TSK_regression_Florian_Huhnd

September 24, 2025

[1]:
```
import numpy as np
from sklearn import datasets
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import␣
↪mean_squared_error,accuracy_score,classification_report
import skfuzzy as fuzz
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
```

[2]:
```
# Regression dataset - diabetes
diabetes = datasets.load_diabetes(as_frame=True)
X = diabetes.data.values
y = diabetes.target.values
```

[3]:
```
#train test spliting
test_size=0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size,␣
↪random_state=42)
```

[4]:
```
# Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)
```

So far, the diabetes dataset has been loaded, randomly divided into 80% training data and 20% percent test data and scaled. Different methods such as mini batch training or stochastic batches could have been used here, but there is really no need to change anything at this point.

In the following, the number of the clusters and the smoothness of each cluster m can be tuned. I decided to use a grid search to find a suitable combination of parameters, which can be found at the end of this notebook.

Note that the targets are included in the training data and therefore act like an additional feature. Since we don't use clustering for unsupervised pattern recognition it is highly recommandable to do so, as it is very useful information and improves the performance, as seen in the comparison: The best result obtained including the targets is an MSE of 2476, without the targets it's only

2492. The effect might increase on bigger or complex data.

[5]:
```
 # Number of clusters
n_clusters = 4
m=1.1

# Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)

# Comparison: without target
# Xexp=Xtr

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

# Fuzzy C-means clustering
centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m=m, error=0.005, maxiter=1000, init=None,
)
```

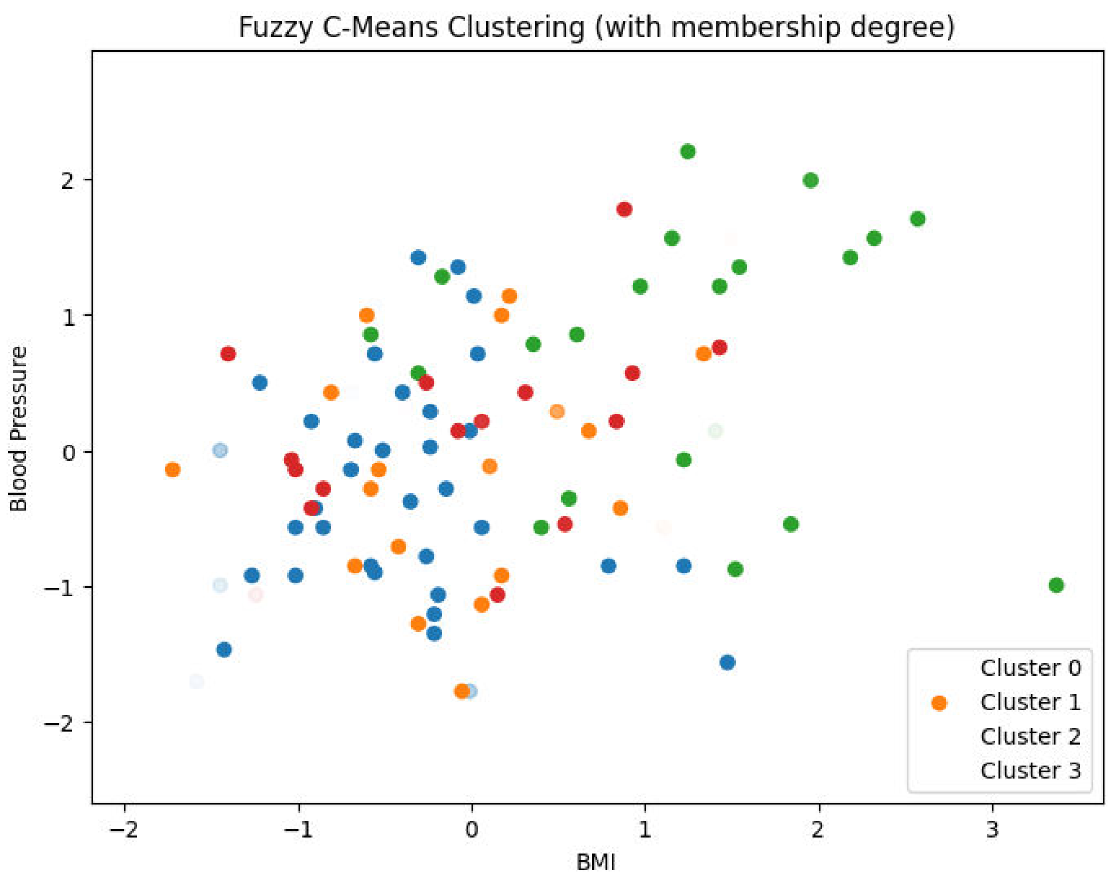
[6]:
```
# Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)
    sigma_j = np.sqrt(var_j)
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)
```

The following plot shows two out of ten features and their clustering. Since we cannot comprehend more than three (let alone ten) dimensions visually, it is obvious that the information in the plot is not complete. However, choosing suitable features, you can get reasonable results: Choosing the features BMI and blood pressure clearly leads to some causality. People with a high BMI seem to have higher blood pressure, whereas people with a low BMI tend to have lower blood pressure. It is likely, that the first group will also have a higher risk of diabetes, so this is indeed a reasonable clustering. Opposing results can be found using non-correlating features such as gender (Feature 1) and age (Feature 0). Since there is no meaningful relation such as “if you are young, you are more likely to be a woman”, the (visualization of the) data (and the clusters) does not give any helpful information.

