UNIVERSIDAD POLITÉCNICA PUERTO RICO

Computer Science

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Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. Given a cubic spline interpolation:

$$S(x) = \begin{cases} S_0(x) = 1 - 2x - x^3 & 0 \le x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^2 & 1 \le x < 2 \end{cases}$$

determine constants b, c, and d so that all conditions for a natural cubic spline hold.

Asg 11 : Part 1

$$S(x) = \begin{cases} S_0(x) = 1 - 2x - x^3 & \text{for } 0 \le x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3 & \text{for } 1 \le x < 2 \end{cases}$$

$x = 1$, $S_0(1) = S_1(1)$:

$1 - 2(1) - (1)^3 = 2 + b(1-1) + c(1-1)^2 + d(1-1)^3$

$-2 = 2$

$S_0'(x) = -2 - 3x^2$

$S_1'(x) = b + 2c(x-1) + 3d(x-1)^2$

$x = 1$, $S_0'(1) = S_1'(1)$:

$S_0'(1) = -2 - 3(1)^2 = -5$

$S_1'(1) = b$

$b = -5$

$S_0''(x) = -6x$

$S_1''(x) = 2c + 6d(x-1)$

$x = 1$, $S_0''(1) = S_1''(1)$

$S_0''(1) = -6(1) = -6$

$S_1''(1) = 2c$

$2c = -6 \quad c = -3$

$S_0''(0) = 0$

$S_0''(0) = -6(0) = 0$

$S_1''(2) = 0$

$S_1''(2) = 2c + 6d(2-1) = -6 + 6d = 0$

$d = 1$

$b = -5 \quad c = -3 \quad d = 1$

2. Given the following data:

x	0	0.25	0.5	1
y	1	1.4	1.6	2

Consider the problem of constructing a natural cubic spline S(x). Form the matrix A and vector of y values which are used to solve the vector containing all coefficients.

Asg 11: Part 2

$x = [0, 0.25, 0.5, 1]$, $y = [1, 1.4, 1.6, 2]$

$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$, $i = 1, 2, 3$

$[0, 0.25]$, $[0.25, 0.5]$, $[0.5, 1]$

$6c_1 + 4c_2 + 1c_3 = \frac{6}{h^2}(y_2 - y_1)$ (at $x_1 = 0.25$)

$1c_2 + 4c_3 + 1c_4 = \frac{6}{h^2}(y_3 - y_2)$ (at $x_2 = 0.5$)

$h = 0.25$ $c_1 = 0$ $c_4 = 0$

$\frac{6}{h^2}(y_2 - y_1) = \frac{6}{0.25^2}(1.4 - 1) = 38.4$

$\frac{6}{h^2}(y_3 - y_2) = \frac{6}{0.25^2}(1.6 - 1.4) = 19.2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 38.4 \\ 19.2 \\ 0 \end{bmatrix} \qquad c_1 = 0 \quad c_4 = 0$$

$4c_2 + c_3 = 38.4 \Rightarrow c_2 + 4c_3 = 19.2 \rightarrow 4c_2 + 16c_3 = 76.8$

$(4c_2 + 16c_3) - (4c_2 + c_3) = 76.8 - 38.4$

$15c_3 = 38.4 \rightarrow c_3 = \frac{38.4}{15} = 2.56$ $\quad c_3 = 2.56 \rightarrow 4c_2 + c_3 = 38.4$

$4c_2 + 2.56 = 38.4 \rightarrow 4c_2 = 38.4 - 2.56 = 35.84$ $\quad c_2 = \frac{35.84}{4} = 8.96$

$c_1 = 0$, $c_2 = 8.96$, $c_3 = 2.56$, $c_4 = 0$

$b_i = \frac{y_{i+1} - y_i}{x_{i+1} + x_i} - \frac{1}{3}(x_{i+1} - x_i)(2c_i + c_{i+1})$ $\quad d_i = \frac{c_{i+1} - c_i}{3(x_{i+1} - x_i)}$

