

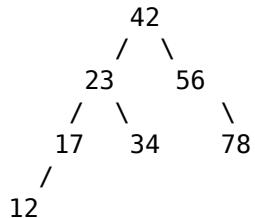
# Binary Search Tree

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## Invalid Date

Ein binärer Suchbaum (Binary Search Tree, BST) ist eine Datenstruktur, die es ermöglicht, Daten in einer hierarchischen Struktur zu speichern. Jeder Knoten im Baum hat maximal zwei Kinder, wobei das linke Kind kleiner und das rechte Kind grösser als der Knoten selbst ist. Dies ermöglicht das effiziente Suchen, Einfügen und Löschen von Elementen.

Sollen beispielsweise die Zahlen 42, 23, 17, 34, 56, 78 und 12 der Reihe nach in einen binären Suchbaum eingefügt werden, geschieht dies wie folgt:



Um einen Knoten zu löschen, gibt es drei Fälle zu beachten: 1. Der Knoten ist ein Blatt (keine Kinder): Der Knoten kann einfach entfernt werden. 2. Der Knoten hat ein Kind: Das Kind ersetzt den Knoten. 3. Der Knoten hat zwei Kinder: Der Knoten wird durch den kleinsten Knoten im rechten Teilbaum ersetzt.

In den folgenden Abschnitten findet sich eine Mögliche Implementierung eines binären Suchbaums in Python.

## Klasse BSTNode

```
1 \KeywordTok{class}\NormalTok{\{} \NormalTok{BSTNode:}\}
  \NormalTok{\{} \NormalTok{def}\}
\FunctionTok{\{}\_\_init\_\_}\NormalTok{\{}\NormalTok{\}} \NormalTok{VariableTok\{self\}}\NormalTok{\{}\NormalTok{, key, value}\}\NormalTok{\}}
\OperatorTok{=}\NormalTok{\{}\NormalTok{VariableTok\{None\}}\NormalTok{\{}\NormalTok{\}}\NormalTok{\}:}\}
  \NormalTok{VariableTok\{self\}}\NormalTok{\{}\NormalTok{.key }\}\NormalTok{OperatorTok{=}\NormalTok{\{}\NormalTok{key}\}\NormalTok{\}}
  \NormalTok{VariableTok\{self\}}\NormalTok{\{}\NormalTok{.value }\}\NormalTok{OperatorTok{=}\NormalTok{\{}\NormalTok{value}\}\NormalTok{\}}
  \NormalTok{VariableTok\{self\}}\NormalTok{\{}\NormalTok{.parent }\}\NormalTok{OperatorTok{=}\NormalTok{\{}\NormalTok{\}}
\NormalTok{\} VariableTok\{None\}}
```

```

6      \VariableTok{self}\NormalTok{.left }\operatorTok{=} \\
7 \VariableTok{None} \\
8     \VariableTok{self}\NormalTok{.right }\operatorTok{=} \\
9 \VariableTok{None} \\
10    \KeywordTok{def} \\
11 \FunctionTok{\_\_str\_\_}\NormalTok{()}\VariableTok{self}\NormalTok{.key}) \\
12 \NormalTok{key }\operatorTok{=} \\
13 \BuiltInTok{str}\NormalTok{()}\VariableTok{self}\NormalTok{.key}) \\
14 \NormalTok{parent }\operatorTok{=} \\
15 \StringTok{\text{if}}\NormalTok{ }\VariableTok{self}\NormalTok{.parent }\KeywordTok{is} \VariableTok{None} \\
16 \ControlFlowTok{else} \\
17 \BuiltInTok{str}\NormalTok{()}\VariableTok{self}\NormalTok{.parent.key}) \\
18 \NormalTok{left }\operatorTok{=} \\
19 \StringTok{\text{if}}\NormalTok{ }\VariableTok{self}\NormalTok{.left }\KeywordTok{is} \VariableTok{None} \\
\ControlFlowTok{else} \\
\BuiltInTok{str}\NormalTok{()}\VariableTok{self}\NormalTok{.left.key}) \\
\NormalTok{right }\operatorTok{=} \\
\StringTok{\text{if}}\NormalTok{ }\VariableTok{self}\NormalTok{.right }\KeywordTok{is} \VariableTok{None} \\
\ControlFlowTok{else} \\
\BuiltInTok{str}\NormalTok{()}\VariableTok{self}\NormalTok{.right.key}) \\
\NormalTok{s }\operatorTok{=} \\
\SpecialStringTok{f\text{'}}\NormalTok{ }\CharTok{\textbackslash} \\
t}\SpecialStringTok{Parent =} \\
}\SpecialCharTok{\{}\NormalTok{parent}\SpecialCharTok{\}}\CharTok{\textbackslash} \\
slash\NormalTok{ }\SpecialStringTok{f\text{'}}\NormalTok{ }\CharTok{\textbackslash} \\
t}\SpecialStringTok{Key =} \\
}\SpecialCharTok{\{}\NormalTok{key}\SpecialCharTok{\}}\CharTok{\textbackslash} \\
slash\NormalTok{ }\SpecialStringTok{f\text{'}}\NormalTok{ }\CharTok{\textbackslash} \\
t}\SpecialStringTok{Left =} \\
}\SpecialCharTok{\{}\NormalTok{left}\SpecialCharTok{\}}\CharTok{\textbackslash} \\
slash\NormalTok{ }\SpecialStringTok{Right =} \\
}\SpecialCharTok{\{}\NormalTok{right}\SpecialCharTok{\}}\SpecialStringTok{\{}\text{'}}\NormalTok{ }\CharTok{\textbackslash} \\
text\NormalTok{ }\SpecialStringTok{f\text{'}}\NormalTok{ }\CharTok{\textbackslash} \\
t}
\NormalTok{) } \\
\ControlFlowTok{return}\NormalTok{ s }

