UNIVERSIDAD POLITÉCNICA
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Computer Science

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Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. Solve the following ODE, $y' + 2y = x + 4, \quad y(0) = 1$ using.

    a. Approximate y (0.4) second order Runge-Kutta with h=0.2 and y(0)=0.

Problem 1: part a:

$x_0 = 0,\ y_0 = 1$

$h = 0.2$

$k_1 = 0.2 \cdot f(0,1) = 0.2 \cdot (0 + 4 - 2 \cdot 1) = 0.2 \cdot 2 = 0.4$

$K_2 = 0.2 \cdot f(0.2, 1 + 0.4) = 0.2 \cdot (0.2 + 4 - 2 \cdot 1.4) = 0.2 \cdot (4.2 - 2.8) = 0.2 \cdot 1.4 = 0.28$

$y_1 = 1 + \frac{1}{2}(0.4 + 0.28) = 1 + \frac{1}{2}(0.68) = 1 + 0.34 = 1.34$

$y_1 = 1.34 \quad x_1 = 0.2$

$K_1 = 0.2 \cdot f(0.2, 1.34) = 0.2 \cdot (0.2 + 4 - 2 \cdot 1.34) = 0.2 \cdot (4.2 - 2.68) = 0.2 \cdot 1.52 = 0.304$

$K_2 = 0.2 \cdot f(0.4, 1.34 + 0.304) = 0.2 \cdot (0.4 + 4 - 2 \cdot 1.644) = 0.2 \cdot (4.4 - 3.288) = 0.2 \cdot 1.112 = 0.2224$

$y_2 = 1.34 + \frac{1}{2}(0.304 + 0.2224) = 1.34 + \frac{1}{2}(0.5264) = 1.34 + 0.2632 = 1.6032$

$y(0.4) = 1.6032$

b. Using the previous results use Adam-Bashforth(predictor) two step explicit method in conjunction with Adam-Moulton two step implicit method (corrector) to approximate y (0.8) with h=0.01

Problem 1: part b

$y_2=1.6032,\ x_2=0.4$

$f(0.4,1.6032)=0.4+4-2\cdot1.6032=4.4-3.2064=1.1936$

$f(0.2,1.34)=0.2+4-2\cdot1.34=4.2-2.68=1.52$

$y_3=1.6032+\frac{0.1}{2}(3\cdot1.1936-1.52=1.6032+\frac{0.1}{2}\cdot(3.5808-1.52)=1.6032+\frac{0.1}{2}\cdot2.0608$

$=1.6032+0.10304=1.70624$

$f(0.5,1.70624)=0.5+4-2\cdot1.70624=4.5-3.41248=1.08752$

$y_3=1.6032+\frac{0.1}{2}\cdot(1.08752+1.1936)=1.6032+\frac{0.1}{2}\cdot2.28112=1.6032+0.11406=1.71726$

$y_3=1.71726\quad x_3=0.5$

$f(0.5),1.71726)=0.5+4-2\cdot1.71726=4.5-3.43452=1.06548$

$f(0.4,1.6032)=1.1936$

$y_4=1.71726+\frac{0.1}{2}(3\cdot1.06548-1.1936)=1.71726+\frac{0.1}{2}(3.19644-1.1936)$

$=1.71726+\frac{0.1}{2}\cdot2.00284=1.71726+0.10014=1.8174$

$f(0.6,1.8174)=0.6+4-2\cdot1.8174=4.6-3.6348=0.9652$

$y_4=1.71726+\frac{0.1}{2}(0.9652+1.06548)=1.71726+\frac{0.1}{2}(2.03068)=1.71726+0.10153$

$=1.81879$

$y_4=1.81879\quad x_4=0.6$

$f(0.6,1.81879)=0.9652$

$f(0.5,1.71726)=1.06548$

$y_5=1.81879+\frac{0.1}{2}(3\cdot0.9652-1.06548)=1.81879+\frac{0.1}{2}(2.8956-1.06548)$

$=1.81879+\frac{0.1}{2}(1.83012)=1.81879+0.09151=1.9103$

$f(0.7,1.9103)=0.7+4-2\cdot1.9103=4.7-3.8206=0.8794$

$y_5=1.81879+\frac{0.1}{2}(0.8794+0.9652)=1.81879+\frac{0.1}{2}(1.8446)=1.81879+0.09223$

$=1.91102$

$y_5=1.91102\quad x_5=0.7$

$f(0.7,1.91102)=0.87996$

$f(0.6,1.81879)=0.9652$

$y_6=1.91102+\frac{0.1}{2}(3\cdot0.87996-0.9652)=1.91102+\frac{0.1}{2}(2.63988-0.9652)$

