Dynamic Portfolio Optimisation in the Australian Stock Market: An Approach Using GARCH Models and Kalman Filters

Chapter 1

Introduction

Portfolio optimisation refers to a means of structuring an asset portfolio in a way that minimizes risk while maximizing returns, a focal point in finance that has led to the development of various models. Harry Markowitz's seminal work established the modern portfolio theory, advocating for diversification to manage risk (Markowitz, 1952). This theory spurred the development of the Capital Asset Pricing Model (CAPM) (Sharpe, 1964) and the Fama & French models (Fama & French, 1993). However, the dynamic nature of financial challenges these traditional models, which are static and unable to adjust in real-time to the dynamic nature of financial markets (Simin, 2008). In response, dynamic asset allocation models using stochastic control theory and other approaches have been developed since the 1960s.

This paper explores a novel dynamic approach using Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models (Bollerslev, 1986) or Multivariate GARCH (MGARCH) (Bollerslev et al., 1988) and Kalman filters (*Kalman 1960*, n.d.). The GARCH models are utilised to estimate the time varying asset volatilities and correlations, which are then fed into a Kalman filter to dynamically estimate a portfolio's optimum weight at every time step.

The first part of this paper discusses existing literature within portfolio optimisation to contextualise the necessity of dynamic optimisation methods. This is followed by a detailed description of the methodology behind the proposed GARCH-Kalman approach. The effectiveness of this GARCH-Kalman approach is demonstrated through empirical analysis using data from the Australian stock exchange (ASX).

Chapter 2

Literature Review

2.1 Portfolio Optimisation

Markowitz's model established a quantitative approach to managing the risk-return trade-off by minimising portfolio variance to achieve desired results. The approach allows the risk-return trade-off to be visualized on a graph as the efficient frontier, a curve depicting the corresponding portfolio variance for given levels of expected returns. This model laid the groundwork for the CAPM model (Sharpe, 1964) which models the expected returns of a portfolio or asset as a function of its risk relative to the market. The Fama-French model (Fama & French, 1993) further refines the approach by incorporating additional market factors like size and value factors. However, these models are static in nature and typically assume a series of 'perfect' assumptions that does not always hold true in reality (Wang, 2023), including constant market conditions and investor preferences.

The weaknesses of these static models in facing real-world conditions where market dynamics are continuously evolving prompted the development of dynamic portfolio models.

These models utilise mathematical methods such as stochastic control theory and multi-period optimisation. Robert Merton's work in 1969 introduced continuous-time optimisation and intertemporal portfolio choice, utilising stochastic differential equations to optimise portfolios across time. The introduction of dynamic optimisation strategies like Merton's portfolio problem led to the development of other dynamic models that better considers the variability in market behaviour and investor circumstances through different mathematical approaches. Some notable examples of dynamic models include Dynamic Mean-Variance Optimisation models (Zhou & Li, 2000) and Reinforcement Learning Models (Deng et al., 2017).

2.2 GARCH models and its variants

Volatility, referring to the degree of variation of a financial instrument over time, is often used as a proxy of risk (Chestnut et al., 2009). GARCH models, developed from Engle's ARCH model (R. F. Engle, 1982) provides a framework for modelling time-varying volatility by considering past variances and returns (Bollerslev, 1986). This addresses the clustering of volatility over time–a phenomenon where high volatility events tend to follow high volatility events and vice versa. The incorporation of a longer memory into the model allows the GARCH model to capture the persistence of volatility over time, which depicts a more realistic representation of volatility dynamics. Extensions like Exponential GARCH (EGARCH) and Threshold GARCH (TGARCH) account for asymmetries in market reactions to price shocks (Nelson, 1991; Zakoian, 1994).

GARCH and its variants plays an important part in Value at Risk (VaR) estimation, as its predictive capabilities is used to assess the likelihood of losses based on historical patterns of volatility. Empirical studies done by Brooks and Persand (2003) suggests that GARCH-based VaR models typically outperforms static models that does not account for dynamic market conditions. This formed one of the primary assumptions for the selection of GARCH to model volatility to be used for a Kalman filter's observation noise covariance matrix. Moreover, the development of Multivariate GARCH (MGARCH) (Bollerslev et al., 1988) models allow for the simultaneous consideration of volatilities and correlations across multiple assets, enhancing the understanding of market dynamics through capturing dynamic interdependencies that single-equation GARCH models cannot.

