



UNIVERSIDAD
POLITÉCNICA
P U E R T O R I C O

Computer Science

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Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. Find the **Lagrange form of interpolating polynomial $p_2(x)$** that **interpolates the function $f(x)=e^{-x^2}$** at the **nodes $x_0 = 1$, $x_1 = 0$ and $x_2 = -1$** . **Further, find the value of $p_2(-0.9)$** (use 6-digit **rounding**). **Compare the value with the true value $f(-0.9)$** (use 6-digit **rounding**). Find the **percentage error** in this **calculation**.

Assg 10: Part 1

$$f(x) = e^{-x^2} \quad x = -0.9$$
$$x_0 = 1 \quad x_1 = 0 \quad x_2 = -1$$

$$P_2(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \quad L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \quad L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$f(x_0) = f(1) = e^{-1^2} = e^{-1} \approx 0.367879$$

$$f(x_1) = f(0) = e^{-0^2} = e^0 = 1$$

$$f(x_2) = f(-1) = e^{-(-1)^2} = e^{-1} \approx 0.367879$$

$$L_0(x) = \frac{(x-0)(x+1)}{(1-0)(1+1)} = \frac{x(x+1)}{2}$$

$$L_1(x) = \frac{(x-1)(x+1)}{(0-1)(0+1)} = -(x^2-1)$$

$$L_2(x) = \frac{(x-1)(x-0)}{(-1-1)(-1-0)} = \frac{x(x-1)}{2}$$

$$P_2(x) = \frac{x(x+1)}{2} * 0.367879 + -(x^2-1) * 1 + \frac{x(x-1)}{2} * 0.367879$$

$$L_0(-0.9) = \frac{-0.9(-0.9+1)}{2} = -0.045$$

$$L_1(-0.9) = -(0.9^2-1) = 0.19$$

$$L_2(-0.9) = \frac{-0.9(-1.9)}{2} = 0.855$$

$$P_2(-0.9) = (-0.045) * 0.367879 + 0.19 * 1 + 0.855 * 0.367879$$

$$P_2(-0.9) \approx -0.016555 + 0.19 + 0.314537 \approx 0.487982$$

$$f(-0.9) = e^{-(-0.9)^2} = e^{-0.81} \approx 0.444858$$

$$\text{Percentage Error} = \left| \frac{0.487982 - 0.444858}{0.444858} \right| \times 100 \approx 9.70\%$$

2. Find the Newton form of interpolating polynomial for the data

x	-3	-1	0	3	5
y	-30	-22	-12	330	3458

and interpolate the following values $f(-2)$, $f(1)$, $f(2)$, $f(4)$.

Ans 10: Part 2

$x = [-3, -1, 0, 3, 5]$
 $y = [-30, -22, -12, 330, 3458]$

$f(-2), f(1), f(2), f(4)$

$f[-3, -1] = \frac{-22 - (-30)}{-1 - (-3)} = \frac{8}{2} = 4$ $f[-3, -1, 0] = \frac{10 - 4}{0 - (-3)} = \frac{6}{3} = 2$
 $f[-1, 0] = \frac{-12 - (-22)}{0 - (-1)} = \frac{10}{1} = 10$ $f[-1, 0, 3] = \frac{114 - 10}{3 - (-1)} = \frac{104}{4} = 26$
 $f[0, 3] = \frac{330 - (-12)}{3 - 0} = \frac{342}{3} = 114$ $f[0, 3, 5] = \frac{1564 - 114}{5 - 0} = \frac{1450}{5} = 290$
 $f[3, 5] = \frac{3458 - 330}{5 - 3} = \frac{3128}{2} = 1564$

$f[-3, -1, 0, 3] = \frac{26 - 2}{3 - (-3)} = \frac{24}{6} = 4$ $f[-3, -1, 0, 3, 5] = \frac{44 - 4}{5 - (-3)} = \frac{40}{8} = 5$
 $f[-1, 0, 3, 5] = \frac{290 - 26}{5 - (-1)} = \frac{264}{6} = 44$

