



UNIVERSIDAD
POLITÉCNICA
P U E R T O R I C O

Computer Science

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Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. The voltage $E = E(t)$ in an electrical circuit obeys the equation

$$E(t) = L \left(\frac{dI}{dt} \right) + RI(t)$$

where R is resistance and L is inductance. Use $L = 0.05$ and $R = 2$ and values for $I(t)$ in the table following.

t	1	1.1	1.2	1.3	1.4
I(t)	8.227 7	7.242 8	5.990 8	4.526 0	2.912 2

Find $dI(t)/dt$ (1.2) using

- Forward difference formula,
- Backward Difference
- Central Difference

and compute $E(1.2)$ for each case. Compare your answer against $I(t) = 10e^{-t/10}\sin(2t)$

Ans: Part 1

$$\frac{dI}{dt} = \frac{I(1.3) - I(1.2)}{t_{1.3} - t_{1.2}} = \frac{4.526 - 5.990}{1.3 - 1.2} = \frac{-1.464}{0.1} = -14.64$$

$$E(1.2) = 0.05 \cdot (-14.64) + 2 \cdot 5.990 = -0.732 + 11.980 = 11.248$$

$$\frac{dI}{dt} = \frac{I(1.2) - I(1.1)}{t_{1.2} - t_{1.1}} = \frac{5.990 - 7.242}{1.2 - 1.1} = \frac{-1.252}{0.1} = -12.52$$

$$E(1.2) = 0.05 \cdot (-12.52) + 2 \cdot 5.990 = -0.626 + 11.980 = 11.354$$

$$\frac{dI}{dt} = \frac{I(1.3) - I(1.1)}{t_{1.3} - t_{1.1}} = \frac{4.526 - 7.242}{1.3 - 1.1} = \frac{-2.716}{0.2} = -13.58$$

$$E(1.2) = 0.05 \cdot (-13.58) + 2 \cdot 5.990 = -0.679 + 11.980 = 11.301$$

- 11.248
- 11.354
- 11.301

2. Compute the second derivative using forward, backward, central difference and Taylor for the following function, $f(x) = e^x - 2x^2 + 3x + 1$. approximate $f''(5)$ and $f''(10)$ using $h=0.1$ and $h=0.01$ for both cases and compare your answers against the analytical solution.

Ans: Part 2

$$f'(x) = e^x - 4x + 3 \quad f''(5) = e^5 - 4 = 147.4132$$

$$f''(x) = e^x - 4 \quad f''(10) = e^{10} - 4 = 22022.4658$$

$$f''(5) = \frac{f(5+0.2) - 2f(5+0.1) + f(5)}{0.1^2} = \frac{182.211 - 2(147.230) + 147.413}{0.01} = 145.9060$$

