Predicates and Quantified Statements

Universal	$\forall x \in D, P(x)$	$\exists x \in D, \sim P(x)$
Existential	$\exists (x,y) \in D, x \neq y \mid P(x,y)$	$\forall (x,y) \in D, x \neq y \mid \sim P(x,y)$
Universal Conditional	$\forall x, P(x) \rightarrow Q(x)$	$\exists x \in D \mid P(x) \wedge \sim Q(x)$

~existential = universal
~universal = existential

More Formal Statements

Formal Contrapositive	$\forall x \in D, \sim Q(x) \rightarrow \sim P(x)$
Formal Converse	$Q(x) \rightarrow P(x)\ \forall x \in D$
Formal Inverse	$\forall x \in D, \sim P(x) \rightarrow \sim Q(x)$

MQ Invalid Arguments

Quantified Converse	$\forall x, P(x) \rightarrow Q(x)$
	$Q(j)$ for a particular j
	$\therefore P(j)$
Quantified Inverse Error	$\forall x, P(x) \rightarrow Q(x)$
	$\sim P(j)$ for a particular j
	$\therefore \sim Q(j)$

Multiple Quantifiers

Existential MQ	$\exists x \in D \mid \forall y \in E, P(x, y)$	
Neg. MQ	$\forall x \in D, \exists y \in E \mid P(x, y)$ [original]	$\exists x \in D, \forall y \in E \mid \sim P(x, y)$ [negation]
Universal Modes Pones		$\forall x \in Z, P(x) \rightarrow Q(x)$
		$P(k)$, for a particular $k \in Z$
		$\therefore \sim Q(k)$
Universal Modus Tones		$\forall x \in D, P(x) \rightarrow Q(x)$
		$\sim Q(j), j \in D$
		$\therefore \sim P(j)$

List of Equivalences

Conditional Statemnts	p→q≡~(p∧~q)	p→q≡~p∨q
contrapositive	p→q≡~q→~p	
Converse	p→q (cond)	q→p (converse)
inverse	p→q (cond)	~p→~p (inverse)

vacuously true = true by absence
converse and inverse are the **SAME**

Useful Symbols

∀	for all (universal operator)
∃	exists (existential operator)
∈	in the set
∧	and
∨	or
~	not
≡	equivalent
⊂	subset
⊃	superset
{}, ∅	empty set
↔	biconditional (both are true)

Statements

Universal	For all, for each
Existential	At least, there exists
Conditional	If → then
Universal Conditional	For all & if-then

Statements (cont)

Universal Existential	For all & there exists
Existential Universal	There exists & for all

Functions

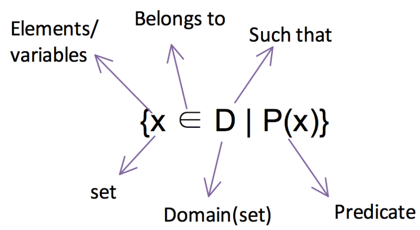
Requirements:
- Arrow coming out of **every element** in domain
- Every element can only have **one** element of *domain* connected to one element of *codomain*

unsatisfied requirement = relation
y can be used repeatedly but x values only have one arrow coming out

Predicates and Quantified Statements

Statement type	original	negated
Universal	∀x∈D, P(x)	∃x∈D, ~P(x)
Existential		
Universal Conditional		

Set-Builder Notation



Set-Roster Notation

A = {1, 2, 3 ... 100}

use ellipses for larger sets

Subsets

B⊆A	B=subset, A=superset

Proper Subsets: elements that belong to superset but NOT subset

Relations

Relations=	subsets of cartesian product
R ⊆ A x B	Relation ⊆ Domain x Codomain
Domain	**SET** that includes every element from source

don't always have to include ordered pairs

DeMorgan's Law

• Tells us how to handle conjunction and disjunction negations
~(p∧q) ≡ ~p∨~q
~(p∨q) ≡ ~p∧~q
"The connector is loose(l) or the machine is unplugged(u)"
l ∨ u -- negation --> ~(l ∨ u) ≡ ~l ∧~u
"The connector is **not** loose *and* the machine is **not** unplugged"
~p∨q is the opposite of p∧~q

When using DeMorgan's law, no need for truth table

Tautologies and Contradictions

Tautologies	Always true statements	t
Contradictions	Always false statements	c
p∧~p≡**c**	T∧F≡**c**	F∧T≡
p∨t≡t	p∧**c**≡**c**	

Absorption law: variable absorbing operator
⇒use truth table to prove law
⇒other variables don't play a role in statement validity
p∨(p∧q)≡ p; p∧(p∨q)≡ p

p→q truth table

Truth Table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Argument Truth Table

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that ar argument of this form have true premises and conclusion. Hence this of argument is invalid.

