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Computer Science

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Numerical analysis for computer science mayors

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1. Suppose that we want to solve the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

and $b = (1,7,16,14)^T$,

a. Do three iterations (by hand) of the Jacobi iteration for this matrix, using x(0) = (0,0,0,0).

Asg 4: Part 1: Part a

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \qquad x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix}$$

$$x_1^{(1)} = \tfrac{1}{4}(1 - 1\cdot 0) = 0.2500$$
$$x_2^{(1)} = \tfrac{1}{5}(7 - 1\cdot 0 - 1\cdot 0) = 1.4000$$
$$x_3^{(1)} = \tfrac{1}{6}(16 - 1\cdot 0 - 1\cdot 0) = 2.6667$$
$$x_4^{(1)} = \tfrac{1}{4}(14 - 1\cdot 0) = 3.5000$$
$$x^{(1)} = \begin{bmatrix} 0.2500 \\ 1.4000 \\ 2.6667 \\ 3.5000 \end{bmatrix}$$

$$x_1^{(2)} = \tfrac{1}{4}(1 - 1\cdot 1.4000) = -0.1000$$
$$x_2^{(2)} = \tfrac{1}{5}(7 - 1\cdot 0.2500 - 1\cdot 2.6667) = 0.8167$$
$$x_3^{(2)} = \tfrac{1}{6}(16 - 1\cdot 1.4000 - 1\cdot 3.5000) = 1.8500$$
$$x_4^{(2)} = \tfrac{1}{4}(14 - 1\cdot 2.6667) = 2.8333$$
$$x^{(2)} = \begin{bmatrix} -0.1000 \\ 0.8167 \\ 1.8500 \\ 2.8333 \end{bmatrix}$$

$$x_1^{(3)} = \tfrac{1}{4}(1 - 1\cdot 0.8167) = 0.0458$$
$$x_2^{(3)} = \tfrac{1}{5}(7 - 1\cdot -0.1000 - 1\cdot 1.8500) = 1.0500$$
$$x_3^{(3)} = \tfrac{1}{6}(16 - 1\cdot 0.8167 - 1\cdot 2.8333) = 2.0583$$
$$x_4^{(3)} = \tfrac{1}{4}(14 - 1\cdot 1.8500) = 3.0375$$
$$x^{(3)} = \begin{bmatrix} 0.0458 \\ 1.0500 \\ 2.0583 \\ 3.0375 \end{bmatrix}$$

b. Do three iterations (by hand) of the Gauss-Seidel iteration for this problem, using the same initial guess.

Asg 4: Part 1: Part b

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \qquad x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix}$$

$$x_1^{(1)} = \tfrac{1}{4}(1 - 1\cdot 0) = 0.2500$$
$$x_2^{(1)} = \tfrac{1}{5}(7 - 1\cdot 0.2500 - 1\cdot 0) = 1.3500$$
$$x_3^{(1)} = \tfrac{1}{6}(16 - 1\cdot 1.3500 - 1\cdot 0) = 2.4417$$
$$x_4^{(1)} = \tfrac{1}{4}(14 - 1\cdot 2.4417) = 2.8896$$
$$x^{(1)} = \begin{bmatrix} 0.2500 \\ 1.3500 \\ 2.4417 \\ 2.8896 \end{bmatrix}$$

$$x_1^{(2)} = \tfrac{1}{4}(1 - 1\cdot 1.3500) = -0.0.875$$
$$x_2^{(2)} = \tfrac{1}{5}(7 - 1\cdot -0.0875 - 1\cdot 2.4417) = 0.9292$$
$$x_3^{(2)} = \tfrac{1}{6}(16 - 1\cdot 0.9292 - 1\cdot 2.8896) = 2.0302$$
$$x_4^{(2)} = \tfrac{1}{4}(14 - 1\cdot 2.0302) = 2.9924$$
$$x^{(2)} = \begin{bmatrix} -0.0875 \\ 0.9292 \\ 2.0302 \\ 2.9924 \end{bmatrix}$$

$$x_1^{(2)} = \tfrac{1}{4}(1 - 1\cdot 0.9292) = 0.0177$$
$$x_2^{(2)} = \tfrac{1}{5}(7 - 1\cdot 0.0177 - 1\cdot 2.0302) = 0.9904$$
$$x_3^{(2)} = \tfrac{1}{6}(16 - 1\cdot 0.9904 - 1\cdot 2.9924) = 2.0029$$
$$x_4^{(2)} = \tfrac{1}{4}(14 - 1\cdot 2.0029) = 2.9993$$
$$x^{(3)} = \begin{bmatrix} 0.0177 \\ 0.9904 \\ 2.0029 \\ 2.9993 \end{bmatrix}$$

c. Do three iterations of SOR for Problem 1, using ω= 1.05.