2.3 The Kalman filter and GARCH-Kalman tandem models

The Kalman filter, developed by Rudy Kalman in 1960 for aeronautical and navigational applications, is a statistical algorithm that predicts, and updates estimates of state variables in uncertain, dynamic systems as new information unfolds. This mechanism is particularly advantageous for dynamic portfolio optimisation through enabling continuous adjustment of portfolio weights based on the continuously updating state estimates of financial variables. In this case, it is particularly useful in assimilating the volatilities modelled by GARCH and using it to refine estimates of optimal portfolio allocations.

The methodology implemented in this paper takes inspiration from several notable financial application of the Kalman filter. One of them is Harvey, Ruiz, and Shepard's (1994) (Harvey et al., 1994)work on multivariate stochastic variance models, where they applied the Kalman Filter to estimate the time-variant betas of stock returns. The estimated betas are then used in the dynamic adjustment of portfolio weights. Another is Alwyn J. Hoffman's (2024) 'Kalman Filtering Applied to Investment Portfolio Management', where the Kalman filter is used to filter out trading signals to optimize risk-adjusted returns.

However, a limitation of the standard Kalman filter is its assumption system linearity, which was addressed by developments like the Extended Kalman filter (EKF) (Mcgee & Schmidt, 1985) that linearizes nonlinear models using Jacobians, and the Unscented Kalman filter model (UKF) (Julier & Uhlmann, n.d.), which employs a deterministic sampling technique for better handling of non-linearities. Additionally, other approaches in using Kalman filters for nonlinear systems would typically use the Kalman filter as a component in stochastic volatility models, such as the Autoregressive-Stochastic volatility with threshold (AR-SVT) model developed by Ghosh et al (2013) (Ghosh et al., 2015). For this paper, we will be utilising the standard Kalman filter model to maintain relevance with course material.

Chapter 3

Data

The dataset provided for this study contains monthly closing share prices of 12 leading ASX-listed companies over a 30-year period from March 1993 to November 2023. In addition to the 12 companies, the monthly closing price for the ASX All Ordinaries (ASXAO) index, the Australian three-month Treasury Bill rate, and the monthly risk-free rate of return have also been included. The 12 companies, their stock ticker, and their sector are listed in the table below.

Table 1 Summary of Australian Companies and Their Sectors

ASX Code	Company Name	Sector
ANZ	ANZ Group Holdings Limited	Financials
BHP	BHP Group Limited	Materials
BXB	Brambles Limited	Industrials
CBA	Commonwealth Bank of Australia	Financials
COH	Cochlear Limited	Health Care
CSL	CSL Limited	Health Care
FMG	Fortescue Metals Group	Materials
QBE	QBE Insurance Group Limited	Financials
SUN	Suncorp Group Limited	Financials
TCL	Transurban Group	Industrials
WES	Wesfarmers Limited	Consumer Discretionary
WOW	Woolworths Group Limited	Consumer Staples

Returns are then calculated from the closing prices, which are then log-transformed to satisfy the stationarity requirement of the GARCH and DCC-GARCH models. The formula to obtain the returns and log-transform them is given below.

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

The log returns data are then re-scaled by a factor of 10 to improve the performance of the optimisation algorithm for the GARCH and DCC-GARCH models. To test for stationarity, the Augmented Dickey-Fuller (ADF) (Dickey & Fuller, 1979) and the Kwiatkowksi-Phillips-Schmidt-Shin (KPSS) (Kwiatkowski et al., 1992) test are conducted on the scaled returns, with the results given in the following table.

Table 2 ADF and KPSS Test Results for Stock Scaled Log Returns

Stock	ADF Statistic	ADF p-value	KPSS Statistic	KPSS p-value
ANZ	-5.88	3.17×10^{-7}	0.304	0.1
BHP	-18.64	2.05×10^{-30}	0.074	0.1
BXB	-14.35	1.03×10^{-26}	0.085	0.1
CBA	-17.93	2.88×10^{-30}	0.181	0.1
COH	-4.67	9.64×10^{-5}	0.343	0.1
CSL	-18.00	2.73×10^{-30}	0.145	0.1
FMG	-4.82	5.03×10^{-5}	0.147	0.1
QBE	-14.78	2.24×10^{-27}	0.200	0.1
SUN	-16.86	1.11×10^{-29}	0.234	0.1
TCL	-14.10	2.63×10^{-26}	0.232	0.1
WES	-6.87	1.53×10^{-9}	0.138	0.1
WOW	-20.25	0.0	0.541	0.032

At 5% significance, it could be determined by the ADF test that all the returns series do not have a unit root. However, the KPSS test suggests that the returns for Woolworths might not be stationary, with a p-value of 0.032. As the returns for Woolworths may be non-stationary, it was removed from the final portfolio.