[7]:
```
# Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

# Plot two features with fuzzy membership
plt.figure(figsize=(8,6))
```

```
for j in range(n_clusters):
    plt.scatter(
        # Xexp[cluster_labels == j, 0],              # Feature 1 = Age
        # Xexp[cluster_labels == j, 1],              # Feature 2 = Gender
        Xexp[cluster_labels == j, 2],              # Feature 3 = BMI
        Xexp[cluster_labels == j, 3],              # Feature 4 = Blood Pressure
        alpha=u[j, :],          # transparency ~ membership
        label=f'Cluster {j}'
    )

plt.title("Fuzzy C-Means Clustering (with membership degree)")
plt.xlabel("BMI")
plt.ylabel("Blood Pressure")
plt.legend()
plt.show()
```

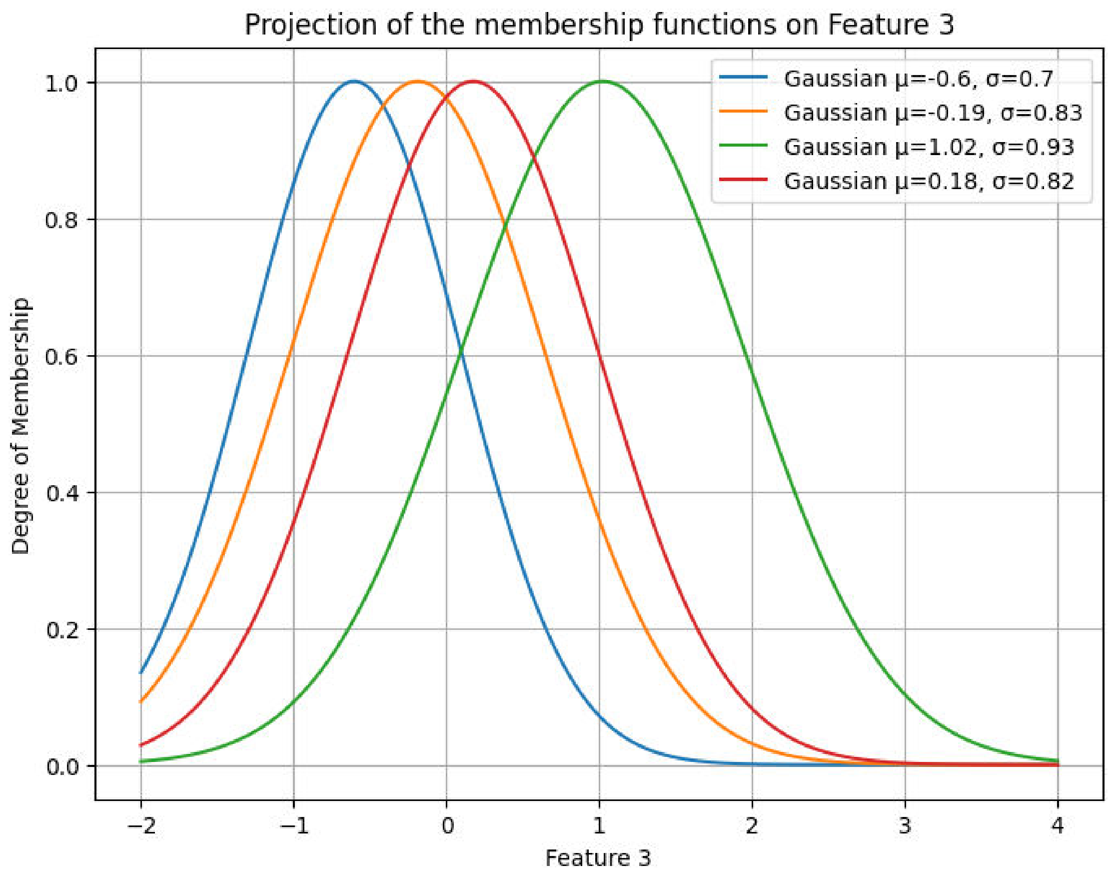
```
Fuzzy partition coefficient (FPC): 0.982773173443856
```



The following plot projects the membership functions on a single feature, which, in other words, shows how easy separatable rules can explain the membership to a certain cluster. Using the BMI again you can easily separate the 4 curves. People with a high BMI have a high membership degree to the cluster on the right, which corresponds to people with a tendency to high blood pressure as well. Remember that there are still 8 (9 including the target) further features that affect the building of the clusters and might reason the choice of 4 clusters.

Using the gender as the chosen feature in this plot yields four almost similar curves, which tells you that the gender does not significantly influence the membership to a certain cluster. Therefore it could be a good idea to remove this feature for further calculations to reduce computational cost.

