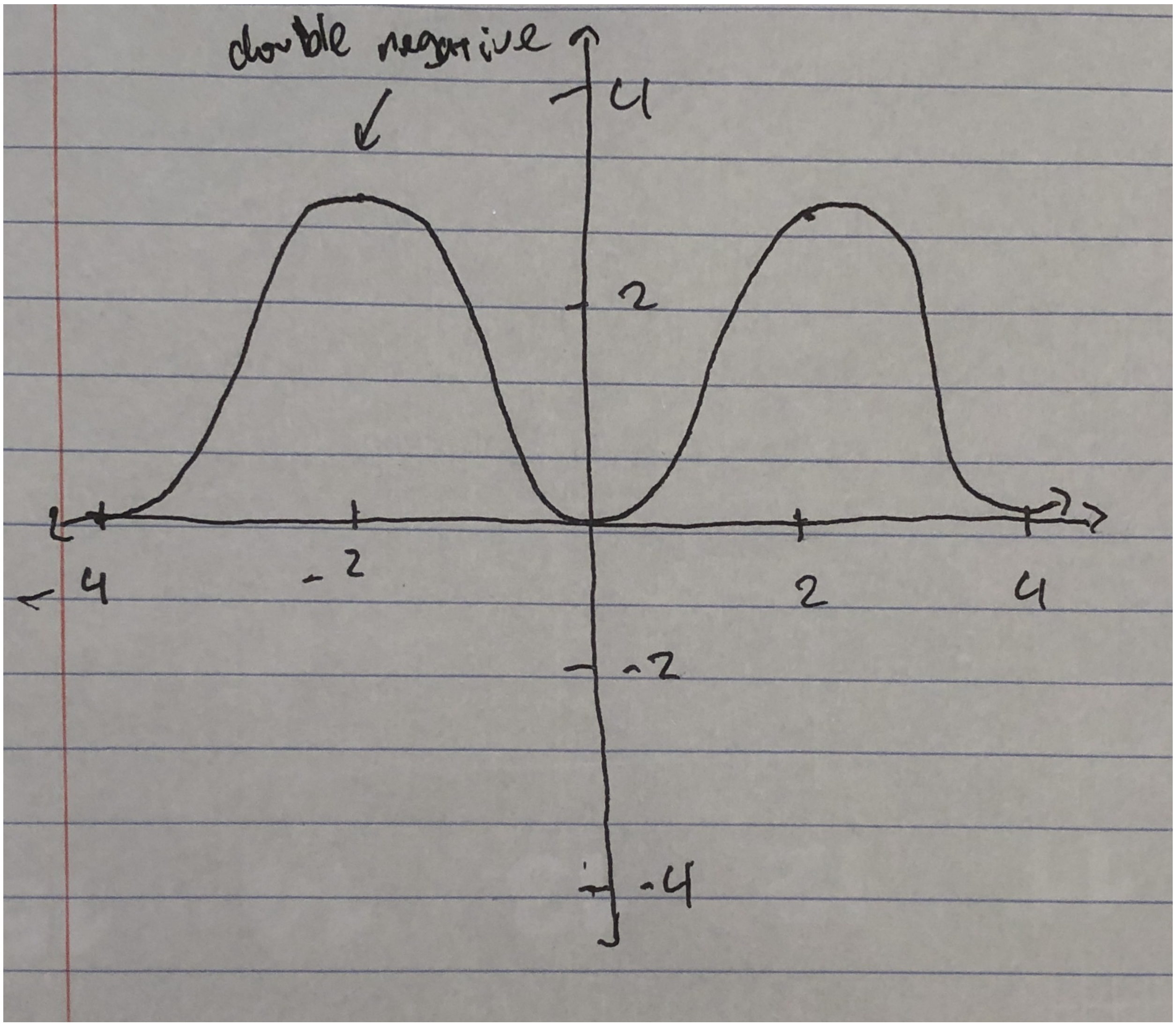
1 | Let $F(x) = \int_0^x f(t)dt$

1.1 | complete the table

x	0	1	2	3	4
F(x)	0	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{4}{3}$	0

