Homework 5

Rosen

36) Show that $A \oplus B = (A-B) \cup (B-A)$

$A \oplus B = (A \cup B) \cap (\bar{A} \cap \bar{B}) = [(A \cup B) \cap U] \cap [U \cap (\bar{B} \cap \bar{A})]$

$[(A \cup B) \cap (\bar{B} \cup \bar{B})] \cap [(A \cup \bar{A}) \cap (\bar{B} \cup \bar{A})]$

$[(A \cap \bar{B}) \cup B] \cap [(A \cap \bar{B}) \cap \bar{A}]$

$(A \cap \bar{B}) \cup (B \cap \bar{A})$

$(A - B) \cup (B - A)$

37) Show that if A is a subset of a universal set U, then

b) $A \oplus \emptyset = A$

$A - B = A \cap \bar{B}$ $\quad \bar{\emptyset} = U$

$(A - \emptyset) \cup (\emptyset - A)$

$(A \cap U) \cup (\emptyset \cap \bar{A})$

$[A \subset U]$

$A \oplus \emptyset = A \cup \emptyset$

d) $A \oplus \bar{A} = U$

$(A - \bar{A}) \cup (\bar{A} - A)$

$(A \cap \bar{\bar{A}}) \cup (\bar{A} \cap \bar{A})$

$(A \cap A) \cup (\bar{A} \cap \bar{A})$

$A \oplus \bar{A} = A \cup \bar{A}$

52) Suppose that the universal set is $U = \{1,2,3,4,5,6,7,8,9,10\}$. Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.

a) $\{3,4,5\} =$ 00 1110 0000

b) $\{1,3,6,10\} =$ 10 1001 0001

c) $\{2,3,4,7,8,9\} =$ 01 1100 1110

57) Show how bitwise operations on bit strings can be used to find these combinations of:

$A = \{a, b, c, d, e\}$
$B = \{b, c, d, g, p, t, v\}$
$C = \{c, e, i, o, u, x, y, z\}$
$D = \{d, e, h, p, n, o, t, u, x, y\}$

a) $A \cup B = \{a, b, c, d, e\} \cup \{b, c, d, g, p, t, v\} =$
$\{a, b, c, d, e, g, p, t, v\}$

b) $A \cap B = \{a, b, c, d, e\} \cap \{b, c, d, g, p, t, v\} =$
$\{b, c, d\}$

c) $(A \cup D) \cap (B \cup C) =$

$A \cup D \rightarrow \{a, b, c, d, e\} \cup \{d, e, h, i, n, o, t, u, x, y\}$
$\{a, b, c, d, e, h, i, n, o, t, u, x, y\}$

$B \cup C \rightarrow \{b, c, d, g, p, t, v\} \cup \{c, e, i, o, u, x, y, z\}$
$\{b, c, d, e, g, i, o, p, t, v, u, x, y, z\}$

$\{b, c, d, e, i, o, t, u, x, y\}$

d) $A \cup B \cup C \cup D$

$\{a, b, c, d, e, g, p, t, v, i, o, u, x, y, z, h, n\}$

9) Prove the complement laws in Table 1 by showing that:

$U = \{a, b, c\}$

$A = \{a, b\}$

$\bar{A} = \{c\}$

a) $A \cup \bar{A} = U$

$\{a, b\} \cup \{c\} = \{a, b, c\}$

b) $A \cap \bar{A} = \emptyset$

$\{a, b\} \cap \{c\} = \emptyset$

16) Let A and B be sets. Show that:

$A = \{1, 2, 3, 4\}$

$B = \{3, 4\}$

c) $A - B \subseteq A$

$\{1, 2, 3, 4\} - \{3, 4\} \subseteq \{1, 2, 3, 4\}$

$\{1, 2\} \subseteq \{1, 2, 3, 4\}$

18) Let A, B and C be sets. Show that

c) $(A - B) - C \subseteq A - B$

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 4\}$

$C = \{5, 6\}$

$(\{1, 2, 3, 4, 5, 6\} - \{3, 4\}) - \{5, 6\} \subseteq \{1, 2, 3, 4, 5, 6\} - \{3, 4\}$

$\{1, 2\} \subseteq \{1, 2, 5, 6\}$

d) $(A - C) \cap (C - B) = \emptyset$

$A = \{1, 2\}$

$B = \{3, 4\}$

$C = \{1, 2, 3, 4\}$

$(\{1, 2\} - \{1, 2, 3, 4\}) \cap (\{1, 2, 3, 4\} - \{3, 4\})$

$\{1, 2\} \cap \{3, 4\} = \emptyset$