  $\begin{pmatrix} 0 \\ 0 \\ x \end{pmatrix}$ )

3.2.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

A column in the matrix is zero? Maybe the columns are linearly dependent. Input is dim 1

3.2.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

Infinite with dim 2?

$$3.3 \mid A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

3.3.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

Only one value of  $x$  makes the product zero.

3.3.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

n/a

3.3.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

column vector has dimension 3, but the vector space has dim 2

$$3.4 \mid A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3.4.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

infinite, same vectors as subproblem 2

3.4.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

dimension 2? column vectors in the matrix are linearly dependent.

3.4.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

infinite, dim 2 (but each vector is dim 3)

$$3.5 \mid A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

3.5.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

infinite, vectors of the form  $\begin{pmatrix} 0 \\ a \\ -a \end{pmatrix}$  (columns linearly dependent)

3.5.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

dimension 2 subspace of  $\mathbb{F}^3$

3.5.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

infinite, dim2 subspace of  $\mathbb{F}^3$

$$3.6 \mid A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

3.6.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

infinite, vectors of the form  $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$  (columns linearly dependent)

3.6.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

dim 2 subspace of  $\mathbb{F}^3$

3.6.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

output has dim 1

$$3.7 \mid A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 3 & 3 & -2 \end{pmatrix}$$

3.7.1 | **How many solutions  $x$  satisfy  $Ax = 0$ ?**

Seems like the rows are linearly independent, so it should be just 1 solution  $x = 0$ ? infinite, vectors of the form  $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$  (columns linearly dependent)

3.7.2 | **When the answer is "infinitely many" what tools might we have to describe the size of that set?**

dim 0

3.7.3 | **How many possible outcomes  $b$  are there for the equation  $Ax = b$  for any  $x$ .**

output should be dim 3

## 4 | **Then we talked about some stuff**

4.1 | **see** `./KBrefHomogeneousEquations.org` **and** `./KBrefColumnSpace.org`

4.2 | **The null space is the stuff that gets sent to zero (responses to subpart 1) definition to expand**

See :q