ST2334 Cheat Sheet

1 Probability

De Morgan's Law

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Permutation & Combination

$$P_r^n = \frac{n!}{(n-r)!} = n(n-1)(n-2)...(n-(r-1))$$

$$C_r^n = \frac{n!}{r!(n-r)!}$$

Properties

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$A \subset B \rightarrow P(A) \leq P(B)$$

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Inverse Probability Formula

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Independence

$$A \perp B \iff P(A \cap B) = P(A)P(B)$$

$$A \perp B \iff P(B|A) = P(B)$$

The knowledge of A doesn't change the probability of B

Law of Total Probability

$$P(B) = P(A)P(B|A) + P(A')(P(B|A')$$

Bayes' Theorem

Probability of an event based on knowledge of related conditions

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A')P(B|A')} = \frac{P(A)P(B|A)}{P(B)}$$

2 Random Variables

Notations

Random variables: X_1, X_2, Y_1, Y_2, Z_1, Z_2
Observed values: x_1, x_2, y_1, y_2, z_1, z_2

$$\{X = x\} = \{s \in S : X(s) = x\}$$

The set $\{X = 0\}$ is the set of outcomes in the sample space when $X(s) = 0$

Discrete Random Variables

The range of X R_X is finite or countable

Probability Mass Function (PMF)

$$f(x) = \begin{cases} P(X = x), & \text{for } x \in R_X \\ 0, & \text{for } x \notin R_X \end{cases}$$

Properties

1. $f(x_i) \geq 0$ for all $x_i \in R_X$
2. $f(x) = 0$ for all $x \notin R_X$
3. Sum of individual probability functions is 1

Continuous Random Variables

R_X is an interval or collection of intervals

Probability Density Function (PDF)

Properties

1. $f(x_i) \geq 0$ for all $x_i \in R_X$
2. $f(x) = 0$ for all $x \notin R_X$
3. $\int_{R_X} f(x)dx = 1$
4. $P(a \leq X \leq b) = \int_a^b f(x)dx$

For any specific value x_0, $P(x_0) = 0$
Also,

$$P(a < X < b) = P(a < X \leq b) = P(a \leq X < b) = P(a \leq X \leq b)$$

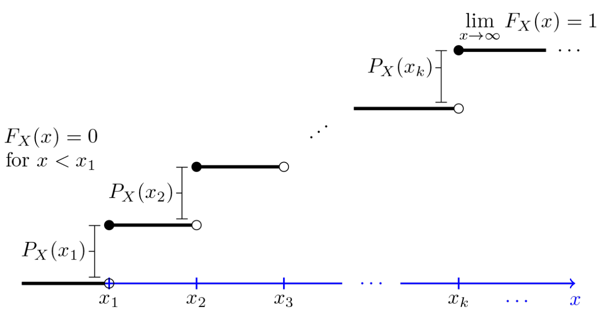
To check if $f(x)$ is a PDF, ensure that 1 & 2 & 3 are true

Culmulative Distribution Function

For both discrete & continuous random variables,

$$F(x) = P(X \leq x)$$

Discrete



Continuous

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$f(x) = \frac{dF(x)}{dx}$$

$-\infty$ can be replaced by the "lower limit" since $f(x) = 0$, for all $x \notin R_X$
Integrate PDF to get CDF, and vice versa

Expectation/Mean

$$\mu_x = E(X)$$

Discrete

$$E(X) = \sum_{x_i \in R_X} x_i f(x_i)$$

Sum of all individual probabilities multipled by the respective values

Continuous

$$E(X) = \int_{-\infty}^{\infty} x f(x)dx$$

Similarly, limits can be replaced

Properties

1. $E(aX + b) = aE(X) + b$
2. $E(X + Y) = E(X) + E(Y)$
3. Discrete: $E[g(X)] = \sum_{x \in R_X} g(x)f(x)$
4. Continuous: $E[g(X)] = \int_{R_X} g(x)f(x)dx$

Variance

$$\sigma_X^2 = V(X) = E(X - \mu_x)^2$$

Computational Formula

$$V(X) = E(X^2) - E(X)^2$$

Standard Deviation

$$\sigma_x = \sqrt{V(X)}$$

In a bell curve, 68% of values are within $\pm 1\sigma$ from the mean