Critical row = row where both premises are true
premises and conclusion = TRUE is a **valid** argument

Arguments

p→q	major premise
p	minor premise
∴q	therefore, conclusion

premises aka assumptions or hypotheses verified using truth table

Argument Forms (VALID)	
Modus Pones	p→q
	p
	∴q
Modus Tollens	p→q
	~q
	∴~p
Gneralization	p
	∴p∨q
Specialization	p∧q
	∴q
Elimination	p∨q
	~q
	∴p
Transitivity	p→q
	q→r
	∴p→r
Proof by div. into cases	p∨q
	p→r
	q→r
	∴r

Fallacy (INVALID ARGUMENTS)	
Converse Error	p→q
	q
⇒	∴p
Inverse Error	q→p
	~p
	∴~q

Common Sets

N	All Natural numbers = {1, 2, 3, 4, ...}
Z	All Integers = {..., −2, −1, 0, 1, 2, ...}
Z^+	All Positive integers = {0, 1, 2, ...}

Common Sets (copy)

N	All Natural numbers = {1, 2, 3, 4, ...}
Z	All Integers = {..., −2, −1, 0, 1, 2, ...}
Z^+	All Positive integers = {0, 1, 2, ...}

Examples

$4 \in A$	4 is an element of A
$2 \notin A$	2 is not an element of A

Examples (copy)

$4 \in A$	4 is an element of A
$2 \notin A$	2 is not an element of A

Examples (copy)

$4 \in A$	4 is an element of A
$2 \notin A$	2 is not an element of A

Examples (copy) (copy)

$4 \in A$	4 is an element of A
$2 \notin A$	2 is not an element of A

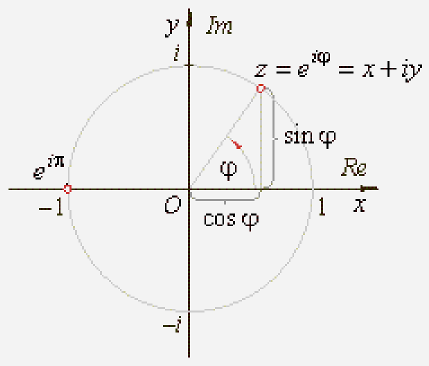
Examples (copy) (copy) (copy)

$4 \in A$	4 is an element of A
$2 \notin A$	2 is not an element of A

DIP Exam 2 Cheat Sheet

by Sawyer McLane (samclane) via cheatography.com/32204/cs/9879/

Complex Unit Circle



Discrete Fourier Transform

$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i k n/N}, \quad k \in \mathbb{Z}$$

Sinc Definition

$$\operatorname{sinc}(x) \equiv \begin{cases} 1 & \text{for } x = 0 \\ \dfrac{\sin x}{x} & \text{otherwise,} \end{cases}$$

2D DFT Definition

$$F[k,l] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m,n] e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)}$$

2D Continuous Fourier Transform

$$F(u,v) = \int\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

Wrap Around Error

Solved by zero padding

If f(x) and h(x) are A and B samples respectively, pad f(x) and h(x) with zeros so both have length P>=A+B-1

If not zero, creates discontinuity called "frequency leakage", equivalent to convolving with sinc() function

Reduced by multiplying with function that tapers smoothly to zero (windowing or apodizing)

Butterworth Lowpass Filter

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)}{D_0}]^{2n}}$$

D0 is cutoff freq and D(u,v) is distribution of (u,v) from centered origin. n is order

DFT Table

2D Convolution

$$x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} x[i,j] \cdot \delta[m-i, n-j]$$

Spatial Shift Theorem

$$\mathcal{F}\{f(t-t_0)\}(s) = e^{-j2\pi s t_0} F(s)$$

Spatial transform only affects FT phase

Conjugate Symetry

F*(u,v) = F(-u, -v) (Conjugate Symmetry)

F*(-u,-v) = -F(u,v) (Conjugate Asymmetry)

Fourier Spectrum and Phase Angle

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$
$$|F(u,v)| = [R^2(u,v) + I^2(u,v)]^{1/2}$$
$$\phi(u,v) = tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

Steps for Filtering

1 + 2. Given f(x,y) is MxN, zero pad to 2Mx2N (PxQ)

