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POLITÉCNICA
P U E R T O R I C O

Computer Science

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Numerical analysis for computer science mayors

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1. Suppose that we want to solve the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

and $b = (1, 7, 16, 14)^T$,

- a. Do three iterations (by hand) of the Jacobi iteration for this matrix, using $x(0) = (0, 0, 0, 0)$.

Assg 4: Part 1: Part a

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \quad x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix}$$

$$\begin{aligned} x_1^{(1)} &= \frac{1}{4} (1 - 1 \cdot 0) = 0.2500 \\ x_2^{(1)} &= \frac{1}{5} (7 - 1 \cdot 0 - 1 \cdot 0) = 1.4000 \\ x_3^{(1)} &= \frac{1}{6} (16 - 1 \cdot 0 - 1 \cdot 0) = 2.6667 \\ x_4^{(1)} &= \frac{1}{4} (14 - 1 \cdot 0) = 3.5000 \end{aligned} \quad x^{(1)} = \begin{bmatrix} 0.2500 \\ 1.4000 \\ 2.6667 \\ 3.5000 \end{bmatrix}$$

$$\begin{aligned} x_1^{(2)} &= \frac{1}{4} (1 - 1 \cdot 1.4000) = -0.1000 \\ x_2^{(2)} &= \frac{1}{5} (7 - 1 \cdot 0.2500 - 1 \cdot 2.6667) = 0.8167 \\ x_3^{(2)} &= \frac{1}{6} (16 - 1 \cdot 1.4000 - 1 \cdot 3.5000) = 1.8500 \\ x_4^{(2)} &= \frac{1}{4} (14 - 1 \cdot 2.6667) = 2.8333 \end{aligned} \quad x^{(2)} = \begin{bmatrix} -0.1000 \\ 0.8167 \\ 1.8500 \\ 2.8333 \end{bmatrix}$$

$$\begin{aligned} x_1^{(3)} &= \frac{1}{4} (1 - 1 \cdot 0.8167) = 0.0458 \\ x_2^{(3)} &= \frac{1}{5} (7 - 1 \cdot -0.1000 - 1 \cdot 1.8500) = 1.0300 \\ x_3^{(3)} &= \frac{1}{6} (16 - 1 \cdot 0.8167 - 1 \cdot 2.8333) = 2.0583 \\ x_4^{(3)} &= \frac{1}{4} (14 - 1 \cdot 1.8500) = 3.0375 \end{aligned} \quad x^{(3)} = \begin{bmatrix} 0.0458 \\ 1.0300 \\ 2.0583 \\ 3.0375 \end{bmatrix}$$

- b. Do three iterations (by hand) of the Gauss-Seidel iteration for this problem, using the same initial guess.

Assg 4: Part 1: Part b

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \quad x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix}$$

$$\begin{aligned} x_1^{(1)} &= \frac{1}{4} (1 - 1 \cdot 0) = 0.2500 \\ x_2^{(1)} &= \frac{1}{5} (7 - 1 \cdot 0.2500 - 1 \cdot 0) = 1.3000 \\ x_3^{(1)} &= \frac{1}{6} (16 - 1 \cdot 1.3000 - 1 \cdot 0) = 2.4167 \\ x_4^{(1)} &= \frac{1}{4} (14 - 1 \cdot 2.4167) = 2.8896 \end{aligned} \quad x^{(1)} = \begin{bmatrix} 0.2500 \\ 1.3000 \\ 2.4167 \\ 2.8896 \end{bmatrix}$$

$$\begin{aligned} x_1^{(2)} &= \frac{1}{4} (1 - 1 \cdot 1.3000) = -0.0875 \\ x_2^{(2)} &= \frac{1}{5} (7 - 1 \cdot -0.0875 - 1 \cdot 2.4167) = 0.9292 \\ x_3^{(2)} &= \frac{1}{6} (16 - 1 \cdot 0.9292 - 1 \cdot 2.8896) = 2.0302 \\ x_4^{(2)} &= \frac{1}{4} (14 - 1 \cdot 2.0302) = 2.9924 \end{aligned} \quad x^{(2)} = \begin{bmatrix} -0.0875 \\ 0.9292 \\ 2.0302 \\ 2.9924 \end{bmatrix}$$

