St. Karen's Secondary School

2020 – 2021

Class – 9 Part – 1 **Subject** – MATHEMATICS

"NUMBER SYSTEMS"

NATURAL NUMBERS:

Counting numbers are known as Natural numbers. They are 1, 2, 3, 4, 5 …… ∞

They are represented by 'N'.

WHOLE NUMBERS:

Natural numbers together with 0 (zero) are known as whole numbers.

They are 0, 1, 2, 3, 4, 5 ……. ∞

They are represented by 'W'.

INTEGERS:

Integers are collection of numbers which consists of set of whole numbers together with negative sign of Natural numbers.

They are ∞ …… -3, -2, -1, 0, 1, 2, 3 ……. to ∞

Here, 0, 1, 2, 3 ….. ∞ are whole numbers. And - ∞ …… -3, -2, -1 are negative sign of Natural numbers.

So, all positive integers are Natural numbers.

All positive integers together with '0' (zero) are whole numbers.

Integers are represented by z.

RATIONAL NUMBERS:

A number which is in the form of "$\frac{p}{q}$" where $p\ \&\ q$ are integers, but $q \neq 0$ is called a Rational number.

Eg. - $\frac{3}{5}, \frac{6}{7}, \frac{-13}{11}$ etc.

Rational numbers are represented by 'Q".

All Integers are Rational numbers because they can be represented in the form of $\frac{p}{q}$.

Eg. – 21 (Integer)

$= \frac{21}{1} \left(\frac{p}{q}\right)$ (A rational number)

-2 (integer)

$= \frac{-2}{1} \left(\frac{p}{q}\right)$ (A rational number)

➔ Finding 5 rational numbers between

Solution: $\frac{3}{5} = \frac{3}{5} \times \frac{10}{10} = \frac{30}{50}$

$\frac{4}{5} = \frac{4}{5} \times \frac{10}{10} = \frac{40}{50}$

So, 5 rational numbers between $\frac{3}{5}$ & $\frac{4}{5}$ are

$$\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$$

2nd METHOD:

$\frac{3}{5} = 0.60$ and $\frac{4}{5} = 0.80$

$\therefore$ Numbers between them are

0.61, 0.62, 0.63, 0.64, 0.65.

Eg. 2 – Finding 5 rational numbers between $\frac{4}{9}$ & $\frac{5}{6}$

First of all we take LCM of their denominators

i.e. LCM of 9 & 6 = 18

Now $\frac{4}{9} = \frac{4}{9} \times \frac{2}{2} = \frac{8}{18}$ and $\frac{5}{6} = \frac{5}{6} \times \frac{3}{3} = \frac{15}{18}$

Now $\frac{8}{18} = \frac{8}{10} \times \frac{10}{10} = \frac{80}{180}$ and $\frac{15}{18} = \frac{15}{18} \times \frac{10}{10} = \frac{150}{180}$

$\therefore$ Rational numbers between them are $\frac{81}{180}, \frac{82}{180}, \frac{83}{180}, \frac{84}{180}....$

So, we find that , there are infinitely many rational numbers between any two given rational numbers.

REAL NUMBERS:

The number whose squaring is non-negative number is called Real numbers.

Eg. - $\frac{5}{7}, \sqrt{3}, \frac{1}{\sqrt{5}}, 5.2$ etc.

Explanation – Squaring of $\sqrt{3} = \left(\sqrt{3}\right)^2 = 3$ (Non negative)

Squaring of $\frac{1}{\sqrt{5}} = \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{1}{5}$ (Non negative)

Every real number is represented by a unique point on the number line.

Also every point on the number line shows a unique real number.

IRRATIONAL NUMBERS:

The real number which is not rational is called an Irrational number.

Eg. - $\sqrt{2}, \frac{5}{\sqrt{7}}, \sqrt{15}, 3\sqrt{9}, \pi$ etc.

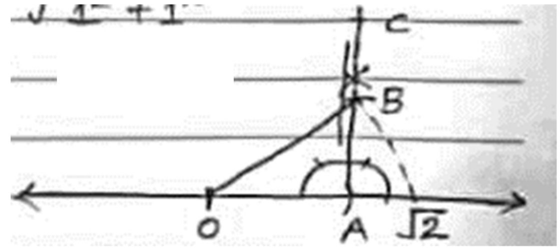
➔ Representation of irrational number on the number line.

