Skoltech
Skolkovo Institute of Science and Technology

Pandemic simulation
A-2

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Basic SIR model

$$\begin{cases} \frac{dS}{dt} = -\beta\frac{SI}{N} \\ \frac{dI}{dt} = \beta\frac{SI}{N} - \gamma I \\ \frac{dR}{dt} = \gamma I \end{cases} \Rightarrow \begin{cases} \frac{S_{i+1}-S_i}{t_{i+1}-t_i} = -\beta\frac{S_i I_i}{N} \\ \frac{I_{i+1}-I_i}{t_{i+1}-t_i} = \beta\frac{S_i I_i}{N} - \gamma I_i \\ \frac{R_{i+1}-R_i}{t_{i+1}-t_i} = \gamma I_i \end{cases} \text{, where}$$

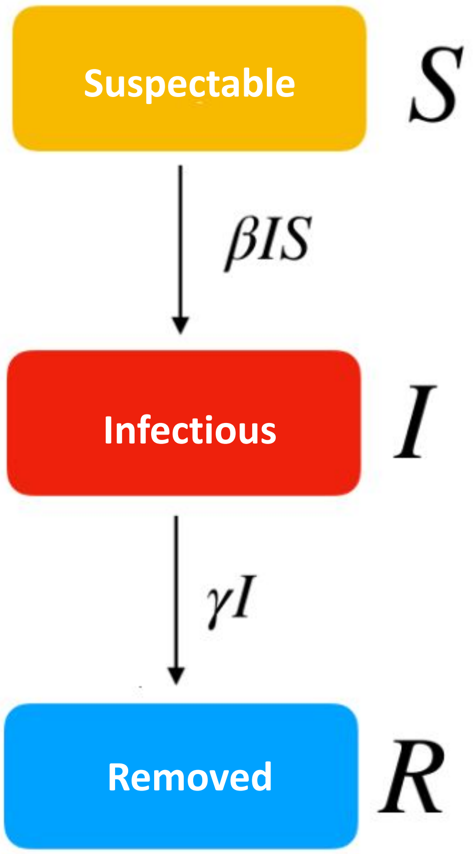
S - suspected individuals;

I - infectious individuals;

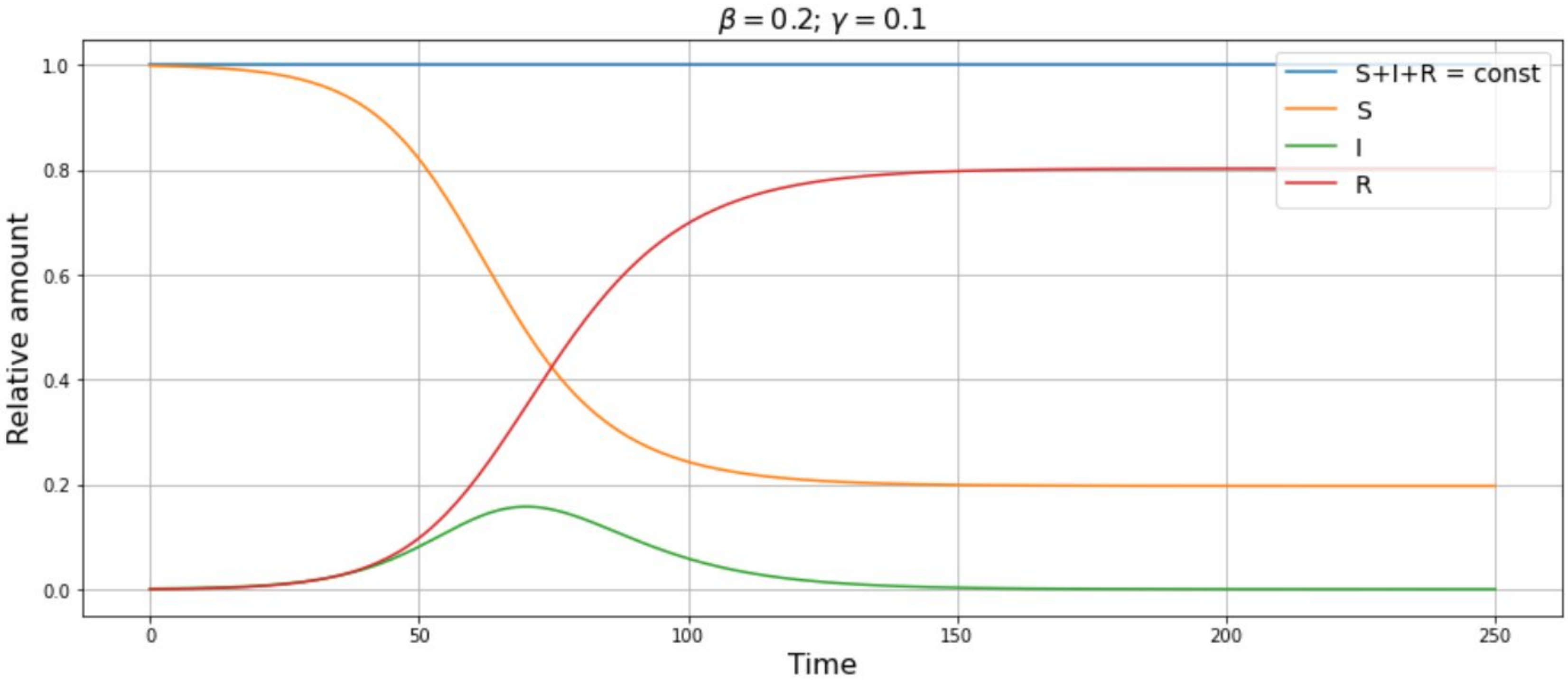
R - removed (unresponsive and dead) individuals.