```

## Klasse BST

```

1 \KeywordTok{class}\NormalTok{ BST:} \\
2   \KeywordTok{def} \\
3 \FunctionTok{\_\_init\_\_}\NormalTok{()}\VariableTok{self}\NormalTok{.key}\operatorTok{=} \\
4 \NormalTok{None}\NormalTok{.value}\operatorTok{=} \\
5 \NormalTok{None} \\
6 \NormalTok{None}\operatorTok{=} \\
7 \NormalTok{None}

```

```

3      \ControlFlowTok{if}\NormalTok{ key }\KeywordTok{is}
4      \VariableTok{None}\NormalTok{:\}}
5          \VariableTok{self}\NormalTok{.root }\OperatorTok{=}
6      \VariableTok{None}
7          \ControlFlowTok{else}\NormalTok{:\}
8      \NormalTok{ node }\OperatorTok{=}\NormalTok{ BSTNode(key, value)
9          \VariableTok{self}\NormalTok{.root }\OperatorTok{=}\NormalTok{ node}\NormalTok{:\}
10     \NormalTok{ node }\OperatorTok{=}\NormalTok{ BSTNode(key, value)}
11     \ControlFlowTok{if} \VariableTok{self}\NormalTok{.root
12 } \KeywordTok{is} \VariableTok{None}\NormalTok{:\}
13     \VariableTok{self}\NormalTok{.root }\OperatorTok{=}\NormalTok{ node}\NormalTok{:\}
14     \ControlFlowTok{return}
15
16     \ControlFlowTok{if}\NormalTok{ root }\KeywordTok{is}
17     \VariableTok{None}\NormalTok{:\}
18     \NormalTok{ root }\OperatorTok{=}
19     \VariableTok{self}\NormalTok{.root}
20
21     \ControlFlowTok{if}\NormalTok{ key
22     }\OperatorTok{\textless}\{}\NormalTok{ root.key }\KeywordTok{and}\NormalTok{ root.left }\KeywordTok{is}
23     \NormalTok{ root.left }\OperatorTok{=}\NormalTok{ node}\NormalTok{:\}
24     \NormalTok{ root.left }\OperatorTok{=}\NormalTok{ node.parent }\OperatorTok{=}\NormalTok{ root}\NormalTok{:\}
25     \ControlFlowTok{return}
26
27
28     \ControlFlowTok{if}\NormalTok{ key
29     }\OperatorTok{\textless}\{}\NormalTok{ root.key:\}
30     \NormalTok{ root }\OperatorTok{=}\NormalTok{ root.left}\NormalTok{:\}
31     \VariableTok{self}\NormalTok{.insert(key, value, root)}
32
33
34     \ControlFlowTok{if}\NormalTok{ key
35     }\OperatorTok{\textgreater}\{}\NormalTok{ root.key:\}
36     \NormalTok{ root }\OperatorTok{=}\NormalTok{ root.right}\NormalTok{:\}
37     \VariableTok{self}\NormalTok{.insert(key, value, root)}