Aside from Woolworths' non-stationarity, the frequency of the data presents itself as a challenge to the model's estimation. Monthly data has fewer data points and smooths out short-run fluctuations, affecting the accuracy and frequency of volatility and correlation estimates in the model, which reduces the model's responsiveness to changing market dynamics.

Chapter 4

Methodology

As previously indicated, our dynamic portfolio optimisation model utilises a Kalman filter model enhanced by a GARCH(1,1) model, or a variant of MGARCH dubbed Dynamic Conditional Correlation (DCC) GARCH (R. Engle, 2002). The Kalman filter itself computes the optimal portfolio weights while the GARCH models computes the conditional variance for the assets in the portfolio, which is then used in the Kalman filter's observation noise covariance matrix.

4.1 GARCH(1,1) model implementation

The choice of the GARCH(1,1) model is primarily due to its simplicity and parsimony. It involves fewer parameters than its other counterparts which makes it less computationally intensive, more stable, and to a certain degree more robust across different asset classes and market conditions (Hansen & Lunde, 2005), which is suitable for the portfolio defined in the previous section. The GARCH(1,1) model is composed of two components: the return equation and the variance equation.

4.1.1 Return Equation

The return equation is given below:

$$r_t = \mu + \varepsilon_t$$

Where r_t is the return of the asset at time t, μ is the expected value of asset returns, and ε_t is the error term at time t.

4.1.2 Variance Equation

The GARCH(1,1) conditional variance specification is given below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Where σ_t^2 is the conditional variance at time t. α_0 is a positive constant term, while α_1 is the coefficient for the lag squared error term ε_{t-1}^2, which measures the sensitivity of current volatility to shocks from previous periods (the 'ARCH' effect) (Campbell et al., 1996). β_1 is the coefficient for the lag squared conditional variance, which measures the persistence of past volatility (the 'GARCH' effect).

4.2 MGARCH/DCC-GARCH model implementation

Meanwhile the choice of a DCC-GARCH model is due to how it extends the GARCH model to model time-varying correlations between multiple assets. This makes it suitable to portfolio optimisation cases such as this one where correlations between assets could also be a significant determinant to asset returns and volatility. In addition, the DCC-GARCH model allows for the residuals to follow a Student-t distribution instead of a normal distribution. The Student-t distribution has thicker tails which can better model the probability of extreme returns, which more realistically reflects real world financial data. The DCC-GARCH model operates similarly to the GARCH model through its two components: the first being a variance equation for each individual asset, and the second being a correlation equation that models the dynamic correlations among the residuals of the first stage.

4.2.1 Variance Equations

$$\sigma_{i,t}^2 = \alpha_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$$

The variance equation for the DCC-GARCH model remains the same as the GARCH(1,1), except the i subscript that denotes the individual asset in the portfolio. The volatility estimates are then used to form a diagonal matrix D_t, where each diagonal element $D_{i,it}$ = σ_{it}, which standardizes the residuals. It is used to calculate the standardised residuals:

$$v_t = D_t^{-1}(r_t - \mu)$$

Where r_t and μ are vectors/time series for the returns and expected returns for each asset.

4.2.2 Correlation Equations

The dynamic correlations between assets are modelled by the equation below:

$$Q_t = (1 - a - b)\bar{Q} + a(v_{t-1}v'_{t-1}) + bQ_{t-1}$$

Q_t is the matrix of conditional correlations of the assets, and $\bar{Q}$ is the unconditional covariance matrix from the standardised residuals. a and b are parameters that control the dynamics of the correlation updates, satisfying $a + b < 1$ for stationarity. Q_t is then normalised to ensure it remains a valid correlation matrix through the process below.

$$R_t = D_t^{-1} Q_t D_t^{-1}$$

R_t captures both individual asset volatilities and the dynamics of correlations between the group of assets over time.

4.3 Kalman Filter Implementation

In this application, the Kalman filter model is adapted to continuously update the weights of N assets in a portfolio based on observed (log) returns and estimated volatilities (conditional variances) calculated by the GARCH(1,1) model or the conditional correlations calculated by the DCC-GARCH model in the previous section. The Kalman filter is utilised in a state-space model that accounts for the time-varying nature of returns and volatilities.