$[0, 0.25]$

$b_1 = \frac{1.4 - 1}{0.25 - 0} - \frac{1}{3}(0.25)(2(0) + 8.96) = 0.853$

$d_1 = \frac{8.96 - 0}{3(0.25)} = 11.947$

$[0.25, 0.5]$

$b_2 = \frac{1.6 - 1.4}{0.5 - 0.25} - \frac{1}{3}(0.25(2(8.96) + 2.56) = -0.9067$

$d_2 = \frac{2.56 - 8.96}{3(0.25)} = -8.533$

$[0.5, 1]$

$b_3 = \frac{2 - 1.6}{1 - 0.5} - \frac{1}{3}(0.5)(2(2.56) + 0) = -0.053$

$d_3 = \frac{0 - 2.56}{3(0.5)} = -1.707$

$[0, 0.25] \Rightarrow S_1(x) = 1 + 0.853x + 11.947x^3$

$[0.25, 0.5] \Rightarrow S_2(x) = 1.4 - 0.9067(x - 0.25) + 8.96(x - 0.25)^2 - 8.533(x - 0.25)^3$

$[0.5, 1] \Rightarrow S_3(x) = 1.6 - 0.053(x - 0.5) + 2.56(x - 0.5)^2 - 1.707(x - 0.5)$

3. Use regression, by hand, to approximate the following data set x = [0 1 8 12 27] and y = [1 2 3 4 5] and plot the results using.,

   a. Linear fit

Asig 11 : Part 3 : Section a

$x = [0, 1, 8, 12, 27], \quad y = [1, 2, 3, 4, 5]$

$y = a + bx$

$$a = \frac{\Sigma y \cdot \Sigma x^2 - \Sigma x \cdot \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{n \Sigma xy - \Sigma x \cdot \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

$\Sigma x = 0 + 1 + 8 + 12 + 27 = 48$

$\Sigma y = 1 + 2 + 3 + 4 + 5 = 15$

$\Sigma x^2 = 0^2 + 1^2 + 8^2 + 12^2 + 27^2 = 0 + 1 + 64 + 144 + 729 = 938$

$\Sigma xy = (0 \times 1) + (1 \times 2) + (8 \times 3) + (12 \times 4) + (27 \times 5) = 0 + 2 + 24 + 48 + 135 = 209$

$$a = \frac{(15)(938) - (48)(209)}{5(938) - (48)^2} = \frac{14070 - 10032}{4690 - 2304} = \frac{4038}{2386} = 1.692$$

$$b = \frac{5(209) - (48)(15)}{5(938) - (48)^2} = \frac{1045 - 720}{4690 - 2304} = \frac{325}{2386} = 0.1363$$

$y = 1.692 + 0.1363x$

b. Quadratic fit

Asig 11: Part 3: Section b

$y = a + bx + cx^2$

$\sum x^3 = 0^3 + 1^3 + 8^3 + 12^3 + 27^3 = 0 + 1 + 512 + 1728 + 19683 = 21924$

$\sum x^4 = 0^4 + 1^4 + 8^4 + 12^4 + 27^4 = 0 + 1 + 4096 + 20736 + 531441 = 556274$

$\sum x^2 y = (0^2 \times 1) + (1^2 \times 2) + (8^2 \times 3) + (12^2 \times 4) + (27^2 \times 5) = 0 + 2 + (64 \times 3) + (144 \times 4) + (729 \times 5)$

