

Source: [\[\[KBPHYS360MasterIndex\]\]](#)

1 | Problem 1

1.1 | (1a)

$$PE = -W$$

Work is force over distance. We will have distance be equal to r .

$$W = \vec{F} * r$$

In our case, force is not constant (and can be treated as a scalar as it is in only 1 direction):

$$W = \int_0^r F(x) dx$$

We know that the force applied to a point mass m by the gravitational field of the earth (with mass M_e) with distance x is modeled by

$$F(x) = \frac{GmM_e}{x^2}$$

. Therefore, our work integral can be modified to be

$$\begin{aligned} W &= \int_0^r \frac{GmM_e}{x^2} dx \\ &= GmM_e \int_0^r \frac{1}{x^2} dx \\ &= GmM_e \left[-\frac{1}{x} \right]_0^r \\ &= -\frac{GmM_e}{r} \\ PE &= \frac{GmM_e}{r} \end{aligned}$$

1.2 | (1b)

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ KE &= PE \\ \frac{1}{2}mv^2 &= \frac{GmM_e}{r} \\ v &= \sqrt{\frac{2GM_e}{r}} \end{aligned}$$