1 | loose definition

$$\int \frac{d}{dx} f(x)dx = f(x)$$

2 | formal definition

The theorem comes in two parts, apparently

2.1 | part 1

If $f(x)$ is continuous over an interval $[a, b]$, and the function $F(x)$ is defined by

$$F(x) = \int_a^x f(t)dt$$

then $F'(x) = f(x)$ over $[a, b]$.

2.1.1 | intuition

Note that its $\int_a^x f(t)dt$ because x is an argument to the function and t is just the iteration variable.

Note that the integral can start anywhere to the left (arbitrary a) because that is removed as a constant when taking the derivative

Proof is by taking the limit form of a derivative of the integrals to x and $x + h$, and seeing that it collapses to the mean value. As the range of the mean value expression goes to zero, the value converges to itself.

2.1.2 | results

1. any integrable function and any continuous function has an anti-derivative

2.2 | part 2: the evaluation theorem

If $f(x)$ is continuous over the interval $[a, b]$ and $F(x)$ is any anti-derivative of $f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

2.2.1 | intuition

If you can find the anti-derivative, then the sum between the regions is just the difference in the anti-derivative, which makes sense. Basically contiguous areas add up.

3 | an example

Imagine a function that has the bound of an integral as an argument:

$$g(x) = \int_0^x t \, dt = \frac{x^2}{2}$$
$$\frac{d}{dx} g(x) = \frac{d}{dx} \int_0^x t \, dt = \frac{d}{dx} \frac{x^2}{2} = x$$