

1 | Problem 1

Differentiate (with respect to x)

1.1 | (a)

$$y = x^2 + x^{74} - \ln x - \log_3 x + 51^x - e^x + \sin x - \cos x$$

$$\frac{d}{dx}[y] = 2x + 74x^{73} - \frac{1}{x} - \frac{1}{x \ln(3)} + \ln(51) * 51^x - e^x + \cos x + \sin x$$

1.2 | (b)

$$g(x) = x^{32} - 7x^{12} + x^{-8} - e^x + 12\sqrt[7]{x+1} + (\cos x)^6$$

$$\frac{d}{dx}[g(x)] = 32x^{31} - 84x^{11} - 8x^{-7} - e^x + \frac{12}{7\sqrt[7]{(x+1)^6}} - 6 \sin x \cos x^5$$

1.3 | (c)

$$f(x) = 7 + x^2 + 6x^3 + 3\sqrt[4]{x} + \frac{1}{x} - \ln x + 5^x$$

$$\frac{d}{dx}[f(x)] = 2x + 18x^2 + \frac{3}{4\sqrt[4]{x^3}} - \frac{1}{x} + \ln(5)5^x$$

1.4 | (d)

$$f(x) = 3x(x^2 + 1)^3 + \cos(\sin x) + \frac{x^9 + x^4}{2x + 5}$$

$$\frac{d}{dx}[f(x)] = 3(x^2 + 1)^3 + 18x^2(x^2 + 1)^2 + -\cos(x) \sin(\sin x) + \frac{45x^8 + 6x^4 + 10x^3}{4x^4 + 10x + 25}$$

1.5 | (e)

$$f(x) = x(x^2 + 2) - \sin(x^4 - x^{90}) + e^{\sin(x)} + \ln(\cos(x^2))$$

$$\frac{d}{dx}[f(x)] = (3x^2 + 2) - (4x^3 - 90x^{89}) \cos(x^4 - x^{90}) + \cos(x) e^{\sin(x)} + \frac{-2x \sin(x^2)}{\cos(x^2)}$$

1.6 | (f)

$$y = \frac{x^5 + x^{25}}{\sin x} + x^5 \sin x + x^3 \sin(x) e^{5x}$$

$$\frac{d}{dx}[y] = \frac{\sin(x)(5d^4 + 25x^{24}) - \cos(x)(x^5 + x^{25})}{\sin^2 x}$$

$$+ (x^5 \cos x + 5x^4 \sin x) + (x^3(5x \sin(x) e^{5x} + \cos(x) e^{5x}) + 3x^2 \sin(x) e^{5x})$$

2 | Problem 2

Sketch the function $f(x) = 2x^5 - 10x^4 - 70x^3$, and label (x, y) of intercepts, maxima, and minima.

3 | Problem 3

Enclosure problem

3.1 | Finding the minimum price:

$$a \times b = 120$$

$$p = 10a + 12b$$

$$b(a) = \frac{120}{a}$$

$$p(a) = 10a + 12b(a) = 10a + \frac{1440}{a}$$

$$\frac{d}{dx}[p(a)] = 10 - \frac{1440}{a^2}$$

$$10 - \frac{1440}{a^2} = 0$$

$$10 = \frac{1440}{a^2}$$

$$1 = \frac{144}{a^2}$$

$$a^2 = 144$$

$$a = 12$$

$$\begin{aligned} b(12) &= \frac{120}{12} \\ &= 10 \end{aligned}$$

$$\begin{aligned} p(12) &= 10(12) + \frac{1440}{12} \\ &= 120 + 120 \\ &= 240 \end{aligned}$$

Price: \$240 **Dimensions:** 12 by 10 feet

3.2 | Finding the maximum price:

As there is no limit as to how long any side can be, it is possible to create a rectangle that is 120 square feet in area, but has two infinitely long sides, resulting in an infinitely large price. Therefore, there is no possible maximum price.

4 | Problem 4