UNIVERSIDAD POLITÉCNICA PUERTO RICO

Computer Science

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Numerical analysis for computer science mayors

FA 2024 CS3010-80

Assignment Problems

1. Use Gauss-elimination with back substitution, by hand, to calculate $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ to solve the following system of equations.

$$2x_1 - x_2 + x_3 = -1$$
$$x_1 + 2x_2 - x_3 = 6$$
$$x_1 - x_2 + 2x_3 = -3$$

Asig 3: Part 1

$$2x_1 - x_2 + x_3 = -1$$
$$x_1 + 2x_2 - x_3 = 6$$
$$x_1 - x_2 + 2x_3 = -3$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 1 & 2 & -1 & 6 \\ 1 & -1 & 2 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 0 & \frac{5}{2} & -\frac{3}{2} & \frac{13}{2} \\ 0 & -\frac{1}{2} & 1 & -2 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 0 & \frac{5}{2} & -\frac{3}{2} & \frac{13}{2} \\ 0 & 0 & \frac{7}{5} & \frac{-1}{5} \end{array}\right]$$

$$\frac{7}{5}x_3 = \frac{-1}{5}$$

$$x_3 = \frac{-1}{5} \cdot \frac{5}{7} = -\frac{1}{7} \cdot 5 = -1$$

$$\frac{5}{2}x_2 - \frac{3}{2}(-1) = \frac{13}{2}$$

$$\frac{5}{2}x_2 + \frac{3}{2} = \frac{13}{2}$$

$$\frac{5}{2}x_2 = \frac{10}{2}$$

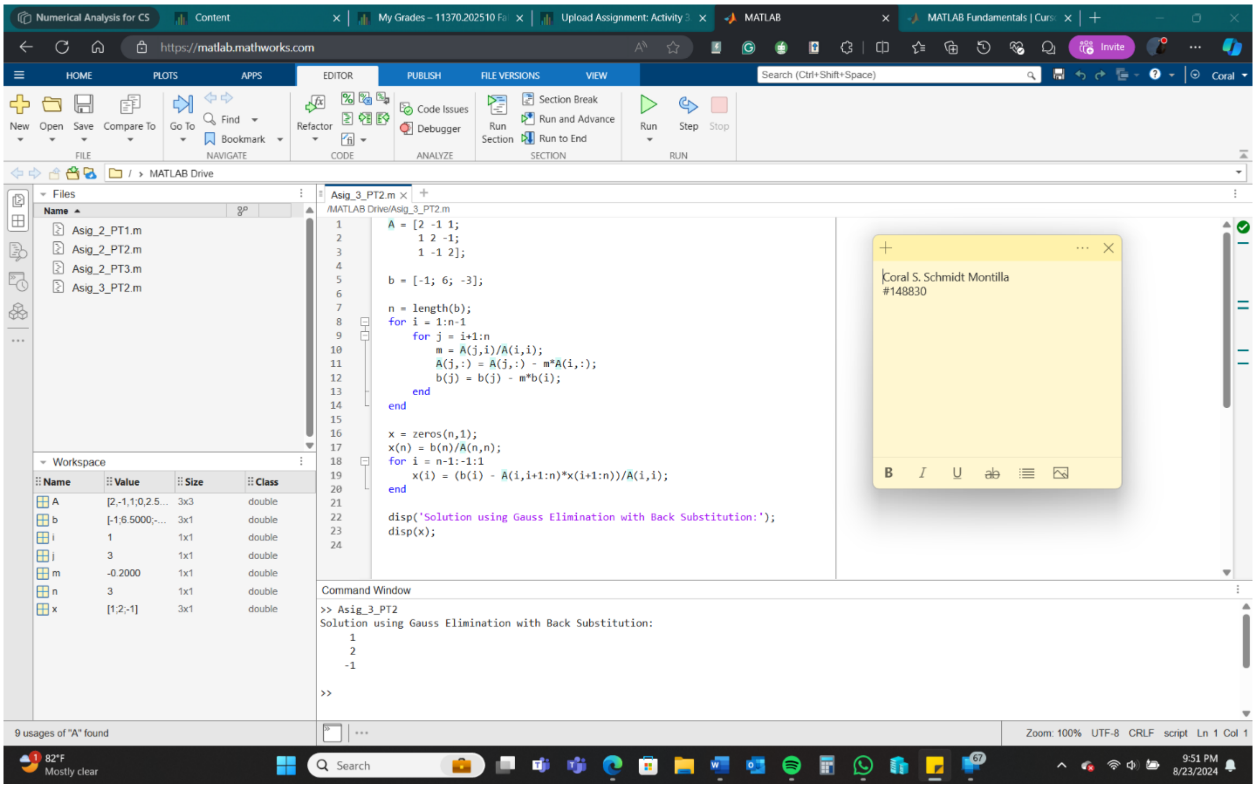
$$x_2 = 2$$

$$2x_1 - 2 + (-1) = -1$$

$$2x_1 - 3 = -1 \rightarrow 2x_1 = 2 \rightarrow x_1 = 1$$

$$x_1 = 1$$
$$x_2 = 2$$
$$x_3 = -1$$

2. Access the following tutorial on solving a system of equations using MATLAB [Matlab Solves System of Equations](#) to corroborate your previous problems solution using MATLAB. Add to the pdf file the MATLAB output for each of the problems.



3. Use LU decomposition, by hand, to calculate $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_3$ to solve the following system of equations using the Doolittle algorithm.

$$\begin{bmatrix} 2 & 4 & 3 \\ -4 & -7 & -5 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix}$$

Asig 3: Part 3

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 3 \\ -4 & -7 & -5 \\ 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix} \quad u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = 2 \quad u_{12} = 4 \quad u_{13} = 3$

$l_{21} = \frac{-4}{2} = -2$

$[-4\ -7\ -5] + 2 \cdot [2\ 4\ 3] = [0\ 1\ 1]$

$u_{22} = 1 \quad u_{23} = 1$

$l_{31} = \frac{6}{2} = 3$