$$f(5.02) = e^{5.02} - 2(5.02)^2 + 3(5.02) + 1 \approx 150.057$$

$$f(5.01) = e^{5.01} - 2(5.01)^2 + 3(5.01) + 1 = 147.230$$

$$f''(5) = \frac{150.057 - 2(147.230) + 147.413}{(0.01)^2} = 145.9060$$

$$f(10.2) = e^{10.2} - 2(10.2)^2 + 3(10.2) + 1 = 26955.616$$

$$f(10.1) = e^{10.1} - 2(10.1)^2 + 3(10.1) + 1 = 24401.635$$

$$f''(10) = \frac{26955.616 - 2(24401.635) + 22026.466}{(0.1)^2} = 24359.3020$$

$$f(10.02) = e^{10.02} - 2(10.02)^2 + 3(10.02) + 1 = 22337.041$$

$$f(10.01) = e^{10.01} - 2(10.01)^2 + 3(10.01) + 1 = 22122.480$$

$$f''(10) = \frac{22337.041 - 2(22122.480) + 22026.466}{(0.01)^2} = 22244.0209$$

$$f(4.9) = e^{4.9} - 2(4.9)^2 + 3(4.9) + 1 = 133.674$$

$$f(4.8) = e^{4.8} - 2(4.8)^2 + 3(4.8) + 1 = 120.105$$

$$f''(5) = \frac{178.413 - 2(133.674) + 120.105}{(0.1)^2} = 130.4017$$

$$f(4.99) = e^{4.99} - 2(4.99)^2 + 3(4.99) + 1 = 143.906$$

$$f(4.98) = e^{4.98} - 2(4.98)^2 + 3(4.98) + 1 = 142.610$$

$$f''(5) = \frac{144.423 - 2(143.906) + 142.610}{(0.01)^2} = 142.9376$$

$$f(9.9) = e^{9.9} - 2(9.9)^2 + 3(9.9) + 1 = 19725.892$$

$$f(9.8) = e^{9.8} - 2(9.8)^2 + 3(9.8) + 1 = 17664.970$$

$$f''(10) = \frac{22026.466 - 2(19725.892) + 17664.970}{(0.1)^2} = 19942.9846$$

$$f(9.99) = e^{9.99} - 2(9.99)^2 + 3(9.99) + 1 = 21901.073$$

$$f(9.98) = e^{9.98} - 2(9.98)^2 + 3(9.98) + 1 = 21776.445$$

$$f''(10) = \frac{22026.466 - 2(21901.073) + 21776.445}{(0.01)^2} = 21803.4806$$

$$f(5.1) = 163.694 \quad f(5.0) = 148.423 \quad f(4.9) = 133.674$$

$$f''(5) = \frac{163.694 - 2(148.423) + 133.674}{(0.1)^2}$$

$$\frac{163.694 - 296.846 + 133.674}{0.01}$$

$$\frac{0.522}{0.01} = 144.5369$$

$$f(5.01) = 147.230 \quad f(5.0) = 144.423 \quad f(4.99) = 143.906$$

$$f''(5) = \frac{147.230 - 2(144.423) + 143.906}{(0.01)^2}$$

$$\frac{147.230 - 288.846 + 143.906}{0.0001}$$

$$\frac{2.310}{0.0001} = 144.4144$$

$$f(10.1) = 24401.635 \quad f(10.0) = 22026.466 \quad f(9.9) = 19725.892$$

$$f''(10) = \frac{24401.635 - 2(22026.466) + 19725.892}{(0.1)^2}$$

$$\frac{24401.635 - 44052.932 + 19725.892}{0.01}$$

$$\frac{0.595}{0.01} = 22040.8273$$

$$f(10.01) = 22121.480 \quad f(10.0) = 22026.466 \quad f(9.99) = 21901.073$$

$$f''(10) = \frac{22121.480 - 2(22026.466) + 21901.073}{(0.01)^2}$$

$$\frac{22121.480 - 44052.932 + 21901.073}{0.0001}$$

$$\frac{2.320}{0.0001} = 22022.6493$$

3. A function f has the values shown below:

x	1	1.25	1.5	1.75	2
$f(x)$	10	8	7	6	5

- a. Use Midpoint rule to approximate $\int_1^2 f(x) dx$
- b. Use Simpson's rule to approximate $\int_1^2 f(x) dx$

Assig 6: Part 3

$$\int_1^2 f(x) dx \approx h \sum f\left(\frac{x_i + x_{i+1}}{2}\right)$$

$$\int_1^2 f(x) dx = 0.25 \cdot (8 + 6) = 3.5000$$

$$\int_1^2 f(x) dx \approx \frac{h}{3} (f(x_1) + 4f(x_2) + f(x_3))$$

$$\int_1^2 f(x) dx = \frac{0.25}{3} \cdot (10 + 4 \cdot 7 + 5) = 3.5833$$

4. Approximate the solution of $\int_0^1 e^{-x^2} dx$ (exact value ≈ 0.7468) by using,
- Trapezoidal with $h=0.2$,
 - 3/8 Simpson methods with $h=1/6$

