Homework 4

Miller 34, 36, 45-48

"Si p entonces q"

34) Sara será liberal cuando los cerdos vuelen

↓

Si Sara es liberal, entonces los cerdos vuelan

36) Un paralelogramo es una figura de cuatro lados con lados opuestos paralelos.

↓

Si un paralelogramo es una figura de cuatro lados, entonces tiene lados opuestos paralelos.

Identifique cada enunciado como verdadero o falso.

45) $6 = 9-3$ si y solo si $8+2=10$
$T \quad \leftrightarrow \quad T = T$

46) $3+1 \neq 7$ si y solo si $8 \neq 8$
$T \quad \leftrightarrow \quad F = F$

47) $8+7 \neq 15$ si y solo si $3 \times 5 \neq 8$
$F \quad \leftrightarrow \quad F = T$

48) $6 \times 2 = 18$ si y solo si $9+7 \neq 16$
$F \quad \leftrightarrow \quad T = F$

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Use una tabla de verdad para determinar si cada uno de los argumentos es válidos o inválido

25) $(\sim p \wedge r) \rightarrow (p \vee q)$ $\quad ((\sim p \wedge r) \rightarrow (p \vee q) \wedge \sim r \rightarrow p) \rightarrow (q \rightarrow r)$

$\sim r \rightarrow p$

$\therefore q \rightarrow r$

p	q	r	~p	(~p ∧ r)	p ∨ q	(~p ∧ r) → (p ∨ q)	~r	~r → p	(~p ∧ r) → (p ∨ q) ∧ ~r → p	(q → r)	((~p ∧ r) → (p ∨ q) ∧ ~r → p) → (q → r)
T	T	T	F	F	T	T	F	T	F	T	T
T	T	F	F	F	T	T	T	T	T	F	F
T	F	T	F	F	T	T	F	T	F	T	T
T	F	F	F	F	T	T	T	T	T	T	T
F	T	T	T	T	T	T	F	T	F	T	T
F	T	F	T	F	T	T	T	F	T	T	T
F	F	T	T	T	F	F	F	T	F	T	T
F	F	F	T	F	F	T	T	F	T	T	T

Susanna

Pág 106 (1, 4, 5)

① A menagerie consists of 7 brown dogs, 2 two black dogs, 6 gray cats, 10 black cats, 5 blue birds, 6 yellow birds, and 1 black bird. Determine which of the following statements are True and which are False.

a) There is an animal in the menagerie that is red. (False)
b) Every animal in the menagerie is a bird or a mammal. (True)
c) Every animal in the menagerie is brown, gray or black. (False)
d) There is an animal in the menagerie that is neither a cat nor a dog. (True)
e) No animal in the menagerie is blue (False)
f) There are in the menagerie a dog, a cat, and bird that all have the same color. (True)

④ Let $Q(n)$ be: "$n^2 \leq 30$"

a) Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$ and indicate which are false

$Q(2) = "2^2 \leq 30" = T$
$Q(-2) = (-2)^2 \leq 30 = T$
$Q(7) = (7)^2 \leq 30 = F$
$Q(-7) = (-7)^2 \leq 30 = F$

b) Find the truth set of $Q(n)$ if the domain is $\mathbf{Z}$, the set of all integers

$Q(n) = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

c) If the domain is the set $\mathbf{Z}^+$ of all positive integers, what is the truth set of $Q(n)$?

$Q(n), Z^+ = \{1, 2, 3, 4, 5\}$

⑤ Let $Q(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$" with domain for both x and y being the set $\mathbf{R}$ of real numbers.

a) Explain why $Q(x, x)$ is false if $x = -2$ and $y = 1$.

if $-2 < 1$ then $(-2)^2 < (1)^2$, is false because $(-2)^2$ is 4 and 4 is bigger than $(1)^2$ which is still 1.

b) Give values different from those in part (a) for which $Q(x, y)$ is false

$x = -4$, $y = 3$; $x = -5$, $y = 4$; $x = -6$, $y = 5$

c) Explain why $Q(x, y)$ is true if $x = 3$ and $y = 8$

if $3 < 8$ then $3^2 < 8^2$ its true because $3^2 = 9$ and $8^2 = 64$ which still keeps the statement true

d) Give values different from part (c) for which $Q(x, y)$ is true

$x = 2, y = 3$; $x = 5, y = 6$; $x = 100, y = 200$.

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14) Consider the following

$\exists x \in R$ such that $x^2 = 2$

Which are equivalent ways of expressing this?

a) The square of each real number is 2
False
b) Some real numbers have square 2.
True
c) The number x has square 2, for some real number x
True
d) If x is a real number, then $x^2 = 2$
False
e) Some real number has square 2
True
f) There is at least one real number whose square is 2
True

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19) Consider the following:

∀ integers n, if n^2 is even then n is even

a) All integers have even squares and are even
False

b) Given any integer whose square is even, that integer is itself even. True

c) For all integers, there are some whose square is even
True

d) Any integer with an even square is even
True

e) If the square of an integer is even, then that integer is even. True

f) All even integers have even squares.
False

18) Let D be the set of all students at your school, and let M(s) be "s is a math major", let C(s) be "is a computer science student", and let E(s) be "is an engineering student". Express each following statements using quantified variables, and the predicates, M(s), C(s), and E(s).

a) There is an engineering student who is a math major.

$\exists s, E(s) \wedge M(s)$

b) Every computer science student is an engineering student. $\forall s, E(s)$

c) No computer science students are engineering students. $\exists s, \neg E(s)$

d) Some computer science students are also math major.

$\exists s, M(s)$

e) Some computer science students are engineering students and some are not.

$\exists s, C(s)$

27) Refer to the picture of Tarki's world given in Example 3.1.13. Let Above(x, y) mean that x is above y (but posibly in a different column). Determine the truth or falsity of each of the following statements. Give reasons for answer

a) $\forall u$, Circle(u) → Gray(u).
False, 1 of the circles is black

b) $\forall u$, Gray(u) → Circle(u).
True, all gray figures are circles

c) $\exists y$ such that Square(y) ∧ Above(y, d).
False, no squares are above "d"

d) $\exists z$ such that Triangle(z) ∧ Above(f, z).
True, 2 triangles are above f.

28) Let the domain of x be the set D of objects discussed in math courses, and let Real(x) be "x is real number", Pos(x) be "x is a positive real number", Neg(x) be "x is a negative real number", and Int(x) be "x is an integer."

a) Pos(0) = "0 is a positive real number".

b) $\forall x$, Int x → Real(x) = "x is an integer if x is a real number".

c) $\forall x$, Real(x) ∧ Neg(x) → Pos(-x) = "x is a real number and is a negative real number if x isn't a positive real number".

d) $\exists x$ such that Real(x) ∧ ~Int(x) = "At least one x exists such that x is a real number and not an integer"