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Emmanuel Jurczenko and Jérôme Teiletche

Emmanuel Jurczenko
is associate dean and professor of finance in the Ecole Hôtelière de Lausanne at the University of Applied Sciences Western Switzerland (HES-SO) in Delémont, Switzerland.
emmanuel.jurczenko@ehl.ch

Jérôme Teiletche
is managing director and the head of Cross-Asset Solutions at Unigestion in Geneva, Switzerland.
jteiletche@unigestion.com

Risk-based investing is attracting increasing interest among the investment community and academics, the most popular approaches being minimum variance (Haugen and Baker [1991]; Clarke, De Silva, and Thorley [2006]), maximum diversification (Choueifaty and Coignard [2008]), and risk parity/equal risk contribution (Qian [2006]; Maillard, Roncalli, and Teiletche [2010]).

The most important feature these portfolio construction methodologies share is that they do not require explicit forecasts of expected returns, only risk forecasts (Clarke, De Silva, and Thorley [2013]), hence the name *risk-based*. The benefit for investors of this *no-views* feature is that it simplifies the portfolio selection process considerably—expected returns are notoriously more difficult to predict than covariances (Merton [1980]). From a practical standpoint, this provides the risk-based methodology with a sizeable advantage over Markowitz mean–variance optimization (MVO), the results of which are strongly dependent on forecasted returns. Nevertheless, the no-views characteristic of risk-based investing has been criticized. Indeed, by preventing the investor from inputting anything other than volatility or correlation information, risk-based solutions may, quite ironically, lead to other forms of risk. One example is the potential expensiveness of low-volatility stocks (Dangl and Kashofer [2013]). A second example is the criticism frequently levelled at multiasset risk parity portfolios: They make a significant allocation to bonds at times when their yields are historically low (Inker [2011]).

How can we reintroduce active views while avoiding the limitations of traditional MVO? A natural solution is to use the Black and Litterman [1992] framework (henceforth, BL), but to make the initial no-views or passive portfolio the risk-based one rather than the market capitalization benchmark. To the best of our knowledge, this idea of active risk-based investing was first investigated by Rappoport and Nottebohm [2012], who proposed to use the most diversified portfolio as a prior to estimate expected returns. This approach is also closely related to the recent work of Haesen et al. [2014], who proposed to use the risk parity portfolio as the reference portfolio in a BL setting.

Our main contribution is to provide a detailed analytical framework that allows for the incorporation of active views in a risk-based portfolio, whereas previous attempts have been empirical only. To this end, we integrate a risk-based approach within the BL analytical framework. This allows us to show that any active risk-based portfolio can be obtained as a simple linear combination of two portfolios: the passive no-views risk-based portfolio and the MVO portfolio consistent with active views. Practical implementation issues are addressed. We apply the solution to an out-of-sample multiasset allocation exercise based on macroeconomic regimes over the period 1974–2016.

A GENERIC RISK-BASED PORTFOLIO

We define the generic risk-based (RB) portfolio so that portfolio weights are inversely proportional to the volatilities:

$$\boldsymbol{w}_{RB} = k\boldsymbol{\sigma}^{-1} \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)^{\mathrm{T}}$ is the vector of volatilities, n is the number of assets, and k is a positive constant we discuss later. Higher volatility assets are thus given lower weights, and vice versa. Although it appears simple, this inverse volatility-weighting scheme is well grounded from several different perspectives.

From a theoretical point of view, Equation (1) corresponds to the risk parity and maximum diversification portfolio solutions when the correlation among assets is constant (Jurczenko, Michel, and Teiletche [2015]). Under the hypothesis of a zero-correlation coefficient, the minimum-variance portfolio is closely related to the generic risk-based portfolio, the difference being that weights are inversely proportional to individual variances and not to volatilities. Finally, the equal-weight (1/N; DeMiguel, Garlappi, and Uppal [2009]) portfolio is obtained as a special case when all volatilities are supposed to be equal and correlation is constant.

From a practical point of view, the risk-based portfolio has also some rationale. For instance, a significant number of prominent systematic strategies, such as trend following or risk parity, are using the approximation in Equation (1) to scale individual exposures (Chaves et al. [2012]; Moskowitz, Ooi, and Pedersen [2012]; Baltas and Kosowski [2015]). For single assets, the risk-based solution is also close to the MSCI risk-weighted indexes, in which individual weights are proportional to the inverse of the stock variances. Finally, empirical studies such as that by Kirby and Ostdiek [2012] have shown that volatility-weighted portfolio strategies have historically outperformed other traditional asset allocation methodologies such as MVO or 1/N.