[8]:
```
# Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)

lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
for j in range(n_clusters):

# Select feature to be shown
    ft=3  # e.g. Feature 3 = BMI; Feature 2 = Gender
    fti=ft-1  # index correction

# Compute curves
    y_aux.append(gaussian(lin, centers[j,fti], sigmas[j,fti]))

# Plot
    plt.plot(lin, y_aux[j], label=f"Gaussian  ={np.round(centers[j,fti],2)}, ={np.round(sigmas[j,fti],2)}")

plt.title(f"Projection of the membership functions on Feature {ft}")
plt.xlabel(f"Feature {ft}")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()
```



[9]:
```python
# ---------------------------
# Gaussian Membership Function
# ---------------------------
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1,␣
↪n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.
↪unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
```

```python
            dist = torch.norm(diff, dim=-1)  # (batch, n_rules) # probablistic intersection
        else:
            dist = torch.max(diff, dim=-1).values  # (batch, n_rules) # min intersection (min instersection of normal funtion is the same as the max on dist)

        return torch.exp(-0.5 * dist ** 2)


# ---------------------------
# TSK Model
# ---------------------------
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )

    def forward(self, x):
        # x: (batch, n_inputs)
        batch_size = x.shape[0]

        # Compute membership values for each input feature
        # firing_strengths: (batch, n_rules)
        firing_strengths = self.mfs(x)

        # Normalize memberships
        # norm_fs: (batch, n_rules)
        norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True) + 1e-9)

        # Consequent output (linear model per rule)
        x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1)  # add bias

        rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents)  # (batch, rules)
```

```
        # Weighted sum
        output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

        return output, norm_fs, rule_outputs
```

[10]:
```
# ---------------------------
# Least Squares Solver for Consequents (TSK)
# ---------------------------
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta= torch.linalg.lstsq(Phi, y).solution


        model.consequents.data = theta.reshape(model.consequents.shape)
```

[11]:
```
# Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:,:-1],␣
 ↪sigmas=sigmas[:,:-1])
# model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers,␣
 ↪sigmas=sigmas) # For comparison without target


Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)
```

[12]:
```
# Training with LS:
train_ls(model, Xtr, ytr.reshape(-1,1))
```

In the previous cells Fuzzy C-Means was used to initialize Gaussian membership functions (MFs) and a TSK (Takagi-Sugeno-Kang) model with Gaussian antecedents and linear consequents was built. The consequents were fit with a Least Squares solver and the model gets trained on the training data set. I did not change anything here, so I keep this comment short.

Finally, the accuracy gets tested as the mean squared error from the performance on the test data set. A lower MSE translates to a better performance.

[13]:
```
y_pred, _, _=model(Xte)
print(f'MSE:{mean_squared_error(yte.detach().numpy(),y_pred.detach().
↪numpy())}') #regression
```

```
MSE:2476.394775390625
```

The following grid search method was implemented by MS Copilot. The results clearly suggest the clustering into 4 clusters.

Best grid-search result (lowest MSE): n_clusters: 4, m: 1.1, mse: 2476.780273, fpc: 0.982749

Top 5 results:

n_clusters / m / mse / fpc

4 / 1.10 / 2476.780273 / 0.982749

4 / 1.50 / 2484.402832 / 0.915329

4 / 1.75 / 2489.548340 / 0.852930

4 / 1.30 / 2495.058350 / 0.952570

2 / 1.10 / 2522.157715 / 0.991280

[14]:
```
# ---------------------------
# Grid search for optimal number of clusters and fuzzifier m
# ---------------------------
# Parameters requested by user
n_clusters_list = [2, 3, 4, 5, 6, 7, 8]
m_list = [1.1, 1.3, 1.5, 1.75, 2.0, 2.25, 2.5]

# Ensure required variables exist: Xtr, ytr, Xte, yte, TSK, train_ls
try:
    Xtr_np = Xtr.detach().numpy() if isinstance(Xtr, torch.Tensor) else np.
↪array(Xtr)
    ytr_np = ytr.detach().numpy().reshape(-1, 1) if isinstance(ytr, torch.
↪Tensor) else np.array(ytr).reshape(-1,1)
    Xte_np = Xte.detach().numpy() if isinstance(Xte, torch.Tensor) else np.
↪array(Xte)
    yte_np = yte.detach().numpy().reshape(-1, 1) if isinstance(yte, torch.
↪Tensor) else np.array(yte).reshape(-1,1)
except Exception as e:
    raise RuntimeError('Required tensors Xtr, ytr, Xte, yte are not defined or␣
↪have wrong type') from e

results = []
import time
start_time = time.time()

# Use concatenated training input+target for clustering as in the notebook
Xexp = np.concatenate([Xtr_np, ytr_np], axis=1)
```

```
Xexp_T = Xexp.T

for n_c in n_clusters_list:
    for m_val in m_list:
        try:
            centers_tmp, u_tmp, u0, d, jm, p, fpc_tmp = fuzz.cluster.cmeans(
                Xexp_T, n_c, m=m_val, error=0.005, maxiter=1000, init=None
            )

            # Compute sigmas per cluster (weighted variance)
            sigmas_tmp = []
            for j in range(n_c):
                u_j = u_tmp[j, :] ** m_val
                var_j = np.average((Xexp - centers_tmp[j])**2, axis=0,␣
 ↪weights=u_j)
                sigma_j = np.sqrt(var_j)
                sigmas_tmp.append(sigma_j)
            sigmas_tmp = np.array(sigmas_tmp)

            # Build TSK using only input dims (exclude appended target column)
            model_tmp = TSK(n_inputs=Xtr.shape[1], n_rules=n_c,␣
 ↪centers=centers_tmp[:,:-1], sigmas=sigmas_tmp[:,:-1])

            # Train consequents with LS
            train_ls(model_tmp, torch.tensor(Xtr_np, dtype=torch.float32),␣
 ↪torch.tensor(ytr_np, dtype=torch.float32))

            # Evaluate on test set
            y_pred_tmp, _, _ = model_tmp(torch.tensor(Xte_np, dtype=torch.
 ↪float32))
            mse = mean_squared_error(yte_np, y_pred_tmp.detach().numpy())

            results.append({'n_clusters': n_c, 'm': m_val, 'mse': float(mse),␣
 ↪'fpc': float(fpc_tmp)})
            print(f'n_clusters={n_c}, m={m_val}, mse={mse:.6f}, fpc={fpc_tmp:.
 ↪6f}')
        except Exception as e:
            print(f'Skipped n_clusters={n_c}, m={m_val} due to error: {e}')

# Summarize results
import pandas as pd
if len(results) == 0:
    print('Grid search produced no results')
else:
    df = pd.DataFrame(results)
    df_sorted = df.sort_values('mse')
```

```python
    best = df_sorted.iloc[0]
    print('\nBest grid-search result (lowest MSE):')
    print(f"n_clusters: {int(best['n_clusters'])}, m: {best['m']}, mse:␣
↪{best['mse']:.6f}, fpc: {best.get('fpc', np.nan):.6f}")

    print('\nTop 5 results:')
    print(df_sorted.head(5).to_string(index=False))

end_time = time.time()
print(f'Grid search finished in {end_time - start_time:.1f}s')
```

```
n_clusters=2, m=1.1, mse=2522.159668, fpc=0.991280
n_clusters=2, m=1.3, mse=2526.612061, fpc=0.967866
n_clusters=2, m=1.5, mse=2534.136230, fpc=0.939724
n_clusters=2, m=1.75, mse=2541.739746, fpc=0.898885
n_clusters=2, m=2.0, mse=2545.299561, fpc=0.855622
n_clusters=2, m=2.25, mse=2547.379395, fpc=0.814105
n_clusters=2, m=2.5, mse=2549.216553, fpc=0.776365
n_clusters=3, m=1.1, mse=2971.822266, fpc=0.986464
n_clusters=3, m=1.3, mse=2939.822266, fpc=0.955597
n_clusters=3, m=1.5, mse=2929.264648, fpc=0.921744
n_clusters=3, m=1.75, mse=2928.722168, fpc=0.868612
n_clusters=3, m=2.0, mse=2933.065430, fpc=0.807699
n_clusters=3, m=2.25, mse=2938.150146, fpc=0.746527
n_clusters=3, m=2.5, mse=2942.795166, fpc=0.689919
n_clusters=4, m=1.1, mse=2476.766357, fpc=0.982750
n_clusters=4, m=1.3, mse=2495.099854, fpc=0.952573
n_clusters=4, m=1.5, mse=2484.392578, fpc=0.915329
n_clusters=4, m=1.75, mse=2489.347656, fpc=0.852928
n_clusters=4, m=2.0, mse=2528.567139, fpc=0.780318
n_clusters=4, m=2.25, mse=2581.445312, fpc=0.707394
n_clusters=4, m=2.5, mse=2635.509277, fpc=0.640102
n_clusters=5, m=1.1, mse=2723.531982, fpc=0.983875
n_clusters=5, m=1.3, mse=2543.464355, fpc=0.946086
n_clusters=5, m=1.5, mse=2562.904541, fpc=0.897049
n_clusters=5, m=1.75, mse=2709.394531, fpc=0.830813
n_clusters=5, m=2.0, mse=2648.436523, fpc=0.743869
n_clusters=5, m=2.25, mse=2688.695557, fpc=0.663031
n_clusters=5, m=2.5, mse=2724.379639, fpc=0.589097
n_clusters=6, m=1.1, mse=2824.114014, fpc=0.981785
n_clusters=6, m=1.3, mse=2813.566650, fpc=0.946266
n_clusters=6, m=1.5, mse=2806.145508, fpc=0.898953
n_clusters=6, m=1.75, mse=2810.644775, fpc=0.825335
n_clusters=6, m=2.0, mse=2797.997314, fpc=0.738494
n_clusters=6, m=2.25, mse=2761.836670, fpc=0.648651
n_clusters=6, m=2.5, mse=2720.177490, fpc=0.565628
n_clusters=7, m=1.1, mse=2892.652832, fpc=0.985260
```

```
n_clusters=7, m=1.3, mse=2889.382812, fpc=0.946551
n_clusters=7, m=1.5, mse=2850.498535, fpc=0.898906
n_clusters=7, m=1.75, mse=2774.795166, fpc=0.819996
n_clusters=7, m=2.0, mse=2759.968506, fpc=0.726035
n_clusters=7, m=2.25, mse=2763.164795, fpc=0.630691
n_clusters=7, m=2.5, mse=2818.480957, fpc=0.542531
n_clusters=8, m=1.1, mse=3083.205811, fpc=0.985915
n_clusters=8, m=1.3, mse=3395.221924, fpc=0.942329
n_clusters=8, m=1.5, mse=2997.790283, fpc=0.887911
n_clusters=8, m=1.75, mse=3765.588135, fpc=0.805320
n_clusters=8, m=2.0, mse=3424.946045, fpc=0.704085
n_clusters=8, m=2.25, mse=2916.573975, fpc=0.616468
n_clusters=8, m=2.5, mse=3298.722168, fpc=0.514249

Best grid-search result (lowest MSE):
n_clusters: 4, m: 1.1, mse: 2476.766357, fpc: 0.982750

Top 5 results:
 n_clusters    m         mse      fpc
          4 1.10 2476.766357 0.982750
          4 1.50 2484.392578 0.915329
          4 1.75 2489.347656 0.852928
          4 1.30 2495.099854 0.952573
          2 1.10 2522.159668 0.991280
Grid search finished in 1.1s
```

