

1 | Review Sheet

1.1 | Problem 1

1.1.1 | (b)

$$g(x) = x^{32} - 7x^{12} + x^{-8} - e^x + 12\sqrt[7]{x+1} + (\cos x)^6$$

$$g'(x) = 32x^{31} - 84x^{11} - 8x^{-9} - e^x + \frac{12}{\sqrt[7]{x+1}^6} + 6 \sin(x)(\cos x)^5$$

1.1.2 | (d)

$$f(x) = 3x(x^2 + 1)^3 + \cos(\sin(x)) + \frac{x^9 + x^4}{2x + 5}$$

$$f'(x) = 18x^2(x^2 + 1)^2 - \cos(x) \sin(\sin(x)) + \frac{19x^9 + 45x^8 + 8x^9 + 10x^3}{4x^2 + 10x + 25}$$

1.2 | Problem 3

$$a \cdot b = 120$$

$$b(a) = \frac{a}{120}$$

$$p(a) = 5 \cdot 2a + 6 \cdot 2b(a)$$

$$= 10a + \frac{1440}{a}$$

$$p'(a) = 10 - \frac{1440}{a^2}$$

$$p'(a_0) = 0$$

$$0 = 10 - \frac{1440}{a_0^2}$$

$$\frac{1440}{a_0^2} = 10$$

$$1440 = 10 \cdot a_0^2$$

$$144 = a_0^2$$

$$a_0 = -12, 12$$

a_0 has to be positive; therefore a_0 is equal to 12.

$$p(a_0) = p(12) = 10(12) + 12 \cdot \frac{12}{120}$$

$$= 120 + \frac{12}{10}$$

$$= 121 \frac{1}{5}$$

The least amount of money would be spent with a dimension of 12x10 ft. As for the most amount of money, as it is possible to create an enclosure with an area of 120 square feet and an infinitely long perimeter, it is impossible to reach an answer.

1.3 | Problem 5

1.3.1 | (b)

$$\int x^3 + 3\sqrt[5]{x} + 8x^{\frac{2}{3}} + bx + a \, dx = \frac{1}{4}x^4 + \frac{5}{2}\sqrt[5]{x^6} + \frac{40}{3}x^{\frac{5}{3}} + \frac{b}{2}x^2 + ax + C$$

1.3.2 | (c)

$$\int 72 + \frac{1}{x} - 4x^7 + (x-2)^3 - \cos x + e^x \, dx = 72x + \ln x - \frac{1}{2}x^8 + \frac{1}{4}(x-2)^4 + \sin x + e^x + C$$

1.4 | **Problem 6**

1.4.1 | (b)

$$\begin{aligned} A &= \int_{-1}^{-2} f(x) \, dx \\ &= \int_{-1}^{-2} 2x^5 - 10x^4 - 70x^3 \, dx \\ &= \left[\frac{1}{3}x^6 - 2x^5 - \frac{35}{2}x^4 \right]_{-1}^{-2} \\ &= \frac{359}{2} \end{aligned}$$

1.4.2 | (c)

The area would be infinite, as after $x = 9$, the function is constantly increasing and not approaching a number.

2 | **Arc Length**

2.1 | #1

$$\begin{aligned} A &= \int_0^2 dr \\ &= \int_0^2 \sqrt{f'(x)^2 + 1} \, dx \\ &= \int_0^2 \sqrt{\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2 + 1} \, dx \\ &= \int_0^2 \sqrt{\frac{9}{4}x + 1} \, dx \\ &= \frac{8}{27} \cdot \int_0^2 \frac{27}{8} \sqrt{\frac{9}{4}x + 1} \, dx \\ &= \frac{8}{27} \cdot \int_0^2 \frac{3}{2} \cdot \frac{9}{4} \sqrt{\frac{9}{4}x + 1} \, dx \\ &= \frac{8}{27} \left[\left(\frac{9}{4}x + 1\right)^{\frac{3}{2}} \right]_0^2 \\ &= \frac{8}{27} \left(\left(\frac{11}{2}\right)^{\frac{3}{2}} - 1 \right) \\ &= \frac{8}{27} \left(\sqrt{\frac{1331}{8}} - 1 \right) \end{aligned}$$