

## Lemma

Prove that the union of two subspaces of  $V$  is a subspace of  $V$  if and only if one of the subspaces is contained in the other.

#incomplete ... this got deleted? I guess see [KBe20math530PremierProofPresentation](#)-export.pdf



## Working it out

1.5 ex 12 on 12 Sep 2020  
 $A, B \text{ sub } V$  Union of  $A, B$  is sub  $V$   
 if  $A \cup B = A$  or  $A \cup B = B$   
 Now, assume  $A \cup B$  is a subspace:  
 then, it must be closed under addition. if they were not contained one within other, assume  $a \in A, b \in B, a \notin B, b \notin A$   
 $\Rightarrow a+b \in A \cup B$  (subspace closed)  $\therefore a+b \in A$  or  $a+b \in B$   
 if  $a+b \in A$  then  $b = (a+b) + (-a) \in A \therefore b \in A$  contradiction  
 if  $a+b \in B$  then  $a = (a+b) + (-b) \in B \therefore a \in B$  contradiction

Figure 1: Scribbles

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