[ ]:
```

```

TSK_classification_Florian_Huhnd

September 24, 2025

Note: My explanations on the regression dataset are a bit more detailed as I started with this one and do not want to repeat too much. Please start with the regression when correcting. Obrigado.

[1]:
```
import numpy as np
from sklearn import datasets
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import␣
↪mean_squared_error,accuracy_score,classification_report
import skfuzzy as fuzz
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
```

[2]:
```
# Classification dataset - diabetes
data = datasets.fetch_openml(name="diabetes",version=1, as_frame=True)

X = data.data.values
y = data.target.values
y = (y == 'tested_positive').astype(float)  # convert to binary 0/1
```

[3]:
```
#train test spliting
test_size=0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size,␣
↪random_state=42)
```

[4]:
```
# Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)
```

So far, the diabetes dataset has been loaded, randomly divided into 80% training data and 20% percent test data and scaled. Since the labels were stored as strings they need to be converted to binary (float) values.

In the following, the number of the clusters and the smoothness of each cluster m can be tuned. I decided to use a grid search to find a suitable combination of parameters, which can be found at the end of this notebook.

Note: I ran the grid search multiple times and somehow the results kept changing a tiny bit. However, I decided to go with a simple model that has only two clusters, which makes it easy to interpret and reduces computational cost but still appeared in the top 3 all the time.

[5]:
```python
 # Number of clusters
n_clusters = 2
m=1.75

# Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

# Fuzzy C-means clustering
centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m=m, error=0.005, maxiter=1000, init=None,
)
```

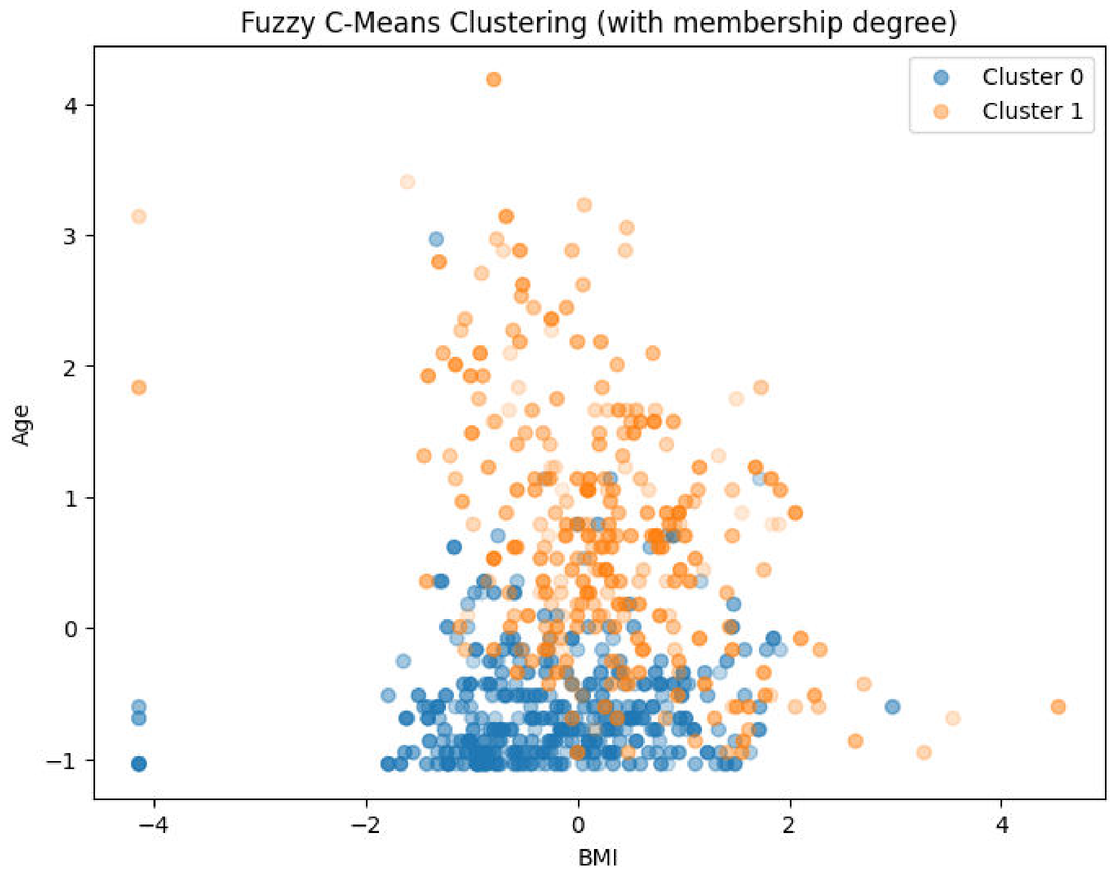
[6]:
```python
# Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)
    sigma_j = np.sqrt(var_j)
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)
```

The following plot shows two out of eight features and their clustering. Clearly age must have a big affect on the clustering, as the clusters can be easily divided into “old” and “young”.