Eg. - $\sqrt{2}$

To represent $\sqrt{2}$ on the number line, Pythagoras theorem is used.

First of all '2' will be splits as sum of the squared of two numbers, like $\sqrt{2}=\sqrt{1^2+1^2}$

Now, a real number line is drawn.



Taking '0' as centre, we cut an arc of radius 1 unit. i.e. 1 cm. At point 'A', a right angle is made. i.e. ∠OAC = 90^0. Now taking A as centre an arc of radius 1 cm is cut along AC. i.e. AB = 1 cm.

Now OB is joined. By Pythagoras theorem OB = $\sqrt{2}$ unit (cm). Now taking '0' as centre, an arc of radius (length OB) is cut along the number line. (positive side, i.e. right side of o)

DECIMAL EXPANSIONS OF REAL NUMBERS:

Decimal expansion of rational numbers:

Let there be two rational numbers

$\frac{1}{2}$ and $\frac{1}{3}$

Now $\frac{1}{2}$ = 2) 10 (·5 → case 1
$\quad$ -10
$\quad$ 00
= 0·5

& $\frac{1}{3}$ = 3) 10 (·333 - - - - → case 2
$\quad$ 9
$\quad$ 10
$\quad$ 9
$\quad$ 10
$\quad$ 9
$\quad$ 1
= 0· 333 · - - - -
= $0\cdot\bar{3}$ (Point 3 over bar)

In case 1, the remainder becomes zero, so decimal expansion of such rational numbers is called 'Terminating'.

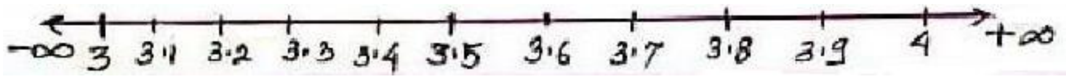
In case 2, the remainder never becomes zero, but in quotient 3 repeats continuously, so such decimal expansion is called 'non terminating repeating'.

So we find that decimal expansion of every rational number is either terminating or non terminating repeating.

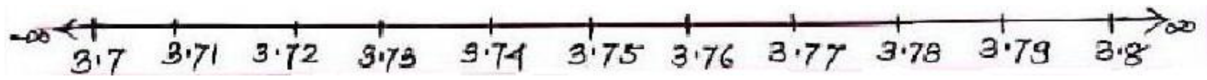
ST. KAREN'S SECONDARY SCHOOL

Class : IX
Sub : MATHEMATICS
Content - 2
NUMBER SYSTEM

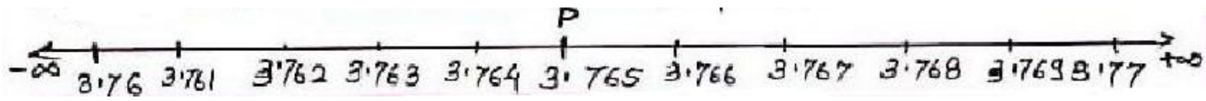
- Visualisation of rational number (in decimal form) on the number line, using successive magnification.
  Let the number is '3.765'
  Now, as the number is 3.765, so it lies between two positive integer 3 & 4.
  A line is drawn, the positive integer 3 & 4 are shown on it. The distance between 3 units and 4 units, (ie. of 1 unit) is divided into 10 equal parts.



  Now, the distance between 3.7 & 3.8 is divided into 10 equal parts.