$= 0 + 2 + 192 + 576 + 3645 = 4415$

$5a + 48b + 938c = 15$

$48a + 938b + 21924c = 209$

$938a + 21924b + 556274c = 4415$

$(48a + 938b + 21924c) - 9.6(5a + 48b + 938c) = 209 - 9.6(15)$

$48a + 938b + 21924c - (48a + 460.8b + 9004.8c) = 209 - 144$

$0 + 477.2b + 12919.2c = 65$

$(938a + 21924b + 556274c) - 187.6(5a + 48b + 938c) = 4415 - 187.6(15)$

$938a + 21924b + 556274c - (938a + 9004.8b + 17594.88c) = 4415 - 2814$

$0 + 12919.2b + 538679.12c = 1601$

$5a + 48b + 938c = 15$

$477.2b + 12919.2c = 65$

$12919.2b + 538679.12c = 1601$

$(12919.2b + 538679.12c) - 27.07(477.2b + 12919.2c) = 1601 - 27.07(65)$

$12919.2b + 538679.12c - (12919.2b + 129855.804c) = 1601 - 1759.55$

$0 + 408823.316c = -158.55$

$c = \frac{-158.55}{408823.316} = -3.878 \times 10^{-4}$

$477.2b + 12919.2(-3.878 \times 10^{-4}) = 64$

$477.2b - 5.012 = 65$

$477.2b = 70.012 \rightarrow b = \frac{70.012}{477.2} = 0.1467$

$5a + 48(0.1467) + 938(-3.878 \times 10^{-4}) = 15$

$5a + 7.0416 - 0.3639 = 15$

$5a + 6.6777 = 15 \rightarrow 5a = 8.3223 \rightarrow a = \frac{8.3223}{5} = 1.6645$

$y = 1.6645 + 0.1467x - 3.878 \times 10^{-4} x^2$

C. Exponential fit

Asig 11 : Part 3 : Section C

$y = Ae^{Bx}$

$\ln(y) = \ln(A) + Bx$

$\ln(y) = [0, 0.6931, 1.0986, 1.3863, 1.6094]$

$x = [0, 1, 8, 12, 27]$, $\ln(y) = [0, 0.6931, 1.0986, 1.3863, 1.6094]$

$\sum \ln(y) = 0 + 0.6931 + 1.0986 + 1.3863 + 1.6094 = 4.7874$

$\sum x\ln(y) = (0 \times 0) + (1 \times 0.6931) + (8 \times 1.0986) + (12 \times 1.3863) + (27 \times 1.6094)$

$= 0 + 0.6931 + 8.7888 + 16.6356 + 43.4548 = 69.5723$

$$B = \frac{5(69.5723) - (48)(4.7874)}{5(938) - (48)^2}$$

$$B = \frac{347.8615 - 229.7952}{4690 - 2304} = \frac{118.0663}{2386} = 0.0495$$

$$\ln(A) = \frac{(4.7874)(938) - (48)(69.5723)}{5(938 - (48)^2}$$

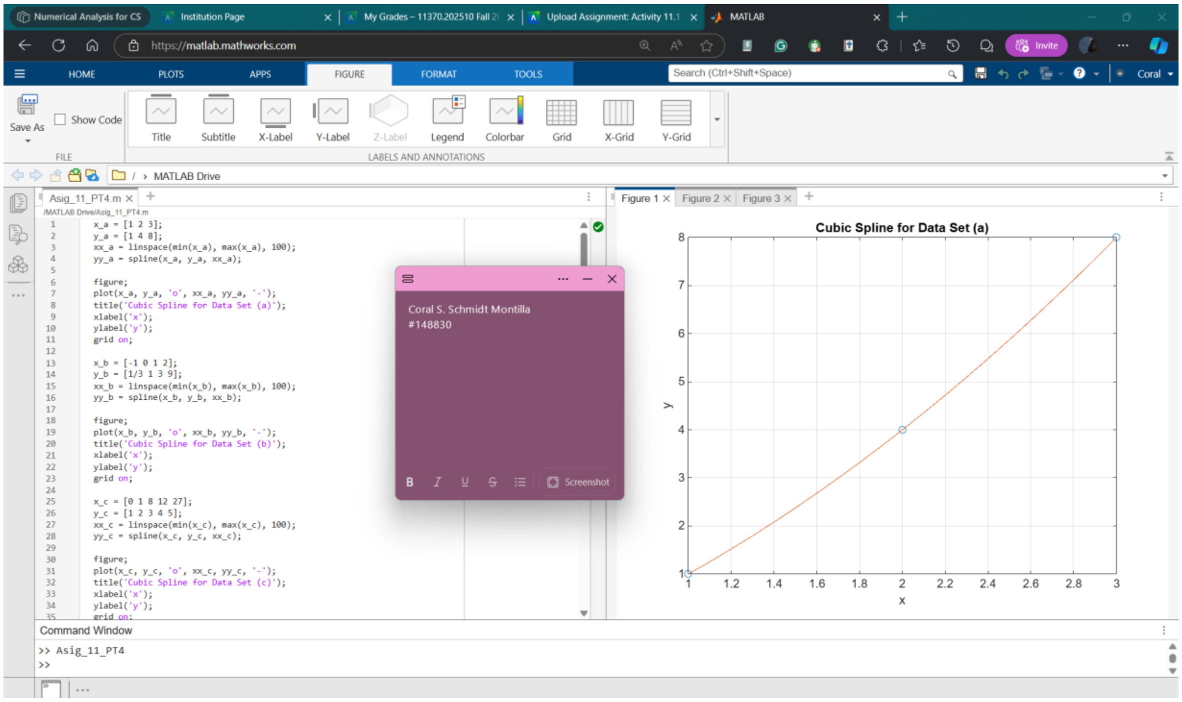
$$\ln(A) = \frac{4488.4737 - 3339.4608}{4690 - 2304} = \frac{1149.0129}{2386} = 0.4816$$

