

## 1 | Axler6.56 Minimizing the distance to a subspace

Suppose  $U$  is a finite-dimensional subspace of  $V$ ,  $v \in V$ , and  $u \in U$ . Then,

$$\|v - P_U v\| \leq \|v - u\|$$

Because we often end up having to find the minimal  $v - u$  where  $u \in U$ , this result makes linear algebra applicable to numerous real-world applications.

### 1.1 | Proof

$$\begin{aligned} \|v - P_U v\|^2 &\leq \|v - P_U v\|^2 + \|P_U v - u\|^2 && \text{by } 0 \leq \|P_U v - u\|^2 \\ &= \|(v - P_U v) + (P_U v - u)\|^2 && \text{by the Pythagorean Theorem} \\ &= \|v - u\|^2 \end{aligned}$$

Inequality is an equality only when  $u = P_U v$ .

### 1.2 | An example