The scaling parameter k can be calibrated in different ways. A general approach consists of targeting a predefined level of volatility for the risk-based portfolio, denoted by σ_T. This strategy is increasingly used in the asset management industry and has proven effective (Hallerbach [2012]). In such a case, we show in the online appendix that

$$k = \frac{\sigma_T}{nCF(\bar{\rho})} \tag{2}$$

where $\bar{\rho} = [n(n-1)]^{-1}(\Sigma_i \Sigma_{j\neq i}\rho_{i,j})$ is the average pairwise correlation across all assets, and $CF(\bar{\rho}) = \sqrt{n^{-1}[1+(n-1)\bar{\rho}]}$ measures the concentration of the risk-based portfolio.[1] The CF parameter varies from 0 (when the average pairwise correlation reaches its lowest bound, $\bar{\rho} = -(n-1)^{-1}$) to 1 (when the average pairwise correlation attains its maximum value, $\bar{\rho} = 1$). Intuitively, k increases when the diversification benefits are important—that is, when $CF(\bar{\rho})$ decreases. In this case, each constituent's weight needs to be increased to reach the desired volatility target σ_T so that the risk-based portfolio becomes potentially leveraged. The opposite mechanism happens when the investment universe consists of highly correlated assets. k is also increasing with the volatility target σ_T and decreasing with the number of assets n.

IS RISK-BASED ALLOCATION REALLY WITHOUT VIEWS?

As previously mentioned, investors tend to associate risk-based investing with no-views. Although it is true that no explicit *active views* on asset returns are incorporated in risk-based investment strategies, the risk-based portfolio (Equation (1)), like any other portfolio, has implied views on the expected excess returns of all its components.

Indeed, following the approach initially developed by Sharpe [1974], it is possible to compute for any unconstrained portfolio the set of implied expected excess returns (henceforth, implied returns) that renders these portfolio weights optimal in the MVO sense. In the case of the risk-based portfolio (Equation (1)), the individual implied returns are equal to (see online Appendix):

$$\mu^*_{\mathrm{RB},i} = \overline{SR}\left[\frac{CF(\bar{\rho}_i)}{CF(\bar{\rho})}\right]^2 \sigma_i \tag{3}$$

where $\overline{SR}$ is a scale parameter equal to the average Sharpe ratio across assets, $\bar{\rho}_i = (n-1)^{-1}(\Sigma_{j\neq i}\rho_{i,j})$ is the average pairwise correlation of asset i, and $CF(\bar{\rho}_i) = \sqrt{n^{-1}[1+(n-1)\bar{\rho}_i]}$ is the concentration factor for asset i. Based on Equation (3), we infer that assets with higher (lower) volatility or poor (high) diversification properties require higher (lower) implied returns for the risk-based portfolio to be held by an MVO investor. Implied Sharpe ratios are given by the ratios between implied returns $\mu^*_{\mathrm{RB},i}$ and individual volatilities σ_i. From Equation (3), we can immediately see that it is optimal for a mean–variance investor to hold the risk-based portfolio if the individual Sharpe ratios, adjusted for diversification characteristics, are equal across assets.

From the relation that exists between implied returns and marginal risk contributions, we infer that the individual risk allocations of the risk-based portfolio are such that (see online Appendix)

$$b_{\mathrm{RB},i} = \left[\frac{CF(\bar{\rho}_i)}{CF(\bar{\rho})}\right]^2 n^{-1} \tag{4}$$

This means that, in the risk-based case, risk allocations are proportional to the average pairwise correlations but independent of the individual risks.

For the special constant correlation case, $\rho_{i,j} = \rho \ \forall i, j$ (Elton and Gruber [1973]), the implied Sharpe ratios are equal for all assets. The risk allocations are also the same across all assets, $b_{\mathrm{RB},i} = n^{-1}$. In other words, the risk-based portfolio becomes risk parity in the usual sense.[2]

INCORPORATING ACTIVE VIEWS

As shown previously, a mean–variance investor optimally allocates wealth according to the risk-based portfolio when the individual Sharpe ratios, adjusted for diversification characteristics, are equal across assets. Let us now assume that the investor has a different opinion and thinks that some assets will have excess returns that differ from their implied risk-based estimates (Equation (3)).

