$$T(n) = 7T\left(\frac{n}{6}\right) + n + 1$$ Expansión

$$\boxed{T(1) = 8}$$

1) $T(n) = n + 1 + \boxed{7}T\left(\frac{n}{6}\right)$

$$T(n) = n + 1 + 7\left(\frac{n}{6} + 1 + 7T\left(\frac{n}{6^2}\right)\right) = n + 1 + \frac{7}{6}n + 7 + 7^2 T\left(\frac{n}{6^2}\right)$$

$$T(n) = 1 + 7 + n + \frac{7}{6}n + 7^2 T\left(\frac{n}{6^2}\right)$$

2) $1 + 7 + n + \frac{7}{6}n + 7^2\left(\frac{n}{6^2} + 1 + 7T\left(\frac{n}{6^3}\right)\right)$

$$1 + 7 + n + \frac{7}{6}n + \left(\frac{7}{6}\right)^2 n + 7^2 + 7^3 T\left(\frac{n}{6^3}\right)$$

$$\underbrace{1}_{7^0} + 7 + 7^{2} + \underbrace{n}_{\left(\frac{7}{6}\right)^0 n} + \frac{7}{6}n + \left(\frac{7}{6}\right)^{\boxed{2}} n + 7^{3} T\left(\frac{n}{6^3}\right)$$

Expansión $k = 3$

$$\boxed{7^0 + 7 + 7^2 + \dots + 7^{k-1}} + \boxed{\left(\frac{7}{6}\right)^0 n + \frac{7}{6}n + \dots \left(\frac{7}{6}\right)^{k-1} n} + 7^k T\left(\frac{n}{6^k}\right)$$

Expansión k

1) ¿Hasta cuando expandimos?

$$\frac{n}{6^k} = 1 \qquad k = \log_6(n) = \frac{\log_2(n)}{\log_2(6)}$$

2) ¿Como es la solución?

$$\boxed{\sum_{i=0}^{\log_6(n)-1} 7^i + n\sum_{i=0}^{\log_6(n)-1} \left(\frac{7}{6}\right)^i + 7^{\log_6(n)} T(1)}$$

$$\boxed{\sum_{i=0}^{\log_6(n)-1} 7^i + n\sum_{i=0}^{\log_6(n)-1} \left(\frac{7}{6}\right)^i + 7^{\log_6(n)} T(1)}$$

$T(1)=8$

$$\sum_{i=0}^{n} ar^i = \frac{ar^{n+1}-a}{r-1} \quad r \neq 1$$

$$T(n) = \frac{7^{\log_6(n)}-1}{6} + n\left(\frac{\left(\frac{7}{6}\right)^{\log_6(n)}-1}{\frac{7}{6}-1}\right) + n^{\log_6(7)} \times 8$$

$$T(n) = \frac{n^{\log_6(7)}-1}{6} + n\left(\frac{n^{\log_6\left(\frac{7}{6}\right)}-1}{1/6}\right) + 8n^{\log_6(7)}$$

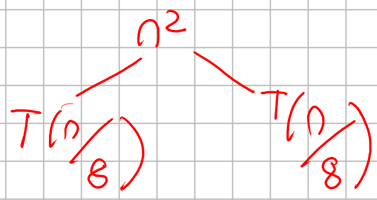
$$T(n) = \frac{n^{\log_6(7)}-1}{6} + 6n\left(n^{\log_6(7)-1} - 1\right) + 8n^{\log_6(7)}$$

Regla $n = 6^x \quad x \geq 0$

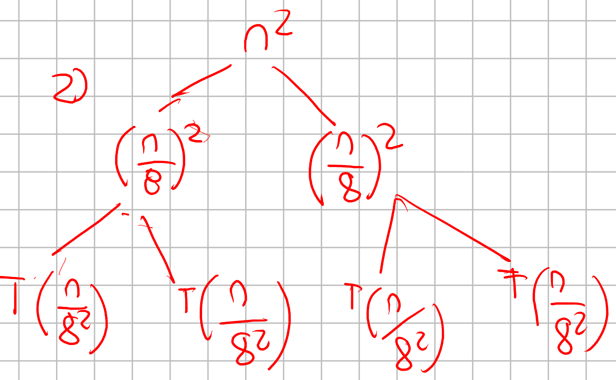
$$T(n) = 2T\left(\frac{n}{8}\right) + n^2$$

