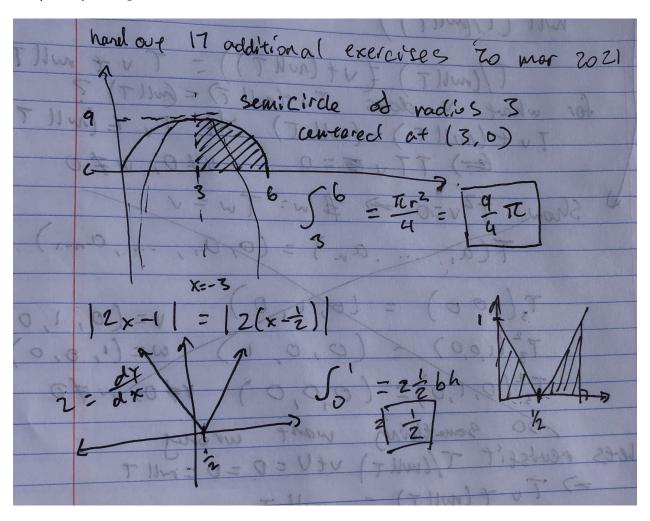
1 | Exercises

1.1 | interpreting in terms of area



1.3 | subtracting integrals

I expect

$$\int_{1}^{2} f(x)dx = \int_{1}^{5} f(x)dx - \int_{2}^{5} f(x)dx = -3 - 4 = -7$$

In fact, I expect

$$\int_{a}^{b} f(x)dx + \int_{b}^{c} f(x)dx = \int_{a}^{c} f(x)dx$$

1.4 | show
$$\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$$

(see attached pages)

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Keep in mind

$$\sum_{k=1}^{n} af(x) = a \sum_{k=1}^{n} f(x)$$

$$\sum_{k=1}^{n} (a + f(x)) = an + \sum_{k=1}^{n} f(x)$$

$$\int_{a}^{b} x^{2} dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{b - a}{n} \left(a + k \frac{b - a}{n} \right)^{2} \right)$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \sum_{k=1}^{n} \left(a + k \frac{b - a}{n} \right)^{2}$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \sum_{k=1}^{n} \left(a^{2} + \left(k \frac{b - a}{n} \right)^{2} + 2ak \frac{b - a}{n} \right)$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \sum_{k=1}^{n} a^{2} + \sum_{k=1}^{n} \left(k \frac{b - a}{n} \right)^{2} + \sum_{k=1}^{n} 2ak \frac{b - a}{n}$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \left(a^{2}n + \sum_{k=1}^{n} k^{2} \left(\frac{b - a}{n} \right)^{2} + 2a \frac{b - a}{n} \sum_{k=1}^{n} k \right)$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \left(a^{2}n + \left(\frac{b - a}{n} \right)^{2} \sum_{k=1}^{n} k^{2} + 2a \frac{b - a}{n} \sum_{k=1}^{n} k \right)$$

$$= \lim_{n \to \infty} \frac{b - a}{n} \left(a^{2}n + \left(\frac{b - a}{n} \right)^{2} \frac{n(n+1)(2n+1)}{6} + 2a \frac{b - a}{n} \frac{n(n+1)}{2} \right)$$

$$= \lim_{n \to \infty} (b - a) \left(a^{2} + \left(\frac{b - a}{n} \right)^{2} \left(n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a \frac{b - a}{n} (n+1) \right)$$

$$= \lim_{n \to \infty} (b - a) \left(a^{2} + \frac{(b - a)^{2}}{n^{2}} \left(n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a \frac{b - a}{n} (n+1) \right)$$

$$= \lim_{n \to \infty} (b - a) \left(a^{2} + \frac{(b - a)^{2}}{n^{2}} \left(n \frac{(2n+1)}{6} + \frac{(2n+1)}{6} \right) + a(b - a) + a \frac{b - a}{n} \right)$$

$$= \lim_{n \to \infty} (b - a) \left(a^{2} + \frac{(b - a)^{2}}{n^{2}} \left(n \frac{(2n+1)}{6} + \frac{(b - a)^{2}}{6} \left(n \frac{(2n+1)}{6} + a(b - a) \right) \right)$$

$$= \lim_{n \to \infty} (b - a) \left(a^{2} + \frac{(b - a)^{2}}{n^{2}} \left(n \frac{(2n+1)}{6} + a(b - a) \right) \right)$$

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