In the asset management industry, BL constitutes the standard approach to incorporating active views on returns around a passive portfolio (Fabozzi, Focardi, and Kolm [2006]). In our framework, the risk-based portfolio constitutes the passive reference that will be modified to reflect the investor's active views, leading to the final active portfolio that we label the *active risk-based portfolio.*

Let μ^*_{RB} and μ_{VIEW} represent the vectors of the implied returns of the passive risk-based portfolio and the investor's views, respectively. Following the BL setup, we assume that the individual excess returns are normally distributed around their implied risk-based estimates; that is, $\mu \sim \mathcal{N}(\mu^*_{\mathrm{RB}}, c\,\Omega)$ with $0 \leq c \leq 1$. We suppose moreover that the investor is able to provide a complete set of absolute views[3] on the individual expected excess returns, such that $\mu|\mu_{\mathrm{VIEW}} \sim \mathcal{N}(\mu_{\mathrm{VIEW}}, (1-c)\Omega)$. The parameter c reflects the relative level of confidence in the active views. Full confidence in the views μ_{VIEW} corresponds to $c = 1$, whereas $c = 0$ means no confidence in the views.

Applying the standard normal multivariate distribution theory, the active risk-based individual expected returns are

$$\mu_{\mathrm{ARB}} = \mu^*_{\mathrm{RB}} + c(\mu_{\mathrm{VIEW}} - \mu^*_{\mathrm{RB}}) \tag{5}$$

Substituting Equation (5) into the solution of the unconstrained MVO program and identifying terms, the resulting active risk-based portfolio weights are equal to

$$\boldsymbol{w}_{\mathrm{ARB}} = (1-c)\boldsymbol{w}_{\mathrm{RB}} + c\,\boldsymbol{w}_{\mathrm{VIEW}} \tag{6}$$

where $\boldsymbol{w}_{\mathrm{RB}} = \dfrac{\sigma_T}{SR_{\mathrm{RB}}}\Omega^{-1}\mu^*_{\mathrm{RB}}$ is the passive risk-based portfolio and $\boldsymbol{w}_{\mathrm{VIEW}} = \dfrac{\sigma_T}{SR_{\mathrm{RB}}}\Omega^{-1}\mu_{\mathrm{VIEW}}$ corresponds to the active-views MVO portfolio. SR_{RB} is the Sharpe ratio of the passive risk-based portfolio, with $SR_{\mathrm{RB}} = \dfrac{\overline{SR}}{CF(\bar{\rho})}$. The active risk-based portfolio is thus a simple linear combination of the passive risk-based portfolio and active-views MVO portfolio.

As a result, the deviations between the active and the passive risk-based portfolios are given by

$$\boldsymbol{w}_{\mathrm{ACT}} = c(\boldsymbol{w}_{\mathrm{VIEW}} - \boldsymbol{w}_{\mathrm{RB}}) = c\left(\frac{\sigma_T}{SR_{\mathrm{RB}}}\right)\Omega^{-1}(\mu_{\mathrm{VIEW}} - \mu^*_{\mathrm{RB}}) \tag{7}$$

where $\boldsymbol{w}_{\mathrm{ACT}}$ represents a long–short portfolio. The vector of active portfolio deviations (Equation (7)) is the result of the risk-adjusted combination of all the pairwise comparisons between the returns associated with the views and the risk-based implied returns. Quite naturally, the higher the level of the volatility target σ_T and the higher the confidence in the views, the more the active portfolio (Equation (6)) will be tilted away from the passive (no-views) one (Equation (1)). If there is no confidence in the views ($c = 0$), we have $\boldsymbol{w}_{\text{ACT}} = 0$; thus, the investor optimally holds the passive risk-based portfolio. At the other extreme, if there is full confidence in the views ($c = 1$), the departures from the passive risk-based portfolio are maximal and the investor optimally holds the active-views MVO portfolio; that is, $\boldsymbol{w}_{\text{ARB}} = \boldsymbol{w}_{\text{VIEW}}$. Hence, Equation (6) encompasses the set of portfolios spanning from the pure risk-based solutions to traditional MVO solutions.[4]

An important remark is that the active portfolio deviations $\boldsymbol{w}_{\text{ACT}}$ do not necessarily sum to zero, meaning that the active portfolio deviations are not cash-neutral. This could be a practical issue for investors, notably if they plan to build fully invested and unleveraged portfolios. Following Grinold and Kahn [1994], this issue can be resolved through the application of a constant shift in the vector of expected returns.[5]

Practical Implementation

The practical implementation of the allocation model (Equation (6)) requires specification of the confidence parameter c and specification of the active views on expected returns μ_{VIEW}. We suggest calibrating c consistently with a predetermined active risk target (i.e., tracking-error volatility) relative to the passive risk-based portfolio. By definition, the tracking-error volatility of an arbitrary portfolio $\boldsymbol{w}_p$ with respect to a reference portfolio $\boldsymbol{w}_0$ is calculated as $TE_p = \sqrt{(\boldsymbol{w}_p - \boldsymbol{w}_0)^{\text{T}}\Omega(\boldsymbol{w}_p - \boldsymbol{w}_0)}$. Substituting the expression of active portfolio deviations (Equation (7)) into the definition of the tracking-error volatility leads to

$$TE_{\text{ARB}} = c\, TE_{\text{VIEW}} \tag{8}$$

where

$$TE_{\text{VIEW}} = (\sigma_{\text{T}}/SR_{\text{RB}})\sqrt{(\mu_{\text{VIEW}} - \mu^*_{\text{RB}})^{\text{T}}\Omega^{-1}(\mu_{\text{VIEW}} - \mu^*_{\text{RB}})}.$$

