Source:

1 | **Section A**

First, we will simplify the problem to a single dimension. We can do this because the object is only moving in a single direction and won't change its direction at any point. Therefore, all vectors will act like scalars or will be replaced by scalars for the moment. Also, the initial starting point for the object will be at $< 0 >$. Given that:

$$F = m \cdot a$$

$$W = \int_0^d F \, dx$$

$$= \int_0^d m \cdot a \, dx$$

Mass is constant so we can extract that from the integral:

$$= m \cdot \int_0^d a \, dx$$

Because acceleration is the derivative of velocity, the next step is trivial:

$$= m \cdot \frac{1}{2} \int_0^d 2a \, dx$$

$$= m \cdot \frac{1}{2} v_f^2$$

$$= \frac{1}{2} m v_f^2$$

2 | **Section B**

We will define d_{2x} as the final distance, and d_{1x} as the initial distance that the object hypothetically would have traveled given constant force. Given this:

$$\delta KE_x = \int_{d_{1x}}^{d_{2x}} F \, dx$$

$$= \int_{d_{1x}}^{d_{2x}} ma \, dx$$

$$= m \cdot \int_{d_{1x}}^{d_{2x}} a \, dx \qquad = \frac{1}{2} m \cdot [v_f^2]_{d_{1x}}^{d_{2x}}$$

By definition, we can replace the result of the integral with $v_{2x}^2 - v_{1x}^2$

$$= \frac{1}{2} m \cdot (v_{2x}^2 - v_{1x}^2)$$

$$= \frac{1}{2} m v_{2x}^2 - \frac{1}{2} m v_{1x}^2$$

3 | **Section C**

We extrapolate from previous sections.

$$W = \int \vec{F} \cdot d\vec{r}$$

$$= \int F_x \, dx + \int F_y \, dy + \int F_z \, dz$$

We can just use our 1D formulas from the other sections.
I trust you to be forgiving enough to let me reuse them.

$$= KE_x + KE_y + KE_z$$
$$= (KE_{xf} + KE_{yf} + KE_{zf}) - (KE_{xo} + KE_{yo} + KE_{zo})$$
$$= (\frac{1}{2}mv_{xf}^2 + \frac{1}{2}mv_{yf}^2 + \frac{1}{2}mv_{zf}^2) - (\frac{1}{2}mv_{xo}^2 + \frac{1}{2}mv_{yo}^2 + \frac{1}{2}mv_{zo}^2)$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

4 | **Calculus is scary**