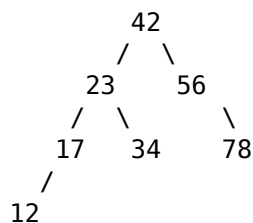# Binary Search Tree

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Ein binärer Suchbaum (Binary Search Tree, BST) ist eine Datenstruktur, die es ermöglicht, Daten in einer hierarchischen Struktur zu speichern. Jeder Knoten im Baum hat maximal zwei Kinder, wobei das linke Kind kleiner und das rechte Kind grösser als der Knoten selbst ist. Dies ermöglicht das effiziente Suchen, Einfügen und Löschen von Elementen.

Sollen beispielsweise die Zahlen 42, 23, 17, 34, 56, 78 und 12 der Reihe nach in einen binären Suchbaum eingefügt werden, geschieht dies wie folgt:

```
        42
       /  \
     23    56
    /  \     \
   17   34    78
  /
12
```

Um einen Knoten zu löschen, gibt es drei Fälle zu beachten: 1. Der Knoten ist ein Blatt (keine Kinder): Der Knoten kann einfach entfernt werden. 2. Der Knoten hat ein Kind: Das Kind ersetzt den Knoten. 3. Der Knoten hat zwei Kinder: Der Knoten wird durch den kleinsten Knoten im rechten Teilbaum ersetzt.

In den folgenden Abschnitten findet sich eine Mögliche Implementierung eines binären Suchbaums in Python.

Klasse BSTNode

```
1  \KeywordTok{class}\NormalTok{ BSTNode:}
2      \KeywordTok{def}
\FunctionTok{\_\_init\_\_}\NormalTok{(}\VariableTok{self}\NormalTok{, key,
value}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
3          \VariableTok{self}\NormalTok{.key }\OperatorTok{=}\NormalTok{ key}
4          \VariableTok{self}\NormalTok{.value }\OperatorTok{=}\NormalTok{
value}
5          \VariableTok{self}\NormalTok{.parent }\OperatorTok{=}
\VariableTok{None}
```

```
6          \VariableTok{self}\NormalTok{.left }\OperatorTok{=}
   \VariableTok{None}
7          \VariableTok{self}\NormalTok{.right }\OperatorTok{=}
   \VariableTok{None}
8
9      \KeywordTok{def}
   \FunctionTok{\_\_str\_\_}\NormalTok{(}\VariableTok{self}\NormalTok{):}
10 \NormalTok{        key }\OperatorTok{=}
   \BuiltInTok{str}\NormalTok{(}\VariableTok{self}\NormalTok{.key)}
11 \NormalTok{        parent }\OperatorTok{=}
   \StringTok{\textquotesingle{}None\textquotesingle{}} \ControlFlowTok{if}
   \VariableTok{self}\NormalTok{.parent }\KeywordTok{is} \VariableTok{None}
   \ControlFlowTok{else}
   \BuiltInTok{str}\NormalTok{(}\VariableTok{self}\NormalTok{.parent.key)}
12 \NormalTok{        left }\OperatorTok{=}
   \StringTok{\textquotesingle{}None\textquotesingle{}} \ControlFlowTok{if}
   \VariableTok{self}\NormalTok{.left }\KeywordTok{is} \VariableTok{None}
   \ControlFlowTok{else}
   \BuiltInTok{str}\NormalTok{(}\VariableTok{self}\NormalTok{.left.key)}
13 \NormalTok{        right }\OperatorTok{=}
   \StringTok{\textquotesingle{}None\textquotesingle{}} \ControlFlowTok{if}
   \VariableTok{self}\NormalTok{.right }\KeywordTok{is} \VariableTok{None}
   \ControlFlowTok{else}
   \BuiltInTok{str}\NormalTok{(}\VariableTok{self}\NormalTok{.right.key)}
14 \NormalTok{        s }\OperatorTok{=}\NormalTok{ (}
15          \SpecialStringTok{f\textquotesingle{}}\CharTok{\textbackslash{}
   t}\SpecialStringTok{Parent =
   }\SpecialCharTok{\{}\NormalTok{parent}\SpecialCharTok{\}}\CharTok{\textback
   slash{}n}\SpecialStringTok{\textquotesingle{}}
16          \SpecialStringTok{f\textquotesingle{}}\CharTok{\textbackslash{}
   t}\SpecialStringTok{Key =
   }\SpecialCharTok{\{}\NormalTok{key}\SpecialCharTok{\}}\CharTok{\textbacksla
   sh{}n}\SpecialStringTok{\textquotesingle{}}
17          \SpecialStringTok{f\textquotesingle{}}Left =
   }\SpecialCharTok{\{}\NormalTok{left}\SpecialCharTok{\}}\CharTok{\textbacksl
   ash{}t}\SpecialStringTok{Right =
   }\SpecialCharTok{\{}\NormalTok{right}\SpecialCharTok{\}}\SpecialStringTok{\
   textquotesingle{}}
18 \NormalTok{        )}
19      \ControlFlowTok{return}\NormalTok{ s}
```

Klasse BST

```
1 \KeywordTok{class}\NormalTok{ BST:}
2      \KeywordTok{def}
   \FunctionTok{\_\_init\_\_}\NormalTok{(}\VariableTok{self}\NormalTok{,