Equation (8) shows that the tracking-error volatility of the active risk-based portfolio is proportional to the tracking error of the active-views MVO portfolio. Hence, given investors' views, the relative confidence parameter can be simply determined by setting $c = TE_{\text{T}}/TE_{\text{VIEW}}$, where TE_{T} corresponds to a prespecified level of portfolio active risk.[6]

For most real-life applications, a more challenging requirement of the framework is the need to formulate a complete set of absolute views on expected returns for all assets. In many cases, investors are solely able to formulate signals or ranks on assets, which then need to be transformed into expected returns to determine portfolio allocations. In such a case, the most common approach is to rely on a Z-score methodology (Grinold [1994]). Let S_i denote the quantitative signal associated with asset i. From the vector of signals $\boldsymbol{S} = (S_1, \ldots, S_n)^T$, we build Z-scores as $Z_i = (S_i - \mu_S)/\sigma_S$, where μ_S and σ_S are the cross-sectional mean and standard deviation of S_i. The individual active return deviations are then inferred as

$$\mu_{\text{VIEW}} - \mu^*_{\text{RB}} = IC\, \Omega^{1/2}\boldsymbol{Z} \tag{9}$$

where $\boldsymbol{Z} = (Z_1, \ldots, Z_n)^T$ is the vector of Z-scores. IC is the ex-ante information coefficient that corresponds to the expected cross-sectional correlation between assets' signals and realized returns, a commonly used measure of forecasting skill. The substitution of Equation (9) into Equation (7) yields the active portfolio deviations expressed in terms of Z-scores:

$$\boldsymbol{w}_{\text{ACT}} = c\, IC\left(\frac{\sigma_{\text{T}}}{SR_{\text{RB}}}\right)\Omega^{-1/2}\boldsymbol{Z} \tag{10}$$

where the active deviations across all assets sum now to zero because the individual scores Z_i have a cross-sectional mean of zero.

It is worth emphasizing that, under the Z-score model, the active risk-based portfolio weights verify the Fundamental Law of Active Management of Grinold and Khan [1994] because multiplying the vector of active portfolio deviations (Equation (7)) by the vector of active returns deviations (Equation (9)) leads to

$$\boldsymbol{E}(R_{\text{ACT}}) = IC\sqrt{n}TE_{\text{ARB}} \tag{11}$$

where $\boldsymbol{E}(R_{\text{ACT}})$ is the expected active return of the active risk-based portfolio.

EMPIRICAL ILLUSTRATION

We now illustrate the theoretical framework with a typical multiasset allocation problem, based on macroeconomic regimes.[7] We assume that the portfolio is fully invested.[8]

Data Analysis

The sample covers the period from January 1974 to December 2016 and spans many different macroeconomic events, such as the oil shock of 1979 and several economic recessions. We consider five key asset classes: U.S. equities (S&P 500; EQ_US), non-U.S. developed equities (MSCI EAFE; EQ_DEV), U.S. Treasuries (Barclays U.S. Aggregate Treasury; GOV), U.S. credit (Barclays U.S. Aggregate Credit Baa; CORP), and commodities (S&P Goldman Sachs Commodity; COMM). Data are retrieved at a monthly frequency from Bloomberg. All returns are in excess of the risk-free rate, which is computed as the one-month Eurodollar deposit rate. The risk-free rate averaged 5.5% over the full sample. In total, we have 516 monthly returns.

In Exhibit 1, we present the asset returns and risk characteristics over the full sample. Panel A reports that excess returns range from 1.7% for government bonds to 6.0% for U.S. equities. Commodities have the highest volatility, followed by equities. Note that most of the asset classes posted similar Sharpe ratios of around 0.3–0.4, with the noticeable exception of commodities, which is much smaller. Commodities' poor individual performance is compensated by their diversifying properties; their average correlation with other assets and associated concentration factor are substantially smaller, as shown in Panel C. From that perspective, Panel D displays the implied returns of the passive risk-based portfolio, as defined in Equation (3) and the associated implied Sharpe ratios. We verify that commodities have the lowest implied Sharpe ratios because they are the most diversifying assets. Their implied risk-based returns are, however, higher than that of bonds because commodities are much more volatile. Treasuries have the second smallest implied Sharpe ratios and the smallest implied returns because they are both low-volatility and low-correlation assets. Interestingly, although they have relatively low implied returns, corporate bonds display the highest implied Sharpe ratios because they have poor diversifying properties. As volatile and poorly diversifying assets, equities require both high implied returns and high implied Sharpe ratios. Another noticeable feature is the similarity that exists between the full-sample individual realized Sharpe ratios and their implied counterparts. This indicates that the passive risk-based portfolio would have been optimal to hold over the period.

