Universidad de San Buenaventura

Facultad ingeniería de sistemas

Taller 1 Corte 2

Análisis de algoritmos

Presenta:

Juan Felipe Hurtado Villani

Cristian Apraez

Samuel Martínez

Punto #1:

① $n^2 + 4$ es $\Theta(n^2)$

$\Omega(n^2)$

$n^2 + 4 \geq C \times n^2$

$n^2 + 4 \geq n^2$

$\lim_{n \to \infty} n^2 + 4 \geq n^2$

$\lim_{n \to \infty} \overset{1}{\frac{n^2}{n^2}} + \overset{0}{\frac{4}{n^2}} \geq \overset{1}{\frac{n^2}{n^2}}$

$1 \geq 1$

V

$O(n^2)$

$n^2 + 4 \leq C \times n^2$

$n^2 + 4 \leq 2n^2$

$\lim_{n \to \infty} n^2 + 4 \leq n^2$

$\lim_{n \to \infty} \overset{1}{\frac{n^2}{n^2}} + \overset{0}{\frac{4}{n^2}} \leq \frac{2n^2}{n^2}$

$1 \leq 2$

V

Punto #2:

(2) $n^2 + 3n$ es $\Omega(n^2 \log(n))$ y no $O(n^2 \log(n))$:

Con Ω:

$n^2 + 3n \geq \Omega(n^2 \log(n))$ $C = 10$

$n^2 + 3n \geq 10n^2 \log(n)$

$$\lim_{n \to \infty} \frac{n^2}{n^2} + \cancel{\frac{3n}{n^2}} \geq \cancel{\frac{10n^2 \log(n)}{n^2}}$$

$\lim_{n \to \infty} 1 \geq 0$ Se cumple | $\lim_{n \to \infty} 1 \geq 10$ No se cumple

Con O:

$n^2 + 3n \leq O(n^2 \log(n))$ $C = 10$

$n^2 + 3n \leq 10n^2 \log(n)$

$$\lim_{n \to \infty} \frac{n^2}{n^2} + \cancel{\frac{3n}{n^2}} \leq \cancel{\frac{10n^2 \log(n)}{n^2}}$$

$\lim_{n \to \infty} 1 \leq 0$ No se cumple | $\lim_{n \to \infty} 1 \leq 10$ Se cumple

Punto #3:

3 $n^3 - n$ es $O(n^3)$

$$n^3 - n \le n^3 \cdot c$$

$$\frac{n^3}{n^3} - \frac{n}{n^3} \le \frac{n^3}{n^3} \cdot c$$

$$1 - \frac{1}{n^2} \le c$$

$$\lim_{n \to \infty} 1 - \overbrace{\frac{1}{\infty^2}}^{0} \le c$$

$$1 \le c$$

$$c \ge 1$$

$$1 - \frac{1}{(2)^2} \le c$$

$$1 - \frac{1}{4} \le c \to c \ge \frac{3}{4} \quad \| \; k = 2$$

Punto #4

$f(n) = ?\quad n$

$g(n) = ?\quad n^2$

$f(n) = O(g(n))$ y $g(n) = \theta(f(n))$

$n \leq c \cdot n^2$

$\lim_{n \to \infty} n \leq n^2$

$\frac{\cancel{n}^{0}}{\cancel{n^2}} \leq \frac{n^2}{n^2}$

$0 \leq 1$

$g(n) = \Omega(f(n))$ $c=1$

$n^2 \geq c \cdot n$

$\lim_{n \to \infty} \frac{n^2 \geq n}{n^2}$

$1 \geq 0$

$g(n) = O(f(n))$ $c=10$

$n^2 \leq c \cdot n$

$\frac{n^2 \leq 10n}{n^2}$

$1 \leq 0$

Punto #5:

a)

a) $f(x) = O(g(x))$ entonces $g(x) = O(f(x))$

Sea $f(x) = x^2 + 2x + 1$

$g(x) = x^2$

$x \geq k$ $\quad c \geq 1$

$x^2 + 2x + 1 \leq c \cdot x^2$

Con $c = 5$

$x^2 + 2x + 1 \leq 5x^2$

$\lim_{x \to \infty} x^2 + 2x + 1 \leq 5x^2$

$\frac{x^2}{x^2} + \cancel{\frac{2x}{x^2}}^{0} + \cancel{\frac{1}{x^2}}^{0} \leq \frac{5x^2}{x^2}$

$\lim_{x \to \infty} \boxed{1 \leq 5}$

$f(x) = O(g(x))$

Verdadera

$c = 5$

$x^2 \leq c \cdot x^2 + 2x + 1$

$x^2 \leq 5(x^2 + 2x + 1)$

$x^2 \leq 5x^2 + 10x + 5$

$\lim_{x \to \infty} x^2 \leq 5x^2 + 10x + 5$

$\lim_{x \to \infty} \frac{x^2}{x^2} \leq \frac{5x^2}{x^2} + \cancel{\frac{10x}{x^2}}^{0} + \cancel{\frac{5}{x^2}}^{0}$