$$T(1) = 10$$

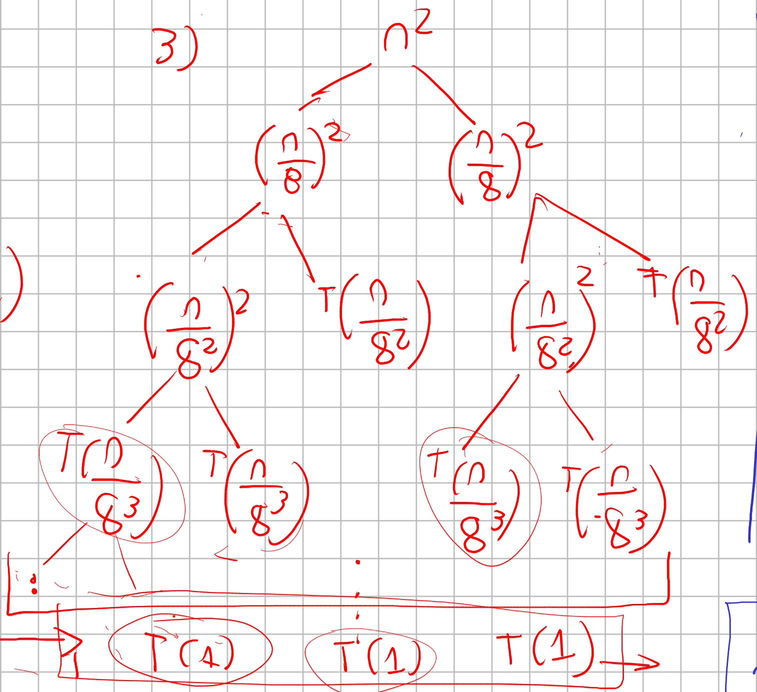
1)

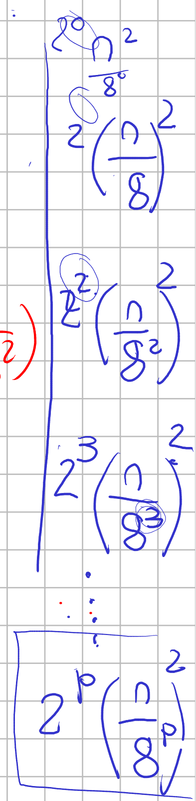


2)



3)





$$\sum_{p=0}^{\log_8(n)-1} 2^k \times \left(\frac{n}{8^k}\right)^2$$

$$\frac{n}{8^k} = 1 \qquad k = \log_8(n)$$

$$\boxed{2^{\log_8(n)}\, T(1)}$$

$$\sum_{k=0}^{\log_8(n)-1} \frac{2^k n^2}{(8^k)^2} = \sum_{k=0}^{\log_8(n)-1} \frac{2^k n^2}{64^k} = \sum_{k=0}^{\log_8(n)-1} \left(\frac{2}{64}\right)^k n^2 = n^2 \sum_{k=0}^{\log_8(n)-1} \left(\frac{1}{32}\right)^k$$

Discreto

$$n^2 \sum_{k=0}^{\log_8(n)-1} \left(\frac{1}{32}\right)^k + n^{\log_8(n)} \times 10$$

$$n^2 \sum_{k=0}^{\log_8(n)-1} \left(\frac{1}{32}\right)^k + n^{\log_8(2)} \times 10$$

$$n^2 \left(\frac{\left(\frac{1}{32}\right)^{\log_8(n)} - 1}{\frac{1}{32} - 1} \right) + n^{\log_8(2)} \times 10$$

$$\log_8\left(\frac{1}{32}\right) = \cancel{\log_8(1)}^{\,0} - \log_8(32)$$

$$= -\log_8(32)$$

$$\boxed{-\frac{32}{31} n^2 \left(n^{\log_8\left(\frac{1}{32}\right)} - 1 \right) + n^{\log_8(2)} \times 10}$$