EXHIBIT 1

Assets Return and Risk Characteristics

	EQ_US	EQ_DEV	GOV	CORP	COMM
Panel A: Descriptive Statistics					
Average excess returns	5.99%	4.36%	1.69%	3.00%	1.02%
Volatility	15.35%	17.19%	5.16%	7.50%	20.05%
Sharpe ratio	0.39	0.25	0.33	0.40	0.05
Panel B: Full Sample Correlation Matrix					
EQ_US	1.00	0.64	0.09	0.40	0.12
EQ_DEV	0.64	1.00	0.05	0.33	0.19
GOV	0.09	0.05	1.00	0.75	–0.09
CORP	0.40	0.33	0.75	1.00	0.03
COMM	0.12	0.19	–0.09	0.03	1.00
Panel C: Diversification Metrics					
Average correlation	0.31	0.30	0.20	0.37	0.06
Concentration factor	0.67	0.67	0.60	0.71	0.50
Panel D: Risk-based Implied Returns and Implied Sharpe Ratios					
Implied returns	4.90%	5.41%	1.32%	2.66%	3.56%
Implied Sharpe ratios	0.32	0.31	0.26	0.35	0.18

Notes: EQ_US, EQ_DEV, GOV, CORP, and COMM stand for U.S. equities, non-U.S. developed equities, U.S. Treasuries, U.S. credit, and commodities, respectively. Returns are in USD and in excess over the one-month LIBOR rate. The sample period is 1974–2016.

We next define our macroeconomic regime indicators. Any investor wanting to design a dynamic asset allocation framework based on economic regimes must be aware of the practical limits associated with macroeconomic data series. First, macroeconomic statistics are published with delays, which can be of several months, even for popular statistics like gross domestic product. Second, most macroeconomic data statistics are frequently revised after their initial publication dates. We voluntarily restrict our analysis to economic indicators that are rapidly available and for which we can obtain vintage data (i.e., historical values) as they were originally released. The macroeconomic factors we consider are U.S. economic activity and inflation because they offer long available samples (our study spans more than 40 years of data) and are widely recognized by investors and academics as having the largest effects on asset returns both in the United States and globally.

Our growth activity indicator is measured through the three-month moving average of the Chicago Fed National Activity Indicator (CFNAI). Our inflation indicator is based on the combination of the monthly changes in inflation with the surprise in inflation.[9] Vintage time-series of the CFNAI and of inflation measures are retrieved from the Chicago Fed and St. Louis Fed websites, respectively. Following Ilmanen, Maloney, and Ross [2014], we model economic regimes as normalized economic variables through Z-scores, in which we subtract the historical median from each observation and divide the result by the historical standard deviation. For the composite inflation indicator, we average the Z-scores of the inflation rate change and of the inflation surprise. In practice, this leads to defining two sets of regimes, Up and Down, for both growth and inflation, each with a 50% probability.

In-Sample Results

We begin with an analysis of the asset macroeconomic sensitivities and study their implications for active risk-based asset allocation. To save space, we restrict the analysis to combinations of growth and inflation signals, leading to four regimes: (1) growth-up and inflation-up, (2) growth-up and inflation-down, (3) growth-down and inflation-up, and (4) growth-down and inflation-down.

Exhibit 2 contrasts the performance of assets during the different regimes with their unconditional (i.e., all periods) performance, normalized by the individual full-sample volatility to facilitate comparisons across assets and regimes. For example, the risk-adjusted excess return of non-U.S. equities during a growth-up/inflation-up period is equal to 0.69, but it is negative (−0.36) during periods in which growth goes down while inflation goes up.

We observe that the best environment for most assets is the combination of strong growth and low inflation. On the contrary, the worst environment for most assets is *stagflation* (i.e., a combination of growth-down and inflation-up). Equities thrive in a high-growth, low-inflation environment. Bonds perform better with lower inflation, whereas commodities perform better with higher inflation. Fixed-income portfolios are the best place to be during periods in which both growth and inflation decline, typically a recession or deflation. Such periods are very unfavorable for commodities, which post the best risk-adjusted returns during periods of stronger inflation and low growth. The reaction of bonds and commodities in specific inflation environments is probably the main explanation for their low average correlation with the other assets.