Asig 6: Part 4

$$x = [0, 0.2, 0.4, 0.6, 0.8, 1.0]$$

$$f(0) = e^{-0^2} = 1$$

$$f(0.2) = e^{-(0.2)^2} = e^{-0.04} = 0.9608$$

$$f(0.4) = e^{-(0.4)^2} = e^{-0.16} = 0.8521$$

$$f(0.6) = e^{-(0.6)^2} = e^{-0.36} = 0.6977$$

$$f(0.8) = e^{-(0.8)^2} = e^{-0.64} = 0.5273$$

$$f(1) = e^{-(1)^2} = e^{-1} = 0.3679$$

$$I \approx \frac{0.2}{2} (f(0) + 2 \cdot (f(0.2) + f(0.4) + f(0.6) + f(0.8)) + f(1))$$

$$\frac{0.2}{2} (1 + 2 \cdot (0.9608 + 0.8521 + 0.6977 + 0.5273) + 0.3679)$$

$$\frac{0.2}{2} (1 + 2 \cdot 3.0379 + 0.3679)$$

$$\frac{0.2}{2} (1 + 6.0758 + 0.3679)$$

$$\frac{0.2}{2} \times 7.4437 = 0.1 \cdot 7.4437 = 0.7444$$

$$x = [0, 0.167, 0.333, 1]$$

$$f(0) = e^{-0^2} = 1$$

$$f(0.167) = e^{-(0.167)^2} = 0.9726$$

$$f(0.333) = e^{-(0.333)^2} = 0.8975$$

$$f(1) = e^{-1^2} = 0.3679$$

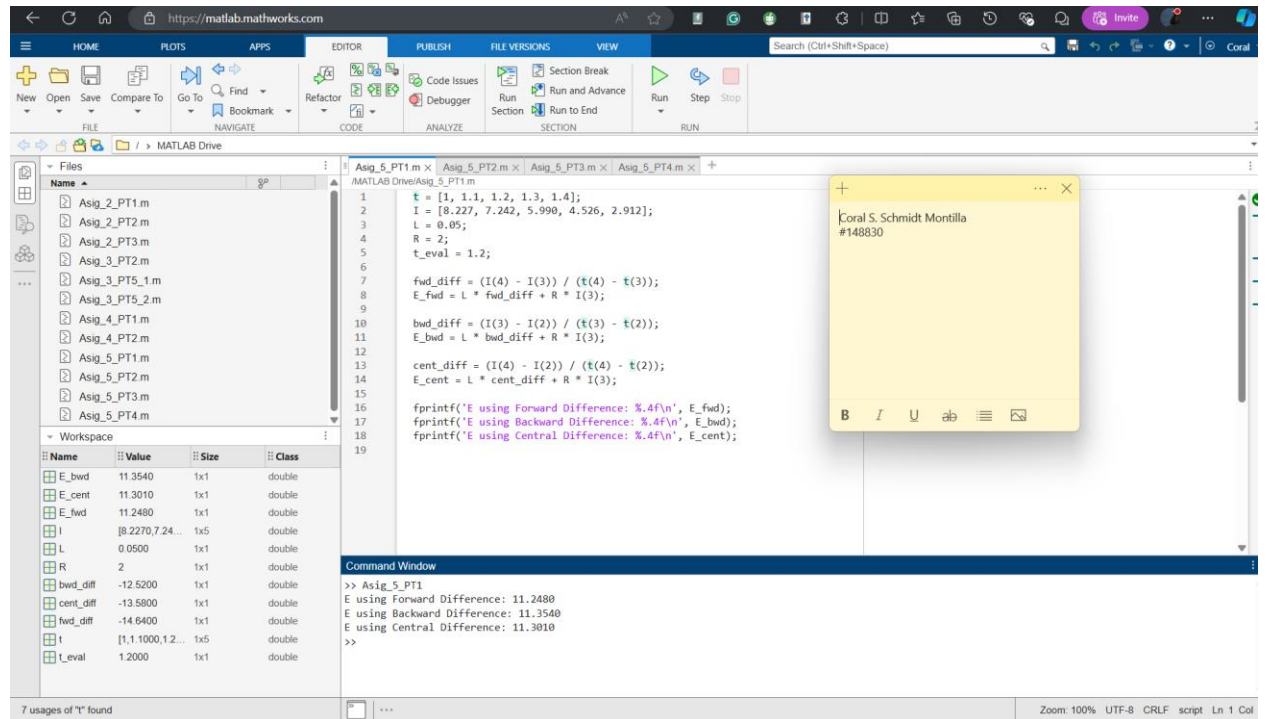
$$I = \frac{2 \cdot \frac{1}{3}}{8} (1 + 3 \cdot 0.9726 + 3 \cdot 0.8975 + 0.3679)$$

$$\frac{0.5}{8} (1 + 2.9178 + 2.6925 + 0.3679)$$

$$0.0625 \cdot 6.9782 = 0.6985$$

5. Validate previous problems results using MATLAB.

Problem 1:



