UNIVERSIDAD POLITÉCNICA PUERTO RICO

Computer Science

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#148830

Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. The voltage E = E(t) in an electrical circuit obeys the equation

$$E(t) = L\left(\frac{dI}{dt}\right) + RI(t)$$

where R is resistance and L is inductance. Use L = 0.05 and R= 2 and values for I(t) in the table following.

t	1	1.1	1.2	1.3	1.4
I(t)	8.2277	7.2428	5.9908	4.5260	2.9122

Find dI(t)/dt (1.2) using

a. Forward difference formula,
b. Backward Difference
c. Central Difference

and compute E (1.2) for each case. Compare your answer against $I(t) = 10e^{-t/10}\sin(2t)$

Asg 6: Part 1

$$\frac{dI}{dt} = \frac{I(1.3) - I(1.2)}{t_{1.3} - t_{1.2}}$$

$$\frac{4.526 - 5.990}{1.3 - 1.2} = \frac{-1.464}{0.1} = -14.64$$

$$E(1.2) = 0.05 \cdot (-14.64) - 2 \cdot 5.990 = -0.732 + 11.980 = 11.248$$

$$\frac{dI}{dt} = \frac{5.990 - 7.242}{1.2 - 1.1} = \frac{-1.252}{0.1} = -12.52$$

$$E(1.2) = 0.05 \cdot (-12.52) + 2 \cdot 5.990 = -0.626 + 11.980 = 11.354$$

$$\frac{dI}{dt} = \frac{4.526 - 7.242}{1.3 - 1.1} = \frac{-2.716}{0.2} = -13.58$$

$$E(1.2) = 0.05 \cdot (-13.58) + 2 \cdot 5.990 = -0.679 + 11.980 = 11.301$$

(a) 11.248
(b) 11.354
(c) 11.301

2. Compute the second derivative using forward, backward, central difference and Taylor for the following function, $f(x) = e^x - 2x^2 + 3x + 1$. approximate f''(5) and f''(10) using h=0.1 and h=0.01 for both cases and compare your answers against the analytical solution.