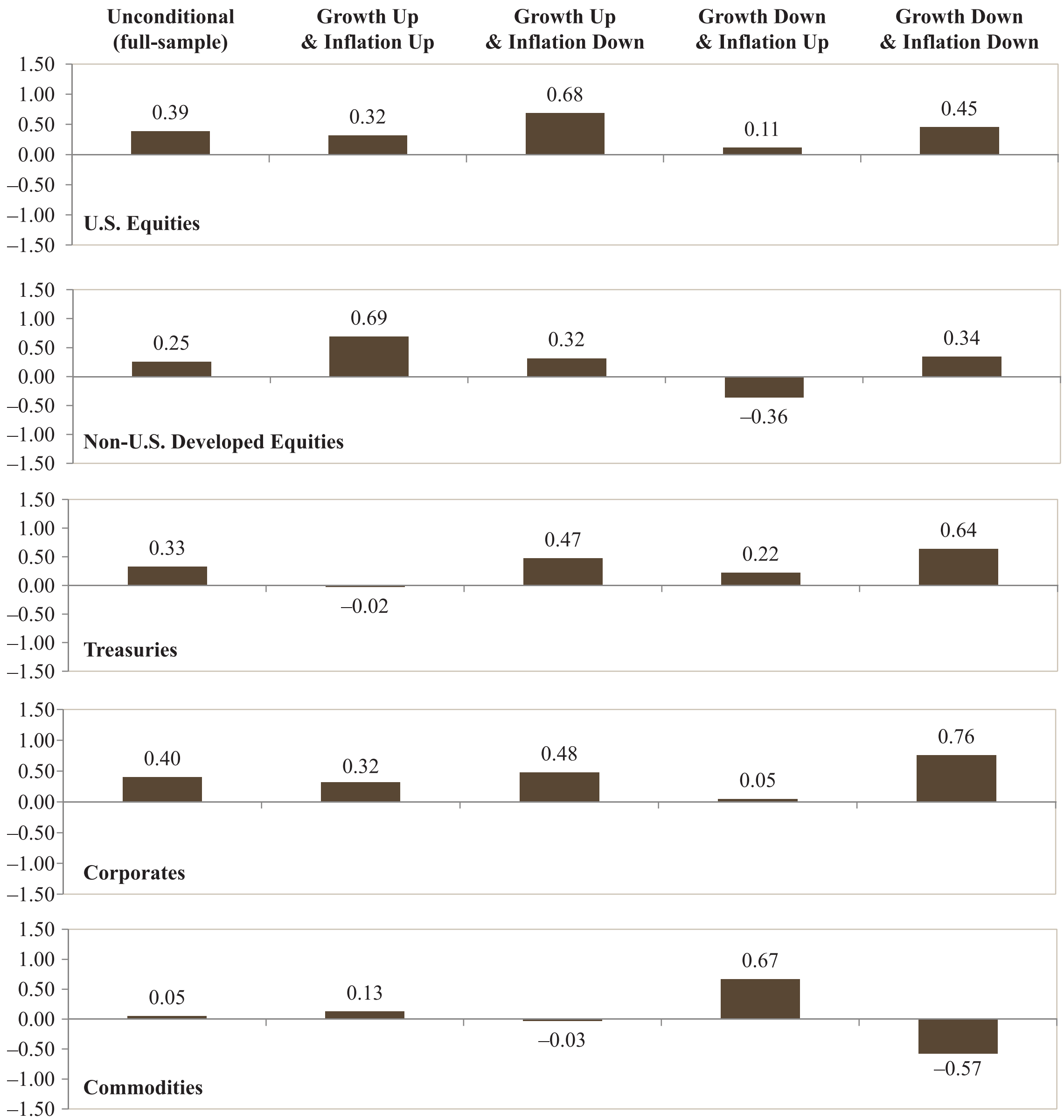
Exhibit 3 analyzes the implications of economic regimes in terms of asset allocations and risk allocations. More precisely, we compare the passive risk-based portfolio (column 2) with the active risk-based portfolios for different growth/inflation environments (columns 3 to 6). The active portfolios are built by setting views consistently with the asset average returns observed during the different regimes and imposing a 2% tracking-error volatility constraint with respect to the passive risk-based portfolio. Panels A and B show that, even with a limited tracking error, the resulting active portfolios differ markedly from one regime to the other, both in terms of capital allocations and risk allocations. Allocations to equities are increased for growth-up scenarios and decreased mainly for growth-down periods. Allocations to fixed income are stronger (weaker) in growth-down (up) and inflation-down (up) environments. Finally, allocations to commodities are larger when inflation is high.

