Coral S. Schmidt **#148830**

Problem:

Using **t**he **pu**mping **Le**mma, **p**rove **t**hat **t**he **fol**lowing **lan**guages **a**re **n**ot **re**gular:

1. L = { a^k | k is a **p**rime **nu**mber }
2. L = { $a^n b^{2n}$ | n ≥ 0 }
3. L = { w | w ∈ **{**a, **b}***, w = w^R }

1. **L = { ak | k is a prime number }**

For contradiction, assume that L is regular. Let p represent the pumping length determined by the pumping lemma for L. Consider the string s = ap, where p represents a prime number. The pumping lemma divides s into three parts: s = xyz, where |xy| ≤ p and |y| > 0, so that xyiz ∈ L for every i ≥ 0.

Because |xy| ≤ p, both x and y must be wholly composed of 'a's. As a result, after pumping, the number of 'a's will increase, but it will no longer be a prime number, which violates the L requirement. Therefore, L cannot be regular.

2. **L = { anb2n | n ≥ 0 }**

Assume, for the sake of contradiction, that L is regular. Let p be the pumping length given by the pumping lemma for L. Consider the string s = apbp2p. According to the pumping lemma, s can be decomposed into three parts: s = xyz, where |xy| ≤ p and |y| > 0, such that xyiz ∈ L for all i ≥ 0.

Since |xy| ≤ p, both x and y must consist entirely of 'a's. After pumping y, the number of 'a's will increase, while the number of 'b's will remain the same. Therefore, the resulting string will not belong to L, as the number of 'b's will not be twice the number of 'a's. Hence, L cannot be regular.

3. **L = { w | w ∈ {a, b}*, w = wR }**

Let me present a hypothetical scenario. Suppose for a moment that L is a regular language. Now, let us examine this assumption with a critical eye and see if it truly holds up under scrutiny. Let p represent the pumping length determined by the pumping lemma for L. Consider the string s = apbap, with |s| ≥ p. The pumping lemma divides s into three parts: s = xyz, where |xy| ≤ p and |y| > 0, so that xyiz ∈ L for every i ≥ 0.

Because |xy| ≤ p, both x and y must be wholly composed of 'a's. After pumping y, the number of 'a's will rise. However, the resulting string will no longer be a palindrome, which violates the L condition. Therefore, L cannot be regular.

As a result, we have demonstrated that none of these languages apply the pumping lemma consistently.