

1 | **Axler6.27 orthonormal basis def**

An *orthonormal basis* of V is an orthonormal list of vectors in V that is also a basis of V .

Pretty self explanatory.

2 | **results**

2.1 | **Axler6.28 orthonormal list of the right length is a basis**

Because it's linearly independent, and linearly independent lists of the right length are bases (Axler2.39).

2.2 | **Axler6.30 vector as a linear combo of orthonormal basis**

Suppose e_1, \dots, e_m is an orthonormal basis of V and $v \in V$. Then,

$$v = \langle v, e_1 \rangle e_1 + \dots + \langle v, e_n \rangle e_n$$

and

$$\|v\|^2 = |\langle v, e_1 \rangle|^2 + \dots + |\langle v, e_n \rangle|^2$$

By taking the inner product of both sides of the equation

$$v = a_1 e_1 + \dots + a_n e_n$$

with e_j for each e_j . And also the Pythagorean theorem

3 | **see also**

3.1 | **orthonormal**