1) $n^2 - n + 2$ es $O(n^2)$

2) $n^2 - 4n + 10$ es $\Omega(n)$

O peor caso

Ω mejor caso

$F(n)$ es $O(g(n))$ $\quad F(n) \leq C \times g(n)$ $\quad n \geq k$

$F(n)$ es $\Omega(g(n))$ $\quad F(n) \geq C \times g(n)$ $\quad n \geq k$

$n^2 - n + 2$ es $O(n^2)$

$n^2 - n + 2 \leq C \cdot n^2$ $\quad n \geq k$

$n^2 - n + 2 \leq 2n^2$

$-n^2 - n + 2 \leq 0$

an^2

$$\frac{1 \pm \sqrt{1 - 4(-1)(2)}}{2(-1)}$$

$$\frac{1 \pm \sqrt{9}}{-2}$$

$$\frac{1 \pm 3}{-2}$$

$\frac{-4}{2} = -2$

$\frac{-2}{-2} = 1$

$\boxed{n \geq 1}$

$\lim_{n \to \infty} n^2 - n + 2 \leq 2n^2$

$\lim_{n \to \infty} \frac{n^2}{n^2} - \cancel{\frac{n}{n^2}}^{0} + \cancel{\frac{2}{n^2}}^{0} \leq \frac{2n^2}{n^2}$

$\lim_{n \to \infty} 1 \leq 2$

$\boxed{1 \leq 2}$ Verdad

2) $n^2 - 4n + 10$ es $\Omega(n)$

$$n^2 + 4n + 10 \geq c \times n$$

$$\lim_{n \to \infty} n^2 + 4n + 10 \geq 8n$$

$$\lim_{n \to \infty} \frac{n^2}{n^2} + \overset{0}{\cancel{\frac{4n}{n^2}}} + \overset{0}{\cancel{\frac{10}{n^2}}} \geq \overset{0}{\cancel{\frac{8}{n^2}}}$$

$$\lim_{n \to \infty} 1 \geq 0$$

$\boxed{1 \geq 0}$ Verdad

$n^2 + 4n$ es $\Omega(n^3)$ ← No es cierto

$n^2 + 4n \geq C \times n^3$

$n^2 + 4n \geq n^3$

$\lim_{n \to \infty} n^2 + 4n \geq n^3$

$\lim_{n \to \infty} \frac{\cancel{n^2}^{\,0}}{n^3} + \frac{\cancel{4n}^{\,0}}{n^3} \geq \frac{n^3}{n^3}$

$\lim_{n \to \infty} 0 \geq 1$

$\boxed{0 \geq 1}$ Falso

$n^2 + 8n - 10$ es $\Theta(n\log(n))$

es $O(n\log(n))$ y $\Omega(n\log(n))$

F y $V = F$

$\frac{\cancel{n\log(n)}}{n^2}$ 0

$O(F(n))$

$n^2 + 8n - 10 \leq C \times n\log(n)$

$n^2 + 8n - 10 \leq 10 \times n\log(n)$

$\lim_{n\to\infty} n^2 + 8n - 10 \leq 10 \times n\log(n)$

$\lim_{n\to\infty} \frac{n^2}{n^2} + \frac{\cancel{8n}}{n^2} - \frac{\cancel{10}}{n^2} \leq \frac{\cancel{10 \times n\log(n)}}{n^2}$

$\lim_{n\to\infty} 1 \leq 0$

$\boxed{1 \leq 0}$ Falso

$\Omega(F(n))$

$n^2 + 8n - 10 \geq C \times n\log(n)$

$\boxed{1 \geq 0}$

Verdadero

$n^2+8n-6 \quad \Theta(n^2)$

$\Omega(n^2)$ y $O(n^2)$

Ω cota inferior

$n^2+8n-6 \geq C \cdot n^2 \qquad C \geq 1$

$n^2+8n-6 \geq n^2$

$\lim\limits_{n\to\infty} n^2+8n-6 \geq n^2$

$\lim\limits_{n\to\infty} \frac{n^2}{n^2} + \cancel{\frac{8n}{n^2}}^{0} - \cancel{\frac{6}{n^2}}^{0} \geq n^2$

$\lim\limits_{n\to\infty} 1 \geq 1$

$\boxed{1 \geq 1}$ Verdad

O cota superior

$n^2+8n-6 \leq C \cdot n^2$

$n^2+8n-6 \leq 4n^2$

$\lim\limits_{n\to\infty} n^2+8n-6 \leq 4n^2$

$\lim\limits_{n\to\infty} \cancel{\frac{n^2}{n^2}}^{1} + \cancel{\frac{8n}{n^2}}^{0} - \cancel{\frac{6}{n^2}}^{0} \leq \cancel{\frac{4n^2}{n^2}}^{4}$

$\lim\limits_{n\to\infty} 1 \leq 4$

$\boxed{1 \leq 4}$ Verdad

$$T(n) = 8T(n/2) + n^2 \qquad \boxed{T(1)} = 4 = \Theta(1)$$

$$\cancel{T(n) = n^2 + 8T(n/2)}$$

1) $$T(n) = n^2 + 8\left(\left(\frac{n}{2}\right)^2 + 8T\left(n/2^2\right)\right)$$

$$T(n) = n^2 + 8\left(\frac{n}{2}\right)^2 + 8^2\,T\left(n/2^2\right)$$

2) $$T(n) = n^2 + 8\left(\frac{n}{2}\right)^2 + 8^2\left(\left(\frac{n}{2^2}\right)^2 + 8T\left(\frac{n}{2^3}\right)\right)$$

$$T(n) = n^2 + 8\left(\frac{n}{2}\right)^2 + 8^2\left(\frac{n}{2^2}\right)^2 + 8^3T\left(\frac{n}{2^3}\right)$$

Iteracion K) $$T(n) = 8^0\left(\frac{n}{2^0}\right)^2 + 8^1\left(\frac{n}{2^1}\right)^2 + 8^2\left(\frac{n}{2^2}\right)^2 + \dots + 8^{K-1}\left(\frac{n}{2^{k-1}}\right)^2 + 8^kT\left(\frac{n}{2^k}\right)$$

Expando hasta $T(1)$

$$\frac{n}{2^k} = 1 \qquad K = \log_2(n)$$

$$T(n) = \underbrace{8^0\left(\frac{n}{2^0}\right)^2 + 8^1\left(\frac{n}{2^1}\right)^2 + 8^2\left(\frac{n}{2^2}\right)^2 + \dots + 8^{\log_2(n)-1}\left(\frac{n}{2^{\log_2(n)-1}}\right)^2} + 8^{\log_2(n)}T(1)$$

$$T(n) = \left(\sum_{k=0}^{\log_2(n)-1} 8^k\left(\frac{n}{2^k}\right)^2\right) + 8^{\log_2(n)}T(1)$$

$$8^k\left(\frac{n}{2^k}\right)^2 = 8^k\frac{n^2}{(2^k)^2} = \frac{8^k}{4^k}n^2 = \left(\frac{8}{4}\right)^k n^2 \qquad (2^k)^2 = \underset{\text{Discreto}}{} = 4^k$$

$$8^{\log_2(n)} = n^{\log_2(8)} = n^3$$

$$\sum_{k=0}^{\log_2(n)-1} \underbrace{n^2}_{\text{cte}} 2^k + 4n^3 = n^3 - n^2 + 4n^3 = \boxed{5n^3 - n^2}$$

$$\frac{n^2 2^{\log_2(n)} - n^2}{2-1} = n^3 - n^2$$