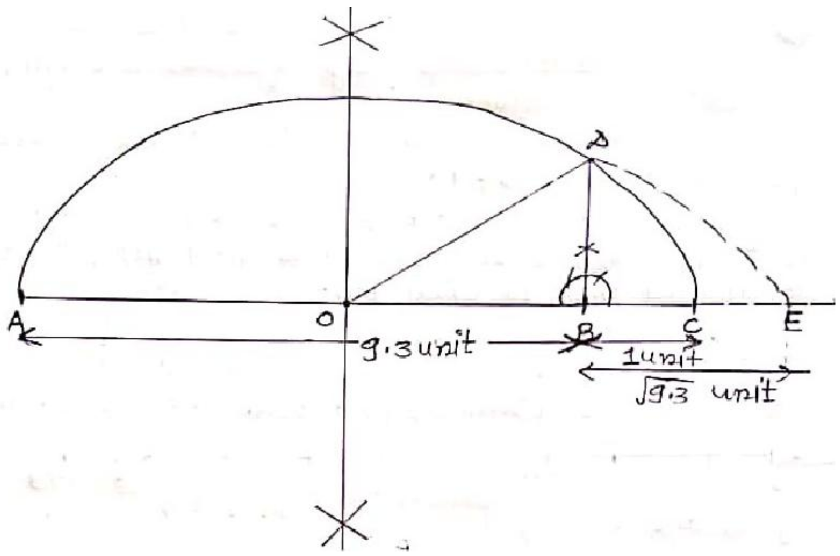
  Again, the distance between 3.76 & 3.77 is divided into 10 equal parts.



  Thus point 'p' shows the number 3.765

- Operations on Real Numbers :
  1) The sum of, or difference of, a rational number and an irrational number is irrational.
  2) The product of, or quotient of, non-zero rational number with an irrational number is irrational.
  3) The sum, difference, product or quotient of two irrational numbers may be rational or irrational.

Representation of $\sqrt{9.3}$ on the number line.



STEPS

1) A line segment of length 9.3 unit is drawn.
2) The line segment AB of length 9.3 unit is produced to 'C' by 1 unit.
3) The perpendicular bisector of line segment AC is drawn, that cuts AC at '0'.
4) Taking 'O' as centre and 'OA' or 'OC' as radius a semicircle is drawn.
5) At point B, right angle is constructed that intersect the semicircle at 'D'.
6) Taking 'B' as centre and BD as radius an arc is drawn that cut the number line at E. Thus $BE = \sqrt{9.3}$ unit.

- Rationalising the denominator :

Eg.1 - Rationalise the denominator of $\frac{1}{\sqrt{3}}$

Solution - The given number is $\frac{1}{\sqrt{3}}$

Multiplying numerator and denominator with $\sqrt{3}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ Ans.

Eg.2 - Rationalise the denominator of $\frac{1}{4+\sqrt{3}}$

Solution - Multiplying numerator and denominator with $4-\sqrt{3}$

$= \frac{1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{4-\sqrt{3}}{(4)^2-(\sqrt{3})^2} = \frac{4-\sqrt{3}}{16-3} = \frac{4-\sqrt{3}}{13}$ Ans.

Eg.3 - Rationalise the denominator of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

Solution - The number is $\frac{1}{\left(\sqrt{2}+\sqrt{3}\right)+\sqrt{5}}$

Now multiplying numerator & denominator by $\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}$

$$= \frac{1}{\left(\sqrt{2}+\sqrt{3}\right)+\sqrt{5}} \times \frac{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}}{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}}$$

$$= \frac{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}}{\left(\sqrt{2}+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{6}-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+3+2\sqrt{6}-5} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{5+2\sqrt{6}-5} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$

Again, Multiplying numerator & denominator by $\sqrt{6}$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\sqrt{6}}{12}$$ Ans.

- <u>Laws of Exponents for Real numbers</u> :

  Let 'a' be any positive real number and 'm' & 'n' be rational numbers then

  1. $a^m \times a^n = a^{m+n}$
  2. $a^m \div a^n = a^{m-n}$
  3. $(a^m)^n = a^{mn}$
  4. $a^m \times b^m = (ab)^m$

  * $a^o = 1$

  * $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

Eg.1 - Simplify $(125)^{-1/3}$

Sol. $(125)^{-1/3} = (5^3)^{-1/3} = (5)^{-1} = \frac{1}{5}$ Ans.

Eg.2 - Simplify $\sqrt{xy^{-1}} \times \sqrt{yz^{-1}} \times \sqrt{zx^{-1}}$

Sol. $\sqrt{xy^{-1}} \times \sqrt{yz^{-1}} \times \sqrt{zx^{-1}}$

$= \sqrt{\frac{x}{y}} \times \sqrt{\frac{y}{z}} \times \sqrt{\frac{z}{x}}$

$= \sqrt{\frac{x}{y} \times \frac{y}{z} \times \frac{z}{x}} = \sqrt{1} = 1$ Ans.

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