$P_4(x) = -30 + 4(x+3) + 2(x+3)(x+1) + 4(x+3)(x+1)x + 5(x+3)(x+1)x(x-3)$

$P_4(-2) = -30 + 4(-2+3) + 2(-2+3)(-2+1) + 4(-2+3)(-2+1)(-2) + 5(-2+3)(-2+1)(-2)(-2-3)$
 $P_4(-2) = -30 + 4(1) + 2(1)(-1) + 4(1)(-1)(-2) + 5(1)(-1)(-2)(-5)$
 $P_4(-2) = -30 + 4 - 2 + 8 - 50 = -70$

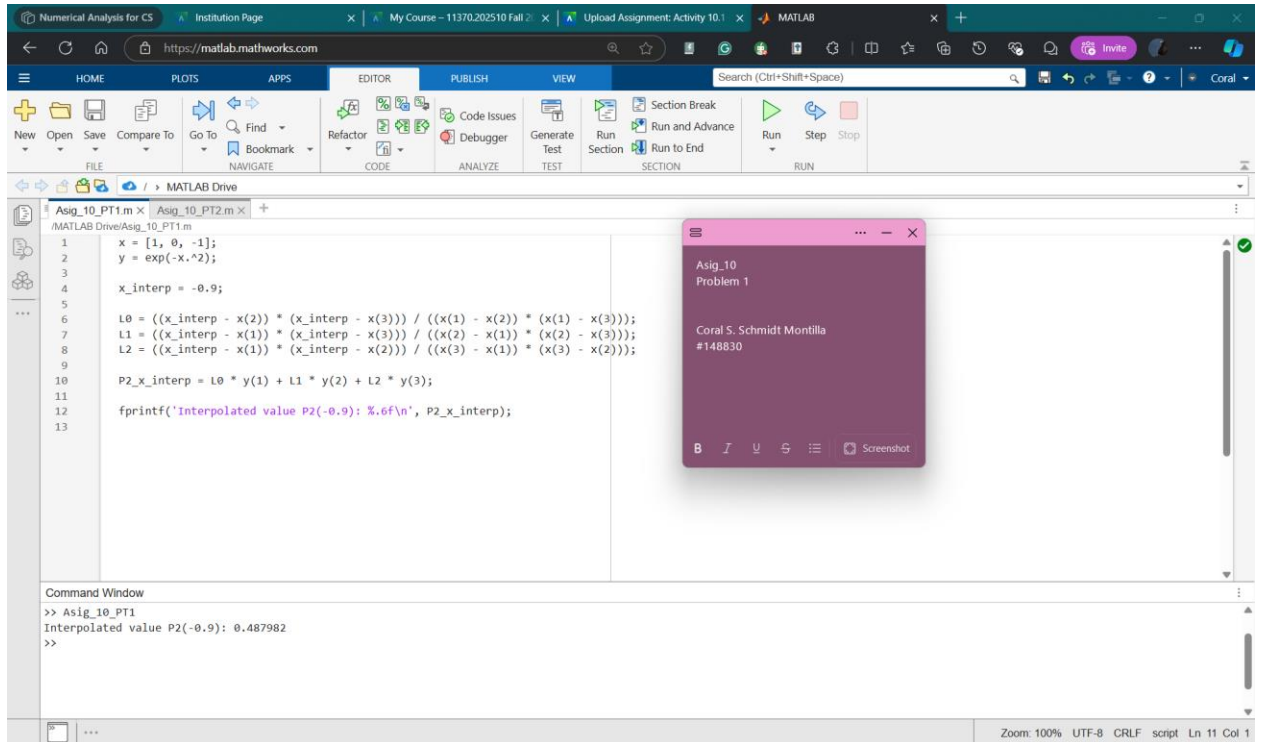
$P_4(1) = -30 + 4(1+3) + 2(1+3)(1+1) + 4(1+3)(1+1)(1) + 5(1+3)(1+1)(1)(1-3)$
 $P_4(1) = -30 + 4(4) + 2(4)(2) + 4(4)(2)(1) + 5(5)(3)(2)(-1)$
 $P_4(1) = -30 + 16 + 16 + 32 - 80 = -46$