Asig 6 : Part2

$f'(x) = e^x - 4x + 3$

$f''(x) = e^x - 4$

$f''(5) = e^5 - 4 = 144.4132$

$f''(10) = e^{10} - 4 = 22022.4658$

$$f''(5) = \frac{f(5+0.2) - 2f(5+0.1) + f(5)}{0.1^2}$$

$$\frac{182.211 - 327.388 + 148.413}{0.01} = 160.1586$$

$f(5.02) = e^{5.02} - 2(5.02)^2 + 3(5.02) + 1 \approx 150.057$

$f(5.01) = e^{5.01} - 2(5.01)^2 + 3(5.01) + 1 = 147.230$

$$f''(5) = \frac{150.057 - 2(147.230) + 144.413}{(0.01)^2} = 145.9060$$

$f(10.2) = e^{10.2} - 2(10.2)^2 + 3(10.2) + 1 = 26955.616$

$f(10.1) = e^{10.1} - 2(10.1)^2 + 3(10.1) + 1 = 24401.635$

$$f''(10) = \frac{26955.616 - 2(24401.635) + 22026.466}{(0.1)^2} = 24359.3020$$

$f(10.02) = e^{10.02} - 2(10.02)^2 + 3(10.02) + 1 = 22337.041$

$f(10.01) = e^{10.01} - 2(10.01)^2 + 3(10.01) + 1 = 22122.480$

$$f''(10) = \frac{22337.041 - 2(22121.48) + 22026.466}{(0.01)^2} = 22244.0209$$

$f(4.9) = e^{4.9} - 2(4.9)^2 + 3(4.9) + 1 = 133.674$

$f(4.8) = e^{4.8} - 2(4.8)^2 + 3(4.8) + 1 = 120.105$

$$f''(5) = \frac{148.413 - 2(133.674) + 120.105}{(0.1)^2} = 130.4017$$

$f(4.99) = e^{4.99} - 2(4.99)^2 + 3(4.99) + 1 = 143.906$

$f(4.98) = e^{4.98} - 2(4.98)^2 + 3(4.98) + 1 = 142.610$

$$f''(5) = \frac{144.423 - 2(143.906) + 142.610}{(0.01)^2} = 142.9376$$

$f(9.9) = e^{9.9} - 2(9.9)^2 + 3(9.9) + 1 = 19725.892$

$f(9.8) = e^{9.8} - 2(9.8)^2 + 3(9.8) + 1 = 17664.970$

$$f''(10) = \frac{22026.466 - 2(19725.892) + 17664.970}{(0.1)^2} = 19942.9846$$

$f(9.99) = e^{9.99} - 2(9.99)^2 + 3(9.99) + 1 = 21901.073$

$f(9.98) = e^{9.98} - 2(9.98)^2 + 3(9.98) + 1 = 21776.445$

$$f''(10) = \frac{22026.466 - 2(21901.073) + 21776.445}{(0.01)^2} = 21803.4805$$

$f(5.1) = 163.694 \quad f(5.0) = 148.413 \quad f(4.9) = 133.674$

$$f''(5) = \frac{163.694 - 2(148.413) + 133.674}{(0.1)^2}$$

$$\frac{163.694 - 296.826 + 133.694}{0.01}$$