[7]:
```python
# Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

# Plot two features with fuzzy membership
plt.figure(figsize=(8,6))
for j in range(n_clusters):
    plt.scatter(
        Xexp[cluster_labels == j, 5],              # BMI
        Xexp[cluster_labels == j, 7],              # Age
        alpha=u[j, :],          # transparency ~ membership
        label=f'Cluster {j}'
    )

plt.title("Fuzzy C-Means Clustering (with membership degree)")
```

```
plt.xlabel("BMI")
plt.ylabel("Age")
plt.legend()
plt.show()
```

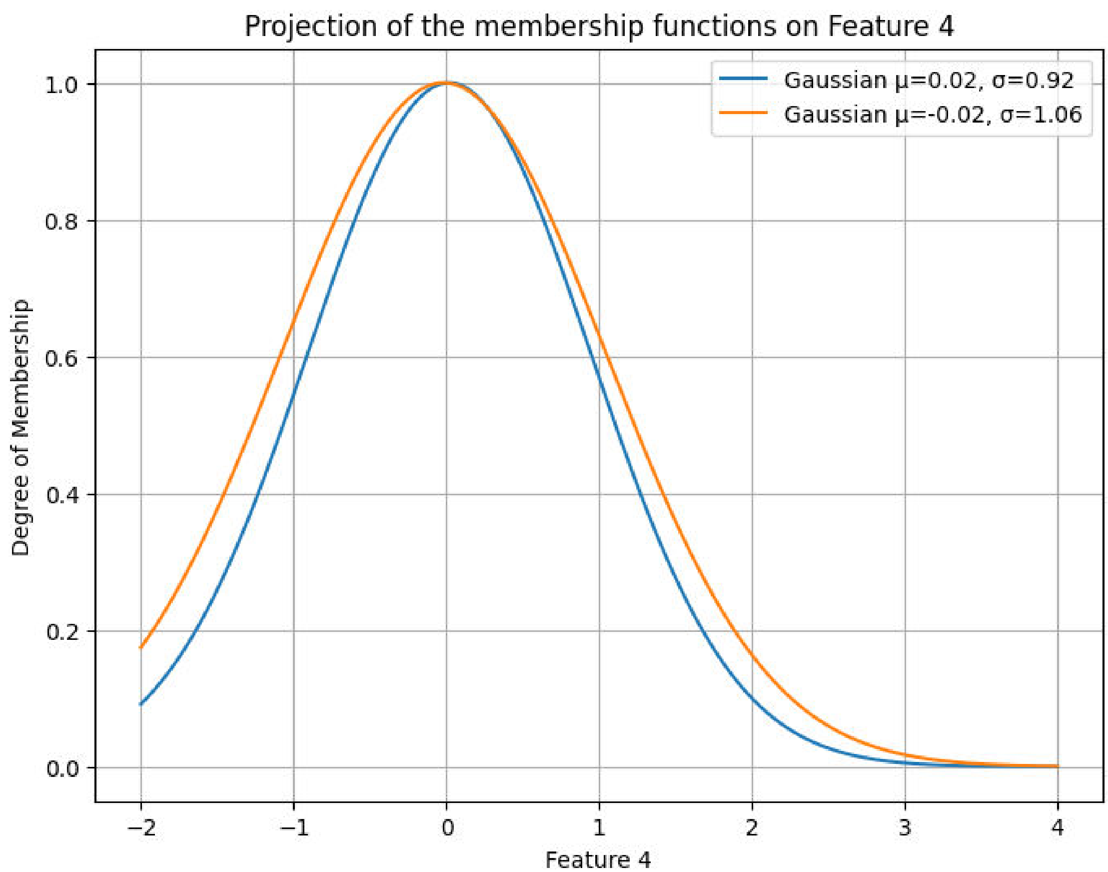
```
Fuzzy partition coefficient (FPC): 0.5562524200642404
```



The following plot proves my previous point, as the feature "age" yields the clearest separation of the curves. In contrast, the feature "Skin thickness" barely influences the clustering, as the membership functions are almost overlapping.

[8]:
```
# Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)

lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
for j in range(n_clusters):
```

```
# Select feature to be shown
    ft=4  # e.g. Feature 8 = Age; Feature 4 = Skin Thickness
    fti=ft-1  # index correction

# Compute curves
    y_aux.append(gaussian(lin, centers[j,fti], sigmas[j,fti]))

# Plot
    plt.plot(lin, y_aux[j], label=f"Gaussian  ={np.round(centers[j,fti],2)}, 
 ↪ ={np.round(sigmas[j,fti],2)}")

plt.title(f"Projection of the membership functions on Feature {ft}")
plt.xlabel(f"Feature {ft}")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()
```