The screenshot shows the MATLAB Editor with a script named 'Asig_5_PT1.m'. The script calculates forward, backward, and central differences for a function $f(t) = [1, 1.1, 1.2, 1.3, 1.4]$ at $t = 1.2$. The workspace shows variables: E_{bwd} (11.3540), E_{cent} (11.3010), E_{fwd} (11.2480), L (0.0500), R (2), bwd_diff (-12.5200), $cent_diff$ (-13.5800), fwd_diff (-14.6400), t ([1, 1.1, 1.2, 1.3, 1.4]), and t_eval (1.2000). The Command Window shows the output of the script.

```
1 t = [1, 1.1, 1.2, 1.3, 1.4];
2 I = [8.227, 7.242, 5.990, 4.526, 2.912];
3 L = 0.05;
4 R = 2;
5 t_eval = 1.2;
6
7 fwd_diff = (I(4) - I(3)) / (t(4) - t(3));
8 E_fwd = L * fwd_diff + R * I(3);
9
10 bwd_diff = (I(3) - I(2)) / (t(3) - t(2));
11 E_bwd = L * bwd_diff + R * I(3);
12
13 cent_diff = (I(4) - I(2)) / (t(4) - t(2));
14 E_cent = L * cent_diff + R * I(3);
15
16 fprintf('E using Forward Difference: %.4f\n', E_fwd);
17 fprintf('E using Backward Difference: %.4f\n', E_bwd);
18 fprintf('E using Central Difference: %.4f\n', E_cent);
19
```

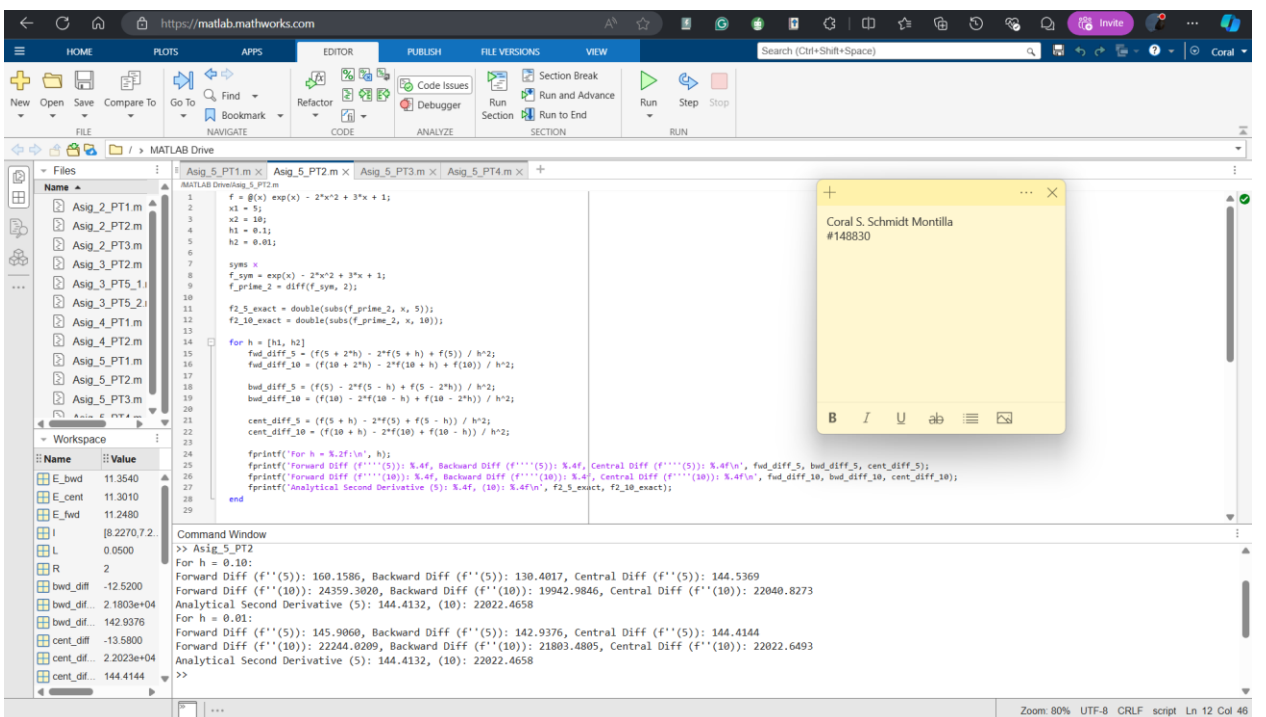
Workspace:

