Desmos graphs

4 | **witch of Maria Agnesi**

Let B be the center of the orange circle with radius a, let D be the closest point to C on the x-axis, and let Q be the closest point to A on the y-axis.

4.1 | $x(t)$

$$\tan\theta = \frac{\overline{CD}}{\overline{OD}}$$
$$\cot\theta = \frac{\overline{OD}}{\overline{CD}}$$
$$\overline{CD}\cot\theta = \overline{OD}$$
$$2a\cot\theta = x$$

4.2 | $y(t)$

First, note that the distances

$$\overline{AB} = \overline{BO} = a$$
$$\overline{PD} = \overline{QO} = \overline{QB} + \overline{BO} = \overline{QB} + a = y$$

Using some geometry:

$$\angle AOB = 90 - \theta$$
$$\angle OAB = 90 - \theta \qquad \text{(isocelese triangle)}$$
$$\angle ABO = 2\theta$$

Which implies:

$$\begin{aligned}
\overline{QB} &= a\cos(2\theta) \\
&= -a\cos(2\theta) \\
&= -a\left(1 - 2\sin^2\theta\right) \\
&= -a + 2a\sin^2\theta
\end{aligned}$$

By going back to the original distance relations, we have

$$\begin{aligned}
y &= \overline{QB} + a \\
&= a - a + 2a\sin^2\theta \quad = 2a\sin^2\theta
\end{aligned}$$

8 | **swallowtail catastrophe curves**

Defined by
$$x = 2ct - 4t^3$$
$$y = -ct^2 + 3t^4$$

8.1 | **features**

8.1.1 | **approaches a parabola-like shape above the y-axis**

8.1.2 | **approaches a parabola-like shape below the x-axis if $c > 0$**

8.1.3 | **has a cross-over in a triangle shape**

1. gets bigger when c gets bigger

8.1.4 | **it looks like a dorito that scales with the value of c**

1. as c approaches zero from the positive direction, the swollowtail gets smaller

9 | **Lissajous Figures**

Defined by
$$x = a \sin(nt)$$
$$y = b \cos t$$

9.1 | **features**

9.1.1 | **spring-like coil shape (almost like standing waves) with tighter "loops" at the ends**

9.1.2 | a, b **control the size of the coil (default $-1 \leq x, y \leq 1$ because of range of** $\sin, \cos$

9.1.3 | **number of y-intercepts is $n + 1$ except in the degenerate cases $n \leq 0$**

11 | **cycloid**

Suppose instead that the circle slides along the surface and the point rotates at one radian per radian traveled. Let's start with the radian rotation...

$$x(t) = \quad\quad r \sin t$$
$$y(t) = r + \quad r \cos t$$

Then, we just have to move the origin as well:

$$x(t) = t + r \sin t$$
$$y(t) = r + r \cos t$$

12 | **first order derivative**

I think I did not come to this conclusion on my own on 30 Aug. because I didn't realize we could assume we had $y(x)$.

$$y = y(x(t))$$

$$\frac{dy}{dt} = y'(x(t))x'(t) = \frac{dy}{dx}\frac{dx}{dt} \qquad \text{(chain rule)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$