$$\begin{aligned} x_1^{(3)} &= \frac{1}{4} (1 - 1 \cdot 0.9292) = 0.0177 \\ x_2^{(3)} &= \frac{1}{5} (7 - 1 \cdot 0.0177 - 1 \cdot 2.0302) = 0.9904 \\ x_3^{(3)} &= \frac{1}{6} (16 - 1 \cdot 0.9904 - 1 \cdot 2.9924) = 2.0029 \\ x_4^{(3)} &= \frac{1}{4} (14 - 1 \cdot 2.0029) = 2.9993 \end{aligned} \quad x^{(3)} = \begin{bmatrix} 0.0177 \\ 0.9904 \\ 2.0029 \\ 2.9993 \end{bmatrix}$$

c. Do three iterations of SOR for Problem 1, using $\omega = 1.05$.

Assg 4: Part 1: Part c

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix} \quad \omega = 1.05$$

$$x_1^{(0)} = (1 - 1.05) \cdot 0 + \frac{1.05}{4} \cdot (1 - 1 \cdot 0) = \frac{1.05}{4} \cdot 1 = 0.2625$$

$$x_2^{(0)} = (1 - 1.05) \cdot 0 + \frac{1.05}{5} \cdot (7 - 1 \cdot 0.2625 - 1 \cdot 0) = \frac{1.05}{5} \cdot 6.7375 = 1.4149$$

$$x_3^{(0)} = (1 - 1.05) \cdot 0 + \frac{1.05}{6} \cdot (16 - 1 \cdot 1.4149 - 1 \cdot 0) = \frac{1.05}{6} \cdot 14.5851 = 2.5524$$

$$x_4^{(0)} = (1 - 1.05) \cdot 0 + \frac{1.05}{4} \cdot (14 - 1 \cdot 2.5524) = \frac{1.05}{4} \cdot 11.4476 = 3.0050$$

$$x^{(0)} = \begin{bmatrix} 0.2625 \\ 1.4149 \\ 2.5524 \\ 3.0050 \end{bmatrix}$$

$$x_1^{(1)} = (1 - 1.05) \cdot 0.2625 + \frac{1.05}{4} \cdot (1 - 1 \cdot 1.4149) = -0.1220$$

$$x_2^{(1)} = (1 - 1.05) \cdot 1.4149 + \frac{1.05}{5} \cdot (7 - 1 \cdot -0.1220 - 1 \cdot 2.5524) = 0.8889$$

$$x_3^{(1)} = (1 - 1.05) \cdot 2.5524 + \frac{1.05}{6} \cdot (16 - 1 \cdot 0.8889 - 1 \cdot 3.0050) = 1.9910$$

$$x_4^{(1)} = (1 - 1.05) \cdot 3.0050 + \frac{1.05}{4} \cdot (14 - 1 \cdot 1.9910) = 3.0021$$

$$x^{(1)} = \begin{bmatrix} -0.1220 \\ 0.8889 \\ 1.9910 \\ 3.0021 \end{bmatrix}$$

$$x_1^{(2)} = (1 - 1.05) \cdot -0.1220 + \frac{1.05}{4} \cdot (1 - 1 \cdot 0.8889) = 0.0353$$

$$x_2^{(2)} = (1 - 1.05) \cdot 0.8889 + \frac{1.05}{5} \cdot (7 - 1 \cdot 0.0353 - 1 \cdot 1.9910) = 1.0000$$

$$x_3^{(2)} = (1 - 1.05) \cdot 1.9910 + \frac{1.05}{6} \cdot (16 - 1 \cdot 1.0000 - 1 \cdot 3.0021) = 2.0001$$

$$x_4^{(2)} = (1 - 1.05) \cdot 3.0021 + \frac{1.05}{4} \cdot (14 - 1 \cdot 2.0001) = 2.9999$$

$$x^{(2)} = \begin{bmatrix} 0.0353 \\ 1.0000 \\ 2.0001 \\ 2.9999 \end{bmatrix}$$

2. Consider the following nonlinear system.

$$2x_1 - x_2 + \frac{1}{9}e^{-x_1} = -1$$

$$-x_1 + 2x_2 + \frac{1}{9}e^{-x_2} = 1$$

Take $x^{(0)} = (1, 1)^T$ and do two iterations of the Newtons method.