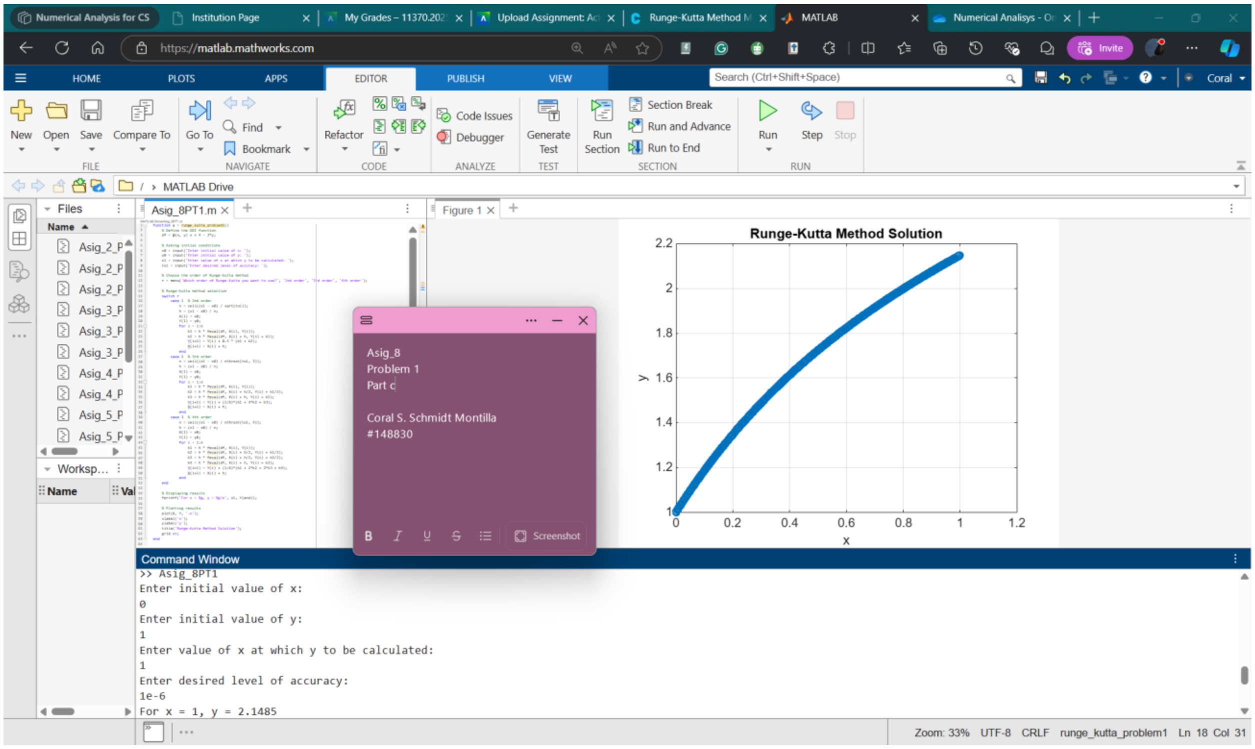
$=1.91102+\frac{0.1}{2}(1.67468)=1.91102+0.08373=1.99475$

$f(0.8,1.99475)=0.8+4-2\cdot1.99475=4.8-3.9895=0.8105$

$f(0.7,1.91102)=0.87996$

$y_6=1.91102+\frac{0.1}{2}\cdot(0.8105+0.87996)=1.91102+\frac{0.1}{2}\cdot1.69046=1.91102+0.08452=1.99554$

c. Use MATLAB to approximate and plot the solution from x=0 to x=1 using 2nd order Runge-Kutta with h=0.2 and y (0) =1



2. Given the following ODE, $y' + 5y = \sin(x)$

a. Approximate by hand y (0.03) using 4th order Runge-Kutta with h=0.01 and y(0)=0.

Problem 2: part a

$x_0 = 0$, $y_0 = 0$

$K_1 = 0.01 \cdot f(0,0) = 0.01 \cdot (\sin(0) - 5 \cdot 0) = 0.01 \cdot 0 = 0$

$K_2 = 0.01 \cdot f(0.005, 0 + \frac{0}{2}) = 0.01 \cdot (\sin(0.005) - 5 \cdot 0) = 0.01 \cdot 0.00499998 = 0.00005$

$K_3 = 0.01 \cdot f(0.005, 0 + \frac{0.00005}{2}) = 0.01 \cdot (\sin(0.005) - 5 \cdot 0.000025)$

$= 0.01 \cdot (0.00499998 - 0.000125) = 0.01 \cdot 0.004875 = 0.00004875$

$K_4 = 0.01 \cdot f(0.01, 0 + 0.00004875) = 0.01 \cdot (\sin(0.01) - 5 \cdot 0.00004875)$

$= 0.01 \cdot (0.00999983 - 0.00024375) = 0.01 \cdot 0.00975608 = 0.00009756$

$y_1 = 0 + \frac{1}{6}(0 + 2 \cdot 0.00005 + 2 \cdot 0.00004875 + 0.00009756) = \frac{1}{6}(0.00029506)$

$= 0.00004918$

$y_1 = 0.00004918 \quad x_1 = 0.01$

$K_1 = 0.01 \cdot f(0.01, 0.00004918) = 0.01 \cdot (\sin(0.01) - 5 \cdot 0.00004918)$