$$\frac{0.542}{0.01} = 144.5369$$

$f(5.01) = 147.230 \quad f(5.0) = 144.413 \quad f(4.99) = 143.906$

$$f''(5) = \frac{147.230 - 2(144.413) + 143.906}{(0.01)^2}$$

$$\frac{147.230 - 288.824 + 143.906}{0.0001}$$

$$\frac{2.320}{0.0001} = 144.4144$$

$f(10.1) = 24401.635 \quad f(10.0) = 22026.466 \quad f(9.9) = 19725.892$

$$f''(10) = \frac{24401.635 - 2(22026.466) + 19725.892}{(0.1)^2}$$

$$\frac{24401.635 - 44052.932 + 19725.892}{0.01}$$

$$\frac{0.595}{0.01} = 22040.8273$$

$f(10.01) = 22121.480 \quad f(10.0) = 22026.466 \quad f(9.99) = 21901.073$

$$f''(10) = \frac{221.480 - 2(22026.466) + 21901.073}{(0.01)^2}$$

$$\frac{22121.480 - 44052.932 + 21901.073}{0.0001}$$

$$\frac{2.320}{0.0001} = 22022.6493$$

3. A function f has the values shown below:

x	1	1.25	1.5	1.75	2
f(x)	10	8	7	6	5

a. Use Midpoint rule to approximate $\int_1^2 f(x)_{dx}$

b. Use Simpson's rule to approximate $\int_1^2 f(x)_{dx}$

Asig 6: Part 3

$$\int_1^2 f(x)\,dx \approx h \sum f\left(\frac{x_i + x_{i+1}}{2}\right)$$

$$\int_1^2 f(x)\,dx = 0.25 \cdot (8+6) = 3.5000$$

$$\int_1^2 f(x)\,dx \approx \frac{h}{3}\left(f(x_1) + 4f(x_3) + f(x_5)\right)$$

$$\int_1^2 f(x)\,dx = \frac{0.25}{3} \cdot (10 + 4 \cdot 7 + 5) = 3.5833$$

4. Approximate the solution of $\int_0^1 e^{-x^2}dx$ $(exact\ value \approx 0.7468)$ by using,

   a. Trapezoidal with h=0.2,

   b. 3/8 Simpson methods with h=1/6

Asig 6: Part 4

$x = [0, 0.2, 0.4, 0.6, 0.8, 1.0]$

$f(0) = e^{-0^2} = 1$

$f(0.2) = e^{-(0.2)^2} = e^{-0.04} = 0.9608$

$f(0.4) = e^{-(0.4)^2} = e^{-0.16} = 0.8521$

$f(0.6) = e^{-(0.6)^2} = e^{-0.36} = 0.6977$

$f(0.8) = e^{-(0.8)^2} = e^{-0.64} = 0.5273$

$f(1) = e^{-(1)^2} = e^{-1} = 0.3679$

$I \approx \frac{0.2}{2}(f(0) + 2\cdot(f(0.2) + f(0.4) + f(0.6) + f(0.8)) + f(1))$