Name	Value	Size	Class
E_{bwd}	11.3540	1x1	double
E_{cent}	11.3010	1x1	double
E_{fwd}	11.2480	1x1	double
I	[8.2270, 7.2420, 5.9900, 4.5260, 2.9120]	1x5	double
L	0.0500	1x1	double
R	2	1x1	double
bwd_diff	-12.5200	1x1	double
$cent_diff$	-13.5800	1x1	double
fwd_diff	-14.6400	1x1	double
t	[1, 1.1, 1.2, 1.3, 1.4]	1x5	double
t_eval	1.2000	1x1	double

Command Window:

```
>> Asig_5_PT1
E using Forward Difference: 11.2480
E using Backward Difference: 11.3540
E using Central Difference: 11.3010
>>
```

Problem 2:



The screenshot shows the MATLAB Editor with a script named 'Asig_5_PT2.m'. The script calculates forward, backward, and central differences for a function $f(x) = \exp(x) - 2x^2 + 3x + 1$ at $x = 10$. The workspace shows variables: E_{bwd} (11.3540), E_{cent} (11.3010), E_{fwd} (11.2480), L (0.0500), R (2), bwd_diff (-12.5200), $cent_diff$ (-13.5800), fwd_diff (-14.6400), t ([1, 1.1, 1.2, 1.3, 1.4]), and t_eval (1.2000). The Command Window shows the output of the script.

```
1 f = @(x) exp(x) - 2*x^2 + 3*x + 1;
2 x1 = 5;
3 x2 = 10;
4 h1 = 0.1;
5 h2 = 0.01;
6
7 sym = x;
8 f_sym = exp(x) - 2*x^2 + 3*x + 1;
9 f_prime_2 = diff(f_sym, 2);
10
11 f2_5_exact = double(subs(f_prime_2, x, 5));
12 f2_10_exact = double(subs(f_prime_2, x, 10));
13
14 for h = [h1, h2]
15     fwd_diff_5 = (f(5 + 2*h) - 2*f(5 + h) + f(5)) / h^2;
16     fwd_diff_10 = (f(10 + 2*h) - 2*f(10 + h) + f(10)) / h^2;
17
18     bwd_diff_5 = (f(5) - 2*f(5 - h) + f(5 - 2*h)) / h^2;
19     bwd_diff_10 = (f(10) - 2*f(10 - h) + f(10 - 2*h)) / h^2;
20
21     cent_diff_5 = (f(5 + h) - 2*f(5) + f(5 - h)) / h^2;
22     cent_diff_10 = (f(10 + h) - 2*f(10) + f(10 - h)) / h^2;
23
24     fprintf('For h = %.2f\n', h);
25     fprintf('Forward Diff (f''(5)): %.4f, Backward Diff (f''(5)): %.4f, Central Diff (f''(5)): %.4f\n', fwd_diff_5, bwd_diff_5, cent_diff_5);
26     fprintf('Forward Diff (f''(10)): %.4f, Backward Diff (f''(10)): %.4f, Central Diff (f''(10)): %.4f\n', fwd_diff_10, bwd_diff_10, cent_diff_10);
27     fprintf('Analytical Second Derivative (5): %.4f, (10): %.4f\n', f2_5_exact, f2_10_exact);
28 end
29
```

Workspace:

