Desmos graphs

4 | witch of Maria Agnesi

Let B be the center of the orange circle with radius a, let D be the closest point to C on the x-axis, and let Q be the closest point to A on the y-axis.

4.1 | x(t)

$$\tan \theta = \frac{\overline{CD}}{\overline{OD}}$$

$$\cot \theta = \frac{\overline{OD}}{\overline{CD}}$$

$$\overline{CD} \cot \theta = \overline{OD}$$

$$2a \cot \theta = x$$

 $4.2 \mid y(t)$

First, note that the distances

$$\overline{AB} = \overline{BO} = a$$

$$\overline{PD} = \overline{QO} = \overline{QB} + \overline{BO} = \overline{QB} + a = y$$

Using some geometry:

$$\angle AOB = 90 - \theta$$
 $\angle OAB = 90 - \theta$ (isocelese triangle) $\angle ABO = 2\theta$

Which implies:

$$\overline{QB} = -a\cos(2\theta)$$

$$= -a\left(1 - 2\sin^2\theta\right)$$

$$= -a + 2a\sin^2\theta$$

By going back to the original distance relations, we have

$$y = \overline{QB} + a$$
$$= a - \alpha + 2a\sin^2\theta = 2a\sin^2\theta$$

5 | parameterization of an elipse

https://www.desmos.com/calculator/wcu1okhjyz

$$x(t) = a\sqrt{c}\sin t$$
$$y(t) = b\sqrt{c}\cos t$$

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6 | mystery curve

it's just $(a\cos t, b\sin t)$ because of how the right triangle aligns with the axes.

8 | swallowtail catastrophe curves

Defined by

$$x = 2ct - 4t^3$$
$$y = -ct^2 + 3t^4$$

- 8.1 | features
- 8.1.1 | approaches a parabola-like shape above the y-axis
- 8.1.2 | approaches a parabola-like shape below the x-axis if c > 0
- 8.1.3 | has a cross-over in a triangle shape
 - 1. gets bigger when c gets bigger
- 8.1.4 | it looks like a dorito that scales with the value of c
 - 1. as c approaches zero from the positive direction, the swollowtail gets smaller

9 | Lissajous Figures

Defined by

$$x = a\sin(nt)$$
$$y = b\cos t$$

- 9.1 | features
- 9.1.1 | spring-like coil shape (almost like standing waves) with tighter "loops" at the ends
- 9.1.2 | a,b control the size of the coil (default $-1 \le x,y \le 1$ because of range of \sin,\cos
- 9.1.3 | number of y-intercepts is n+1 except in the degenerate cases $n \le 0$
- 11 | cycloid

Suppose instead that the circle slides along the surface and the point rotates at one radian per radian traveled. Let's start with the radian rotation...

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$$x(t) = r \sin t$$
$$y(t) = r + r \cos t$$

Then, we just have to move the origin as well:

$$x(t) = t + r \sin t$$
$$y(t) = r + r \cos t$$

12 | first order derivative

I think I did not come to this conclusion on my own on 30 Aug. because I didn't realize we could assume we had y(x).

$$y=y(x(t))$$

$$\frac{dy}{dt}=y'(x(t))x'(t)=\frac{dy}{dx}\frac{dx}{dt}$$
 (chain rule)
$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

13 | second order derivative

$$x = f(t)$$

$$y = g(t) = g(f(t))$$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dx}\frac{d}{dt}\frac{dx}{dt} + \frac{dx}{dt}\frac{d}{dt}\frac{dy}{dx}$$

$$= \frac{dy}{dx}\frac{d^2x}{dt^2} + \frac{dx}{dt}\frac{d^2y}{dxdt(??)}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\frac{dx}{dt}$$

um... that seems like it didn't actually do anything. I'm kind of stuck... lets try working backwards:

$$\frac{d^2y}{dx^2} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x})^3}$$
$$= \dot{x}\frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right)$$

why should the \dot{x} in the bottom be cubed?

13.1 | in class review

$$\frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right) = \frac{d}{dx}u = \frac{\frac{du}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}u}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\frac{\dot{y}}{\dot{x}}}{\dot{x}}$$

$$= \frac{\frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}}}{\dot{x}}$$

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