$\frac{0.2}{2}(1 + 2\cdot(0.9608 + 0.8521 + 0.6977 + 0.5273) + 0.3679)$

$\frac{0.2}{2}(1 + 2\cdot 3.0379 + 0.3679)$

$\frac{0.2}{2}(1 + 6.0758 + 0.3679)$

$\frac{0.2}{2} \times 7.4437 = 0.1 \cdot 7.4437 = 0.7444$

$x = [0, 0.167, 0.333, 1]$

$f(0) = e^{-(0)^2} = 1$

$f(0.167) = e^{-(0.167)^2} = 0.9726$

$f(0.333) = e^{-(0.333)^2} = 0.8975$

$f(1) = e^{-(1)^2} = 0.3679$

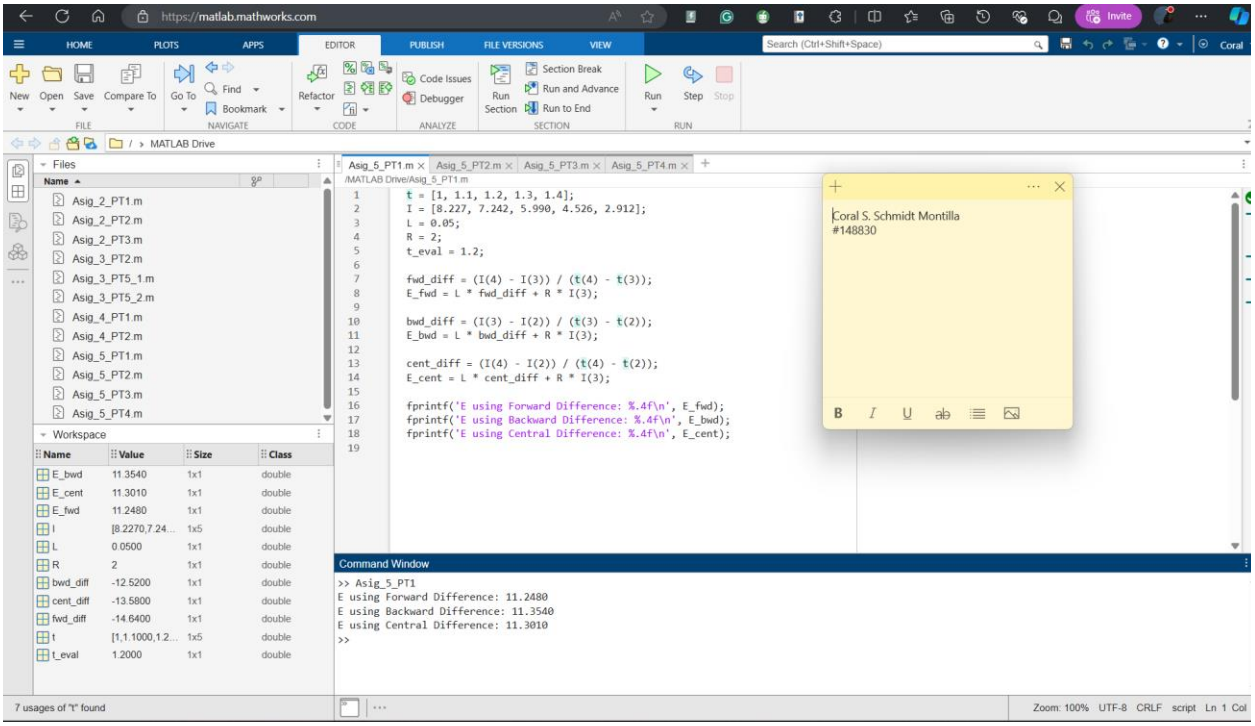
$I = \frac{3\cdot\frac{1}{6}}{8}(1 + 3\cdot 0.9726 + 3\cdot 0.8975 + 0.3679)$