β - effective contact rate of the disease $\beta \sim P(S, I \text{ contact})$.

γ - removing rate $\sim I$.



Basic SIR model simulation



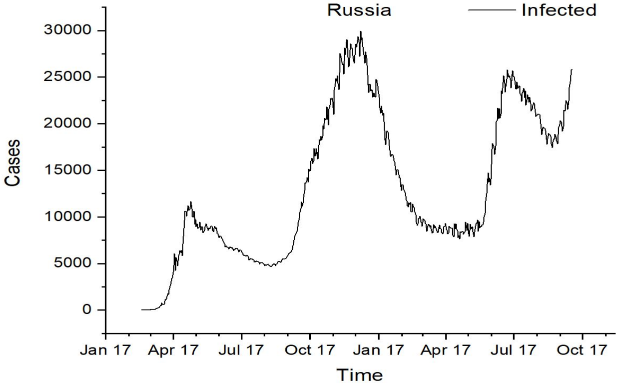
Data acquisition

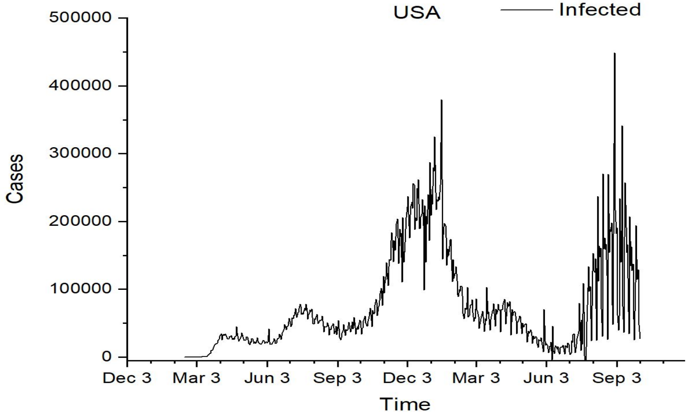
Дата	Заражений	Смертей	Выздоровлений	Заражено на дату
24.10.2021	46312782	756362	36052614	9503806
23.10.2021	46294210	756205	36033886	9504119
22.10.2021	46263935	755705	36000281	9507949

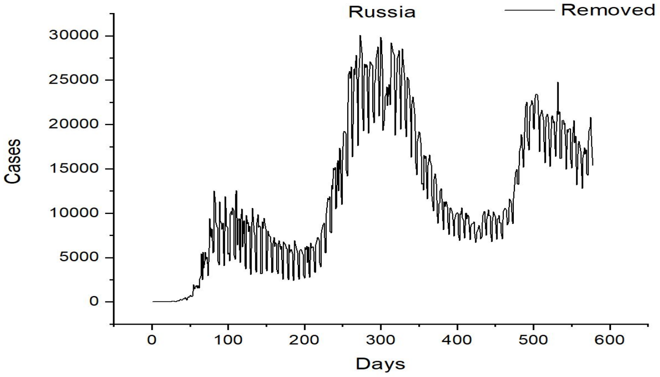
https://russian-trade.com/coronavirus/usa/