```

```

37     \KeywordTok{def}
38     \BuiltInTok{min}\NormalTok{()}\VariableTok{self}\NormalTok{, bst}\OperatorTok{=}\VariableTok{None}\NormalTok{():}
39         \ControlFlowTok{if}\NormalTok{ bst }\KeywordTok{is}
40         \VariableTok{None}\NormalTok{::}
41         \NormalTok{minimum }\OperatorTok{=}
42         \VariableTok{self}\NormalTok{.root}
43         \ControlFlowTok{else}\NormalTok{::}
44         \NormalTok{minimum }\OperatorTok{=}\NormalTok{bst.root}
45
46         \ControlFlowTok{while}\NormalTok{ minimum.left }\KeywordTok{is}
47         \KeywordTok{not} \VariableTok{None}\NormalTok{::}
48         \NormalTok{minimum }\OperatorTok{=}\NormalTok{ minimum.left}
49
50     \KeywordTok{def}
51     \BuiltInTok{max}\NormalTok{()}\VariableTok{self}\NormalTok{, bst}\OperatorTok{=}\VariableTok{None}\NormalTok{():}
52         \ControlFlowTok{if}\NormalTok{ bst }\KeywordTok{is}
53         \VariableTok{None}\NormalTok{::}
54         \NormalTok{maximum }\OperatorTok{=}
55         \VariableTok{self}\NormalTok{.root}
56         \ControlFlowTok{else}\NormalTok{::}
57         \NormalTok{maximum }\OperatorTok{=}\NormalTok{bst.root}
58
59         \ControlFlowTok{while}\NormalTok{ maximum.right }\KeywordTok{is}
60         \KeywordTok{not} \VariableTok{None}\NormalTok{::}
61         \NormalTok{maximum }\OperatorTok{=}\NormalTok{ maximum.right}
62
63         \ControlFlowTok{return}\NormalTok{ maximum}
64
65     \KeywordTok{def}\NormalTok{ search()\VariableTok{self}\NormalTok{, key, node}\OperatorTok{=}\VariableTok{None}\NormalTok{():}
66         \CommentTok{\# If initial call or we've hit None in recursion}
67         \ControlFlowTok{if}\NormalTok{ node }\KeywordTok{is}
68         \VariableTok{None}\NormalTok{::}
69             \ControlFlowTok{if}\NormalTok{ \VariableTok{self}\NormalTok{.root }
70                 \KeywordTok{is} \VariableTok{None}\NormalTok{::}
71                     \CommentTok{\# Empty tree}
72                     \ControlFlowTok{return}\NormalTok{ DecValTok{1}}
73
74         \NormalTok{node }\OperatorTok{=}\NormalTok{ \VariableTok{self}\NormalTok{.root}
75
76         \CommentTok{\# Found the key}
77         \ControlFlowTok{if}\NormalTok{ key }\OperatorTok{==}\NormalTok{ node.key}
78             \ControlFlowTok{return}\NormalTok{ node}
79
80         \CommentTok{\# Key doesn't exist in this path}
81         \ControlFlowTok{if}\NormalTok{ key }\OperatorTok{less}\NormalTok{ node.key}
82             \ControlFlowTok{return}\NormalTok{ NormalTok{ node.key}}

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72          \ControlFlowTok{if}\NormalTok{ node.left }\KeywordTok{is}
73      \VariableTok{None}\NormalTok{:\}}
74          \ControlFlowTok{return} \OperatorTok{{-}}\DecValTok{1}
75          \ControlFlowTok{return}
76      \VariableTok{self}\NormalTok{.\search(key, node.left)}
77          \ControlFlowTok{else}\NormalTok{:\# key}
78          \textgreater{} node.key
79          \ControlFlowTok{if}\NormalTok{ node.right }\KeywordTok{is}
80      \VariableTok{None}\NormalTok{:\}
81          \ControlFlowTok{return} \OperatorTok{{-}}\DecValTok{1}
82          \ControlFlowTok{return}
83      \VariableTok{self}\NormalTok{.\search(key, node.right)}
84
85          \KeywordTok{def}\NormalTok{ delete()\VariableTok{self}\NormalTok{.\NormalTok{key,:}}
86          \CommentTok{\# Find the node to delete}
87      \NormalTok{node }\OperatorTok{=}
88      \VariableTok{self}\NormalTok{.\search(key)}
89
90          \CommentTok{\# If node not found, return}
91          \ControlFlowTok{if}\NormalTok{ node }\OperatorTok{==}
92          \OperatorTok{{-}}\DecValTok{1}\NormalTok{:\}
93          \ControlFlowTok{return}
94