4.3.1 State-Space Representation

The portfolio's state-space system is composed of two components, with the observation component accounting for the log returns and the state component accounting for the portfolio weights.

4.3.1.1 Observation Equation

$$y_t = A + Bs_t + u_t$$
$$u_t \sim N(0, R)$$

In this equation y_t is the observation vector at time t, which represents the log returns of the assets. A is a vector of intercepts, while B is the observation matrix that links the observations with the state vector s_t. Lastly u_t is the observation disturbance term, which captures measurement error in observation.

4.3.1.2 State Equation

$$s_t = \Phi s_{t-1} + v_t$$
$$v_t \sim N(0, Q)$$

Here, s_t is the state vector at time t, which represents the portfolio weights. Φ is the state transition matrix, which models how the portfolio weights evolve over time. v_t is then the state disturbance term, which captures the 'random' fluctuations in the state vector.

4.3.2 Kalman Filter Process

The Kalman filter progresses through recursive steps of prediction, observation, and updating. The Kalman filter's parameters are initialised based on historical data, with the initial state $s_{0|0}$ of the portfolio weights being equal across all assets. The rationale behind opting for an initial equal weighting lies in its simplicity, as well as avoiding overfitting of the model to historical data. The Kalman filter's steps then optimises the weights based on new observations.

4.3.2.1 Prediction

This step predicts the state vector at time t, based on the information available at up to time $t-1$.

$$s_{t|t-1} = \Phi s_{t-1|t-1}$$
$$P_{t|t-1} = \Phi P_{t-1|t-1} \Phi' + \mathrm{Q}$$

$s_{t|t-1}$ is the predicted state vector given all information up to time $t-1$, while $P_{t|t-1}$ is the predicted state covariance matrix.

4.3.2.2 Observation

This step uses the state predictions to calculate the expected returns of the assets and their uncertainties.

$$y_{t|t-1} = \mathrm{B}s_{t-1|t-1}$$
$$V_{t|t-1} = \mathrm{B}P_{t-1|t-1}\mathrm{B}' + \mathrm{R}$$

$y_{t|t-1}$ is the predicted observation vector, while $V_{t|t-1}$ is the predicted observation covariance matrix.

4.3.2.3 Update

This step updates the state vector using the new observations at time t, optimising the asset weights.

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}B'V_{t|t-1}^{-1}(y_t - y_{t|t-1})$$
$$P_{t|t} = P_{t|t-1} - P_{t|t-1}B'V_{t|t-1}^{-1}BP_{t|t-1}$$

$s_{t|t}$ is the updated state vector incorporating the new observation y_t, while $P_{t|t}$ is the updated state covariance matrix.

In the equations above Φ is the state transition matrix initialised to an identity matrix, indicating stability in weights unless influenced by new observations. B is the observation matrix which links the state to the observations. Q is the process noise covariance matrix, and R is the observation noise covariance matrix, which is constructed with estimates from the GARCH(1,1) model or from the correlation matrix R_t from the DCC-GARCH model. The methodology revolves around the magnitude of volatility in the case of the GARCH(1,1) model, or the magnitude of volatility and asset correlation for the DCC-GARCH model.

In practice the lagged log-returns are being used to predict current and future log-returns through the observation equation. In times where the observed log-returns are consistently higher than its predicted values, the Kalman filter increases the allocated weight to that asset in the state vector $s_{t|t}$ and vice versa. The adjustment mechanism also relies on the 'Kalman gain' portion of the update equation, denoted by $P_{t|t-1}B'V_{t|t-1}^{-1}$, which determines the magnitude of adjustment. This term is where the noise covariance matrix containing the estimated volatilities (R) is encapsulated. If volatility is high (and if the change in correlation is significant in the DCC-GARCH based model), then the observation noise covariance matrix R will be larger, which would result in a smaller Kalman gain. The smaller Kalman gain means that the filter 'trusts' new observations less, which leads to smaller adjustments to the state estimates (weights) and vice versa.