Name	Value	Size	Class
E_{bwd}	11.3540	1x1	double
E_{cent}	11.3010	1x1	double
E_{fwd}	11.2480	1x1	double
L	0.0500	1x1	double
R	2	1x1	double
bwd_diff	-12.5200	1x1	double
$cent_diff$	-13.5800	1x1	double
fwd_diff	-14.6400	1x1	double
t	[1, 1.1, 1.2, 1.3, 1.4]	1x5	double
t_eval	1.2000	1x1	double

Command Window:

```
>> Asig_5_PT2
For h = 0.10:
Forward Diff (f''(5)): 160.1586, Backward Diff (f''(5)): 130.4017, Central Diff (f''(5)): 144.5369
Forward Diff (f''(10)): 24359.3020, Backward Diff (f''(10)): 19942.9846, Central Diff (f''(10)): 22040.8273
Analytical Second Derivative (5): 144.4132, (10): 22022.4658
For h = 0.01:
Forward Diff (f''(5)): 145.9060, Backward Diff (f''(5)): 142.9376, Central Diff (f''(5)): 144.4144
Forward Diff (f''(10)): 22244.0209, Backward Diff (f''(10)): 21803.4805, Central Diff (f''(10)): 22022.6493
Analytical Second Derivative (5): 144.4132, (10): 22022.4658
>>
```

Problem 3:

The MATLAB interface displays the code for the Midpoint Rule approximation in the editor. The workspace shows variables for the function, interval, and results. The command window shows the output of the function.

```

1  x = [1, 1.25, 1.5, 1.75, 2];
2  f_x = [10, 8, 7, 6, 5];
3
4  h_mid = (x(5) - x(1)) / (length(x) - 1);
5  mid_approx = h_mid * (f_x(2) + f_x(4));
6
7  h_simp = (x(5) - x(1)) / (length(x) - 1);
8  simp_approx = (h_simp / 3) * (f_x(1) + 4*f_x(3) + f_x(5));
9
10 fprintf('Midpoint Rule Approximation: %.4f\n', mid_approx);
11 fprintf('Simpson's Rule Approximation: %.4f\n', simp_approx);
12

```

Name	Value
E_bwd	11.3540
E_cent	11.3010
E_fwd	11.2480
l	[8 2270.7.2]
L	0.0500
R	2
bwd_diff	-12.5200
bwd_diff...	2.1803e+04
bwd_diff...	142.9376
cent_diff	-13.5800
cent_diff...	2.2023e+04
cent_diff...	144.4144

```

>> Asig_5_PT3
Midpoint Rule Approximation: 3.5000
Simpson's Rule Approximation: 3.5833
>>

```

Problem 4:

The MATLAB interface displays the code for the Trapezoidal and Simpson's 3/8 Rule approximations in the editor. The workspace shows variables for the function, interval, and results. The command window shows the output of the function.

```

1  f_gauss = @(x) exp(-x.^2);
2
3  h_trap = 0.2;
4  x_trap = 0:h_trap:1;
5  f_vals_trap = f_gauss(x_trap);
6  trap_approx = h_trap * (sum(f_vals_trap) - (f_vals_trap(1) + f_vals_trap(end)) / 2);
7
8  h_simp38 = 1/6;
9  x_simp38 = 0:h_simp38:1;
10 f_vals_simp38 = f_gauss(x_simp38);
11 simp38_approx = (3*h_simp38 / 8) * (f_vals_simp38(1) + 3*(sum(f_vals_simp38(2:2:end-1))) + 2*(sum(f_vals_simp38(3:2:end-2))) + f_vals_simp38(end));
12
13 fprintf('Trapezoidal Rule Approximation: %.4f\n', trap_approx);
14 fprintf('Simpson's 3/8 Rule Approximation: %.4f\n', simp38_approx);
15

```

Name	Value
E_bwd	11.3540
E_cent	11.3010
E_fwd	11.2480
l	[8 2270.7.2]
L	0.0500
R	2
bwd_diff	-12.5200
bwd_diff...	2.1803e+04
bwd_diff...	142.9376
cent_diff	-13.5800
cent_diff...	2.2023e+04
cent_diff...	144.4144

```

>> Asig_5_PT4
Trapezoidal Rule Approximation: 0.7444
Simpson's 3/8 Rule Approximation: 0.6995
>>

```