$[6\ 8\ 2] - 3 \cdot [2\ 4\ 3] = [0\ -4\ -7]$

$l_{32} = \frac{-4}{1} = -4$

$u_{33} = -7 - (-4) \cdot 1 = -7 + 4 = -3$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 9 \end{bmatrix}$$

$y_1 = 5$

$-2y_1 + y_2 = -8 \rightarrow -2(5) + y_2 = -8 \rightarrow y_2 = 10 - 8 = 2$

$3y_1 - 4y_2 + y_3 = 9 \rightarrow 3(5) - 4(2) + y_3 = 9 \rightarrow 15 - 8 + y_3 = 9 \rightarrow y_3 = 2$

$$\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$$

$x_1 = \frac{-3.6667}{2} \approx -1.8333$

$x_2 = \frac{8}{3} \approx 2.66667$

$x_3 = -\frac{2}{3} \approx -0.66667$

$-3x_3 = 2 \Rightarrow x_3 = -\frac{2}{3}$

$x_2 + x_3 = 2 \rightarrow x_2 - \frac{2}{3} = 2 \rightarrow x_2 = \frac{8}{3}$

$2x_1 + 4x_2 + 3x_3 = 5 \rightarrow 2x_1 + 4 \cdot \frac{8}{3} + 3 \cdot (-\frac{2}{3}) = 5$

$2x_1 + 10.6668 - 2.0001 = 5 \rightarrow 2x_1 + 8.6667 = 5 \rightarrow 2x_1 = 5 - 8.6667 \rightarrow 2x_1 = -3.6667$

$x_1 = \frac{-3.6667}{2}$

4. Use LU decomposition, by hand, to calculate $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ to solve the following system of equations using the Crout algorithms.

$$\begin{bmatrix} 2 & 1 & 9 \\ 7 & -2 & -8 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

Asig 3: Part 4

$$\begin{bmatrix} 2 & 1 & 9 \\ 7 & -2 & -8 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$l_{11} = 2 \quad l_{21} = \frac{7}{2} = 3.5 \quad l_{31} = \frac{3}{2} = 1.5 \quad u_{12} = \frac{1}{l_{11}} = \frac{1}{2} = 0.5 \quad u_{13} = \frac{9}{l_{11}} = \frac{9}{2} = 4.5$

$l_{22} = -2 - l_{21} \cdot u_{12} = -2 - 3.5 \cdot 0.5 = -2 - 1.75 = -3.75$

$u_{23} = \frac{-8 - l_{21} \cdot u_{13}}{l_{22}} = \frac{-8 - 3.5 \cdot 4.5}{-3.75} = \frac{-8 - 15.75}{-3.75} = \frac{-23.75}{-3.75} = 6.3333$

$l_{32} = 6 - l_{31} \cdot u_{13} - l_{32} \cdot u_{23} = 2 - 1.5 \cdot 4.5 - 5.25 \cdot 6.3333 \rightarrow 2 - 6.75 - 33.333$ $\boxed{-38.0833}$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 3.5 & -3.75 & 0 \\ 1.5 & 5.25 & -38.0833 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 0.5 & 4.5 \\ 0 & 1 & 6.3333 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 3.5 & -3.75 & 0 \\ 1.5 & 5.25 & -38.0833 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$y_1 = \frac{-1}{2} = -0.5$