$P_4(2) = -30 + 4(2+3) + 2(2+3)(2+1) + 4(2+3)(2+1)(2) + 5(2+3)(2+1)(2)(2-3)$
 $P_4(2) = -30 + 4(5) + 2(5)(3) + 4(5)(3)(2) + 5(5)(3)(2)(-1)$
 $P_4(2) = -30 + 20 + 30 + 120 + 150 = -10$

$P_4(4) = -30 + 4(4+3) + 2(4+3)(4+1) + 4(4+3)(4+1)(4) + 5(4+3)(4+1)(4)(4-3)$
 $P_4(4) = -30 + 4(7) + 2(7)(5) + 4(7)(5)(4) + 5(7)(5)(4)(1)$
 $P_4(4) = -30 + 28 + 70 + 560 + 700 = 1328$

$f(-2) = -70$ $f(1) = -46$ $f(2) = -10$ $f(4) = 1328$

3. Corroborate your previous problems solution using MATLAB. Add to the pdf file the MATLAB output for each of the problems.

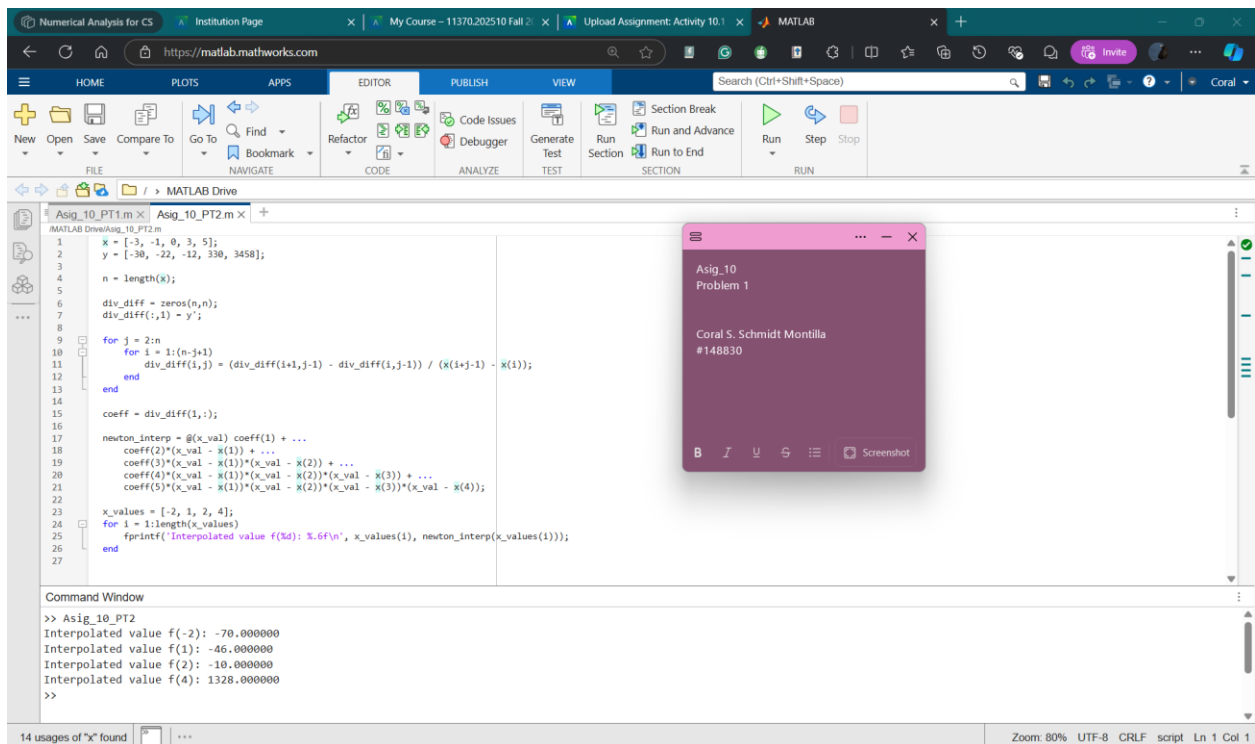


The MATLAB Editor window displays a script for linear interpolation. The script defines a function `Asig_10_PT1.m` that takes a vector `x` and a scalar `x_interp` as input. It calculates the interpolated value `P2_x_interp` using the formula $P_2(x_{interp}) = L_0(x_{interp}) + L_1(x_{interp}) + L_2(x_{interp})$, where L_0, L_1, L_2 are the Lagrange basis polynomials. The Command Window shows the output: `>> Asig_10_PT1` and `Interpolated value P2(-0.9): 0.487982`.

```
1 x = [1, 0, -1];
2 y = exp(-x.^2);
3
4 x_interp = -0.9;
5
6 L0 = ((x_interp - x(2)) * (x_interp - x(3))) / ((x(1) - x(2)) * (x(1) - x(3)));
7 L1 = ((x_interp - x(1)) * (x_interp - x(3))) / ((x(2) - x(1)) * (x(2) - x(3)));
8 L2 = ((x_interp - x(1)) * (x_interp - x(2))) / ((x(3) - x(1)) * (x(3) - x(2)));
9
10 P2_x_interp = L0 * y(1) + L1 * y(2) + L2 * y(3);
11
12 fprintf('Interpolated value P2(-0.9): %.6f\n', P2_x_interp);
13
```

Command Window

```
>> Asig_10_PT1
