

1 | Let  $F(x) = \int_0^x f(t)dt$

1.1 | complete the table

x	0	1	2	3	4
$F(x)$	0	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{4}{3}$	0

1.2 | sketch the function



2 |  $G(x) = \int_2^x f(t)dt$

2.1 | complete the table

x	0	1	2	3	4
G(x)	$-\frac{8}{3}$	$-\frac{4}{3}$	0	$-\frac{4}{3}$	$-\frac{8}{3}$

## 2.2 | sketch the function



$$3 \mid H(x) = \int_4^x f(t)dt$$

## 3.1 | complete the table

x	0	1	2	3	4
H(x)	0	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{4}{3}$	0

## 3.2 | sketch the function

same as 1b

## 4 | complete the table

	$F(x)$	$G(x)$	$H(x)$
maximum	4n+2		
minimum	4n		
increases	4n, 4n + 2		
decreases	4n - 2, 4n		

The other columns are the same, because  $H(x) = F(x)$  and  $G(x) = H(x) - \frac{8}{3}$ .

## 5 | why does it work over the entire domain?

The argument for each cell is the same: it should work across the domain because those are the points where the derivative of  $F(x)$  (which is  $f(x)$ ) is zero and function is periodic.

$F(x)$  increases when  $f(x)$  is positive.

## 6 | family of functions

Changing the 'zero point' of the derivative just shifts the graph up and down, by up to the range of the function.

## 7 | sketching more functions

$$7.1 | F_1(x) = \int_x^0 f(t)dt$$

$F(x)$  but reflected across the x-axis (negated)

$$7.2 | F_2(x) = \int_0^x f(-t)dt$$

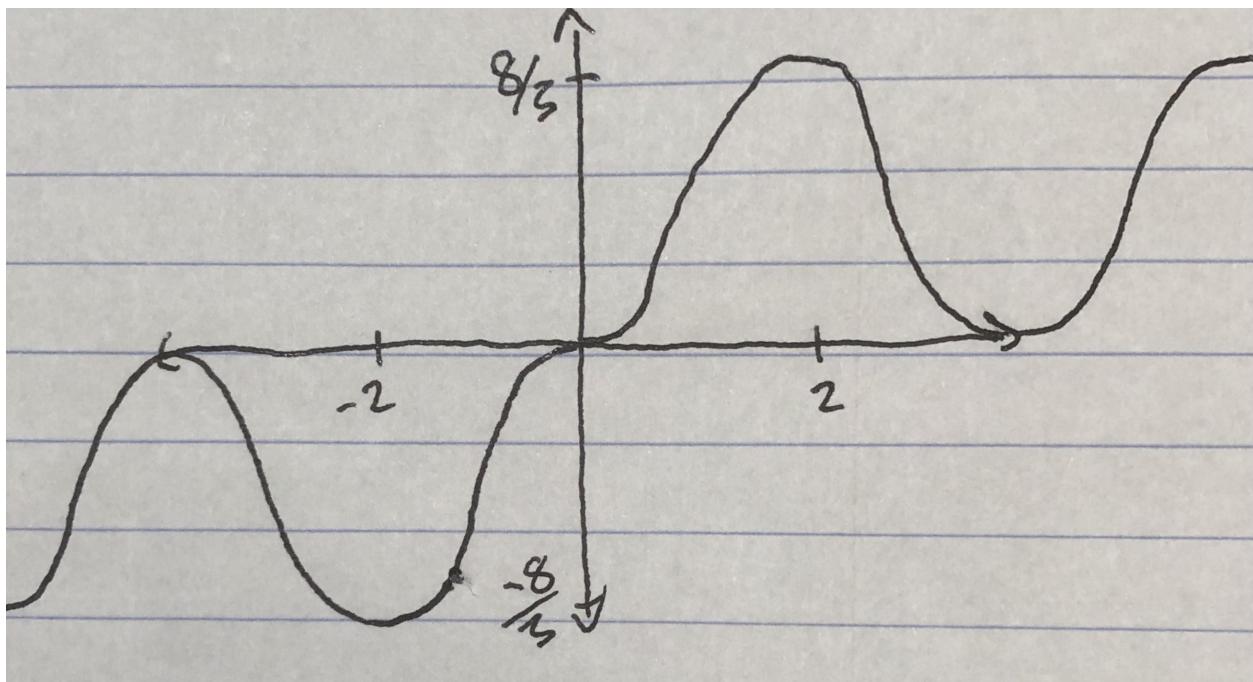
$f$  is an odd function so  $f(-t) = -f(t)$  so  $F_2 = F_1$

$$7.3 | F_3(x) = \int_0^{2x} f(t)dt$$

$F_3(x) = F(2x)$  which is a parent function transformation which compresses the graph along the x-axis.