95          \VariableTok{self}\NormalTok{.\_delete\_node(node)}
96
97          \KeywordTok{def}\NormalTok{_delete\_node()\VariableTok{self}\NormalTok{.\NormalTok{node,:}}
98          \CommentTok{\# Case 1: Node has no children (leaf node)}
99          \ControlFlowTok{if}\NormalTok{ node.left }\KeywordTok{is}
100         \VariableTok{None}\NormalTok{ \KeywordTok{and}\NormalTok{ node.right }\KeywordTok{is}
101         \VariableTok{None}\NormalTok{:\}
102         \ControlFlowTok{if}\NormalTok{ node.parent.left }
103         \OperatorTok{==}\NormalTok{ node:\}
104         \NormalTok{node.parent.left }\OperatorTok{=}
105         \VariableTok{None}\NormalTok{:\}
106         \ControlFlowTok{else}\NormalTok{:\}
107         \ControlFlowTok{if}\NormalTok{ node.parent.left }
108         \OperatorTok{==}\NormalTok{ node:\}
109         \NormalTok{node.parent.left }\OperatorTok{=}
110         \VariableTok{None}\NormalTok{:\}
111         \ControlFlowTok{else}\NormalTok{:\}
112         \NormalTok{node.parent.right }\OperatorTok{=}
113         \VariableTok{None}\NormalTok{:\}
114
115         \CommentTok{\# Case 2: Node has only one child}
116         \ControlFlowTok{elif}\NormalTok{ node.left }\KeywordTok{is}
117         \VariableTok{None}\NormalTok{:\# Has only right child}
118         \ControlFlowTok{if}\NormalTok{ node.parent.left }
119         \OperatorTok{==}\NormalTok{ node:\}
120         \VariableTok{self}\NormalTok{.\root:\}
121         \VariableTok{self}\NormalTok{.\root }
122         \OperatorTok{=}\NormalTok{ node.right}

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105 \NormalTok{          node.right.parent }\operatorTok{=}
106 \VariableTok{None}      \ControlFlowTok{else}\NormalTok{:}
107             \ControlFlowTok{if}\NormalTok{ node.parent.left
108 }\operatorTok{==}\NormalTok{ node:}
109 \NormalTok{          node.parent.left }\operatorTok{=}\NormalTok{ node.parent}
110 \NormalTok{          node.right }\operatorTok{=}\NormalTok{ node.parent}
111 \NormalTok{          node.right.parent }\operatorTok{=}\NormalTok{ node.parent}
112
113             \ControlFlowTok{else}\NormalTok{:}
114 \VariableTok{None}\NormalTok{: }\CommentTok{# Has only left child}
115             \ControlFlowTok{if}\NormalTok{ node }\operatorTok{==}
116 \VariableTok{self}\NormalTok{.root:}
117                 \VariableTok{self}\NormalTok{.root
118 }\operatorTok{=}\NormalTok{ node.left}
119 \NormalTok{          node.left.parent }\operatorTok{=}
120 \VariableTok{None}      \ControlFlowTok{else}\NormalTok{:}
121             \ControlFlowTok{if}\NormalTok{ node.parent.left
122 }\operatorTok{==}\NormalTok{ node:}
123 \NormalTok{          node.parent.left }\operatorTok{=}\NormalTok{ node.left}
124             \ControlFlowTok{else}\NormalTok{:}
125                 \CommentTok{# Case 3: Node has two children}
126                 \ControlFlowTok{else}\NormalTok{:}
127 \NormalTok{          successor }\operatorTok{=} \VariableTok{None}
128 \NormalTok{          current }\operatorTok{=} \NormalTok{ node.right}
129
130             \ControlFlowTok{while}\NormalTok{ current.left }\KeywordTok{is}
131 \KeywordTok{not} \VariableTok{None}\NormalTok{:}
132 \NormalTok{          current }\operatorTok{=} \NormalTok{ current.left}
133 \NormalTok{          successor }\operatorTok{=} \NormalTok{ current}
134
135             \CommentTok{# Copy successor\textquotesingle{}s key and value to
136             the node}
137 \NormalTok{          node.key }\operatorTok{=} \NormalTok{ successor.key}
138 \NormalTok{          node.value }\operatorTok{=} \NormalTok{ successor.value}
139
140             \CommentTok{# Delete the successor (which has at most one right
child)}
```