$A = e^{0.4816} = 1.618$

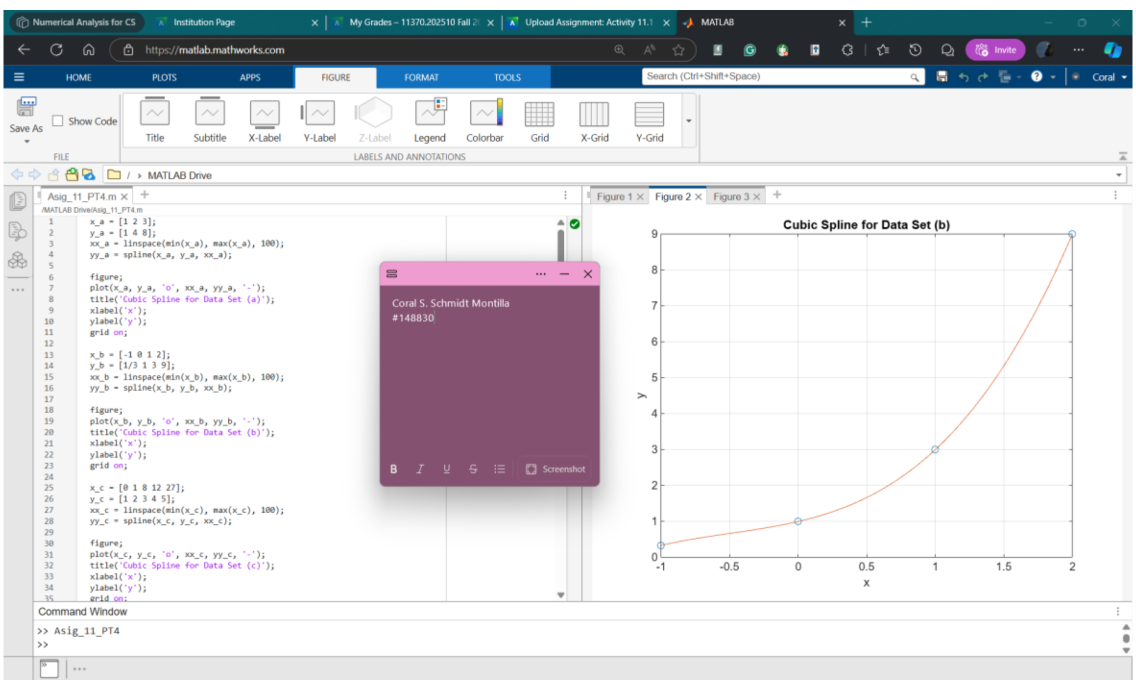
$y = 1.618e^{0.0495x}$

4. Create a MATLAB Script to do a cubic spline using the MATLAB spline function and plot the points and the curve. Test it for the following data sets.