1.2 | sketch the function

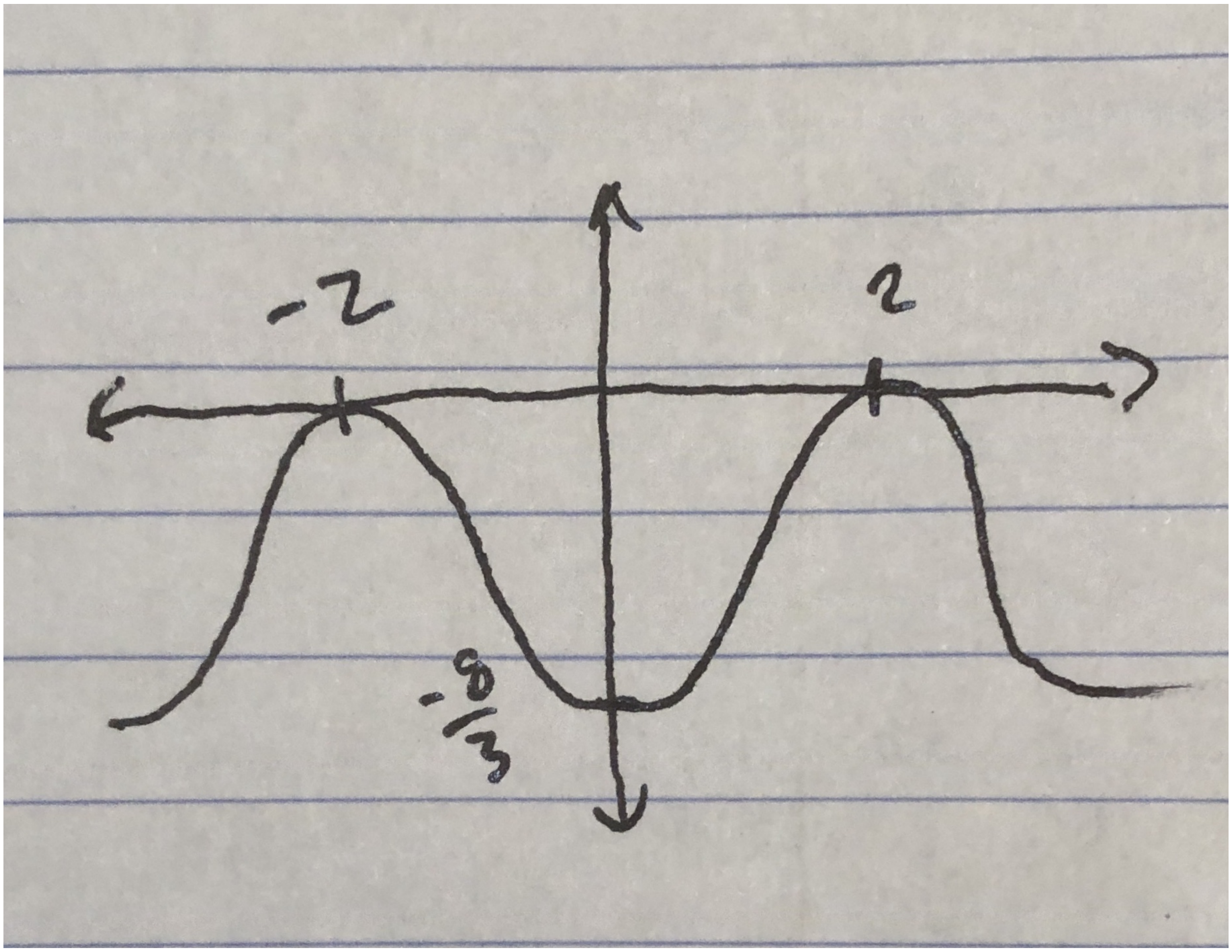


2 | $G(x) = \int_2^x f(t)dt$

2.1 | complete the table

x	0	1	2	3	4
G(x)	$-\frac{8}{3}$	$-\frac{4}{3}$	0	-$\frac{4}{3}$	$-\frac{8}{3}$

2.2 | **sketch the function**



3 | **3**

3.1 | **complete the table**

x	0	1	2	3	4
H(x)	0	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{4}{3}$	0

3.2 | **sketch the function**

same as 1b

4 | complete the table

	$F(x)$	$G(x)$	$H(x)$
maximum	4x+2		
minimum	4x		
increases	4x, 4x+2		
decreases	4x-2, 4x		

The other columns are the same, because $H(x) = F(x)$ and $G(x) = H(x) - \frac{8}{3}$.

5 | why does it work over the entire domain?

The argument for each cell is the same: it should work across the domain because those are the points where the derivative of $F(x)$ (which is $f(x)$) is zero and function is periodic.

$F(x)$ increases when $f(x)$ is positive.

6 | family of functions

Changing the 'zero point' of the derivative just shifts the graph up and down, by up to the range of the function.