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140         \VariableTok{self}\NormalTok{\_delete\_node(successor)}
141
142     \KeywordTok{def}\NormalTok{ iterate()\VariableTok{self}\NormalTok{\_NormalTok{\_iterate}}
143     \operatorname{node}\operatorname{OperatorTok}{=}\VariableTok{None}\NormalTok{\_NormalTok{\_iterate}
144     \operatorname{result}\operatorname{OperatorTok}{=}\VariableTok{None}\NormalTok{\_NormalTok{\_iterate}():
145         \CommentTok{\# Initialize result list on first call}
146         \ControlFlowTok{if}\NormalTok{ result }\operatorname{KeywordTok}{is}
147     \VariableTok{None}\NormalTok{\_NormalTok{\_iterate}:
148     \NormalTok{\_NormalTok{\_iterate}\operatorname{result}\operatorname{OperatorTok}{=}\NormalTok{ []}}
149
150         \CommentTok{\# Use root if no starting node provided}
151         \ControlFlowTok{if}\NormalTok{ node }\operatorname{KeywordTok}{is}
152     \VariableTok{None}\NormalTok{\_NormalTok{\_iterate}:
153         \ControlFlowTok{if}\NormalTok{ \VariableTok{self}\NormalTok{\_root}
154     \operatorname{is}\VariableTok{None}\NormalTok{\_NormalTok{\_iterate}\operatorname{:}\CommentTok{\# Empty tree}
155         \ControlFlowTok{return}\NormalTok{\_NormalTok{\_iterate}\operatorname{result}}
156     \NormalTok{\_NormalTok{\_iterate}\operatorname{node}\operatorname{OperatorTok}{=}}
157     \VariableTok{self}\NormalTok{\_root}
158
159         \CommentTok{\# In{-}order traversal: left {-}\textgreater{} current
160         \operatorname{-}\textgreater{}\right\} right}
161         \ControlFlowTok{if}\NormalTok{ node.left }\operatorname{KeywordTok}{is}
162     \KeywordTok{not}\VariableTok{None}\NormalTok{\_NormalTok{\_iterate}:
163         \VariableTok{self}\NormalTok{\_NormalTok{\_iterate(node.left, result})}
164
165     \NormalTok{\_NormalTok{\_iterate}\operatorname{result.append(node)}}
166
167         \ControlFlowTok{if}\NormalTok{ node.right }\operatorname{KeywordTok}{is}
168     \KeywordTok{not}\VariableTok{None}\NormalTok{\_NormalTok{\_iterate}:
169         \VariableTok{self}\NormalTok{\_NormalTok{\_iterate(node.right, result})}
170
171         \ControlFlowTok{return}\NormalTok{\_NormalTok{\_iterate}\operatorname{result}}

```