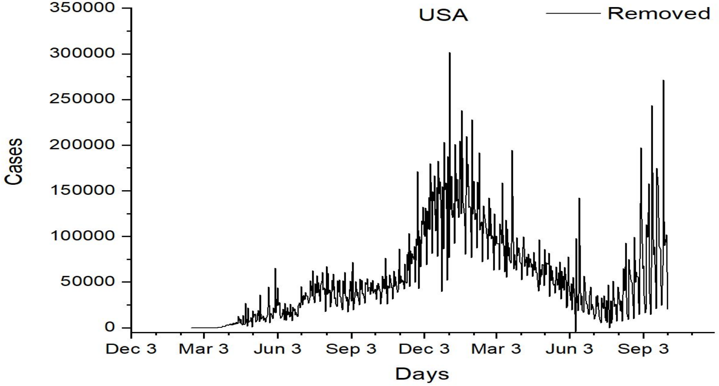
https://yandex.ru/covid19/stat

Waves identification







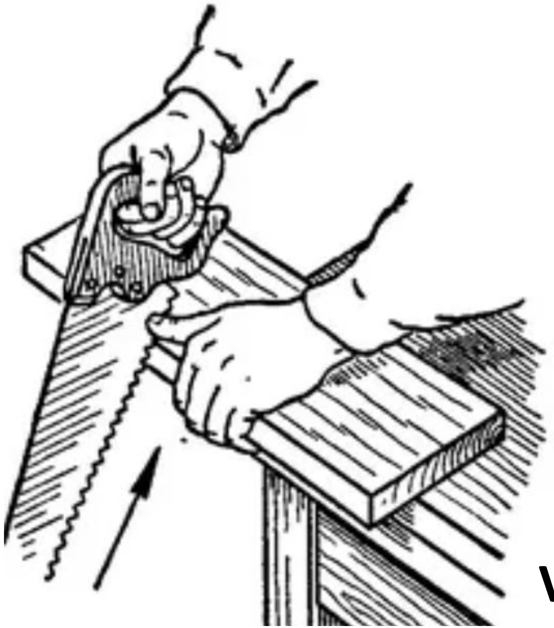


An attempt to fit coefficients

Manually

Open source COVID-19 statistics

Using optimization

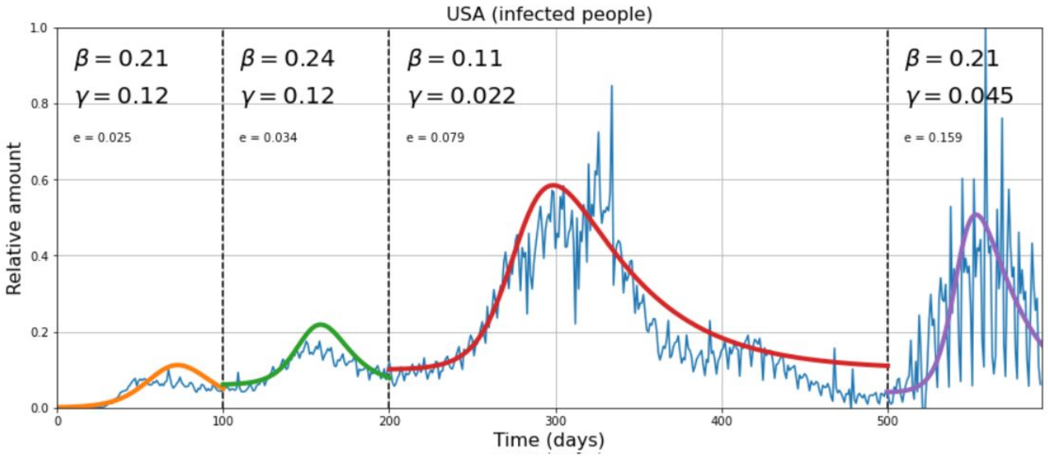


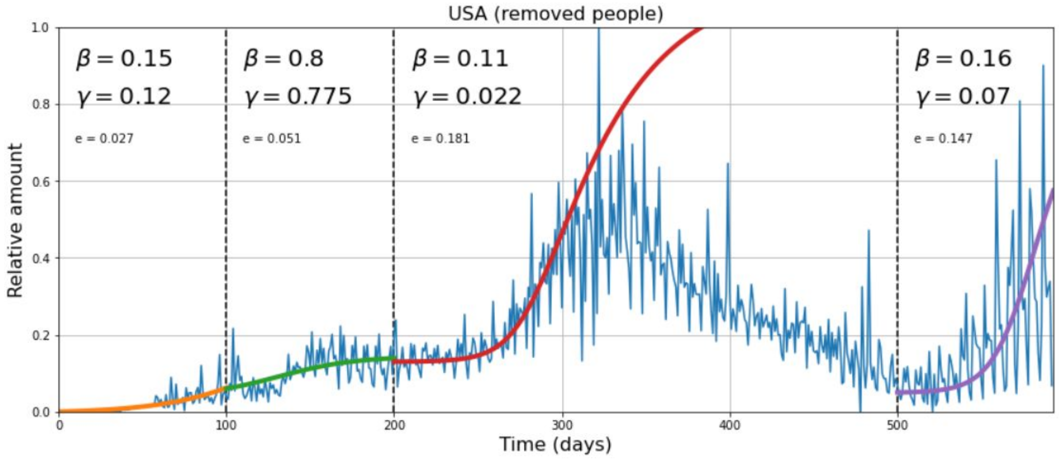
Visual estimation

Target metric minimization

Manual coefficients fitting

Infected





Removed

Infected & Removed (USA)

Optimization-based adjustment

Minimization problem formalization:

$$\beta, \gamma, \delta = argmin\left[\sum_{t=0}^{T}(I_{real}(t_i) - I_{model}(\beta, \gamma, \delta, t_i))^2\right]$$

Error estimation metric (RMSE) :

$$e = \sqrt{\frac{1}{T}\sum_{t=0}^{T}(I_{real}(t_i) - I_{model}(t_i))^2}$$

where:

I_{real} - real data;

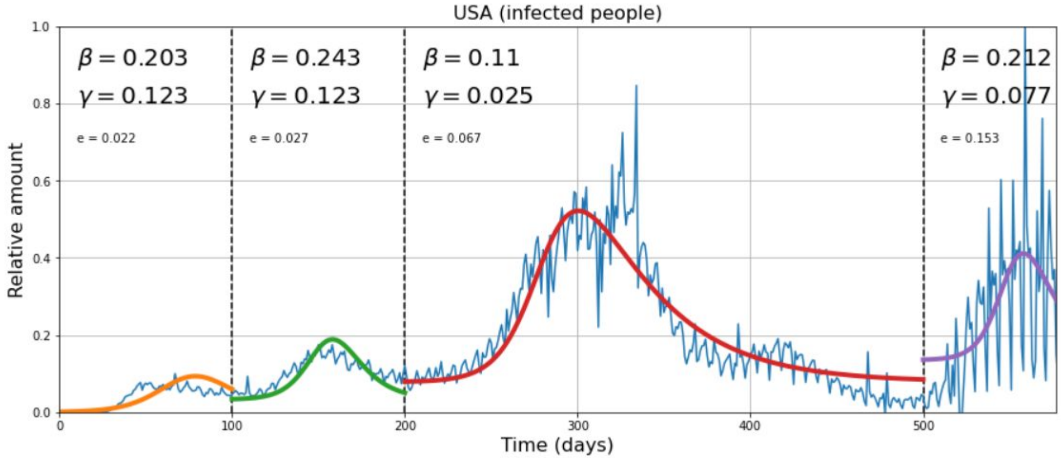
I_{model} - estimated curve;