   key}\OperatorTok{=}\VariableTok{None}\NormalTok{,
   value}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
```

```
3          \ControlFlowTok{if}\NormalTok{ key }\KeywordTok{is}
  \VariableTok{None}\NormalTok{:}
4              \VariableTok{self}\NormalTok{.root }\OperatorTok{=}
  \VariableTok{None}
5          \ControlFlowTok{else}\NormalTok{:}
6 \NormalTok{            node }\OperatorTok{=}\NormalTok{ BSTNode(key, value)
          }
7              \VariableTok{self}\NormalTok{.root }\OperatorTok{=}\NormalTok{
  node}
8
9      \KeywordTok{def}\NormalTok{ insert(}\VariableTok{self}\NormalTok{, key,
  value}\OperatorTok{=}\VariableTok{None}\NormalTok{,
  root}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
10 \NormalTok{          node }\OperatorTok{=}\NormalTok{ BSTNode(key, value)}
11         \ControlFlowTok{if} \VariableTok{self}\NormalTok{.root
  }\KeywordTok{is} \VariableTok{None}\NormalTok{:}
12             \VariableTok{self}\NormalTok{.root }\OperatorTok{=}\NormalTok{
  node}
13             \ControlFlowTok{return}
14
15         \ControlFlowTok{if}\NormalTok{ root }\KeywordTok{is}
  \VariableTok{None}\NormalTok{:}
16 \NormalTok{            root }\OperatorTok{=}
  \VariableTok{self}\NormalTok{.root}
17
18         \ControlFlowTok{if}\NormalTok{ key
  }\OperatorTok{\textless{}}\NormalTok{ root.key }\KeywordTok{and}\NormalTok{
  root.left }\KeywordTok{is} \VariableTok{None}\NormalTok{:}
19 \NormalTok{            root.left }\OperatorTok{=}\NormalTok{ node}
20 \NormalTok{            node.parent }\OperatorTok{=}\NormalTok{ root}
21             \ControlFlowTok{return}
22
23         \ControlFlowTok{if}\NormalTok{ key
  }\OperatorTok{\textless{}}\NormalTok{ root.key:}
24 \NormalTok{            root }\OperatorTok{=}\NormalTok{ root.left}
25             \VariableTok{self}\NormalTok{.insert(key, value, root)}
26
27
28         \ControlFlowTok{if}\NormalTok{ key
  }\OperatorTok{\textgreater{}}\NormalTok{ root.key
  }\KeywordTok{and}\NormalTok{ root.right }\KeywordTok{is}
  \VariableTok{None}\NormalTok{:}
29 \NormalTok{            root.right }\OperatorTok{=}\NormalTok{ node}
30 \NormalTok{            node.parent }\OperatorTok{=}\NormalTok{ root}
31             \ControlFlowTok{return}
32
33         \ControlFlowTok{if}\NormalTok{ key
  }\OperatorTok{\textgreater{}}\NormalTok{ root.key:}
34 \NormalTok{            root }\OperatorTok{=}\NormalTok{ root.right}
35             \VariableTok{self}\NormalTok{.insert(key, value, root)}
36
```

```
37      \KeywordTok{def}
\BuiltInTok{min}\NormalTok{(}\VariableTok{self}\NormalTok{,
bst}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
38          \ControlFlowTok{if}\NormalTok{ bst }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
39  \NormalTok{            minimum }\OperatorTok{=}
\VariableTok{self}\NormalTok{.root}
40          \ControlFlowTok{else}\NormalTok{:}
41  \NormalTok{            minimum }\OperatorTok{=}\NormalTok{bst.root}
42
43          \ControlFlowTok{while}\NormalTok{ minimum.left }\KeywordTok{is}
\KeywordTok{not} \VariableTok{None}\NormalTok{:}
44  \NormalTok{            minimum }\OperatorTok{=}\NormalTok{ minimum.left}
45
46          \ControlFlowTok{return}\NormalTok{ minimum}
47
48      \KeywordTok{def}
\BuiltInTok{max}\NormalTok{(}\VariableTok{self}\NormalTok{,
bst}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
49          \ControlFlowTok{if}\NormalTok{ bst }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
50  \NormalTok{            maximum }\OperatorTok{=}
\VariableTok{self}\NormalTok{.root}
51          \ControlFlowTok{else}\NormalTok{:}
52  \NormalTok{            maximum }\OperatorTok{=}\NormalTok{ bst.root}
53
54          \ControlFlowTok{while}\NormalTok{ maximum.right }\KeywordTok{is}
\KeywordTok{not} \VariableTok{None}\NormalTok{:}
55  \NormalTok{            maximum }\OperatorTok{=}\NormalTok{ maximum.right}
56
57          \ControlFlowTok{return}\NormalTok{ maximum}