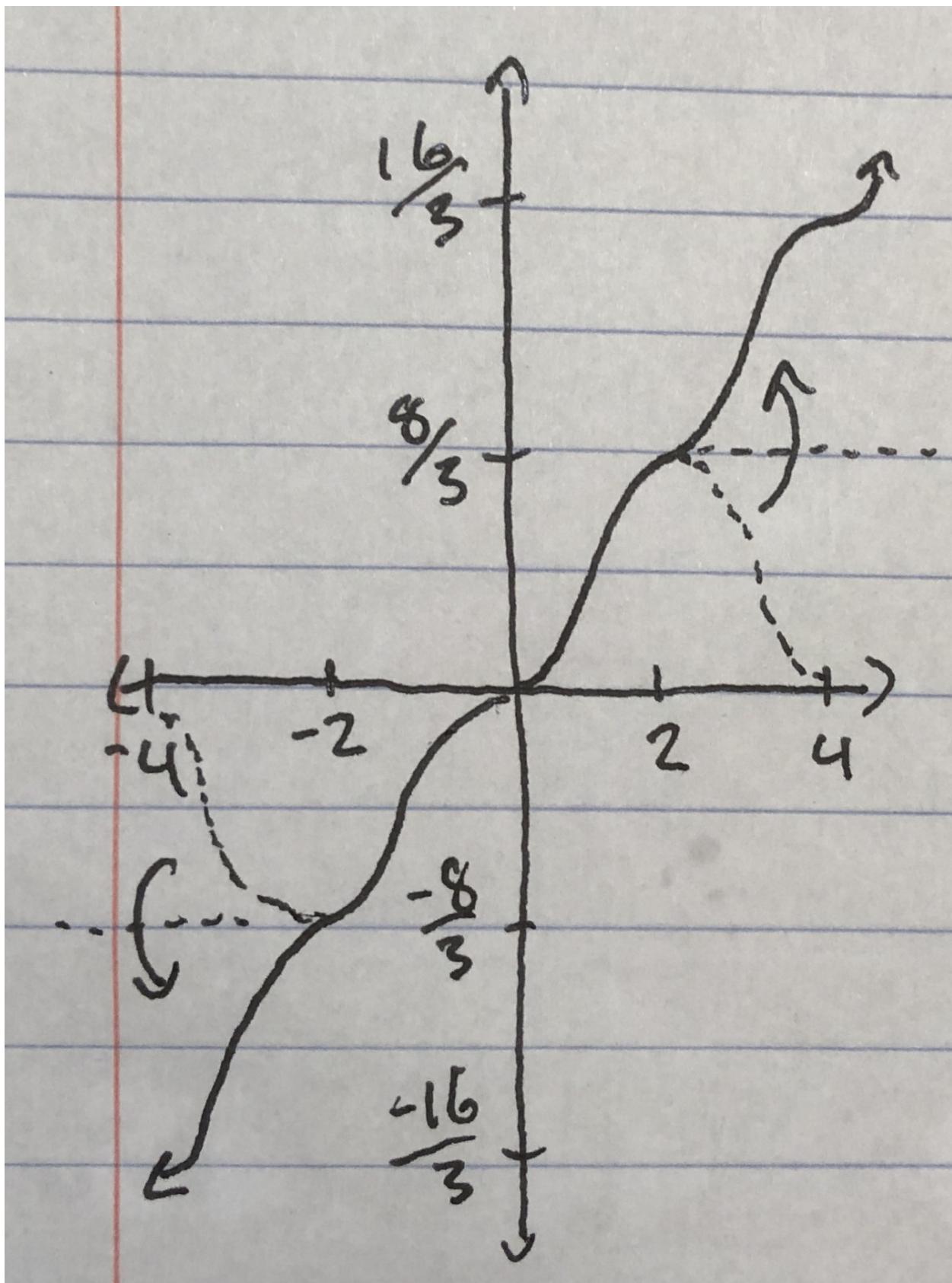
$$7.4 | F_4(x) = \int_0^x f(|t|)dt$$

For  $x \geq 0$ ,  $F_4(x) = F(x)$ . However, for  $x < 0$ , the function will be the negative of the  $x \geq 0$  case because the integral is from right to left.



$$7.5 \mid F_5(x) = \int_0^x |f(t)| dt$$

Instead of being a periodic function, this function will be even (all the decreasing parts of  $F(x)$  become increasing with the same shape)



## 8 | derivatives of integral functions

8.1 |  $F(x) = \int_{-1}^{x^2} \sin(t^3 - 1) dt$

$$\begin{aligned}f(x) &= \int_{-1}^x \sin(t^3 - 1) dt \\F(x) &= f(x^2) \\ \frac{d}{dx} F(x) &= \frac{d}{dx} f(x^2) \\&= f'(x^2)(2x) \\&= 2x \sin(x^{2^3} - 1)\end{aligned}$$

8.2 |  $F(x) = \int_0^{2x} \ln(t - 3) dt$

$$\begin{aligned}\frac{d}{dx} \left( \int_0^{2x} \ln(t - 3) dt \right) &= 2 \frac{d}{dx} \int_0^{2x} \ln(t - 3) dt \\&= 2 \ln(2x - 3)\end{aligned}$$