Out-of-Sample Results

We next perform an out-of-sample allocation exercise over the last 13 years of the sample (2004–2016). The first 30 years of data (1974–2003) are used to initialize the model, and then we use rolling samples of 30 years. Parameters (volatilities, correlation, and expected returns) are re-estimated every month, based on data available at that precise date. For the estimation of economic regimes, we use vintage time-series of macroeconomic indicators (i.e., collecting them as they come out originally). To deal with delays in publications and implementation, we impose a lag of two months at each monthly rebalancing period. As an example, the estimation of the prevailing regime for the month of March 2006 is based on macroeconomic indicators available up to and including the month of January 2006. Once the economic regime is determined, we compute the expected returns associated with the perceived regime through historical empirical averages during similar episodes in the past. To limit turnover, we impose the criterion that regime change must be confirmed two months in a row before being considered a change.

Exhibit 2

Impact of Economic Regimes on Average Excess Returns (scaled by unconditional full-sample volatility)



We consider four competing asset allocation methodologies for fully invested portfolios. We compare passive and active risk-based portfolios, in which the latter uses mixed growth–inflation signals under a 2% tracking-error constraint. We extend the comparison to two other popular portfolio construction solutions: the equally weighted portfolio (so-called 1/*N*; DeMiguel, Garlappi, and Uppal [2009]) and the maximum Sharpe ratio (MSR; Martellini [2008]). The MSR portfolio corresponds to the fully invested active-views MVO portfolio. To remain consistent, views are based on the same economic regime return forecasts used in the active risk-based approach.

Performance results are summarized in Exhibit 4. Panel A compares the performance before costs of the passive (no-views) strategies and active ones. No-views strategies underperform both in absolute and risk-adjusted terms. This is most spectacular for the

Exhibit 3

Active Portfolios for Different Growth/Inflation Regimes: In-Sample Results

	Passive Risk-Based	Active Risk-Based			
	Unconditional (All Periods)	Growth/Inflation Regimes Up/Up	Up/Down	Down/Up	Down/Down
Panel A: Capital Allocation					
Equities	24.7%	30.4%	31.5%	20.9%	23.7%
EQ_US	13.0%	6.3%	25.4%	18.8%	12.1%
EQ_DEV	11.6%	24.1%	6.1%	2.1%	11.6%
Fixed Income	65.4%	55.9%	65.2%	62.3%	74.8%
GOV	38.7%	40.5%	45.4%	52.1%	26.1%
CORP	26.7%	15.5%	19.8%	10.1%	48.7%
Commodities	10.0%	13.7%	3.3%	16.9%	1.5%
Panel B: Risk Allocation					
Equities	44.6%	57.2%	59.8%	34.9%	38.7%
EQ_US	22.5%	9.0%	49.3%	31.7%	19.2%
EQ_DEV	22.1%	48.2%	10.5%	3.1%	19.5%
Fixed Income	42.9%	23.3%	38.4%	32.2%	60.7%
GOV	18.0%	12.7%	20.7%	23.7%	12.7%
CORP	24.9%	10.6%	17.7%	8.5%	48.1%
Commodities	12.5%	19.5%	1.8%	32.9%	0.6%

Note: The active risk-based portfolios are built under the constraints of being long-only and of not exceeding more than 2% tracking error relative to the passive risk-based portfolio.

Exhibit 4

Active Portfolios for Different Growth/Inflation Regimes: Out-of-Sample Results

	Passive Risk-Based	Active Risk-Based	Maximum Sharpe Ratio	Equal Weight
Panel A: Risk–Return Profile				
Average excess return	3.03%	3.38%	3.13%	2.84%
Volatility	6.28%	5.43%	5.83%	9.70%
Sharpe ratio	0.48	0.62	0.54	0.29
Maximum drawdown	25.0%	17.8%	17.0%	38.4%
Calmar ratio	0.12	0.19	0.18	0.07
Panel B: Turnover and Breakeven Transaction Costs				
Turnover (yearly; two-way)	12.4%	147.5%	258.6%	17.0%
Breakeven t-costs Sharpe ratio	NA	0.56%	0.13%	–39.95%
Breakeven t-costs Calmar ratio	NA	0.90%	0.44%	–39.39%

Notes: Active risk-based and maximum Sharpe ratio are using the same expected returns as inputs. Sharpe (Calmar) ratio is computed as the ratio of average excess return with volatility (maximum drawdown). Breakeven t-costs (transactions costs) are computed as the unitary transaction costs such that the different portfolio construction methodologies would post the same risk-adjusted returns as the passive risk-based solution.