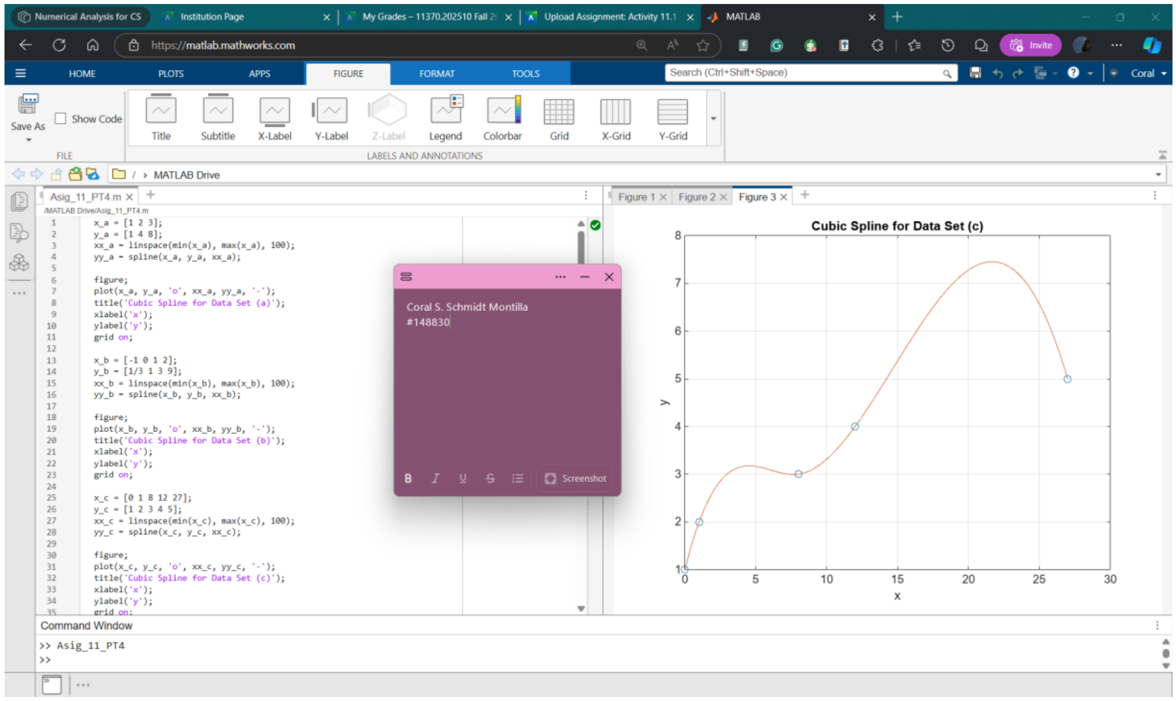
   a. x = [1 2 3] and y = [1 4 8]



   b. x = [-1 0 1 2] and y = [1/3 1 3 9]