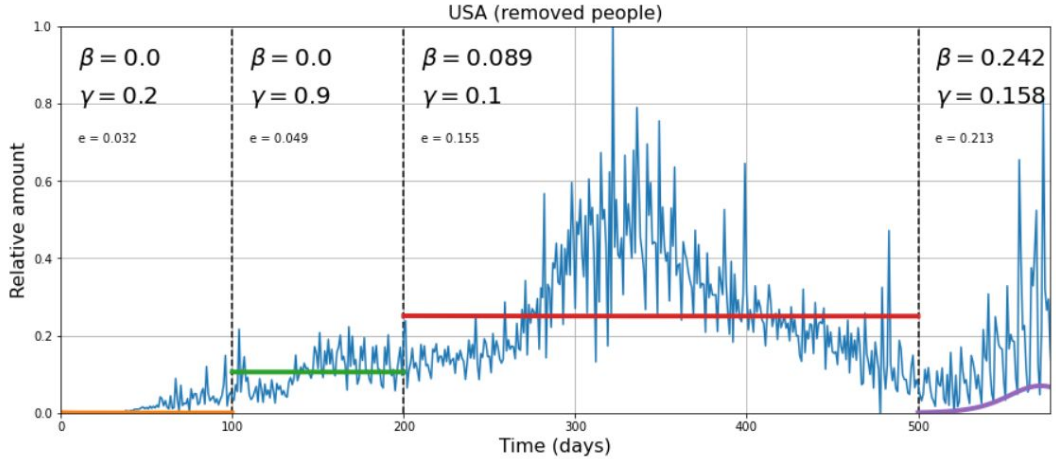
i - time moment, $i \in [0 \ldots T]$.

Nelder-Mead optimization method has been used (scipy.optimize)

Optimization-based fitting

Infected



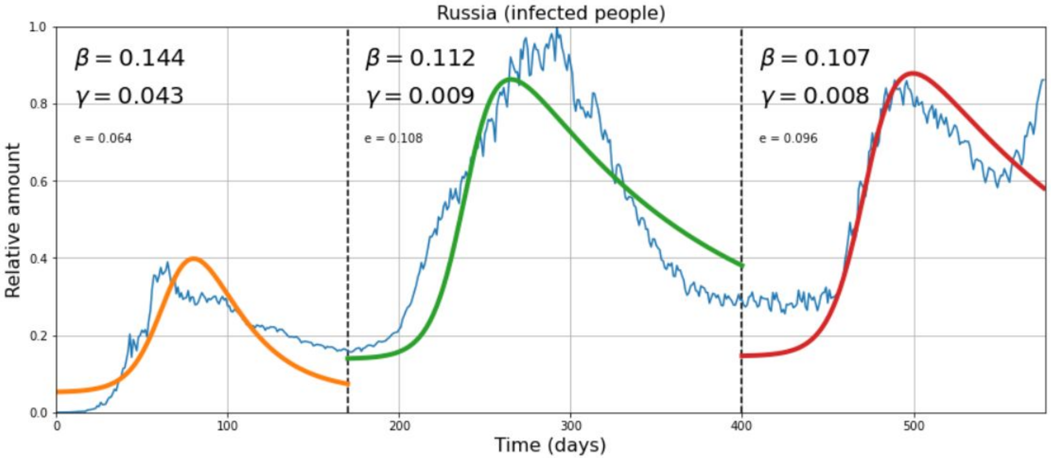


Removed

Infected & Removed (USA)

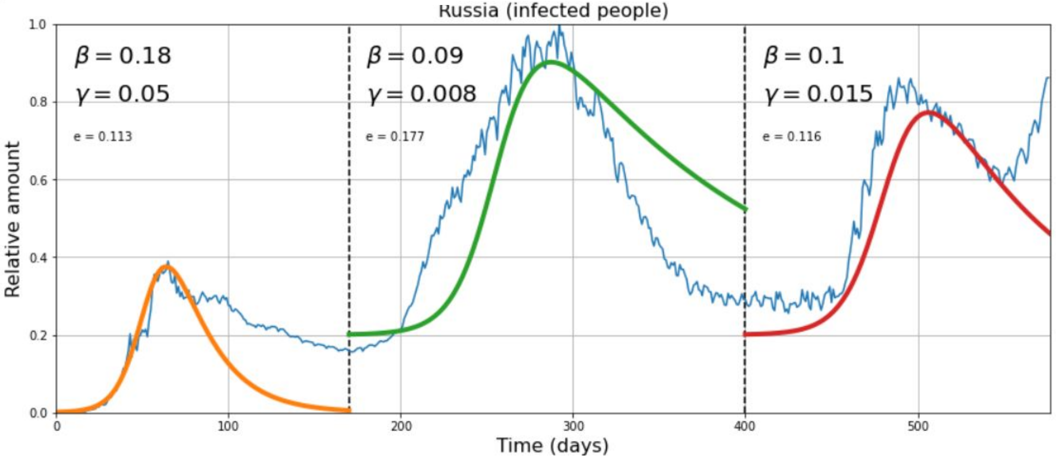
Manual vs optimization

Manual



Infected (Russia)