7 | sketching more functions

7.1 | $F_1(x) = \int_x^0 f(t)dt$

$F(x)$ but reflected across the x-axis (negated)

7.2 | $F_2(x) = \int_0^x f(-t)dt$

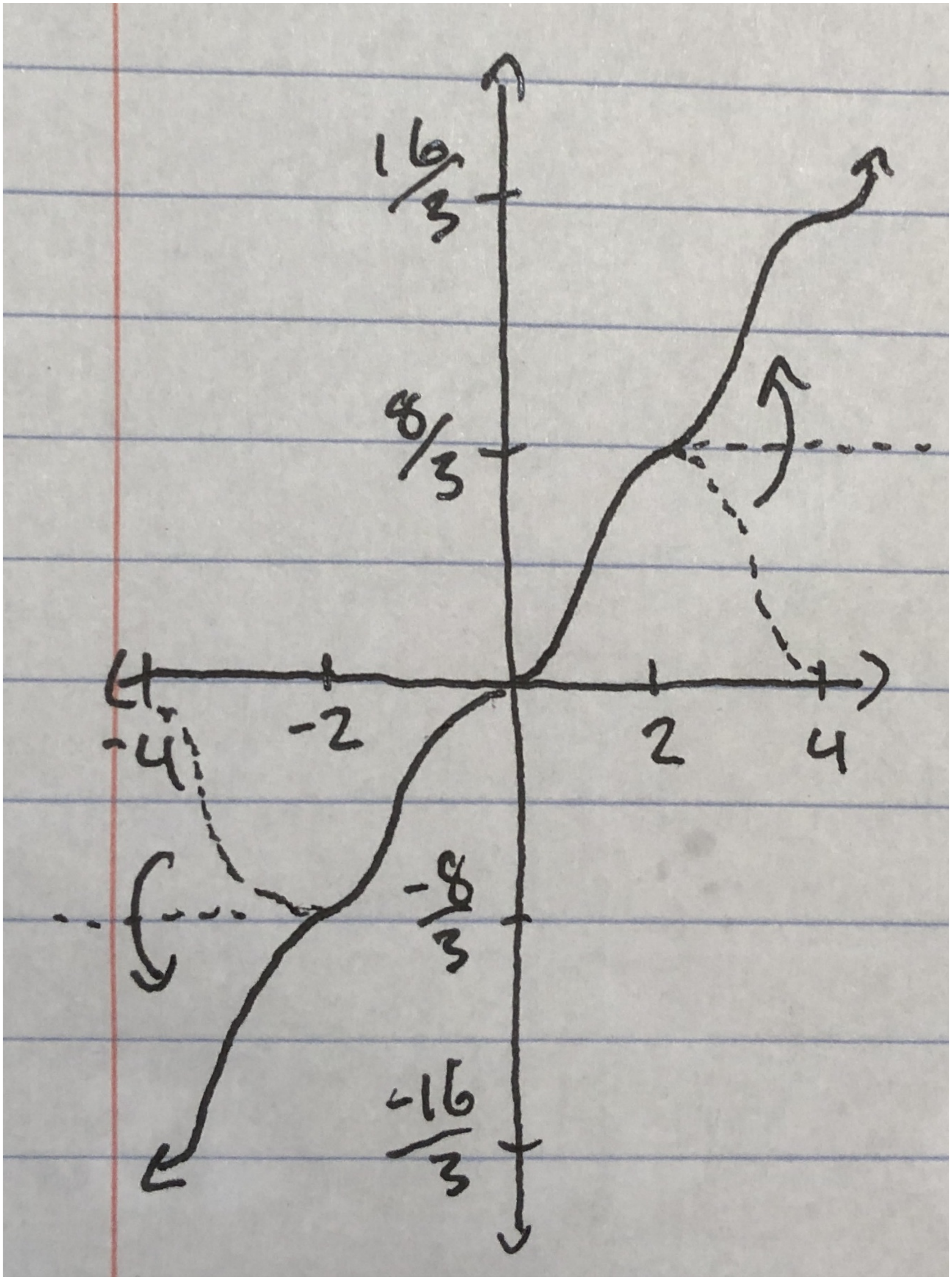
f is an odd function so $f(-t) = -f(t)$ so $F_2 = F_1$

7.3 | $F_3(x) = \int_0^{2x} f(t)dt$

$F_3(x) = F(2x)$ which is a parent function transformation which compresses the graph along the x-axis.