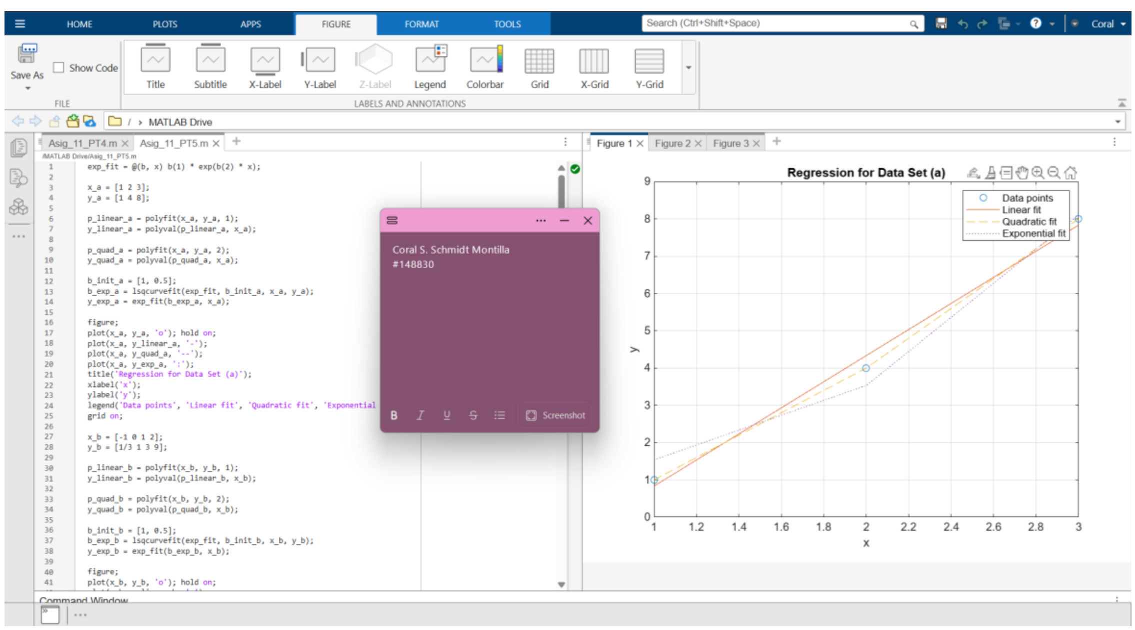


c. x = [0 1 8 12 27] and y = [1 2 3 4 5]

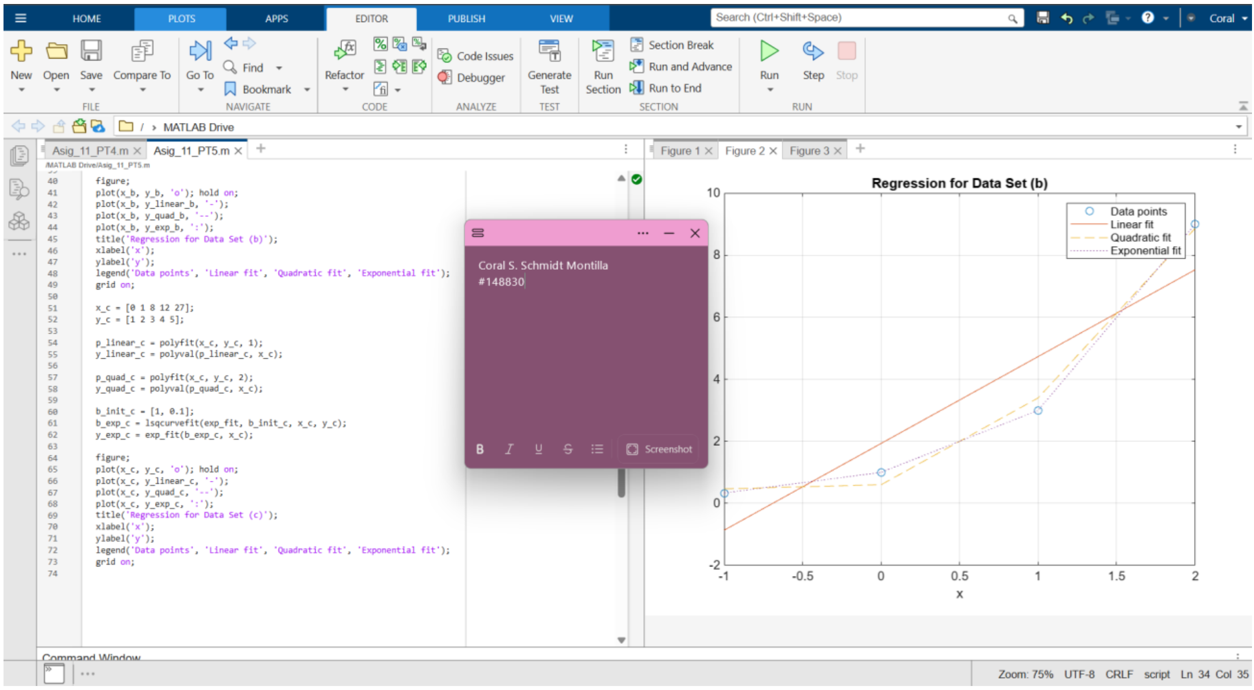


5. Create a MATLAB Script to do a linear, quadratic and exponential regression (using polyfit and lsqcurvefit MATLAB regression functions) and plot the points and the curve. Test it for the following data sets.

a. x = [1 2 3] and y = [1 4 8]



b. x = [-1 0 1 2] and y = [1/3 1 3 9]



c. x = [0 1 8 12 27] and y = [1 2 3 4 5]