Optimization



Error comparison

GOOD

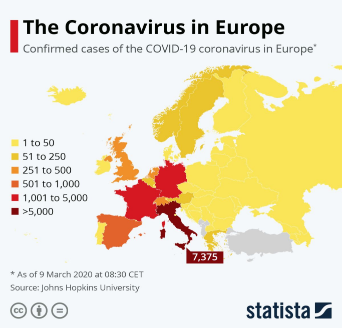
BAD

Fitting RMSE for Infectious (x 1e3)				
	Russia		USA	
Wave	Manually	Optimization	Manually	Optimization
1	113	64	25	22
2	177	108	34	27
3	116	96	79	67
4	-\\-	-\\-	159	153

Cons of Basic SIR model:

This model could be improved (to better fits reality) by following ways:

- 1) to consider vaccination effects;
- 2) to consider a latent period (when individuals are already infected by the pathogens but not yet infectious);
- 3) to consider time-varying parameters;
- 4) to consider additional parameters to describe the specific rate of immunity loss;
- 5) to consider the possibility to be infected several times.



Examples of SIR models for different diseases: Ebola, Flu, Measles, COVID-19

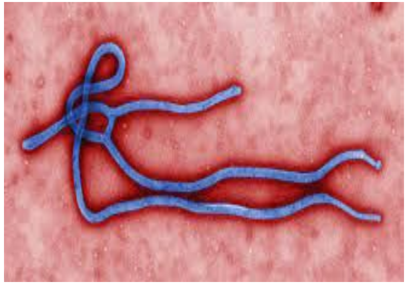
1) Temporary immunity: the SIRS model for endemic diseases.
2) The SIR model with demography
3) SIR model with vaccination (this SIR model is for vaccine for newborn)
4) SEIR model with a latent period
5) Age structured SIR model
6) SIR model with time-varying parameters and periodic forcing

Examples for different diseases: Ebola, Measles, COVID-19

Ebola and Measles in Africa during current outbreaks:

$$\begin{cases} \frac{dS}{dt} = -\beta \frac{SI}{N} + \delta R \\ \frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I \\ \frac{dR}{dt} = \gamma I - \delta R \end{cases}$$

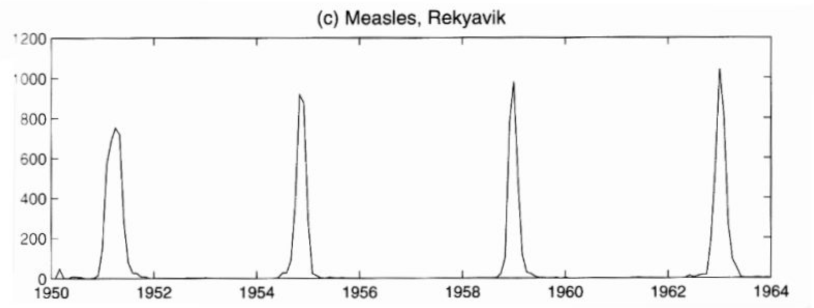
An additional parameter δ is introduced in order to represent the specific rate of immunity loss.



Measles before invention of vaccine:

$$\begin{cases} \frac{dS}{dt} = -\beta \frac{SI}{N} \\ \frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I \quad \beta(t) = \beta_0[1 + \beta_1 cos(2\pi t)] \\ \frac{dR}{dt} = \gamma I \end{cases}$$

with the time t in years and β1 = 0.2, (i.e. a 20% seasonal variation).



Examples for different diseases: Ebola Measles, COVID-19

Measles in first world countries:

$$\begin{cases} \frac{dS}{dt} = (\mu - \sigma)N - \beta\frac{SI}{N} - \mu S \\ \frac{dI}{dt} = \beta\frac{SI}{N} - \gamma I - \mu I \\ \frac{dR}{dt} = \gamma I - \mu R \end{cases}$$

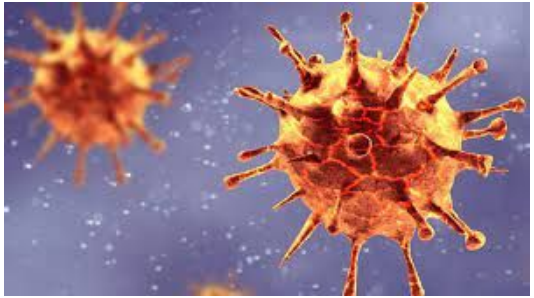
σ is represents the specific vaccination rate of the newborns

μ is accountable for deaths (or emigration)

COVID-19 ?

$$\begin{cases} \frac{dS}{dt} = -\nu N - \beta IS - \mu S \\ \frac{dI}{dt} = \beta IS - \gamma I - \mu I \\ \frac{dR}{dt} = \gamma I - \mu R \end{cases}$$

$$\begin{cases} \frac{dS}{dt} = (\mu - \sigma)N - \beta\frac{SI}{N} - \mu S \\ \frac{dI}{dt} = \beta\frac{SI}{N} - \gamma I - \mu I \\ \frac{dR}{dt} = \gamma I - \mu R \end{cases}$$



