1 | shoestring loop

$$\begin{aligned} x &= t^2 \\ y &= t^3 - 3t \\ \frac{dx}{dt} &= 2t \\ \frac{dy}{dt} &= 3t^2 - 3 \\ \frac{dy}{dx} &= \frac{3t^2-3}{2t} \end{aligned}$$

1.1 | tangents are horizontal or vertical

1.1.1 | horizontal

$$\begin{aligned} 3t^2 - 3 &= 0 \\ 3t^2 &= 3 \\ t^2 &= 1 \\ t &= \pm 1 \end{aligned}$$

1.1.2 | vertical

$$\begin{aligned} 2t &= 0 \\ t &= 0 \end{aligned}$$

1.2 | concave up

$$\begin{aligned} \frac{d}{dx}\frac{dy}{dx} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3} &= \frac{2t(6t) - (6t^2-3)(2)}{8t^3} \\ &= \frac{6t^2 - 6t^2 + 3}{4t^3} = \frac{3}{4t^3} > 0 \\ &\therefore \text{concave up for } t > 0 \end{aligned}$$

1.3 | concave down

Using similar logic, the curve is concave down for $t \leq 0$.

2 | polar curves + converting to cartesian

polar sketches
Also see the desmos.

3 | cardiod

3.1 | **sketch**

Oops I thought cosine was sine

3.2 | **crosses the origin**

Only hapens when $\theta = 0$.

$$\begin{aligned} r = 1 + 2\cos\theta &= 0 \\ 2\cos\theta &= -1 \\ \cos\theta &= -\frac{1}{2} \\ \theta &= \cos^{-}\left(-\frac{1}{2}\right) \\ &= \frac{2\pi}{3}, -\frac{2\pi}{3} \end{aligned}$$

3.3 | **derivatives to verify crossing**

$$\frac{dy}{d\theta}$$