[9]:

```python
# ---------------------------
# Gaussian Membership Function
# ---------------------------
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1,␣
↪n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.
↪unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
            dist = torch.norm(diff, dim=-1)  # (batch, n_rules) # probablistic␣
↪intersection
        else:
            dist = torch.max(diff, dim=-1).values  # (batch, n_rules) # min␣
↪intersection (min instersection of normal funtion is the same as the max on␣
↪dist)

        return torch.exp(-0.5 * dist ** 2)


# ---------------------------
# TSK Model
# ---------------------------
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )
```

```
    def forward(self, x):
        # x: (batch, n_inputs)
        batch_size = x.shape[0]

        # Compute membership values for each input feature
        # firing_strengths: (batch, n_rules)
        firing_strengths = self.mfs(x)

        # Normalize memberships
        # norm_fs: (batch, n_rules)
        norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True)␣
↪+ 1e-9)

        # Consequent output (linear model per rule)
        x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1)  # add bias

        rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents)  #␣
↪(batch, rules)
        # Weighted sum
        output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

        return output, norm_fs, rule_outputs
```

[10]:
```
# ---------------------------
# Least Squares Solver for Consequents (TSK)
# ---------------------------
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta= torch.linalg.lstsq(Phi, y).solution


        model.consequents.data = theta.reshape(model.consequents.shape)
```

[11]:
```
# Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:,:-1],␣
↪sigmas=sigmas[:,:-1])
```

```
# model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers,␣
↪sigmas=sigmas) # For comparison without target


Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)
```

[12]:
```
# Training with LS:
train_ls(model, Xtr, ytr.reshape(-1,1))
```

The accuracy of a classification is simply given by the percentage of correctly classified data. Since the target labels are crisp, a threshold needs to be defined (typically 0.5).

[13]:
```
y_pred, _, _=model(Xte)
print(f'ACC:{accuracy_score(yte.detach().numpy(),y_pred.detach().numpy()>0.
↪5)}') #classification
```

```
ACC:0.7857142857142857
```

The following grid search method was implemented by MS Copilot. Several parameters lead to very similar accuracy around 79%. Thus, a simpler method with only two clusters is preferred.