Age-structured SIR

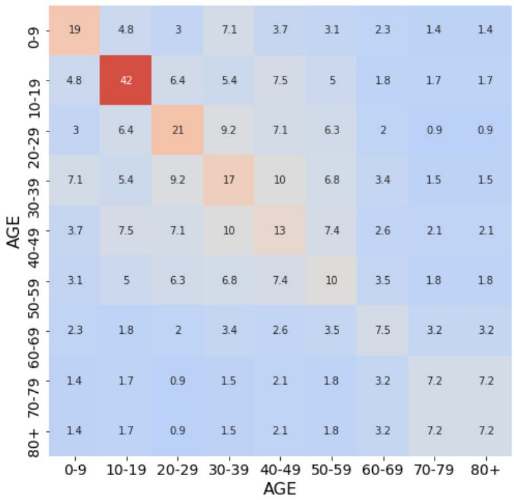
$$\begin{cases} \frac{dS_i}{dt} = -\beta \frac{S_i}{N} \cdot \sum_{j=1}^{L} M_{ij} I_j \\ \frac{dI_i}{dt} = \beta \frac{S_i}{N} \cdot \sum_{j=1}^{L} M_{ij} I_j - \gamma I_i \\ \frac{dR_i}{dt} = \gamma I_i \end{cases} \Rightarrow \begin{cases} \frac{d}{dt}\mathbf{S} = -\beta \frac{\mathbf{S}}{N} \cdot \mathbf{MI} \\ \frac{d}{dt}\mathbf{I} = \beta \frac{\mathbf{S}}{N} \cdot \mathbf{MI} - \gamma \mathbf{I} \\ \frac{d}{dt}\mathbf{R} = \gamma \mathbf{I} \end{cases}, \text{ where}$$

$\mathbf{S} \in \mathbb{R}^{Lx1}$ - the vector of susceptible individuals;

$\mathbf{I} \in \mathbb{R}^{Lx1}$ - the vector of infectious individuals;

$\mathbf{R} \in \mathbb{R}^{Lx1}$ - the vector of removed individuals;