3. Multiply by $(-1)^{x+y}$ to center

4. Take DFT of f(x,y) to get F(u,v)

5. Generate symmetric filter H(u,v) of size PxQ

6. Get processed image gp(x,y)={real[$\mathcal{F}^{-1}$-{G(u,v)}} * $(-1)^{x+y}$

Laplacian in Freq. Domain

$$H(u,v) = -4\pi^2(u^2+v^2)$$
$$H(u,v) = -4\pi^2\left[\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2\right]$$
$$H(u,v) = -4\pi^2 D^2 * (u,v)$$
$$\rightarrow \nabla^2 f(x,y) = \mathfrak{F}^{-1}\{H(u,v)F(u,v)\}$$

$$\text{Enhancement: } g(x,y) = f(x,y) + c\nabla^2 f(x,y), \; c = -1$$
$$g(x,y) = \mathfrak{F}^{-1}\{F(u,v)H(u,v) - F(u,v)\}$$
$$g(x,y) = \mathfrak{F}^{-1}\{[1 - H(u,v)]F(u,v)\}$$
$$g(x,y) = \mathfrak{F}^{-1}\{[1 + 4\pi^2 D^2(u,v)]F(u,v)\}$$

Impulse Train Definition

$$\psi_P(t) \triangleq \sum_{m=-\infty}^{\infty} \delta(t - mP)$$

Convolution Definition

$$(f * g)[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} f[m]\, g[n-m] = \sum_{m=-\infty}^{\infty} f[n-m]\, g[m].$$

2D Sampling

$$\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - nX)\delta(y - mY)$$

Frequency Shift Theorem

$$\mathcal{L}\left[e^{-at} f(t)\right] = F(s+a)$$

DC Component

$$F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) = MN\overline{f}(x,y)$$

Gaussian Filter

$$\text{Low-Pass: } H(u) = Ae^{\frac{-u^2}{2\sigma^2}}$$
$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2 x^2}$$

$$\text{High-Pass (wide-narrow band): } H(u) = Ae^{\frac{-u^2}{2\sigma_1^2}} - Be^{\frac{-u^2}{2\sigma_2^2}}$$
$$h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2}$$

Unsharp, Highboost, High-Emphasis

$$g(x,y) = \mathfrak{F}^{-1}\{[1 + kH_{HP}(u,v)]F(u,v)\}$$

gmask(x,y) = f(x,y) - flp(x,y)

g(x,y) = f(x,y) + k*gmask(x,y)

k=1, unsharp

k>1, highboost

Table C.2 Fourier Transform Pairs

	$x(t)$		$X(f)$
1.	$\Pi(t/\tau)$		$\tau\,\mathrm{sinc}(\tau f)$
2.	$\Lambda(t/\tau)$		$\tau\,\mathrm{sinc}^2(\tau f)$
3.	$\mathrm{sinc}(at)$	$a > 0$	$\frac{1}{a}\Pi(f/a)$
4.	$e^{-at}u(t)$	$a > 0$	$\frac{1}{a + j2\pi f}$
5.	$te^{-at}u(t)$	$a > 0$	$\frac{1}{(a + j2\pi f)^2}$
6.	$e^{-a\lvert t\rvert}$	$a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
7.	$e^{-a^2t^2}$		$\frac{\sqrt{\pi}}{a}e^{-(\pi f/a)^2}$
8.	1		$\delta(f)$
9.	$\delta(t)$		1
10.	$\mathrm{sgn}\,(t)$		$1/j\pi f$
11.	$u(t)$		$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
12.	$\cos(2\pi f_0 t + \theta)$		$\frac{1}{2}e^{j\theta}\delta(f - f_0) + \frac{1}{2}e^{-j\theta}\delta(f + f_0)$
13.	$\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$		$f_s\sum_{m=-\infty}^{\infty}\delta(f - mf_s) \quad f_s = \frac{1}{T_s}$

Table C.1 Fourier Transform Theorems

	Name	Transform Pair
1.	Linearity	$ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$
2.	Scale change	$x(at) \leftrightarrow \frac{1}{\lvert a\rvert}X(f/a)$
3.	Time reversal	$x(-t) \leftrightarrow X(-f)$
4.	Complex conjugation	$x^*(t) \leftrightarrow X^*(-f)$
5.	Duality	$X(t) \leftrightarrow x(-f)$
6.	Time shift	$x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
7.	Frequency translation	$x(t)e^{j2\pi f_0 t} \leftrightarrow X(f - f_0)$
8.	Modulation	$x(t)\cos 2\pi f_0 t \leftrightarrow \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
9.	Time differentiation	$\frac{d^n x(t)}{dt^n} \leftrightarrow (j2\pi f)^n X(f)$
10.	Time integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow (j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
11.	Convolution	$x(t) * y(t) \leftrightarrow X(f)Y(f)$
12.	Multiplication	$x(t)y(t) \leftrightarrow X(f) * Y(f)$