Interpolated value P2(-0.9): 0.487982
>>
```



The MATLAB Editor window displays a script for Newton's method. The script defines a function `Asig_10_PT2.m` that takes a vector `x` and a scalar `x_val` as input. It calculates the interpolated value `newton_interp` using the formula $newton_interp = f(x_{val}) + \sum_{i=1}^n \frac{f^{(i)}(x_{val})}{i!} (x_{val} - x(i))^i$. The Command Window shows the output: `>> Asig_10_PT2` and `Interpolated value f(-2): -70.000000`, `Interpolated value f(1): -46.000000`, `Interpolated value f(2): -10.000000`, and `Interpolated value f(4): 1328.000000`.

```
1 x = [-3, -1, 0, 3, 5];
2 y = [-30, -22, -12, 330, 3458];
3
4 n = length(x);
5
6 div_diff = zeros(n,n);
7 div_diff(:,1) = y';
8
9 for j = 2:n
10     for i = 1:(n-j+1)
11         div_diff(i,j) = (div_diff(i+1,j-1) - div_diff(i,j-1)) / (x(i+j-1) - x(i));
12     end
13 end
14
15 coeff = div_diff(1,:);
16
17 newton_interp = @(x_val) coeff(1) + ...
18     coeff(2)*(x_val - x(1)) + ...
19     coeff(3)*(x_val - x(1))*(x_val - x(2)) + ...
20     coeff(4)*(x_val - x(1))*(x_val - x(2))*(x_val - x(3)) + ...
21     coeff(5)*(x_val - x(1))*(x_val - x(2))*(x_val - x(3))*(x_val - x(4));
22
23 x_values = [-2, 1, 2, 4];
24 for i = 1:length(x_values)
25     fprintf('Interpolated value f(%d): %.6f\n', x_values(i), newton_interp(x_values(i)));
26 end
27
```

Command Window

```
>> Asig_10_PT2
Interpolated value f(-2): -70.000000
Interpolated value f(1): -46.000000
Interpolated value f(2): -10.000000
Interpolated value f(4): 1328.000000
>>
```