$\mathbf{M} \in \mathbb{R}^{LxL}$ - weighted age-contact matrix



Age-contact matrix scaling

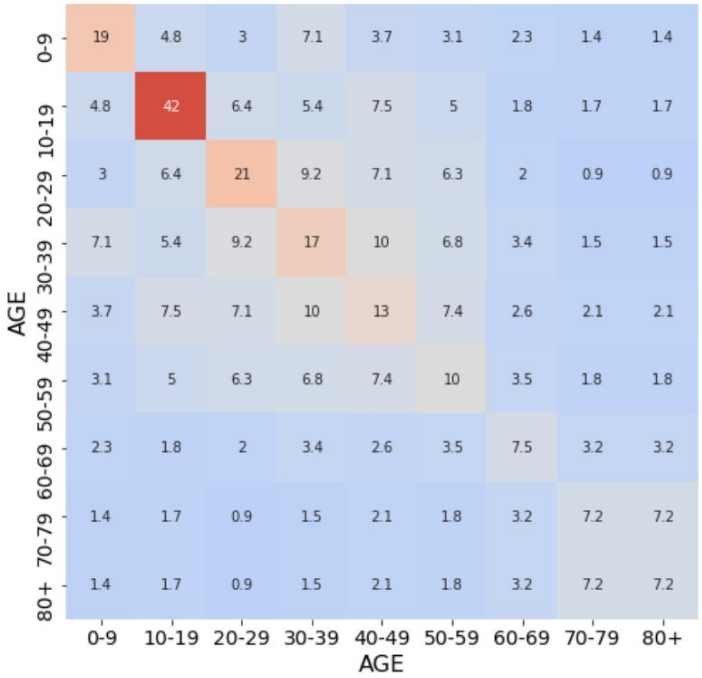
$M = M_0 \cdot diag(\mathbf{p})$, where

$\mathbf{p}$ - age distribution for current region

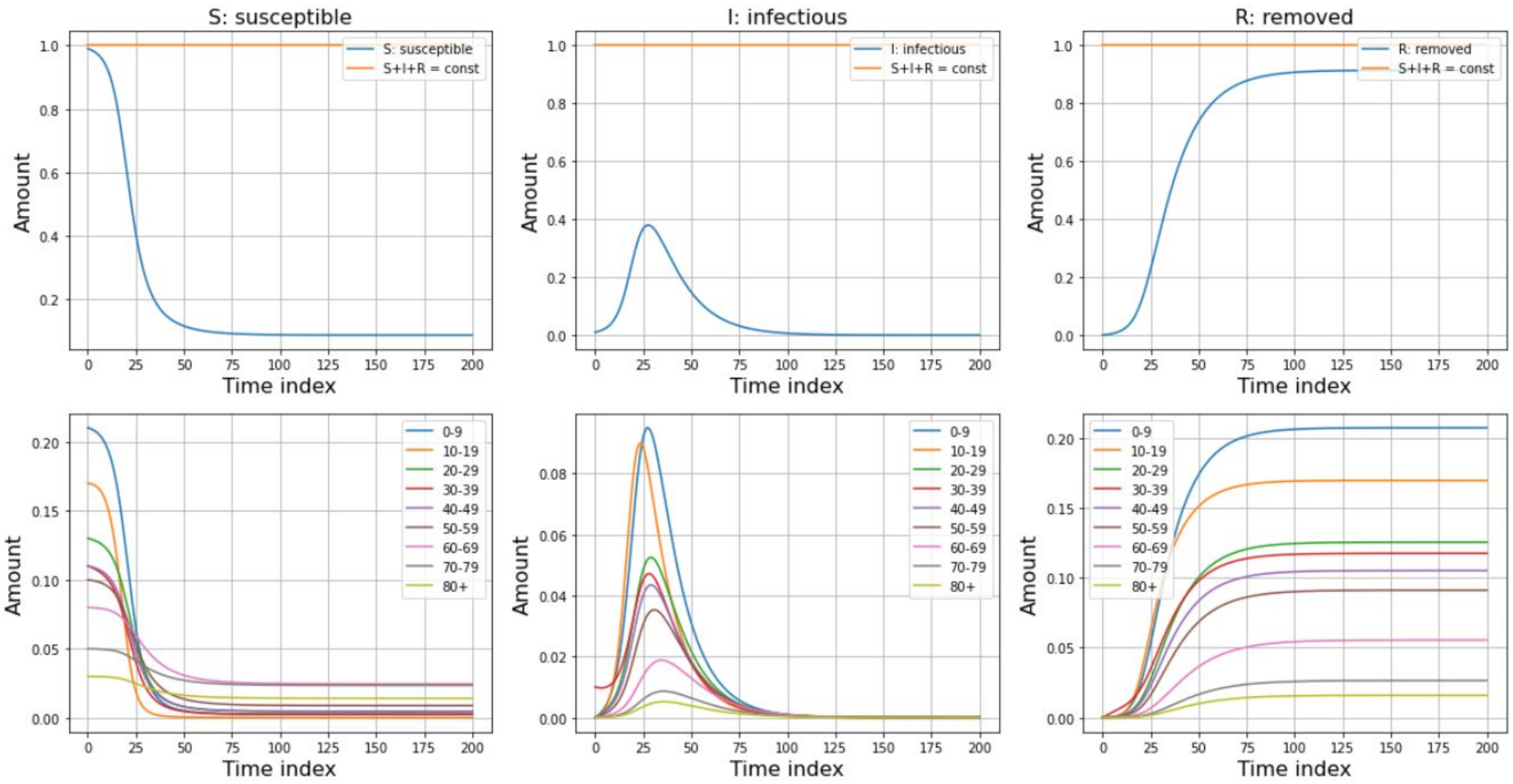
$(\mathbf{p} \in \mathbb{R}^{L\mathrm{x}1}, |p| = 1)$;

M_0 - initial age-contact matrix;

M - weighted age-contact matrix.



Age-structured SIR - Current results



Age-targeted vaccination

$$\begin{cases} \frac{d}{dt}\mathbf{S} = -\beta\frac{\mathbf{S}}{N}\cdot\mathbf{MI} - \mu \\ \frac{d}{dt}\mathbf{I} = \beta\frac{\mathbf{S}}{N}\cdot\mathbf{MI} - \gamma\mathbf{I} \\ \frac{d}{dt}\mathbf{R} = \gamma\mathbf{I} + \mu \end{cases} \text{, where}$$

$\mathbf{S} \in \mathbb{R}^{Lx1}, \mathbf{I} \in \mathbb{R}^{Lx1}, \mathbf{R} \in \mathbb{R}^{Lx1}, \beta, \gamma, \mathbf{M}$ - the same;

$\mu = T\frac{\omega\circ\mathbf{S}}{|\omega\circ\mathbf{S}|}$ - vaccination rate vector;

ω - age-structured vaccination priority;

T - vaccination capability.

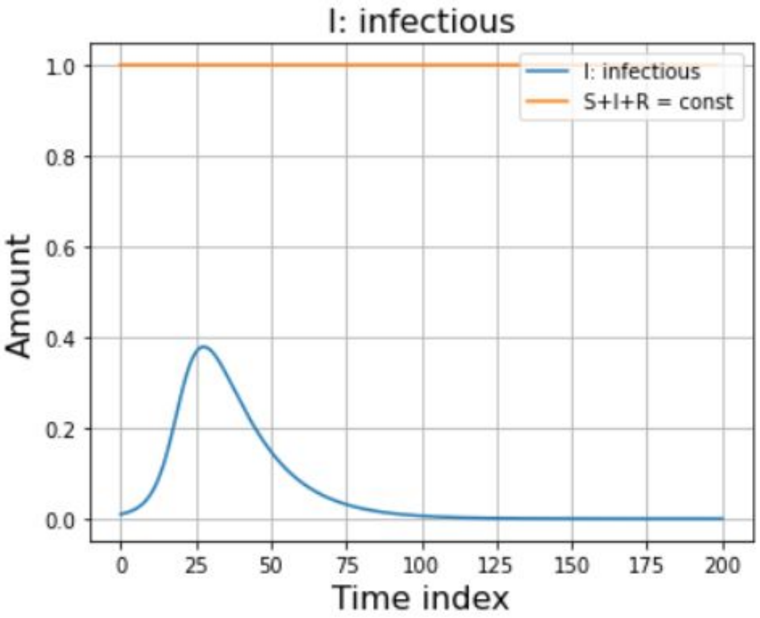
Age

	w_c	w_s
0-9	1	1
10-19	1	1
20-29	1	1
30-39	1	1
40-49	1	2
50-59	1	4
60-69	1	8
70-79	1	16
80+	1	16