$3.5y_1 - 3.75y_2 = -3 \rightarrow 3.5(-0.5) - 3.75y_2 = -3 \rightarrow -1.75 - 3.75y_2 = -3$

$y_2 = \frac{-1.25}{-3.75} = \frac{1.25}{3.75} = 0.3333$

$1.5y_1 + 5.25y_2 - 38.0833y_3 = 4 \rightarrow 1.5(-0.5) + 5.25(0.3333) - 38.0833y_3 = 4$

$-0.75 + 1.75 - 38.0833y_3 = 4 \rightarrow y_3 = \frac{3.0}{38.0833} = 0.0788$

$$y = \begin{bmatrix} -0.5 \\ 0.3333 \\ 0.0788 \end{bmatrix}$$

$x_1 = -0.7714$

$x_2 = -0.1664$

$x_3 = 0.0788$

$$\begin{bmatrix} 1 & 0.5 & 4.5 \\ 0 & 1 & 6.3333 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.3333 \\ 0.0788 \end{bmatrix}$$

$x_3 = 0.0788$

$x_2 + 6.3333x_3 = 0.3333 \rightarrow x_2 = 0.3333 - 6.3333(0.0788) = 0.3333 - 0.4997 = -0.1664$

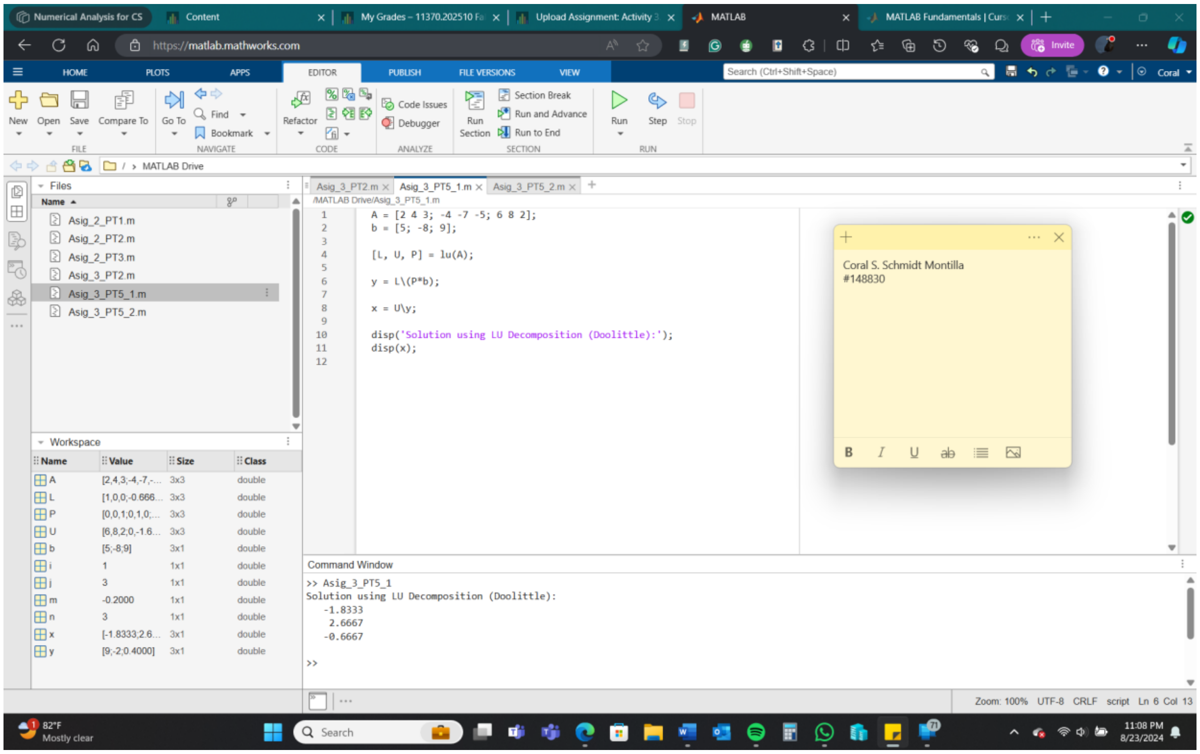
$x_1 + 0.5x_2 + 4.5x_3 = -0.5 \rightarrow x_1 = -0.5 - 0.5(-0.1664) - 4.5(0.0788) =$

$-0.5 + 0.0832 - 0.3546$

-0.7724

5. Watch the following video LU Decomposition Using Crout's Method in MatLab and use MATLAB to validate the results from previous problem.

Exercise 3:



Exercise 4