equal-weight strategy, which totally fails in terms of risk control, defined either in terms of volatility or maximum drawdown. Active (views) strategies lead to both better risk control and higher returns, but the best results in terms of absolute and risk-adjusted returns are obtained for the active risk-based strategy.[10] In Panel B, we observe that the dynamic tilts associated with the inflation and growth signals result in a substantial increase in turnover.[11] Hence, the question arises as to whether the performance for active strategies covers trading costs. To get a more precise assessment of this question, we follow the approach described by Kritzman, Page, and Turkington [2012] in terms of breakeven transaction costs. From Panel B, the main conclusion is that, although an active risk-based solution is likely to benefit on a net basis from the economic regime signals, MSR is less likely to do so. Indeed, as long as trading costs are less than 56 (for the Sharpe ratio) or 90 bps (for the Calmar ratio) per two-way transaction, the active risk-based strategy will outperform the passive risk-based allocation net of trading costs. By opposition, the MSR portfolio faces more stringent breakeven transaction costs that limit its ability to survive trading implementation.

CONCLUSION

Risk-based investing is experiencing growing success among investors, although some critics contend that the implicit no-views characteristic of these solutions might trigger other forms of risk, such as valuation risk. In this article, we introduce an analytical framework that allows investors to add active views on top of a risk-based solution. Portfolio composition is based on the assumption that the risk-based solution is the neutral (no-views) portfolio in the Black and Litterman [1992] setup.

We obtain closed-form expressions for the active risk-based portfolio. We show that any active risk-based portfolio can be obtained as a simple linear combination of two portfolios: the passive no-views risk-based portfolio and the active-views MVO portfolio. We discuss how to calibrate the model in terms of active risk and how to incorporate views in the case in which the investor is unable to formulate absolute views on returns but instead has an idea about the relative ranking of assets. Finally, we show that our active risk-based framework satisfies the Fundamental Law of Active Management.

We illustrate the methodology with a typical multiasset allocation exercise over more than 40 years of data, using views generated from macroeconomic regime signals. We show how investors can design active risk-based investment strategies, which outperform empirically both passive risk-based strategies and popular methodologies such as equal-weight or MSR. It would be interesting to investigate whether these promising results are robust when applied to other asset universes, notably to highly correlated assets such as regional equity indexes. We leave this for future research.

ENDNOTES

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[1]Note that the inverse of $CF(\bar{\rho})$ is equivalent to the leverage-correlation factor proposed by Baltas and Kosowski [2015] and also corresponds to the Choueifaty and Coignard [2008] diversification ratio of the risk-based portfolio.

[2]Although the literature frequently advocates for the constant correlation and equality hypotheses of Sharpe ratios to justify the mean–variance efficiency of the risk-parity portfolio (Maillard, Roncalli, and Teiletche [2010]), our analysis shows that these restrictions constitute a sufficient but unnecessary condition for the optimality of the risk-based solution (Equation (1)).

[3]Contrary to the original BL model, here the views are not required to be uncorrelated, so that the matrix of uncertainty of μ_{VIEW} is a full covariance matrix.

[4]Note that the active weights can also be seen as the approximate solution of a constrained robust portfolio optimization problem in the presence of estimation errors in mean returns (see Heckel et al. [2016]).

[5]Applied to our framework, the cash-neutral equivalent shift is given by $\mu_{\mathrm{VIEW}}^{\mathrm{mod}} - \mu_{\mathrm{VIEW}} = -\frac{1}{\varphi}\mathbf{1}^{\mathrm{T}}\Omega^{-1}(\mu_{\mathrm{VIEW}} - \mu_{\mathrm{RB}}^{*})\mathbf{1}$, where $\varphi = \mathbf{1}^{\mathrm{T}}\Omega^{-1}\mathbf{1}$. By using these modified views, the full investment constraint becomes automatically fulfilled without requiring a formal budgetary constraint in the allocation program.

[6]Following Martellini, Milhau, and Tarelli [2015], an alternative approach consists of calibrating the confidence parameter c as a function of the ratio between the variances of the estimators for implied and views returns. Another approach suggested by Rappoport and Nottebohm [2012] is to set the confidence parameter as $c = (1 - p)$, where p is the P-value of a multivariate test of equality of the forecasted returns with their risk-based implied estimates.

[7]A numerical illustration is provided in the online appendix.

[8]As discussed previously, the framework is also valid for an investor who can implement long–short leveraged strategies (e.g., through derivatives).

[9]We define *inflation surprises* as the spread between realized inflation and expected inflation. We measure inflation expectations as one-year lagged inflation, meaning that we assume sticky inflation expectations. Alternative measures of expectations such as the Survey of Professional Forecasters are not available at the beginning of the sample.

[10]The Calmar ratio (Young [1991]) is calculated as the average excess returns divided by the maximum drawdown over the same time period.

[11]Turnover is measured at each monthly rebalancing and integrates the drift in allocations between consecutive months.

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