[14]:
```
# ---------------------------
# Grid search for highest classification accuracy (no MSE)
# Treat the continuous target as binary by thresholding at the training-set␣
↪median.
# This cell searches n_clusters in [2..8] and m in [1.1,1.3,1.5,1.75,2.0,2.25,2.
↪5]
# Primary metric: accuracy (higher is better). Pure classification, MSE removed.
# ---------------------------

n_clusters_list = [2, 3, 4, 5, 6, 7, 8]
m_list = [1.1, 1.3, 1.5, 1.75, 2.0, 2.25, 2.5]

# Prepare numpy arrays
Xtr_np = Xtr.detach().numpy() if isinstance(Xtr, torch.Tensor) else np.
↪array(Xtr)
ytr_np = ytr.detach().numpy().reshape(-1, 1) if isinstance(ytr, torch.Tensor)␣
↪else np.array(ytr).reshape(-1,1)
Xte_np = Xte.detach().numpy() if isinstance(Xte, torch.Tensor) else np.
↪array(Xte)
yte_np = yte.detach().numpy().reshape(-1, 1) if isinstance(yte, torch.Tensor)␣
↪else np.array(yte).reshape(-1,1)

# Binarize targets using training median (pure classification)
thr = np.median(ytr_np)
```

```
ytr_bin = (ytr_np > thr).astype(float)
yte_bin = (yte_np > thr).astype(float)

results = []
import time
start_time = time.time()

for n_c in n_clusters_list:
    for m_val in m_list:
        try:
            # Use binary target in concatenated space for clustering
            Xexp = np.concatenate([Xtr_np, ytr_bin], axis=1)
            Xexp_T = Xexp.T

            centers_tmp, u_tmp, u0, d, jm, p, fpc_tmp = fuzz.cluster.cmeans(
                Xexp_T, n_c, m=m_val, error=0.005, maxiter=1000, init=None
            )

            # Compute sigmas for each cluster (weighted variance)
            sigmas_tmp = []
            for j in range(n_c):
                u_j = u_tmp[j, :] ** m_val
                var_j = np.average((Xexp - centers_tmp[j])**2, axis=0, weights=u_j)
                sigma_j = np.sqrt(var_j)
                sigmas_tmp.append(sigma_j)
            sigmas_tmp = np.array(sigmas_tmp)

            # Build TSK model using input dims only (exclude appended target column)
            model_tmp = TSK(n_inputs=Xtr.shape[1], n_rules=n_c, centers=centers_tmp[:,:-1], sigmas=sigmas_tmp[:,:-1])

            # Train consequents with LS on binary labels
            train_ls(model_tmp, torch.tensor(Xtr_np, dtype=torch.float32), torch.tensor(ytr_bin, dtype=torch.float32))

            # Evaluate on test set
            y_pred_tmp, _, _ = model_tmp(torch.tensor(Xte_np, dtype=torch.float32))
            y_pred_np = y_pred_tmp.detach().numpy().reshape(-1,1)

            # Binarize predictions at 0.5
            y_pred_bin = (y_pred_np >= 0.5).astype(float)

            acc = accuracy_score(yte_bin, y_pred_bin)
```

```
            results.append({'n_clusters': n_c, 'm': m_val, 'accuracy':␣
↪float(acc), 'fpc': float(fpc_tmp)})
            print(f'n_clusters={n_c}, m={m_val}, acc={acc:.4f}, fpc={fpc_tmp:.
↪6f}')
        except Exception as e:
            print(f'Skipped n_clusters={n_c}, m={m_val} due to error: {e}')

# Summarize results
import pandas as pd
if len(results) == 0:
    print('Grid search produced no results')
else:
    df = pd.DataFrame(results)
    # Sort by accuracy descending (pure classification)
    df_sorted = df.sort_values('accuracy', ascending=False)

    best = df_sorted.iloc[0]
    print('\nBest grid-search result (highest accuracy):')
    print(f"n_clusters: {int(best['n_clusters'])}, m: {best['m']}, accuracy:␣
↪{best['accuracy']:.4f}, fpc: {best.get('fpc', np.nan):.6f}")

    print('\nTop 5 results:')
    print(df_sorted.head(5).to_string(index=False))

end_time = time.time()
print(f'Grid search finished in {end_time - start_time:.1f}s')
```

```
n_clusters=2, m=1.1, acc=0.7662, fpc=0.925030
n_clusters=2, m=1.3, acc=0.7662, fpc=0.776379
n_clusters=2, m=1.5, acc=0.7792, fpc=0.654217
n_clusters=2, m=1.75, acc=0.7857, fpc=0.556284
n_clusters=2, m=2.0, acc=0.7468, fpc=0.504949
n_clusters=2, m=2.25, acc=0.7403, fpc=0.500004
n_clusters=2, m=2.5, acc=0.7532, fpc=0.500000
n_clusters=3, m=1.1, acc=0.7662, fpc=0.918832
n_clusters=3, m=1.3, acc=0.7468, fpc=0.719213
n_clusters=3, m=1.5, acc=0.7597, fpc=0.547274
n_clusters=3, m=1.75, acc=0.7597, fpc=0.403685
n_clusters=3, m=2.0, acc=0.7468, fpc=0.337674
n_clusters=3, m=2.25, acc=0.7532, fpc=0.333336
n_clusters=3, m=2.5, acc=0.7468, fpc=0.333335
n_clusters=4, m=1.1, acc=0.7662, fpc=0.893701
n_clusters=4, m=1.3, acc=0.7727, fpc=0.656839
n_clusters=4, m=1.5, acc=0.7597, fpc=0.465725
n_clusters=4, m=1.75, acc=0.7662, fpc=0.311210
n_clusters=4, m=2.0, acc=0.7468, fpc=0.253739
n_clusters=4, m=2.25, acc=0.7468, fpc=0.250005
```

```
n_clusters=4, m=2.5, acc=0.7532, fpc=0.250001
n_clusters=5, m=1.1, acc=0.7403, fpc=0.919223
n_clusters=5, m=1.3, acc=0.7597, fpc=0.632663
n_clusters=5, m=1.5, acc=0.7792, fpc=0.409118
n_clusters=5, m=1.75, acc=0.7597, fpc=0.252851
n_clusters=5, m=2.0, acc=0.7468, fpc=0.203295
n_clusters=5, m=2.25, acc=0.6623, fpc=0.200005
n_clusters=5, m=2.5, acc=0.6688, fpc=0.200001
n_clusters=6, m=1.1, acc=0.7727, fpc=0.928281
n_clusters=6, m=1.3, acc=0.7403, fpc=0.659877
n_clusters=6, m=1.5, acc=0.7792, fpc=0.354735
n_clusters=6, m=1.75, acc=0.7143, fpc=0.212255
n_clusters=6, m=2.0, acc=0.7273, fpc=0.169587
n_clusters=6, m=2.25, acc=0.6688, fpc=0.166671
n_clusters=6, m=2.5, acc=0.6688, fpc=0.166668
n_clusters=7, m=1.1, acc=0.7532, fpc=0.908058
n_clusters=7, m=1.3, acc=0.7468, fpc=0.597743
n_clusters=7, m=1.5, acc=0.7922, fpc=0.327176
n_clusters=7, m=1.75, acc=0.7013, fpc=0.182333
n_clusters=7, m=2.0, acc=0.7273, fpc=0.145466
n_clusters=7, m=2.25, acc=0.7662, fpc=0.142862
n_clusters=7, m=2.5, acc=0.7532, fpc=0.142858
n_clusters=8, m=1.1, acc=0.7208, fpc=0.906780
n_clusters=8, m=1.3, acc=0.7468, fpc=0.631106
n_clusters=8, m=1.5, acc=0.7857, fpc=0.292501
n_clusters=8, m=1.75, acc=0.7078, fpc=0.159619
n_clusters=8, m=2.0, acc=0.7013, fpc=0.125056
n_clusters=8, m=2.25, acc=0.6948, fpc=0.125005
n_clusters=8, m=2.5, acc=0.6688, fpc=0.125001

Best grid-search result (highest accuracy):
n_clusters: 7, m: 1.5, accuracy: 0.7922, fpc: 0.327176

Top 5 results:
 n_clusters    m  accuracy      fpc
          7 1.50  0.792208 0.327176
          8 1.50  0.785714 0.292501
          2 1.75  0.785714 0.556284
          2 1.50  0.779221 0.654217
          5 1.50  0.779221 0.409118
Grid search finished in 1.9s
```

[ ]: