1 | Escape Velocity and Gravitational Potential Energy

1.1 | Newton's Universal Gravitation Law

$$\vec{F_g} = -\frac{GM_1M_2}{r^2}\hat{r} \tag{1}$$

where, $\vec{F_g}$ is the force of gravity on M_2; M_1 and M_2 are two point masses; G the universal gravitation constant; r the magnitude of the vector $\vec{r}$ from M_1 to M_2 and $\hat{r}$ the unit vector in the $\vec{r}$ direction.

1.2 | Equation for Gravitational Potential Energy

1.2.1 | Needed Definitions

To begin, we need to modify the ***Newton's Universal Gravitation Law*** to fit the parameters of the scenario. Namely, we need to treat both Earth and our object as point masses, and assign M_1 to be Earth and M_2 to be our object.

Also, it is necessary to define the coordinate system: that our object, M_2, is defined to be to the ***left///more negative*** side of the coordinate compared to the location occupied by the Earth, $M_e = M_1$ (as, per the problem, the "zero" point is set at $r = \infty$.)

With this assumption, we could therefore claim $\vec{r}$ to be pointing from the origin to the ***negative*** side of the axis, rendering it represented by the value -1 for this system.

Hence, with the necessary variable substitutions as highlighted before, we arrive at the following equation:

$$\vec{F_{em}}(r) = \frac{GM_eM_2}{r^2} \tag{2}$$

1.2.2 | Deducing Gravitational Potential energy

The general equation for work is as follows:

$$W = F(x)dx \tag{3}$$

In this case, as we will be deducing the total gravitational potential energy as per the setup above, we need to be integrating upon $\vec{F_{em}}(r)dr$. Hence, the integral — with bounds $[0, \infty]$ — is therefore:

$$W = \int_0^\infty \frac{GM_eM_2}{r^2}dr \tag{4}$$

Determining the total energy