



UNIVERSIDAD
POLITÉCNICA
P U E R T O R I C O

Computer Science

Coral S. Schmidt Montilla

#148830

Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. Use Gauss-elimination with back substitution, by hand, to calculate x_1, x_2 and x_3 to solve the following system of equations.

$$2x_1 - x_2 + x_3 = -1$$

$$x_1 + 2x_2 - x_3 = 6$$

$$x_1 - x_2 + 2x_3 = -3$$

Asig 3: Part 1

$$\begin{array}{l} 2x_1 - x_2 + x_3 = -1 \\ x_1 + 2x_2 - x_3 = 6 \\ x_1 - x_2 + 2x_3 = -3 \end{array} \quad \left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 1 & 2 & -1 & 6 \\ 1 & -1 & 2 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 0 & \frac{5}{2} & -\frac{3}{2} & \frac{13}{2} \\ 0 & -\frac{3}{2} & 1 & -2 \end{array} \right] \quad \left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 0 & \frac{5}{2} & -\frac{3}{2} & \frac{13}{2} \\ 0 & 0 & \frac{7}{5} & \frac{-1}{5} \end{array} \right]$$

$$\frac{7}{5}x_3 = \frac{-1}{5}$$

$$x_3 = \frac{-1}{5} \cdot \frac{5}{7} = -\frac{1}{7} \cdot 5 = -1$$

$$\frac{5}{2}x_2 - \frac{3}{2}(-1) = \frac{13}{2}$$

$$x_1 = 1$$

$$\frac{5}{2}x_2 + \frac{3}{2} = \frac{13}{2}$$

$$x_2 = 2$$

$$x_3 = -1$$

$$\frac{5}{2}x_2 = \frac{10}{2}$$

$$x_2 = 2$$

$$2x_1 - 2 + (-1) = -1$$

$$2x_1 - 3 = -1 \rightarrow 2x_1 = 2 \rightarrow x_1 = 1$$

2. Access the following tutorial on solving a system of equations using MATLAB [Matlab Solves System of Equations](#) to corroborate your previous problems solution using MATLAB. Add to the pdf file the MATLAB output for each of the problems.

The screenshot displays the MATLAB R2023a environment. The main window shows a script named 'Asig_3_PT2.m' with the following code:

```
1 A = [2 -1 1;
2     1 2 -1;
3     1 -1 2];
4
5 b = [-1; 6; -3];
6
7 n = length(b);
8 for i = 1:n-1
9     for j = i+1:n
10        m = A(j,i)/A(i,i);
11        A(j,:) = A(j,:) - m*A(i,:);
12        b(j) = b(j) - m*b(i);
13    end
14 end
15
16 x = zeros(n,1);
17 x(n) = b(n)/A(n,n);
18 for i = n-1:-1:1
19    x(i) = (b(i) - A(i,i+1:n)*x(i+1:n))/A(i,i);
20 end
21
22 disp('Solution using Gauss Elimination with Back Substitution:');
23 disp(x);
24
```

The Workspace panel on the left shows the following variables:

Name	Value	Size	Class
A	[2,-1,0,2,5...]	3x3	double
b	[-1,6,5000,...]	3x1	double
i	1	1x1	double
j	3	1x1	double
m	-0.2000	1x1	double
n	3	1x1	double
x	[1,2,-1]	3x1	double

The Command Window shows the output of the script:

```
>> Asig_3_PT2
Solution using Gauss Elimination with Back Substitution:
     1
     2
    -1
>>
```

A yellow sticky note is overlaid on the right side of the editor, containing the text:

Coral S. Schmidt Montilla
#148830

3. Use LU decomposition, by hand, to calculate x_1, x_2 and x_3 to solve the following system of equations using the Doolittle algorithm.

$$\begin{bmatrix} 2 & 4 & 3 \\ -4 & -7 & -5 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix}$$

Assg 3: Part 3

$$\begin{bmatrix} 2 & 4 & 3 \\ -4 & -7 & -5 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$u_{11} = 2 \quad u_{12} = 4 \quad u_{13} = 3$$

$$l_{21} = \frac{-4}{2} = -2$$

$$\begin{bmatrix} -4 & -7 & -5 \end{bmatrix} + 2 \cdot \begin{bmatrix} 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

$$u_{22} = 1 \quad u_{23} = 1$$

$$l_{31} = \frac{6}{2} = 3$$

$$\begin{bmatrix} 6 & 8 & 2 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -7 \end{bmatrix}$$

$$l_{32} = \frac{-4}{1} = -4$$

$$u_{33} = -7 - (-4) \cdot 1 = -7 + 4 = -3$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix}$$

$$y_1 = 5$$

$$-2y_1 + y_2 = -8 \rightarrow -2(5) + y_2 = -8 \rightarrow y_2 = 10 - 8 = 2$$

$$3y_1 - 4y_2 + y_3 = 9 \rightarrow 3(5) - 4(2) + y_3 = 9 \rightarrow 15 - 8 + y_3 = 9 \rightarrow y_3 = 2$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$$

$$-3x_3 = 2 \rightarrow x_3 = -\frac{2}{3}$$

$$x_2 + x_3 = 2 \rightarrow x_2 - \frac{2}{3} = 2 \rightarrow x_2 = \frac{8}{3}$$

$$2x_1 + 4x_2 + 3x_3 = 5 \rightarrow 2x_1 + 4 \cdot \frac{8}{3} + 3 \cdot \left(-\frac{2}{3}\right) = 5$$

$$2x_1 + 10.6668 - 2.0001 = 5 \rightarrow 2x_1 + 8.6667 = 5 \rightarrow 2x_1 = 5 - 8.6667 \rightarrow 2x_1 = -3.6667$$

$$x_1 = \frac{-3.6667}{2}$$

$$x_1 = \frac{-3.6667}{2} \approx -1.8333$$

$$x_2 = \frac{8}{3} \approx 2.6667$$

$$x_3 = -\frac{2}{3} \approx -0.6667$$

4. Use LU decomposition, by hand, to calculate x_1, x_2 and x_3 to solve the following system of equations using the Crout algorithms.