Asig 4: Part 1: Part c

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 6 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 7 \\ 16 \\ 14 \end{bmatrix} \qquad \omega = 1.05$$

$$x_1^{(1)} = (1-1.05)\cdot 0 + \frac{1.05}{4}\cdot(1-1\cdot 0) = \frac{1.05}{4}\cdot 1 = 0.2625$$
$$x_2^{(1)} = (1-1.05)\cdot 0 + \frac{1.05}{5}\cdot(7-1\cdot 0.2625-1\cdot 0) = \frac{1.05}{5}\cdot 6.7375 = 1.4149$$
$$x_3^{(1)} = (1-1.05)\cdot 0 + \frac{1.05}{6}\cdot(16-1\cdot 1.4149-1\cdot 0) = \frac{1.05}{6}\cdot 14.5851 = 2.5524$$
$$x_4^{(1)} = (1-1.05)\cdot 0 + \frac{1.05}{4}\cdot(14-1\cdot 2.5524) = \frac{1.05}{4}\cdot 11.4476 = 3.0050$$

$$x^{(1)} = \begin{bmatrix} 0.2625 \\ 1.4149 \\ 2.5524 \\ 3.0050 \end{bmatrix}$$

$$x_1^{(2)} = (1-1.05)\cdot 0.2625 + \frac{1.05}{4}\cdot(1-1\cdot 1.4149) = -0.1220$$
$$x_2^{(2)} = (1-1.05)\cdot 1.4149 + \frac{1.05}{5}\cdot(7-1\cdot -0.1220-1\cdot 2.5524) = 0.8889$$
$$x_3^{(2)} = (1-1.05)\cdot 2.5524 + \frac{1.05}{6}\cdot(16-1\cdot 0.8889-1\cdot 3.0050) = 1.9920$$
$$x_4^{(2)} = (1-1.05)\cdot 3.0050 + \frac{1.05}{4}\cdot(14-1\cdot 1.9920) = 3.0021$$

$$x^{(2)} = \begin{bmatrix} -0.1220 \\ 0.8889 \\ 1.9920 \\ 3.0021 \end{bmatrix}$$

$$x_1^{(3)} = (1-1.05)\cdot -0.1220 + \frac{1.05}{4}\cdot(1-1\cdot 0.8889) = 0.0353$$
$$x_2^{(3)} = (1-1.05)\cdot 0.8889 + \frac{1.05}{5}\cdot(7-1\cdot 0.0353-1\cdot 1.9920) = 1.0000$$
$$x_3^{(3)} = (1-1.05)\cdot 1.9920 + \frac{1.05}{6}\cdot(16-1\cdot 1.0000-1\cdot 3.0021) = 2.0001$$
$$x_4^{(3)} = (1-1.05)\cdot 3.0021 + \frac{1.05}{4}\cdot(14-1\cdot 2.0001) = 2.9999$$

$$x^{(3)} = \begin{bmatrix} 0.0353 \\ 1.0000 \\ 2.0001 \\ 2.9999 \end{bmatrix}$$

2. Consider the following nonlinear system.

$$2x_1 - x_2 + \frac{1}{9}e^{-x_1} = -1$$
$$-x_1 + 2x_2 + \frac{1}{9}e^{-x_2} = 1$$

Take $x^{(0)} = (1,1)^T$ and do two iterations of the Newtons method.

Asig 4: Part 2

$$2x_1 - x_2 + \frac{1}{9}e^{-x_1} = -1 \qquad x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$-x_1 + 2x_2 + \frac{1}{9}e^{-x_2} = 1$$

$$f(x^{(0)}) = \begin{bmatrix} 2(1)-1+\frac{1}{9}e^{-1}+1 \\ -1+2(1)+\frac{1}{9}e^{-1}-1 \end{bmatrix} = \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix}$$

$$J(x^{(0)}) = \begin{bmatrix} 1.8966 & -1 \\ -1 & 2.8966 \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix} = \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix}$$

$$f(x^{(1)}) = \begin{bmatrix} 2(-0.4028) - 0.2926 + \frac{1}{9}e^{0.4028} + 1 \\ -(-0.4028) + 2(0.2926) + \frac{1}{9}e^{-0.2926} - 1 \end{bmatrix} = \begin{bmatrix} -0.4782 \\ 0.2224 \end{bmatrix}$$

$$J(x^{(1)}) = \begin{bmatrix} 1.7328 & -1 \\ -1 & 2.7856 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} -0.4028 \\ 0.2926 \end{bmatrix} + \begin{bmatrix} -0.4782 \\ 0.2224 \end{bmatrix} = \begin{bmatrix} -0.8810 \\ 0.5150 \end{bmatrix}$$

3. Use MATLAB to validate the results from previous problem.