$= 0.01 \cdot (0.00999983 - 0.0002459) = 0.01 \cdot 0.00975393 = 0.00009754$

$K_2 = 0.01 \cdot f(0.015, 0.00004918 + \frac{0.00009754}{2}) = 0.01 \cdot (\sin(0.015) - 5 \cdot 0.00009795)$

$= 0.01 \cdot (0.01499944 - 0.00048975) = 0.01 \cdot 0.01450969 = 0.00014510$

$K_3 = 0.01 \cdot f(0.015, 0.00004918 + \frac{0.0001451}{2}) = 0.01 \cdot (\sin(0.015) - 5 \cdot 0.00012173)$

$= 0.01 \cdot (0.01499944 - 0.00060865) = 0.01 \cdot 0.01439079 = 0.00014391$

$K_4 = 0.01 \cdot f(0.02, 0.00004918 + 0.00014391) = 0.01 \cdot (\sin(0.02) - 5 \cdot 0.00019309)$

$= 0.01 \cdot (0.01999867 - 0.00096545) = 0.01 \cdot 0.01903322 = 0.00019033$

$y_2 = 0.00004918 + \frac{1}{6}(0.00009754 + 2 \cdot 0.00014510 + 2 \cdot 0.00014391 + 0.00019033)$

$= 0.00004918 + \frac{1}{6}(0.00086589) = 0.00004918 + 0.00014431 = 0.00019349$

$x_2 = 0.02, y_2 = 0.00019349$

$K_1 = 0.01 \cdot f(0.02, 0.00019349) = 0.01 \cdot (\sin(0.02) - 5 \cdot 0.00019349)$

$= 0.01 \cdot (0.01999867 - 0.00096745) = 0.01 \cdot 0.01903122 = 0.00019031$

$K_2 = 0.01 \cdot f(0.025, 0.00019349 + \frac{0.00019031}{2}) = 0.01 \cdot (\sin(0.025) - 5 \cdot 0.00028865)$

$= 0.01 \cdot (0.02499974 - 0.00144323) = 0.01 \cdot 0.02355417 = 0.00023554$

$K_3 = 0.01 \cdot f(0.025, 0.00019349 + \frac{0.00023554}{2}) = 0.01 \cdot (\sin(0.025) - 5 \cdot 0.00031126)$

$= 0.01 \cdot (0.02499974 - 0.0015563) = 0.01 \cdot 0.02344411 = 0.00023441$

$K_4 = 0.01 \cdot f(0.03, 0.00019349 + 0.00023441) = 0.01 \cdot (\sin(0.03) - 5 \cdot 0.0004279)$

$= 0.01 \cdot (0.02999955 - 0.0021395) = 0.01 \cdot 0.027856 = 0.00027856$

$y_3 = 0.00019349 + \frac{1}{6}(0.00019031 + 2 \cdot 0.00023554 + 2 \cdot 0.00023441 + 0.00027856)$

$= 0.00019349 + \frac{1}{6}(0.00140978) = 0.00019349 + 0.00023496 = 0.00042845$

$y(0.03) = 0.00042845$

b. Using the previous results use Adam-Bashforth three step explicit method to approximate y (0.05) with h=0.01

Problem 2: part: b

$y_2 = 0.00019349$

$y_3 = 0.00042845$

$f(0.03, 0.00042845) = \sin(0.03) - 5 \cdot 0.00042845 = 0.0299955 - 0.00214225 = 0.02785325$

$f(0.02, 0.00019349) = 0.01903122$

$y_4 = 0.00042845 + \frac{0.01}{12}(23 \cdot 0.02785325 - 16 \cdot 0.01903122 + 5 \cdot 0.00975393)$

$y_4 = 0.00042845 + \frac{0.01}{12}(0.64064475 - 0.30449952 + 0.04876965) = 0.00042845 + \frac{0.01}{12}(0.38491488)$

$= 0.00042845 + 0.00003208 = 0.00046053$

$y_4 = 0.00046053 \quad x_4 = 0.04$

$f(0.04, 0.00046053) = \sin(0.04) - 5 \cdot 0.00046053 = 0.03998933 - 0.00230265$

$= 0.03768668$

$y_5 = 0.00046053 + \frac{0.01}{12}(23 \cdot 0.03768668 - 16 \cdot 0.02785325 + 5 \cdot 0.01903122)$

$y_5 = 0.00046053 + \frac{0.01}{12}(0.86779364 - 0.44565192 + 0.0951561) = 0.00046053 + \frac{0.01}{12}(0.51729782)$

$0.00046053 + 0.00004311 = 0.00050364$

$y_5 = 0.00050364 \quad x_5 = 0.05$

$y(0.05) = 0.00050364$

c. Use MATLAB to approximate and plot the solution from x=0 to x=1 using 4th order Runge-Kutta with h=0.01 and y (0) =1.