Fourier Series Definition

$$F_n(x) = a_0 + \sum_{k=1}^{k=n}\Big(a_k\cos(kx) + b_k\sin(kx)\Big).$$

Convolution Theorem

$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$$
$$\mathcal{F}\{f \cdot g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\}$$
$$f * g = \mathcal{F}^{-1}\big\{\mathcal{F}\{f\} \cdot \mathcal{F}\{g\}\big\}$$
$$f \cdot g = \mathcal{F}^{-1}\big\{\mathcal{F}\{f\} * \mathcal{F}\{g\}\big\}$$

Space convolution = frequency multiplication

Center DC

To shift **F(0,0)** (DC Component) to center, multiply by $\mathbf{(-1)^{x+y}}$

Power Spectrum

$$P(u,v) = |F(u,v)|^2$$

Total power of image is just sum of P(u,v) over P-1,Q-1

a = 100[doublesum P(u,v)/Pt]

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EECS 203 Exam 2 Cheat Sheet

by Kalbi via cheatography.com/19660/cs/2790/

Permutations, no repetition

If n and r are integers with $0 \le r \le n$, then $P(n, r) = \frac{n!}{(n-r)!}$.

permutation formula, ORDER MATTERS (i.e. ways to sort 5 of 10 students in a line)

Permutations, repetition

The number of r-permutations of a set of n objects with repetition allowed is n^r.

very easy, just use product rule as shown

Combinations, no repetition

The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \le r \le n$, equals

$$C(n, r) = \frac{n!}{r!\,(n-r)!}.$$

combination formula, ORDER does NOT matter (i.e committee of 3 out of 5 students)

Combinations, repetition

There are $C(n + r - 1, r) = C(n + r - 1, n - 1)$ r-combinations from a set with n elements when repetition of elements is allowed.

Bars and stars! Order does not matter, ways to select bills/fruit and place in a container

C/P Quick table

TABLE 1 Combinations and Permutations With and Without Repetition.

Type	*Repetition Allowed?*	*Formula*
r-permutations	No	$\frac{n!}{(n-r)!}$
r-combinations	No	$\frac{n!}{r!\,(n-r)!}$
r-permutations	Yes	n^r
r-combinations	Yes	$\frac{(n+r-1)!}{r!\,(n-1)!}$

quick reference

Binomial Theorem

THE BINOMIAL THEOREM Let x and y be variables, and let n be a nonnegative integer. Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n.$$

binomial theorem... coefficient is a Combination.

Pascal's identity

PASCAL'S IDENTITY Let n and k be positive integers with $n \ge k$. Then

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

binomial coefficients, a recursive definition

Finite probability

If S is a finite nonempty sample space of equally likely outcomes, and E is an event, that is, a subset of S, then the *probability* of E is $p(E) = \frac{|E|}{|S|}$.

event over sample space. event is a subset of sample space

Compliment of probability event

Let E be an event in a sample space S. The probability of the event $\overline{E} = S - E$, the complementary event of E, is given by

$$p(\overline{E}) = 1 - p(E).$$

technique to calculate some probabilities

Probability of union of 2 events

Let E_1 and E_2 be events in the sample space S. Then

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2).$$

useful for proving things

Conditional Probability

Let E and F be events with $p(F) > 0$. The *conditional probability* of E given F, denoted by $p(E \mid F)$, is defined as

$$p(E \mid F) = \frac{p(E \cap F)}{p(F)}.$$

probability of E given F E|F

Definition of independent event

The events E and F are *independent* if and only if $p(E \cap F) = p(E)p(F)$.

use for proofs

Pigeonhole Principle

THE GENERALIZED PIGEONHOLE PRINCIPLE If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

if k is a positive integer and k+1 or more objects are placed into boxes, at least 1 box has 2+ objects

Bernoulli trials probability of success

The probability of exactly k successes in n independent Bernoulli trials, with probability of success p and probability of failure $q = 1 - p$, is

$$C(n, k) p^k q^{n-k}.$$

.