Assg 4: Part 2

$$2x_1 - x_2 + \frac{1}{9}e^{-x_1} = -1 \quad x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-x_1 + 2x_2 + \frac{1}{9}e^{-x_2} = 1$$

$$f(x^{(0)}) = \begin{bmatrix} 2(1) - 1 + \frac{1}{9}e^{-1} - 1 \\ -1 + 2(1) + \frac{1}{9}e^{-1} - 1 \end{bmatrix} = \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix}$$

$$J(x^{(0)}) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix} = \begin{bmatrix} 0.5972 \\ 1.2926 \end{bmatrix}$$

$$f(x^{(1)}) = \begin{bmatrix} 2(-0.4028) - 0.2926 + \frac{1}{9}e^{-0.5972} - 1 \\ -(-0.4028) + 2(1.2926) + \frac{1}{9}e^{-1.2926} - 1 \end{bmatrix} = \begin{bmatrix} -0.4782 \\ 0.2224 \end{bmatrix}$$

$$J(x^{(1)}) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} 0.5972 \\ 1.2926 \end{bmatrix} + \begin{bmatrix} -0.4782 \\ 0.2224 \end{bmatrix} = \begin{bmatrix} 0.1190 \\ 1.5150 \end{bmatrix}$$

3. Use MATLAB to validate the results from previous problem.

The MATLAB interface shows the execution of the Jacobi and Gauss-Seidel iterative methods. The workspace displays the following variables:

Name	Value	Size	Class
A	4x4 double	4x4	double
D	4x4 double	4x4	double
L	4x4 double	4x4	double
U	4x4 double	4x4	double
b	[17;16;14]	4x1	double
omega	1.0500	1x1	double
x0	[0;0;0]	4x1	double
x_gs	[0.0177;0.9]	4x1	double
x_jacobi	[0.0458;1.0]	4x1	double
x_new	3.0000	1x1	double
x_sor	[0.0353;1.2]	4x1	double

The Command Window shows the following output:

```
>> Asig_4_PT1
Jacobi iteration 1: x = [0.2500 1.4000 2.6667 3.5000]
Jacobi iteration 2: x = [-0.1000 0.8167 1.9500 2.0333]
Jacobi iteration 3: x = [0.0458 1.0500 2.0583 3.0375]
Gauss-Seidel iteration 1: x = [0.2500 1.3500 2.4417 2.8896]
Gauss-Seidel iteration 2: x = [-0.0075 0.9292 2.0302 2.9924]
Gauss-Seidel iteration 3: x = [0.0177 0.9904 2.0029 2.9993]
SOR iteration 1: x = [0.2625 1.4149 2.5524 3.0050]
SOR iteration 2: x = [-0.1220 0.8889 1.9910 2.0021]
SOR iteration 3: x = [0.0353 1.0000 2.0001 2.9999]
```

The MATLAB interface shows the execution of the Newton-Raphson method. The workspace displays the following variables:

Name	Value	Size	Class
A	4x4 double	4x4	double
D	4x4 double	4x4	double
L	4x4 double	4x4	double
U	4x4 double	4x4	double
b	[17;16;14]	4x1	double
omega	1.0500	1x1	double
x0	[0;0;0]	4x1	double
x_newton	[-0.4028 0.2926]	2x1	double
x	[-0.4782 0.2224]	2x1	double

The Command Window shows the following output:

```
>> Asig_4_PT2
Newton iteration 1: x = [-0.4028 0.2926]
Newton iteration 2: x = [-0.4782 0.2224]
```