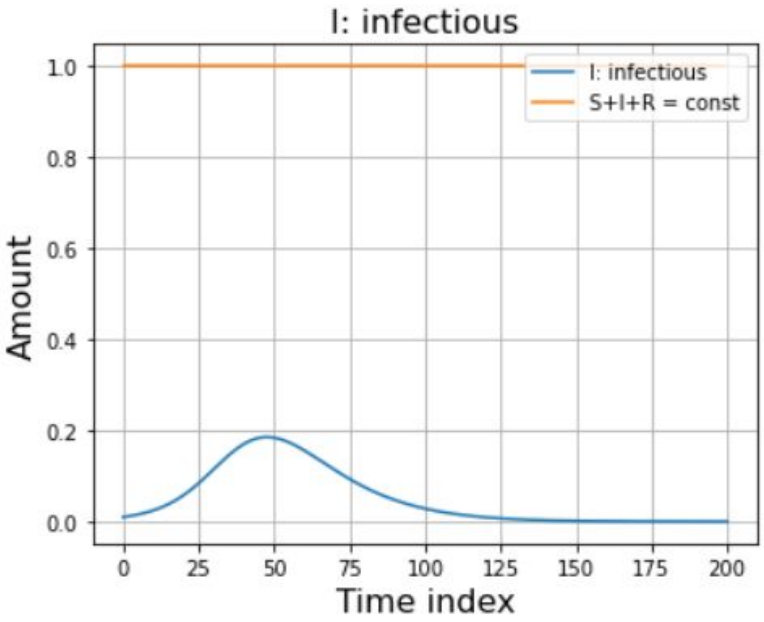
Results comparison

Beta (0.25) and gamma (1/14) - the same

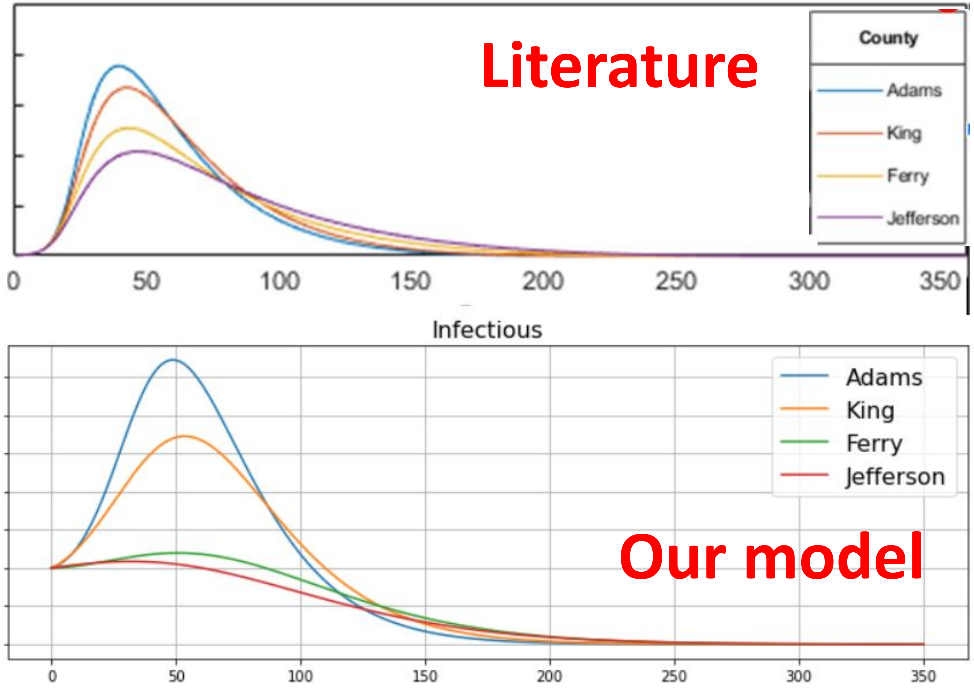
Without vaccination

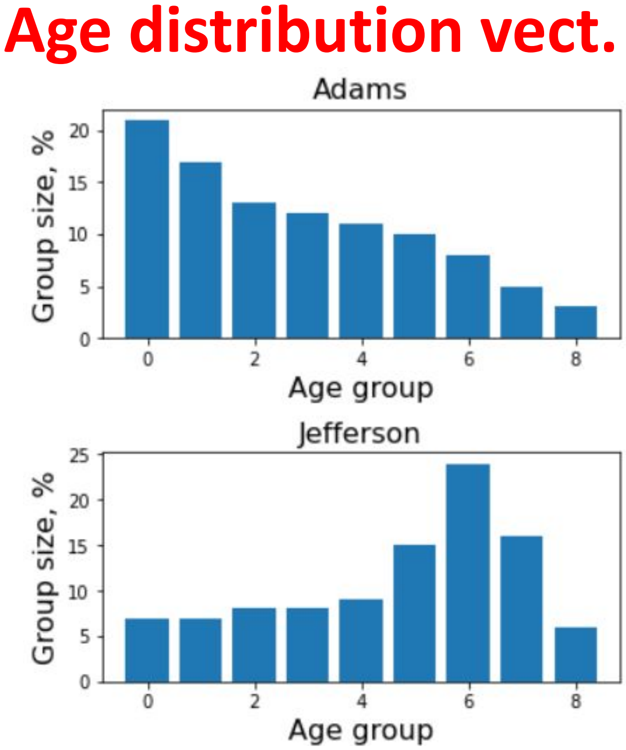


With vaccination (fair and old age)



Age-targeted vaccination - literature comparison





https://www.nature.com/articles/s41598-021-94609-3.pdf

Conclusions

1. Vaccinate ASAP

2. Keep the distance

3. Wash your hands

4. Wear a mask

5. Make others do all of the above!

Thank you for your attention!

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