

1 | Review Sheet

1.1 | Problem 1

1.1.1 | (e)

$$f(x) = x(x^2 + 2) - \sin(x^4 - x^{90}) + e^{\sin(x)} + \ln \cos(x^2)$$

$$f'(x) = 3x^2 + 2 - (4x^3 - 90x^{89}) \cos(x^4 - x^{90}) + \cos(x)e^{\sin(x)} + -\frac{2x \sin(x^2)}{\cos(x^2)}$$

1.1.2 | (f)

$$y = \frac{x^5 + x^{25}}{\sin(x)} + x^5 \sin(x) + x^3 \sin(x)e^{5x}$$

$$\frac{d}{dx}[y] = \frac{\sin(x)(5x^4 + 25x^{24}) - \cos(x)(x^4 + x^{25})}{\sin^2(x)} + (5x^4 \sin(x) + x^5 \cos(x)) + ((3x^2 \sin(x) + x^3 \cos(x))e^{5x} + 5x^4 \sin(x)e^{5x})$$

1.2 | Problem 4

1.2.1 | (a)

$$V = 24.0 L mol^{-1}$$

$$V(t) = 24t$$

$$R(t) = \sqrt[3]{\frac{3}{4}V(t)}$$

$$= \sqrt[3]{18t}$$

$$t = 3$$

$$V(3) = 72L$$

$$R(3) = 3\sqrt[3]{2} * 10cm = 30\sqrt[3]{2}cm$$

$$V'(t) = 24$$

$$R'(t) = \frac{18}{\sqrt[3]{18t^2}}$$

$$V'(3) = 24 L s^{-1}$$

$$R'(3) = \frac{18}{\sqrt[3]{18(3)^2}}$$

$$= \frac{18}{6\sqrt[3]{2}} * 10cms^{-1} = \frac{30}{\sqrt[3]{2}}cms^{-1}$$

1.2.2 | (b)

Assuming that the question is asking how much time would have passed when the radius is 3m:

$$\begin{aligned}
 3m &= 30 * 10cm \\
 R(t) &= 30 \\
 \sqrt[3]{18t} &= 30 \\
 18t &= 30^3 \\
 t &= \frac{30^3}{18} \\
 &= 1500
 \end{aligned}$$

1.3 | Problem 5

1.3.1 | (e)

$$\begin{aligned}
 \int (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy \\
 &= 3 \int \frac{1}{3} (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy \\
 &= 3(4y - y^2 + 4y^3 + 1)^{1/3} + C
 \end{aligned}$$

1.3.2 | (f)

$$\begin{aligned}
 \int 2x \cos(x) dx &= 2x \sin(x) - \int 2 \sin(x) dx \\
 &= 2x \sin(x) - 2 \int \sin(x) dx \\
 &= 2x \sin(x) + 2 \cos(x)
 \end{aligned}$$

2 | Arc Length

2.1 | Problem 2

$$\begin{aligned}f(x) &= \frac{x^2}{8} - \ln x \\f'(x) &= \frac{1}{4}x - \frac{1}{x} \\L &= \int_1^2 \sqrt{1 + f'(x)^2} dx \\&= \int_1^2 \sqrt{1 + \left(\frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2}\right)} dx \\&= \int_1^2 \sqrt{\frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}} dx \\&= \left[\frac{\sqrt{\frac{(x^2+4)^2}{x^2}} (x^3 + 8x \log(x))}{8(x^2 + 4)} \right]_1^2 \\&= \frac{3}{8} + \log 2\end{aligned}$$

2.2 | Problem 8

$$\begin{aligned}f(x) &= x^2 \\f'(x) &= 2x \\f'(x) &= 6 \\2x &= 6 \\x &= 3\end{aligned}$$