Computer Science

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Numerical analysis for computer science mayors

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Assignment Problems

1. Find the Lagrange form of interpolating polynomial p2(x) that interpolates the function f(x)=e-$^{x^2}$ at the nodes x0 = 1, x_1= 0 and x_2 =1. Further, find the value of p2(-0.9) (use 6-digit rounding). Compare the value with the true value f(-0.9) (use 6-digit rounding). Find the percentage error in this calculation.

Asig 10: Part 1

$f(x) = e^{-x^2}$ $\qquad$ $x = -0.9$

$x_0 = 1 \qquad x_1 = 0 \qquad x_2 = -1$

$P_2(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2)$

$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \qquad L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \qquad L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

$f(x_0) = f(1) = e^{-1^2} = e^{-1} \approx 0.367879$

$f(x_1) = f(0) = e^{-0^2} = e^0 = 1$

$f(x_2) = f(-1) = e^{-(-1)^2} = e^{-1} \approx 0.367879$

$L_0(x) = \frac{(x-0)(x+1)}{(1-0)(1+1)} = \frac{x(x+1)}{2}$

$L_1(x) = \frac{(x-1)(x+1)}{(0-1)(0+1)} = -(x^2-1)$

$L_2(x) = \frac{(x-1)(x-0)}{(-1-1)(-1-0)} = \frac{x(x-1)}{2}$

$P_2(x) = \frac{x(x+1)}{2} * 0.367879 + -(x^2-1) * 1 + \frac{x(x-1)}{-2} * 0.367879$

$L_0(-0.9) = \frac{-0.9(-0.9+1)}{2} = -0.045$

$L_1(-0.9) = -(0.9^2-1) = 0.19$

$L_2(-0.9) = \frac{-0.9(-1.9)}{-2} = 0.855$

$P_2(-0.9) = (-0.045) * 0.367879 + 0.19 * 1 + 0.855 * 0.367879$

$P_2(-0.9) \approx -0.016555 + 0.19 + 0.314537 \approx 0.487982$

$f(-0.9) = e^{-(-0.9)^2} = e^{-0.81} \approx 0.444858$

$\text{Percentage Error} = \left| \frac{0.487982 - 0.444858}{0.444858} \right| \times 100 \approx 9.70\%$

2. **F**ind **t**he **Ne**wton **f**orm of **inte**rpolating **pol**ynomial **f**or **t**he **d**ata

x	-3	-1	0	3	5
y	-30	-22	-12	330	3458

and **int**erpolate **t**he **fol**lowing **va**lues **f(**-2), **f**(1), **f**(2), **f**(4).

Asig 10: Part 2

$x = [-3, -1, 0, 3, 5]$

$y = [-30, -22, -12, 330, 3458]$

$f(-2), f(1), f(2), f(4)$

$f[-3,-1] = \frac{-22-(-30)}{-1-(-3)} = \frac{8}{2} = 4$

$f[-1,0] = \frac{-12-(-22)}{0-(-1)} = \frac{10}{1} = 10$

$f[0,3] = \frac{330-(-12)}{3-0} = \frac{342}{3} = 114$

$f[3,5] = \frac{3458-330}{5-3} = \frac{3128}{2} = 1564$

$f[-3,-1,0] = \frac{10-4}{0-(-3)} = \frac{6}{3} = 2$

$f[-1,0,3] = \frac{114-10}{3-(-1)} = \frac{104}{4} = 26$

$f[0,3,5] = \frac{1564-114}{5-0} = \frac{1450}{5} = 290$

$f[-3,-1,0,3] = \frac{26-2}{3-(-3)} = \frac{24}{6} = 4$

$f[-1,0,3,5] = \frac{290-26}{5-(-1)} = \frac{264}{6} = 44$

$f[-3,-1,0,3,5] = \frac{44-4}{5-(-3)} = \frac{40}{8} = 5$

$P_4(x) = -30 + 4(x+3) + 2(x+3)(x+1) + 4(x+3)(x+1)x + 5(x+3)(x+1)x(x-3)$

$P_4(-2) = -30 + 4(-2+3) + 2(-2+3)(-2+1) + 4(-2+3)(-2+1)(-2) + 5(-2+3)(-2+1)(-2)(-2-3)$

$P_4(-2) = -30 + 4(1) + 2(1)(-1) + 4(1)(-1)(-2) + 5(1)(-1)(-2)(-5)$

$P_4(-2) = -30 + 4 - 2 + 8 - 50 = -70$

$P_4(1) = -30 + 4(1+3) + 2(1+3)(1+1) + 4(1+3)(1+1)(1) + 5(1+3)(1+1)(1)(1-3)$

$P_4(1) = -30 + 4(4) + 2(4)(2) + 4(4)(2)(1) + 5(5)(3)(2)(-1)$

$P_4(1) = -30 + 16 + 16 + 32 - 80 = -46$

$P_4(2) = -30 + 4(2+3) + 2(2+3)(2+1) + 4(2+3)(2+1)(2) + 5(2+3)(2+1)(2)(2-3)$

$P_4(2) = -30 + 4(5) + 2(5)(3) + 4(5)(3)(2) + 5(5)(3)(2)(-1)$

$P_4(2) = -30 + 20 + 30 + 120 + 150 = -10$

$P_4(4) = -30 + 4(4+3) + 2(4+3)(4+1) + 4(4+3)(4+1)(4) + 5(4+3)(4+1)(4)(4-3)$

$P_4(4) = -30 + 4(7) + 2(7)(5) + 4(7)(5)(4) + 5(7)(5)(4)(1)$

$P_4(4) = -30 + 28 + 70 + 560 + 700 = 1328$

$f(-2) = -70 \quad f(1) = -46 \quad f(2) = -10 \quad f(4) = 1328$

3. Corroborate your previous problems solution using MATLAB. Add to the pdf file the MATLAB output for each of the problems.