$\lim_{x \to \infty} \boxed{1 \leq 5}$

b)

b) $f(x) = O(f(\frac{x}{2}))$ $\quad c = 1 \quad x \geq k$

Sea $f(x) = x^2 + 2x + 1$

$f(x/2) = \left(\frac{x}{2}\right)^2 + \cancel{2}\left(\frac{x}{\cancel{2}}\right) + 1 = \left(\frac{x}{2}\right)^2 + x + 1$

$x^2 + 2x + 1 \leq \left(\frac{x}{2}\right)^2 + x + 1$

$x^2 + 2x + 1 \leq \frac{x^2}{4} + x + 1$

$x^2 + \cancel{2x} \cancel{+1} - \frac{x^2}{4} \cancel{-x} \cancel{-1} \leq 0$

$x^2 - \frac{x^2}{4} + x \leq 0$

$\frac{3}{4}x^2 + x \leq 0$

$\cancel{4} \cdot \frac{3}{\cancel{4}}x^2 + 4x \leq 0$

$3x^2 + 4x \leq 0$

$x(3x + 4) \leq 0$

$x_1 \Rightarrow x \geq -4/3$

$\boxed{-4/3 \leq 0}$

Verdadero

c)

c)

$f(x) + O(f(x)) = \Theta(f(x))$

Sea $C_x \geq 1$

$x \geq k$

Sea $f(x) = x^2$

$$x^2 + C_x x^2 \geq x^2$$

$$\cancel{x^2} + C_x x^2 - \cancel{x^2} \geq 0$$

$$C_x x^2 \geq 0 \quad \text{Verdadero}$$

$$x^2 + C_x x^2 \leq x^2$$

$$\lim_{x \to \infty} x^2 + C_x x^2 \leq x^2$$

$$\lim_{x \to \infty} \frac{\cancel{x^2}}{\cancel{x^2}} + \frac{C_x \cancel{x^2}}{\cancel{x^2}} \leq \frac{\cancel{x^2}}{\cancel{x^2}}$$

$$1 + C_x \leq 1$$

$$C_x \leq 0 \quad \text{Falso}$$

Ya que las 2 preposiciones no se cumplen, la p es Falso

d)

d) $f(x) = \Theta\left((f(n))^2\right)$

Sea $f(x) = x^2 + 1$

$$(f(x))^2 = x^4 + 2x^2 + 1$$

$$x^2 + 1 \geq x^4 + 2x^2 + 1$$

$$\lim_{x \to \infty} \frac{x^2}{x^2} + \frac{\cancel{1}^0}{\cancel{x^2}} \geq \frac{x^4}{x^2} + \frac{2x^2}{x^2} + \frac{\cancel{1}^0}{\cancel{x^2}}$$

$$\lim_{x \to \infty} \frac{\cancel{1}^0}{\cancel{x^2}} \geq \frac{\cancel{x^2}}{\cancel{x^2}} + \frac{\cancel{2}^0}{\cancel{x^2}}$$

$$\lim_{x \to \infty} 0 \geq 1$$

Falso

$$x^2 + 1 \leq x^4 + 2x^2 + 1$$

$$\lim_{x \to \infty} \frac{x^2}{x^2} + \frac{\cancel{1}^0}{\cancel{x^2}} \leq \frac{x^4}{x^2} + \frac{2x^2}{x^2} + \frac{\cancel{1}^0}{\cancel{x^2}}$$

$$\lim_{x \to \infty} \frac{\cancel{1}^0}{\cancel{x^2}} \leq \frac{x^2}{x^2} + \frac{\cancel{2}^0}{\cancel{x^2}}$$

$$\lim_{x \to \infty} 0 \leq 1$$

Verdadero

Ya que las 2 no son verdaderas, significa que la pr teoria es falsa

e)

e)

$f(n) = O(g(n))$ implicando que $g(n) = \Omega(f(n))$ $\quad k \geq 1$

$\Omega(f(x)) = k \cdot f(x)$

sea $f(x) = x^2 + 2x + 1$

y $g(x) = x^2$ $\quad c \geq 1$

$$x^2 + 2x + 1 \leq C_x\, x^2$$

$$\lim_{x \to \infty} \frac{x^2}{x^2} + \cancel{\frac{2x}{x^2}}^{0} + \cancel{\frac{1}{x^2}}^{0} \leq \frac{C_x\, x^2}{x^2}$$

$$\lim_{x \to \infty} 1 \leq C_x$$

Verdadero

$$x^2 \leq x^2 + 2x + 1 \cdot k$$

$$\lim_{x \to \infty} \cancel{\frac{x^2}{x^2}} \leq \frac{(x^2 + 2x + 1) \cdot k}{x^2}$$

$$\lim_{x \to \infty} 1 \leq \cancel{\frac{x^2}{x^2}}^{1} + \cancel{\frac{2x}{x^2}}^{0} + \cancel{\frac{1}{x^2}}^{0} \cdot k$$

$$1 \leq 1 \cdot k$$

$$1 \leq k$$

$$k \geq 1$$

Verdadero