Desing & Analysis of Algorithms
CECS 4206
Sec: 20

Examen 2

Cada pregunta es independiente.

Muestre los pasos para resolver los problemas.

Preguntas:

1. $F(n) = F(n-1) + F(n-2)$

   donde:

   $F(0) = 0$

   $F(1) = 1$

2. $t(n) = 6t(n-1) + 4t(n-2)$

   donde:

   $t(0) = 0$

   $t(1) = 4\sqrt{5}$

3. $t(n) = 6t(n-1) + 4t(n-2) + 4(3^n)$

4. $F(n) = \begin{cases} 1 & , n=1 \\ 3F\left(\frac{2n}{3}\right) + 2, & n>1 \end{cases}$

5. $F(n) = 3F\left(\frac{n}{2}\right) + n,$ use el Teorema Maestro

① Alfredo Jiménez Olivares #116518

$$F(n) = F(n-1) + F(n-2)$$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$F(n) = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$F(0) = 0$$

$$F(0) = A\left(\frac{1+\sqrt{5}}{2}\right)^0 + B\left(\frac{1-\sqrt{5}}{2}\right)^0$$

$$F(0) = A+B = A = -B$$

$$F(1) = 1$$

$$F(1) = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$$

$$= \frac{1}{2}(A+B) + \frac{\sqrt{5}}{2}(-B-B)$$

$$= -2B\left(\frac{\sqrt{5}}{2}\right) \quad B = -\frac{1}{\sqrt{5}} \quad A = \frac{1}{\sqrt{5}}$$

$$\therefore \boxed{F(n) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

2.) Alfredo Jiménez Oliveras #116518

$$t(n) = 6t(n-1) + 4t(n-2)$$

$$t(0) = 0$$

$$t(1) = 4\sqrt{5}$$

$$x = \frac{6 \pm \sqrt{36+16}}{2} = \left(\frac{6+2\sqrt{13}}{2}, \frac{6-2\sqrt{13}}{2}\right)$$

$$x = (3+\sqrt{13}, 3-\sqrt{13})$$

$$t(0) = A(3+\sqrt{13})^0 + B(3-\sqrt{13})^0 = A+B = 0 \Rightarrow A = -B$$

$$t(1) = A(3+\sqrt{13})^1 + B(3-\sqrt{13})^1$$

$$= t(1) = (-B)(3+\sqrt{13}) + B(3-\sqrt{13}) = 4\sqrt{5}$$

$$= 2*(-B)*\sqrt{13} = 4\sqrt{5}$$

$$4\sqrt{5} = -2\sqrt{13}\,B \Rightarrow B = -\frac{2\sqrt{5}}{\sqrt{13}}$$

$$A = -B = A = \frac{2\sqrt{5}}{13}$$

$$\therefore \boxed{t(n) = \frac{2\sqrt{5}}{\sqrt{13}}(3+\sqrt{13})^n - \frac{2\sqrt{5}}{\sqrt{13}}(3-\sqrt{13})^n}$$

Coral S. Schmidt #148830

③ $t(n) = 6t(n-1) + 4t(n-2) + 4(3^n)$

$t(0) = 1$
$t(1) = 2$

$t(2) = 6t(1) + 4t(0) + 4(3^n)$
$6(2) + 4(1) + 4(3)$
$12 + 4 + 12$
$\boxed{28}$

$t(3) = 6t(2) + 4t(1) + 4(3^n)$
$6(28) + 4(2) + 4(3)$
$168 + 8 + 12$
$\boxed{188}$

$\therefore$ Patrón: Coeficientes:
$t(n-1) = 6$
$t(n-2) = 4$
Constante:
$4(3^n)$

④ $F(n) = \begin{cases} 1 & , n = 1 \\ 3F\left(\frac{2n}{3}\right) + 2 & , n > 1 \end{cases}$

$F\left(\frac{2n}{3}\right) = 3F\left(\frac{4n}{9}\right) + 2$

$F\left(\frac{2^n \cdot n}{3^n}\right) = 3F(1) + 2 \cdot \left(1 + \frac{2}{3} + \frac{2^2}{3^2} + \ldots + \frac{2^{n-1}}{3^{n-1}}\right)$

$$\sum_{k=0}^{n=1} \left(\frac{2}{3}\right)^k = \frac{1 - \left(\frac{2}{3}\right)^n}{1 - \left(\frac{2}{3}\right)} = \frac{3 - 2^n}{1} = 3 - 2^n$$

$F(n) = 3^{1 - \log_3 n} + 2\left(3 - 2^{1 - \log_3 n}\right)$
$3 - 3\log_3 n + 6 - 2 - 2\log_3 n$
$4 - 5\log_3 n$

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$F(n) 3F\left(\frac{2n}{3}\right)+2$

$4-5\log_3 n = 3\left(4-5\log_3\left(\frac{2n}{3}\right)\right)+2$

$12-15\log_3\left(\frac{2n}{3}\right)+2$

$5\log_3 n = 10 - 15\log_3\left(\frac{2n}{3}\right)$

$\log_3 n = 2 - 3\log_3\left(\frac{2n}{3}\right)$

$\log_3 n = 2-3\left(\log_3 2 + \log_3 n - \log_3 3\right)$

$\log_3 n = 2 - 3\log_3 2 - 3\log_3 n + 3$

$4\log_3 n = 5 - 3\log_3 2 \quad \frac{5-3\log_3 2}{4}$

$n = 3^{\frac{5-3\log_3 2}{4}} = 3^{\frac{5}{4}} \cdot 3^{-\frac{3}{4}\log_3 2}$

$n = \sqrt[4]{81} \cdot 3^{-\log_3 2} = 3 \cdot 3^{-\log_3 2}$

$\boxed{\frac{3}{2}}$

$\boxed{F(n) = 4-5\log_3 n}$

#5 Daniel Vicente #105491

$F(n) = 3F\left(\frac{n}{2}\right) + n$, Use el teorema Maestro

fase# 1

$a = 3$ $b = 2$ $f(n) = n$

Preguntas: respuestas:

¿Son a y b constantes? Si

¿$a \geq 1$? Si

¿$b > 1$? Si

¿es f(n) una función positiva? Si

Fase#2

Caso #1

¿$F(n) = O(n^{\log_b a - \epsilon})$?, $\epsilon > 0$, $f(n) = n$

¿$n^{\log_b a - \epsilon} \geq F(n)$?

¿$n^{\log_2 3 - \epsilon}$ $\Rightarrow$ $n^{\frac{\log 3}{\log 2}} \approx \boxed{n^{1.5849}} \Rightarrow n^{\frac{\log 3}{\log 2}} \geq n$?

¿$\log_2 3 - \epsilon \geq 1$? Si, $\epsilon \neq 0$