58
59      \KeywordTok{def}\NormalTok{ search(}\VariableTok{self}\NormalTok{, key,
node}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
60          \CommentTok{\# If initial call or we\textquotesingle{}ve hit None in
recursion}
61          \ControlFlowTok{if}\NormalTok{ node }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
62              \ControlFlowTok{if} \VariableTok{self}\NormalTok{.root
}\KeywordTok{is} \VariableTok{None}\NormalTok{:  }\CommentTok{\# Empty tree}
63                  \ControlFlowTok{return} \OperatorTok{{-}}\DecValTok{1}
64  \NormalTok{            node }\OperatorTok{=}
\VariableTok{self}\NormalTok{.root}
65
66          \CommentTok{\# Found the key}
67          \ControlFlowTok{if}\NormalTok{ key }\OperatorTok{==}\NormalTok{
node.key:}
68              \ControlFlowTok{return}\NormalTok{ node}
69
70          \CommentTok{\# Key doesn\textquotesingle{}t exist in this path}
71          \ControlFlowTok{if}\NormalTok{ key
}\OperatorTok{\textless{}}\NormalTok{ node.key:}
```

```
72          \ControlFlowTok{if}\NormalTok{ node.left }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
73              \ControlFlowTok{return} \OperatorTok{{-}}\DecValTok{1}
74          \ControlFlowTok{return}
\VariableTok{self}\NormalTok{.search(key, node.left)}
75      \ControlFlowTok{else}\NormalTok{:  }\CommentTok{\# key
\textgreater{} node.key}
76          \ControlFlowTok{if}\NormalTok{ node.right }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
77              \ControlFlowTok{return} \OperatorTok{{-}}\DecValTok{1}
78          \ControlFlowTok{return}
\VariableTok{self}\NormalTok{.search(key, node.right)}
79
80      \KeywordTok{def}\NormalTok{ delete(}\VariableTok{self}\NormalTok{,
key):}
81          \CommentTok{\# Find the node to delete}
82  \NormalTok{        node }\OperatorTok{=}
\VariableTok{self}\NormalTok{.search(key)}
83
84          \CommentTok{\# If node not found, return}
85          \ControlFlowTok{if}\NormalTok{ node }\OperatorTok{==}
\OperatorTok{{-}}\DecValTok{1}\NormalTok{:}
86              \ControlFlowTok{return}
87
88          \VariableTok{self}\NormalTok{.\_delete\_node(node)}
89
90      \KeywordTok{def}\NormalTok{
\_delete\_node(}\VariableTok{self}\NormalTok{, node):}
91          \CommentTok{\# Case 1: Node has no children (leaf node)}
92          \ControlFlowTok{if}\NormalTok{ node.left }\KeywordTok{is}
\VariableTok{None} \KeywordTok{and}\NormalTok{ node.right }\KeywordTok{is}
\VariableTok{None}\NormalTok{:}
93              \ControlFlowTok{if}\NormalTok{ node }\OperatorTok{==}
\VariableTok{self}\NormalTok{.root:}
94                  \VariableTok{self}\NormalTok{.root }\OperatorTok{=}
\VariableTok{None}
95              \ControlFlowTok{else}\NormalTok{:}
96                  \ControlFlowTok{if}\NormalTok{ node.parent.left
}\OperatorTok{==}\NormalTok{ node:}
97  \NormalTok{                    node.parent.left }\OperatorTok{=}
\VariableTok{None}
98                  \ControlFlowTok{else}\NormalTok{:}
99  \NormalTok{                    node.parent.right }\OperatorTok{=}
\VariableTok{None}
100
101          \CommentTok{\# Case 2: Node has only one child}
102          \ControlFlowTok{elif}\NormalTok{ node.left }\KeywordTok{is}
\VariableTok{None}\NormalTok{:  }\CommentTok{\# Has only right child}
103              \ControlFlowTok{if}\NormalTok{ node }\OperatorTok{==}
\VariableTok{self}\NormalTok{.root:}
104                  \VariableTok{self}\NormalTok{.root
}\OperatorTok{=}\NormalTok{ node.right}
```

```
105    \NormalTok{                    node.right.parent }\OperatorTok{=}
       \VariableTok{None}
106            \ControlFlowTok{else}\NormalTok{:}
107                \ControlFlowTok{if}\NormalTok{ node.parent.left
       }\OperatorTok{==}\NormalTok{ node:}
108    \NormalTok{                    node.parent.left }\OperatorTok{=}\NormalTok{
       node.right}
