Rosen

Pg 125. (1, 2, 9, 10, 11)

① List the members of these sets.

a) $\{x/x$ is a real number such that $x^2 = 1\}$

$\{1, -1\}$

b) $\{x/x$ is a positive integer less than $12\}$

$\{1,2,3,4,5,6,7,8,9,10,11\}$

c) $\{x/x$ is the square of an integer and $x < 100\}$

$\{1,2,3,4,5,6,7,8,9\}$

d) $\{x/x$ is an integer such that $x^2 = 2\}$

No integer # exists such that $x^2 = 2$

② Set builder notation

① Use set builder notation to give a description of each of these sets.

a) $\{0,3,6,9,12\}$

$\{x/x$ is a positive integer multiple of 3 and $x < 12\}$

b) $\{-3,-2,-1,0,1,2,3\}$

$\{x/x$ is an integer, $x < 3$ and $x > -3\}$

c) $\{m,n,o,p\}$

$\{x/x$ is a letter of the alphabet that comes after l, but before Q$\}$

⑨ Determine whether each of these statements is true or false.

a) $0 \in \emptyset$ = F

b) $\emptyset \in \{0\}$ F

c) $\{0\} \subset \emptyset$ = F

d) $\emptyset \subset \{0\}$ T

e) $\{0\} \in \{0\}$ = T

f) $\{0\} \subset \{0\}$ T

g) $\{\emptyset\} \subseteq \{\emptyset\}$ = F

⑩ Determine whether each of these statements is true or false.

a) $\emptyset \in \{\emptyset\}$ = T

b) $\emptyset \in \{0\}$ = T

c) $\{\emptyset\} \in \{\emptyset\}$ = T

d) $\emptyset \subset \{0\}$ = T

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ = T

f) $\{0\} \subset \{0\}$ = T

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ = F

⑪ Determine whether each of these statements is true or false.

a) $x \in \{x\}$ = T

d) $\{x\} \in \{\{x\}\}$ = T

b) $\{x\} \subseteq \{x\}$ = F

e) $\emptyset \subseteq \{x\}$ = T

c) $\{x\} \in \{x\}$ = T

f) $\emptyset \in \{x\}$ = T

Rossen

Pg 126 (19, 20, 21, 27)

(19) What is the cardinality of these sets?

a) $\{a\} = n(1) = 1$ b) $\{\{a\}\} = n(1) = 1$

c) $\{a, \{a\}\} = n(2) = 2$ d) $\{a, \{a\}, \{a, \{a\}\}\} = n(3) = 3$

(20) What is the cardinality of these sets?

a) $\emptyset = n(0) = 0$ b) $\{\emptyset\} = n(1) = 1$

c) $\{\emptyset, \{\emptyset\}\} = n(2) = 2$ d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = n(3) = 3$

(21) Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\} = P(\{a\}) = \{\emptyset, \{a\}\}$

b) $\{a, b\} = P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{(a, b)\}\}$

c) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = P(\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\})$

$= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset\}, \{\{\emptyset\}\}\},$

$\{\{\emptyset\}, \{\{\emptyset\}\}\}, \{\{\{\emptyset\}\}, \{\{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}\}\}$

(27) Let $A = \{a, b, c, d\}$ and $B = \{x, y\}$. Find:

a) $A \times B = \{(a, x), (a, y), (b, x), (b, y)$

$(c, x), (c, y), (d, x), (d, y)\}$

b) $B \times A = \{(x, a), (y, a), (x, b), (y, b)$

$(x, c), (y, c), (x, d), (y, d)\}$

Rossen

Pág 126 (32, 42, 43)

(32) Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$. Find

a) $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1)$
$(b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1)$
$(c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

b) $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c)$
$(0, y, a), (0, y, b), (0, y, c)$
$(1, x, a), (1, x, b), (1, x, c)$
$(1, y, a), (1, y, b), (1, y, c)\}$

c) $C \times A \times B = \{(0, a, x), (0, a, y), (1, a, x), (1, a, y)$
$(0, b, x), (0, b, y), (1, b, x), (1, b, y)$
$(0, c, x), (0, c, y), (1, c, x), (1, c, y)\}$

d) $B \times B \times B = \{(x, x, x), (x, x, y), (y, x, x), (y, x, y)$
$(x, y, x), (x, y, y), (y, y, x), (y, y, y)\}$

(42) Translate each of these quantifications into English and determine its truth value.

a) $\forall x \in \mathbb{R}\,(x^2 \neq -1)$ = for all Real numbers $x^2 \neq -1$; True

b) $\exists x \in \mathbb{Z}\,(x^2 = 2)$ = an integer exist such that $x^2 = 2$; False

c) $\forall x \in \mathbb{Z}\,(x^2 > 0)$ = for all integers $x^2 > 0$; False $(0^2 = 0)$

d) $\exists x \in \mathbb{R}\,(x^2 = x)$ = a Real number exists such that $x^2 = x$; True $(0^2 = 0)$

(43) Find the truth set of each of these predicates where the domain is the set of integers

a) $P(x): x^2 < 3$ = $\{-1, 0, 1\}$

b) $Q(x): x^2 > x$ = $\{-2, -1, 0, 1, 2, 3\}$

c) $R(x): x < x^2$ = $\{x$ does not exist such that $x < x^2\}$