$$\begin{bmatrix} 2 & 1 & 9 \\ 7 & -2 & -8 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

Asig 3: Part 4

$$\begin{bmatrix} 2 & 1 & 9 \\ 7 & -2 & -8 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$$l_{11} = 2 \quad l_{21} = \frac{7}{2} = 3.5 \quad l_{31} = \frac{3}{2} = 1.5 \quad u_{12} = \frac{1}{2} = 0.5 \quad u_{13} = \frac{9}{2} = 4.5$$

$$l_{22} = -2 - l_{21} \cdot u_{12} = -2 - 3.5 \cdot 0.5 = -2 - 1.75 = -3.75$$

$$u_{23} = \frac{-8 - l_{21} \cdot u_{13}}{l_{22}} = \frac{-8 - 3.5 \cdot 4.5}{-3.75} = \frac{-8 - 15.75}{-3.75} = \frac{-23.75}{-3.75} = 6.3333$$

$$l_{32} = 6 - l_{31} \cdot u_{12} - l_{32} \cdot u_{23} = 2 - 1.5 \cdot 4.5 - 5.25 \cdot 6.3333 \rightarrow 2 - 6.75 - 33.3333$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 3.5 & -3.75 & 0 \\ 1.5 & 5.25 & -38.0833 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 0.5 & 4.5 \\ 0 & 1 & 6.3333 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 3.5 & -3.75 & 0 \\ 1.5 & 5.25 & -38.0833 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$$y_1 = \frac{-1}{2} = -0.5$$

$$3.5y_1 - 3.75y_2 = -3 \rightarrow 3.5(-0.5) - 3.75y_2 = -3 \rightarrow -1.75 - 3.75y_2 = -3$$

$$y_2 = \frac{-1.25}{-3.75} = \frac{1.25}{3.75} = 0.3333$$

$$1.5y_1 + 5.25y_2 - 38.0833y_3 = 4 \rightarrow 1.5(-0.5) + 5.25(0.3333) - 38.0833y_3 = 4$$

$$-0.75 + 1.75 - 38.0833y_3 = 4 \rightarrow y_3 = \frac{3.0}{-38.0833} = 0.0788$$

$$y = \begin{bmatrix} -0.5 \\ 0.3333 \\ 0.0788 \end{bmatrix}$$

$$x_1 = -0.7714$$

$$x_2 = -0.1664$$

$$x_3 = 0.0788$$

$$\begin{bmatrix} 1 & 0.5 & 4.5 \\ 0 & 1 & 6.3333 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.3333 \\ 0.0788 \end{bmatrix}$$

$$x_3 = 0.0788$$

$$x_2 + 6.3333x_3 = 0.3333 \rightarrow x_2 = 0.3333 - 6.3333(0.0788) = 0.3333 - 0.4997 = -0.1664$$

$$x_1 + 0.5x_2 + 4.5x_3 = -0.5 \rightarrow x_1 = -0.5 - 0.5(-0.1664) - 4.5(0.0788) = -0.5 + 0.0832 - 0.3546 = -0.7714$$

5. Watch the following video [LU Decomposition Using Crout's Method in MatLab](#) and use MATLAB to validate the results from previous problem.

Exercise 3:

The screenshot shows the MATLAB environment with the following details:

- Editor:** Contains a script for `Asig_3_PT5_1.m` with the following code:

```
1 A = [2 4 3; -4 -7 -5; 6 8 2];  
2 b = [5; -8; 9];  
3  
4 [L, U, P] = lu(A);  
5  
6 y = L\(P*b);  
7  
8 x = U\y;  
9  
10 disp('Solution using LU Decomposition (Doolittle):');  
11 disp(x);  
12
```
- Workspace:** Lists variables: A (3x3 double), L (3x3 double), P (3x3 double), U (3x3 double), b (3x1 double), i (1x1 double), j (1x1 double), m (-0.2000), n (3), x (3x1 double), and y (3x1 double).
- Command Window:** Shows the output of the script:

```
>> Asig_3_PT5_1  
Solution using LU Decomposition (Doolittle):  
-1.8333  
2.6667  
-0.6667  
>>
```
- Overlay:** A yellow sticky note with the text "Coral S. Schmidt Montilla #148830".

Exercise 4

The screenshot shows the MATLAB environment with the following details:

- Editor:** Contains a script for `Asig_3_PT5_2.m` with the following code:

```
1 A = [2 1 9; 7 -2 -8; 3 6 2];  
2 b = [-1; -3; 4];  
3  
4 [L, U, P] = lu(A);  
5  
6 y = L\(P*b);  
7  
8 x = U\y;  
9  
10 disp('Solution using LU Decomposition (Crout):');  
11 disp(x);  
12
```
- Workspace:** Lists variables: A (3x3 double), L (3x3 double), P (3x3 double), U (3x3 double), b (3x1 double), i (1x1 double), j (1x1 double), m (-0.2000), n (3), x (3x1 double), and y (3x1 double).
- Command Window:** Shows the output of the script:

```
>> Asig_3_PT5_2  
Solution using LU Decomposition (Crout):  
-0.3320  
0.8776  
-0.1349  
>>
```
- Overlay:** A yellow sticky note with the text "Coral S. Schmidt Montilla #148830".