Baye's theorem

BAYES' THEOREM Suppose that E and F are events from a sample space S such that $p(E) \neq 0$ and $p(F) \neq 0$. Then

$$p(F \mid E) = \frac{p(E \mid F)p(F)}{p(E \mid F)p(F) + p(E \mid \overline{F})p(\overline{F})}.$$

calculate probability of i.e diseases/diagnosis, probability of spam...

table 1

TABLE 7 **Logical Equivalences Involving Conditional Statements.**

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 **Logical Equivalences Involving Biconditional Statements.**

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

kk

Proof Laws

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

k

Inference

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $\underline{p \to q}$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\underline{\neg p}$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
$\underline{p}$ $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$\underline{p \wedge q}$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p $\underline{q}$ $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\underline{\neg p \vee r}$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

k

Set ID's

TABLE 1 **Set Identities.**

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

wow

DeMorgans Quant

TABLE 2 **De Morgan's Laws for Quantifiers.**

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

k

Quant Inference

TABLE 2 **Rules of Inference for Quantified Statements.**

Rule of Inference	*Name*
$\underline{\forall x P(x)}$ $\therefore P(c)$	Universal instantiation
$\underline{P(c) \text{ for an arbitrary } c}$ $\therefore \forall x P(x)$	Universal generalization
$\underline{\exists x P(x)}$ $\therefore P(c) \text{ for some element } c$	Existential instantiation
$\underline{P(c) \text{ for some element } c}$ $\therefore \exists x P(x)$	Existential generalization

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TABLE 1 **Quantifications of Two Variables.**

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

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Union/Intersect Collection

The *union* of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

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Complex Numbers

$j^2 = -1$	$j^3 = -j$
$j^4 = 1$	$z = a + bj$
$z = r(\sin θ + j\sin θ)$	$z = re^{jθ}$
$\tan^{-1} b/a = θ$	$\cos^{-1} a/r = θ$
$\sin^{-1} b/r = θ$	$(a + bj)^* = a - bj$
$\lvert z\rvert = r = \mathrm{sqrt}(a^2 + b^2)$	$\lvert z\rvert^x = \lvert z^x\rvert$
$\arg(z)^x = x \arg(z)$	$\arg(z) = θ + 2kπ$
$(\cos θ + j\sin θ)^k$	$= \cos kθ + j\sin kθ$
$= (e^{jθ})^k = e^{jkθ}$	< DeMoivre's Theorum

* means conjugate

j = i = sqrt(-1) = imaginary unit

Find roots example:

$z^2 = -4j$

Convert to exponential form first:

$z^2 = 4e^{-jÏ€/2}$

$|z^2| = r^2 = \mathrm{sqrt}(0^2 + 4^2) = 4$

$|z| = r = 2$

k = (0, 1 ...n where n = expon' of z) = 0, 1

$\arg(z^2)$ = 2 arg(z) = -Ï€/2 + 2kÏ€

arg(z) = -Ï€/4 + kÏ€

Substitute values of k (0, 1) for $z = |z|e^{j\arg(z)}$ = $2e^{-jÏ€/4}$, $2e^{j3Ï€/4}$

Discrete Probability & Sets & Whatever

Probability

1. $P(x) = {}^nCx \, . \, p^x \, . \, (1-p)^{n-x}$
2. $P(x) = ({}^XCk)(({}^{N-X})C(n-k))/ {}^NCn$

Set Theory

A = B when A subset of B & B subset of A

A - B = A n B'

A u (A n B) = A

A n (A u B) = A

A u A' = U

A n A' = nullset or {}

Power set of S is the set of ALL SUBSETS of S e.g. S = {1,2} , P(S) = { {}, {1}, {2}, {1,2}}

$|A| = n$, $|P(A)| = 2^n$

Sets A and B are disjoint iff A n B = {}

Cardinality of union: |A u B| = |A| + |B| - |A n B|

Proof by induction:

Show that when p(k) is true, p(k + 1) follows.

1. Binomial Distribution

n = trials, x = successes, p = probability of success

2. Hypergeometric Distribution

N = deck size, n = draws, X = copies of card, k = successes

Matrix Manipulations

A^T: Transpose of A - Switch Rows with Columns (R1 becomes C1, R2 becomes C2 etc.)

-A = -1 . A

A^{-1}: Inverse of A

$A^{-1} . I = I = A . I$

$A^{-1}A=I$

Augment Identity matrix to matrix and perform Guass-Jordon elimination on both to get change Identity matrix to the Inverse.

EROs:

Switch Rows

Scale Row (Multiply entire row)

Add multiple of different row to another

A matrix A is in row echelon form if

1. The nonzero rows in A lie above all zero rows (when there is at least a nonzero row and a zero row).
2. The first nonzero entry in a nonzero row (called a pivot) lies to the right of the pivot in the row immediately above it.