109                \ControlFlowTok{else}\NormalTok{:}
110    \NormalTok{                    node.parent.right }\OperatorTok{=}\NormalTok{
       node.right}
111    \NormalTok{                    node.right.parent }\OperatorTok{=}\NormalTok{
       node.parent}
112
113        \ControlFlowTok{elif}\NormalTok{ node.right }\KeywordTok{is}
       \VariableTok{None}\NormalTok{:  }\CommentTok{\# Has only left child}
114            \ControlFlowTok{if}\NormalTok{ node }\OperatorTok{==}
       \VariableTok{self}\NormalTok{.root:}
115                \VariableTok{self}\NormalTok{.root
       }\OperatorTok{=}\NormalTok{ node.left}
116    \NormalTok{                    node.left.parent }\OperatorTok{=}
       \VariableTok{None}
117            \ControlFlowTok{else}\NormalTok{:}
118                \ControlFlowTok{if}\NormalTok{ node.parent.left
       }\OperatorTok{==}\NormalTok{ node:}
119    \NormalTok{                    node.parent.left }\OperatorTok{=}\NormalTok{
       node.left}
120                \ControlFlowTok{else}\NormalTok{:}
121    \NormalTok{                    node.parent.right }\OperatorTok{=}\NormalTok{
       node.left}
122    \NormalTok{                    node.left.parent }\OperatorTok{=}\NormalTok{
       node.parent}
123
124        \CommentTok{\# Case 3: Node has two children}
125        \ControlFlowTok{else}\NormalTok{:}
126            \CommentTok{\# Find successor (smallest node in right subtree)}
127    \NormalTok{            successor }\OperatorTok{=} \VariableTok{None}
128    \NormalTok{            current }\OperatorTok{=}\NormalTok{ node.right}
129
130            \ControlFlowTok{while}\NormalTok{ current.left }\KeywordTok{is}
       \KeywordTok{not} \VariableTok{None}\NormalTok{:}
131    \NormalTok{                current }\OperatorTok{=}\NormalTok{ current.left}
132
133    \NormalTok{            successor }\OperatorTok{=}\NormalTok{ current}
134
135            \CommentTok{\# Copy successor\textquotesingle{}s key and value to
       the node}
136    \NormalTok{            node.key }\OperatorTok{=}\NormalTok{ successor.key}
137    \NormalTok{            node.value }\OperatorTok{=}\NormalTok{
       successor.value}
138
139            \CommentTok{\# Delete the successor (which has at most one right
       child)}
```

```
140              \VariableTok{self}\NormalTok{.\_delete\_node(successor)}
141
142      \KeywordTok{def}\NormalTok{ iterate(}\VariableTok{self}\NormalTok{,
    node}\OperatorTok{=}\VariableTok{None}\NormalTok{,
    result}\OperatorTok{=}\VariableTok{None}\NormalTok{):}
143          \CommentTok{\# Initialize result list on first call}
144          \ControlFlowTok{if}\NormalTok{ result }\KeywordTok{is}
    \VariableTok{None}\NormalTok{:}
145 \NormalTok{              result }\OperatorTok{=}\NormalTok{ []}
146
147          \CommentTok{\# Use root if no starting node provided}
148          \ControlFlowTok{if}\NormalTok{ node }\KeywordTok{is}
    \VariableTok{None}\NormalTok{:}
149              \ControlFlowTok{if} \VariableTok{self}\NormalTok{.root
    }\KeywordTok{is} \VariableTok{None}\NormalTok{:  }\CommentTok{\# Empty tree}
150                  \ControlFlowTok{return}\NormalTok{ result}
151 \NormalTok{              node }\OperatorTok{=}
    \VariableTok{self}\NormalTok{.root}
152
153          \CommentTok{\# In{-}order traversal: left {-}\textgreater{} current
    {-}\textgreater{} right}
154          \ControlFlowTok{if}\NormalTok{ node.left }\KeywordTok{is}
    \KeywordTok{not} \VariableTok{None}\NormalTok{:}
155              \VariableTok{self}\NormalTok{.iterate(node.left, result)}
156
157 \NormalTok{          result.append(node)}
158
159          \ControlFlowTok{if}\NormalTok{ node.right }\KeywordTok{is}
    \KeywordTok{not} \VariableTok{None}\NormalTok{:}
160              \VariableTok{self}\NormalTok{.iterate(node.right, result)}
161
162          \ControlFlowTok{return}\NormalTok{ result}
```