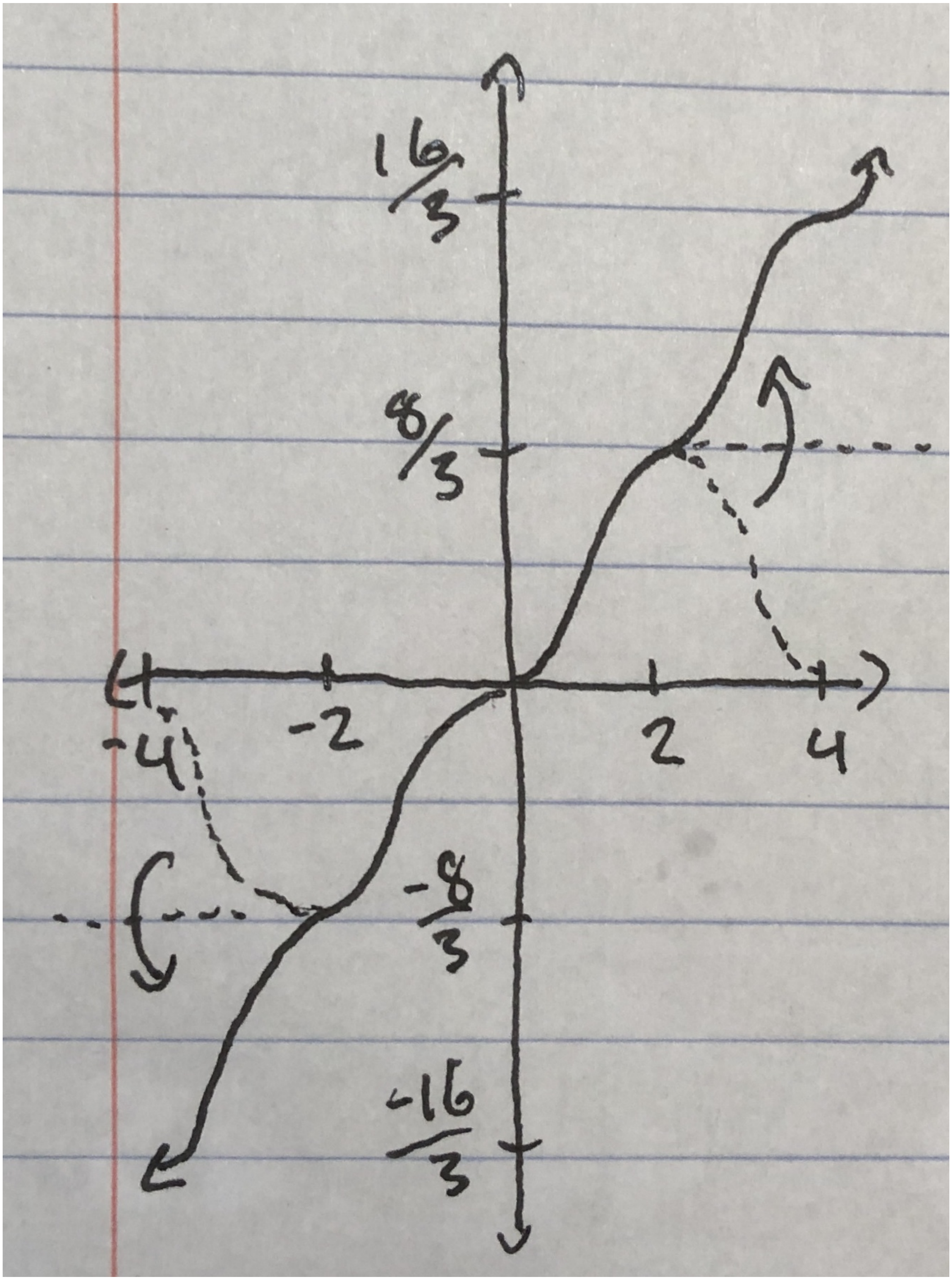
7.4 | $F_4(x) = \int_0^x f(|t|)dt$

For $x \geq 0$, $F_4(x) = F(x)$. However, for $x < 0$, the function will be the negative of the $x \geq 0$ case because the integral is from right to left.


16/3
8/3
-4
-2
2
4
-8/3
-16/3

7.5 | $F_5(x) = \int_0^x |f(t)| dt$

Instead of being a periodic function, this function will be even (all the decreasing parts of $F(x)$ become increasing with the same shape)


16/3
8/3
-4
-2
2
4
-8/3
-16/3

8 | derivatives of integral functions

8.1 | $F(x) = \int_{-1}^{x^2} \sin(t^3 - 1)dt$

$$\begin{aligned} f(x) &= \int_{-1}^{x} \sin(t^3 - 1)dt \\ F(x) &= f(x^2) \\ \frac{d}{dx}F(x) &= \frac{d}{dx}f(x^2) \\ &= f'(x^2)(2x) \\ &= 2x\sin(x^{2^3} - 1) \end{aligned}$$

8.2 | $F(x) = \int_{0}^{2x} \ln(t - 3)dt$

$$\begin{aligned} \frac{d}{dx}\left(\int_0^{2x} \ln(t-3)dt\right) &= 2\frac{d}{dx}\int_0^{2x} \ln(t-3)dt \\ &= 2\ln(2x - 3) \end{aligned}$$