Susanna ✓p. 38, 53, 54 12/7/2022

✓ p. 49, 20 e, f, g, 23 g

✓ p. 50, 46, e, f, 48, 50

✓ p. 62, 27, 20

p. 63, 42, 44, 37

p. 38

(N) Use Theorem 2.1.1. to verify the logical equivalences. Supply a reason for each step.

$\cancel{p \wedge (\sim q \vee p) \equiv p}$

(53) $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$

$\equiv \neg(\neg p \wedge q) \wedge \neg(\neg p \neg q) \vee (p \wedge q)$ (by 9 eq)

$\equiv [(\neg(\neg p) \vee \neg q) \wedge (\neg(\neg p) \vee \neg(\neg q))] \vee (p \wedge q)$ (by 9 property)

$\equiv [(p \vee \neg q) \wedge (p \vee q)] \vee (p \wedge q)$ (by 8 property)

$\equiv [p \vee (\neg q \wedge q)] \vee (p \wedge q)$ (by 3 iii property)

$\equiv (p \vee c) \vee (p \wedge q)$ (by 5 iii property)

$\equiv p \vee (p \wedge q)$ (by 4 iii property)

$\boxed{\equiv p}$ (by 10 property)

P. 49 20 e,f,g. 23g

(20) Write negations for each of the following statements. (assume that all variables represent fixed quantities or entities as appropiate

(e) If x is nonnegative, then x is positive or x is 0.

= If x is not nonnegative, then x is not positive and x is not 0.

(f) If Tom is Ann's father, then Jim is her Uncle and Sue is her aunt.

= If Tom is not Ann's father, then Jim is not her uncle or Sue is not her aunt.

(g) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

= If n is not divisible by 6, then n is not divisible by 2 or n is divisible by 3.

(23)(g) Write the converse and inverse for each statement of exercise 20.

(g) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.
p q

(1) Converse: Then n is divisible by 2 and n is divisible by 3, if n is divisible by 6. $p \to q$
p q

(2) Inverse: If n is not divisible by 6, then n is not divisible by 2 and n is not divisible by 3. ~~p→q~~

p.50) 46, e, f; 48, 50

(46) "If compound X is boiling, then its temperature must be at least 150°C". Assuming that this statement is true, which of the following must also be true?

(e) A necessary condition for compound X to boil is that its temperature be at least 150°C. **True**

(f) A sufficient condition for compound X to boil is that its temperature be at least 150°C. **True**

(48) In 48 use the logical equivalences $p \to q \equiv \neg p \vee q$ and $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\leftrightarrow$ and (b) use the logical equivalence $p \vee q \equiv \neg(\neg p \wedge \neg q)$ to rewrite each statement form using only $\wedge$ and $\neg$.

(48) $p \vee \neg q \to r \vee q = \boxed{p \vee \neg q \wedge r \vee q}$

(50) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) = \boxed{(p \wedge (q \wedge r)) \wedge ((p \wedge q) \wedge r)}$

P.62) 27, 20

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(27) Some of the arguments is 27 are valid, whoreas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

(27) If this number is larger than 2, then its square is larger than 4
This number is not larger than 2.
∴ The square of this number is not larger than 4

Modus Tollen rule

$p \to q$
$\neg q$
$\therefore \neg p$

hence argument is invalid and inverse error.

P. 38) 54, 20

(54) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$\equiv (p \wedge (\neg(\neg p) \wedge \neg q)) \vee (p \wedge q)$ (by de Morgan's Law)

$\equiv (p \wedge (p \wedge \neg q)) \vee (p \wedge q)$ (by double negation Law)

$\equiv (p \wedge \neg q) \vee (p \wedge q)$ (by absorption law)

$\equiv p \wedge (\neg q \vee q)$ (by distributive Law)

$\equiv p \wedge T$ (by Tautology)

$\equiv p$ (by identity law)

(20) Use Truth tables to show that the argument forms referred to in (20) are valid.

p	q	$p \vee q$	$\neg p$	q	
		premises		conclusion	
T	T	T	F	T	
T	F	T	F	F	
F	T	T	T	T	← Critical row
F	F	F	T	F	

Argument is Valid

P.63) 42, 44, 37

(42) In 43-44, a set of premises and a conclusions are given, use the valid argument form listed in Table 2.3.1 to deduce the conclusion from the premises giving a reason for each step as in Example 2.3.5. Assume all variables are statement variables

(42)
(a) $p \vee q$
(b) $q \to r$
(c) $p \wedge s \to t$
(d) $\neg r$
(e) $\neg q \to u \wedge s$
(f) $\therefore t$

(1) $p \to q$
(2) $r \vee s$
(3) $\neg s \to \neg t$
(4) $\neg q \vee s$
(5) $\neg s$
(6) $\neg p \wedge r \to u$
(7) $w \vee t$
(8) $s \vee \neg q$ (using commutative law)
(9) $\neg q$ (using rs on 8 & s if p vq is ~~true~~ $\neg p$)
(10) $\neg p$ (using MT (modus Tollens) on 1 & 9]
(11) $s \vee r$ (using commutative law)
(12) r [using DS on 11 and 5]
(13) $(\neg p \wedge r)$ (using conjunction on 10 and r if p is true)
(14) u (using mp (modus pollens) on 6 and 13]
(15) $\neg t$ (using mp on 3 and 5)
(16) $t \vee w$ (using commutative law)
(17) w (using DS on 15 and 16)
(18) $u \wedge w$ (using conjunction on 14 and 17)

44) a) $p \to q$

b) $r \vee s$

c) $\neg s \to \neg t$

d) $\neg q \vee s$

e) $\neg s$

f) $\neg p \wedge r \to u$

g) $w \vee t$

h) $\therefore u \wedge w$ (using commutative law)

9) $\neg q$ (using DS on 8 and 9)

10) $\neg p$ (using modus tollens)

11) $s \vee r$ (using cumulative)

12) r (using DS)

13) $(\neg p \wedge r)$ (using conjunction)

14) u (using modus ponens)

15) $\neg t$ (using modus ponens)

16) $t \vee w$ (using cumulative)

17) w (using DS)

18) $u \wedge w$ (conjuction)

So conclusion is true.

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