1 | shoestring loop

$$\begin{aligned}
x &= t^2 \\
y &= t^3 - 3t \\
\frac{dx}{dt} &= 2t \\
\frac{dy}{dt} &= 3t^2 - 3 \\
\frac{dy}{dx} &= \frac{3t^2 - 3}{2t}
\end{aligned}$$

1.1 | tangents are horizontal or vertical

1.1.1 | horizontal

$$\begin{aligned}
3t^2 - 3 &= 0 \\
3t^2 &= 3 \\
t^2 &= 1 \\
t &= \pm 1
\end{aligned}$$

1.1.2 | vertical

$$\begin{aligned}
2t &= 0 \\
t &= 0
\end{aligned}$$

1.2 | concave up

$$\begin{aligned}
\frac{d}{dx}\frac{dy}{dx} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3} &= \frac{2t(6t) - (6t^2 - 3)(2)}{8t^3} \\
&= \frac{6t^2 - 6t^2 + 3}{4t^3} = \frac{3}{4t^3} > 0 \\
&\therefore \text{concave up for } t > 0
\end{aligned}$$

1.3 | concave down

Using similar logic, the curve is concave down for $t \leq 0$.

2 | polar curves + converting to cartesian

polar sketches
Also see the desmos.

3 | cardiod

3.1 | **sketch**

Oops I thought cosine was sine

3.2 | **crosses the origin**

Only hapens when $\theta = 0$.

$$
\begin{aligned}
r = 1 + 2\cos\theta &= 0 \\
2\cos\theta &= -1 \\
\cos\theta &= -\frac{1}{2} \\
\theta &= \cos^{-}\left(-\frac{1}{2}\right) \\
&= \frac{2\pi}{3}, -\frac{2\pi}{3}
\end{aligned}
$$

3.3 | **derivatives to verify crossing**

$$
\begin{aligned}
y &= r\sin\theta = (1 + 2\cos\theta)\sin\theta = \sin\theta + 2\cos\theta\sin\theta = \sin\theta + \sin 2\theta \\
x &= r\cos\theta = (1 + 2\cos\theta)\cos\theta = \cos\theta + 2\cos^2\theta \\
\frac{dy}{d\theta} &= \cos\theta + 2\cos 2\theta \implies -\frac{1}{2} + 2(-\frac{1}{2}) = -\frac{3}{2}\frac{dx}{d\theta} \qquad = -\sin\theta - 2(2\cos\theta)\sin\theta = -\sin\theta - 2\sin 2\theta = -\frac{\sqrt{3}}{2} \\
\frac{d^2y}{d\theta^2} &= -\sin\theta - 4\sin 2\theta \\
\frac{d^2x}{d\theta^2} &= -\cos\theta - 4\cos 2\theta \\
\frac{dy}{dx} &= \frac{\dot{y}}{\dot{x}} = -\frac{\cos\theta + 2\cos 2\theta}{\sin\theta + 2\sin 2\theta} \\
\frac{dy^2}{dx^2} &= \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3} =
\end{aligned}
$$