$\frac{0.5}{8}(1 + 2.9178 + 2.6925 + 0.3679)$

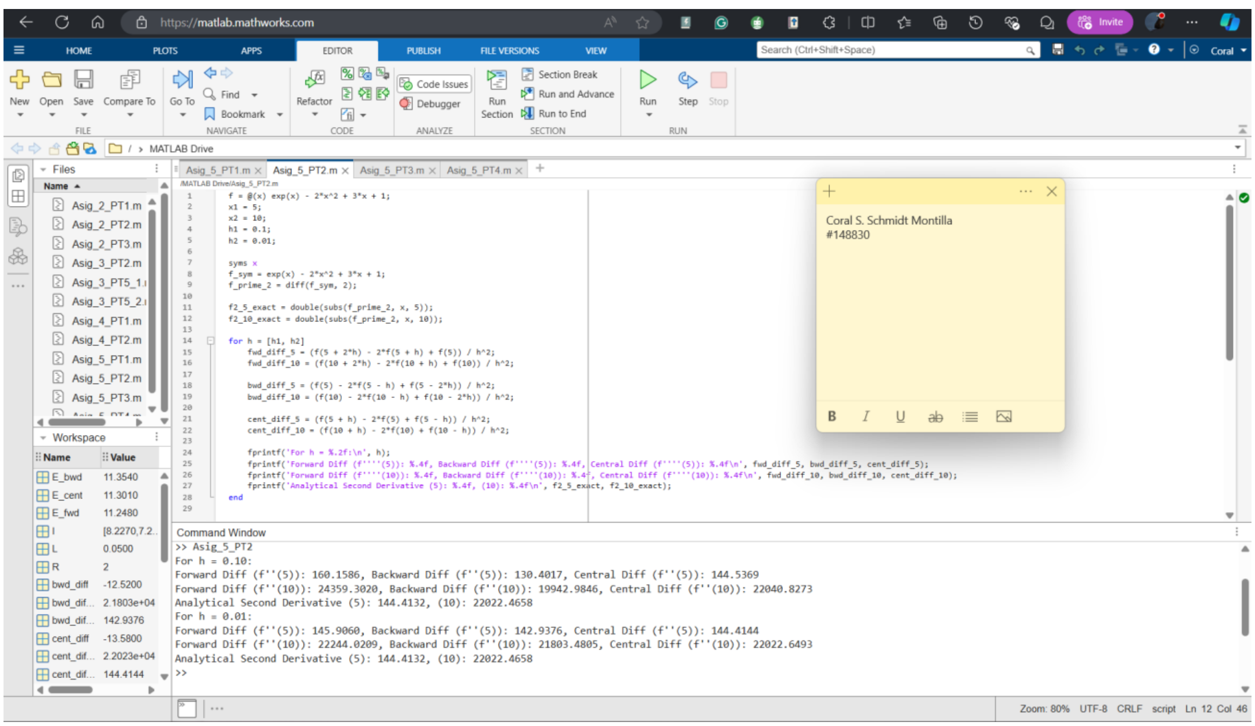
$0.0625 \cdot 6.9782 = 0.6985$

5. Validate previous problems results using MATLAB.

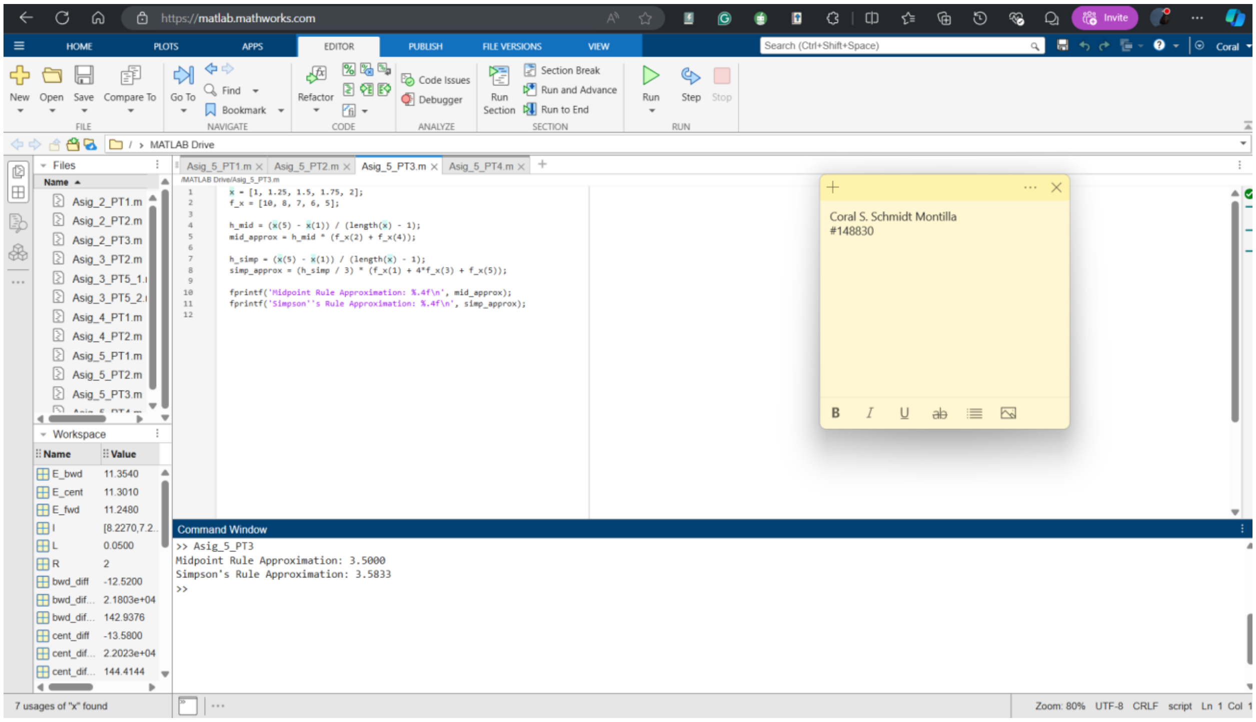
Problem 1:



Problem 2:



Problem 3:


https://matlab.mathworks.com
Asig_5_PT1.m Asig_5_PT2.m Asig_5_PT3.m Asig_5_PT4.m
/MATLAB Drive/Asig_5_PT3.m
x = [1, 1.25, 1.5, 1.75, 2];
f_x = [10, 8, 7, 6, 5];
h_mid = (x(5) - x(1)) / (length(x) - 1);
mid_approx = h_mid * (f_x(2) + f_x(4));
h_simp = (x(5) - x(1)) / (length(x) - 1);
simp_approx = (h_simp / 3) * (f_x(1) + 4*f_x(3) + f_x(5));
fprintf('Midpoint Rule Approximation: %.4f\n', mid_approx);
fprintf('Simpson''s Rule Approximation: %.4f\n', simp_approx);
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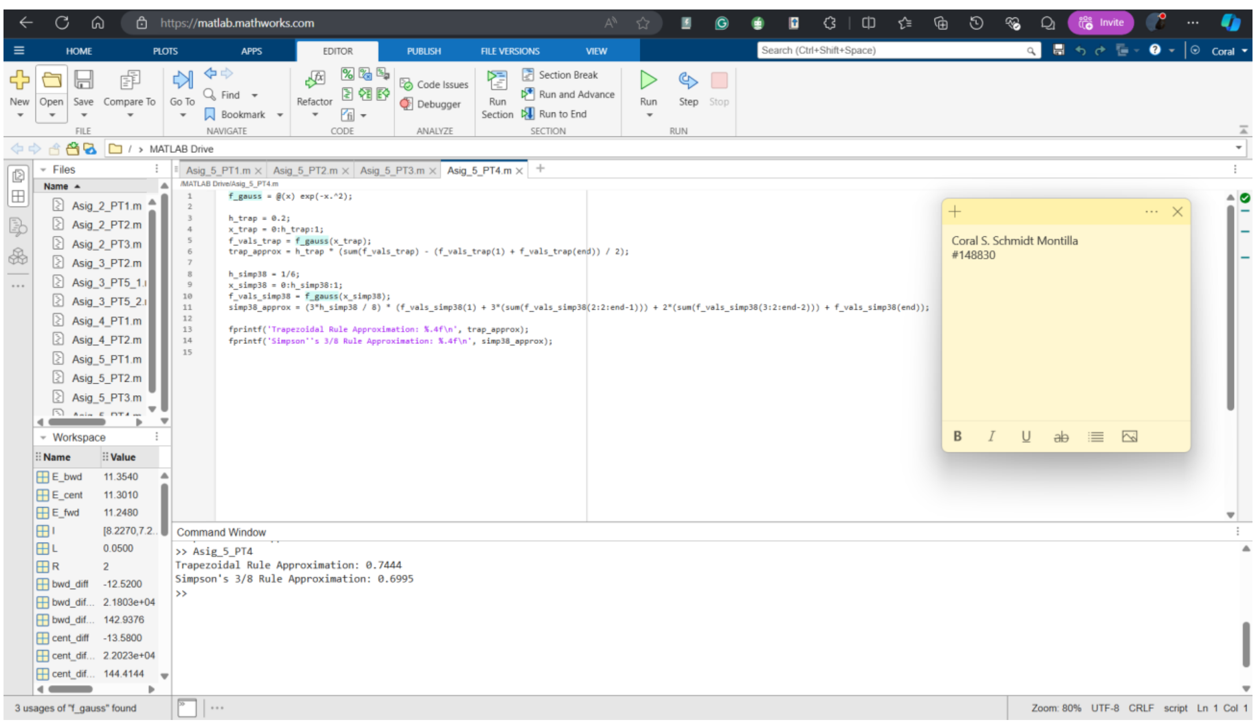
#148830
Command Window
>> Asig_5_PT3
Midpoint Rule Approximation: 3.5000
Simpson's Rule Approximation: 3.5833
>>
7 usages of "x" found

Problem 4:


https://matlab.mathworks.com
Asig_5_PT1.m Asig_5_PT2.m Asig_5_PT3.m Asig_5_PT4.m
/MATLAB Drive/Asig_5_PT4.m
f_gauss = @(x) exp(-x.^2);
h_trap = 0.2;
x_trap = 0:h_trap:1;
f_vals_trap = f_gauss(x_trap);
trap_approx = h_trap * (sum(f_vals_trap) - (f_vals_trap(1) + f_vals_trap(end)) / 2);
h_simp38 = 1/6;
x_simp38 = 0:h_simp38:1;
f_vals_simp38 = f_gauss(x_simp38);
simp38_approx = (3*h_simp38 / 8) * (f_vals_simp38(1) + 3*(sum(f_vals_simp38(2:2:end-1))) + 2*(sum(f_vals_simp38(3:2:end-2))) + f_vals_simp38(end));
fprintf('Trapezoidal Rule Approximation: %.4f\n', trap_approx);
fprintf('Simpson''s 3/8 Rule Approximation: %.4f\n', simp38_approx);
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Command Window
>> Asig_5_PT4
Trapezoidal Rule Approximation: 0.7444
Simpson's 3/8 Rule Approximation: 0.6995
>>
3 usages of "f_gauss" found