The asset weights computed from the tandem model are then used in the computation of returns by multiplying the historic returns of each asset by their respective lagged weights (estimated weights from previous period). This approach reflects the practical scenario in portfolio management where investment decisions and asset allocations are made through using past data and applying them to future periods. While the model provides a robust dynamic asset allocation framework, it has several limitations that may affect its practical effectiveness. The parameters of GARCH and DCC-GARCH might not remain stable over different market conditions, which could lead to inaccuracies in the estimation of volatilities and correlations. This instability is more pronounced in periods of high volatility or market crises (,). The model in question also does not consider transaction costs, which plays a significant part in determining the efficiency and effectivity of rebalancing a portfolio to the optimum weights determined through the model. Other issues with the model primarily pertain to its sensitivity to input data, whose weakness was discussed in chapter 3.

Chapter 5

Results

In this section, we present the estimation results of the tandem GARCH/DCC-GARCH Kalman filter model, evaluating them against the ASXAO index as well as a static Markowitz portfolio re-balanced every 12 months. The models' robustness is assessed over several time frames to analyse its performance while addressing the concerns addressed in the previous sections.

5.1 Full Sample Period Performance

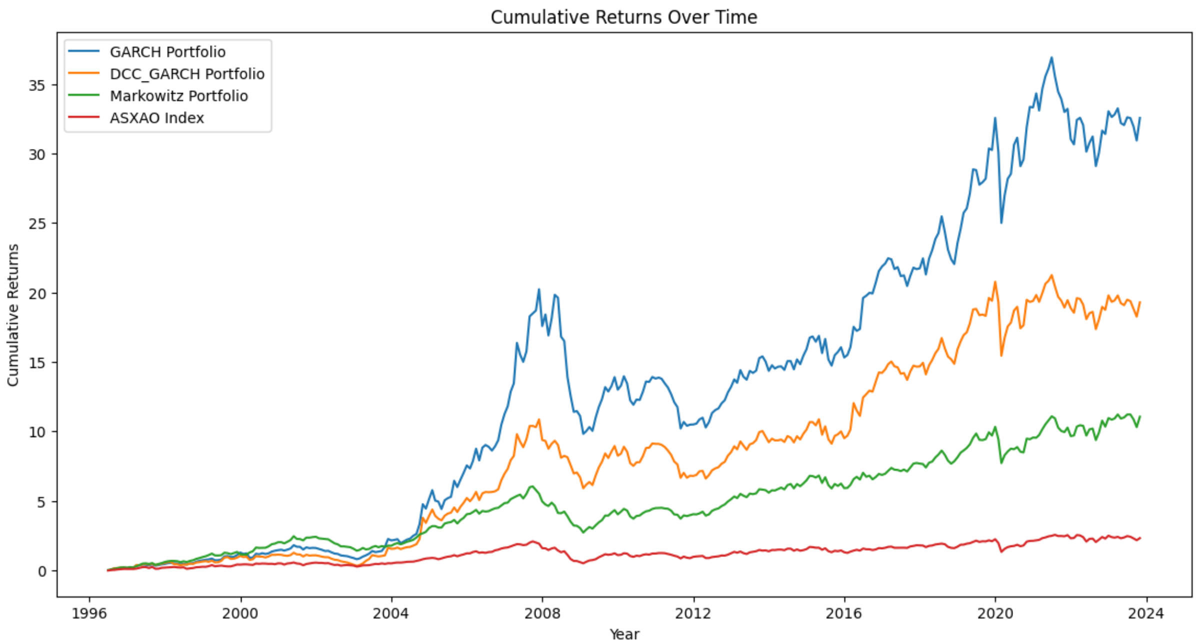

Figure 1 Comparison of portfolios and index March 1996- November 2023

Table 3 Performance comparison of portfolios during the full sample period

Portfolio	Cumulative Returns	Annualized Returns	Sharpe Ratio	Volatility
DCC-GARCH	19.2721	0.1164	0.5848	0.0572
GARCH	32.5482	0.1371	0.6909	0.0562
Markowitz	11.0433	0.0950	0.6535	0.0384
ASXAO	2.3057	0.0446	0.2963	0.0390

Over the full sample period we could see that the GARCH(1,1) based Kalman filter dynamic portfolio performed better than its DCC-GARCH counterpart and the static Markowitz by a significant margin. Over the 27 year period between March 1996 to November 2023, the GARCH(1,1) Kalman filter portfolio achieved a cumulative return of 3254.82%, and annualized return of 13.72% compared to the ASXAO's index 230.57% cumulative return and 2.77% annualized returns. All portfolios and the index shows similar performance from 1996 up to 2004, where they began to diverge.

The Sharpe ratio, a generally accepted measure of risk-adjusted returns, was 0.69 for the GARCH(1,1) based Kalman filter dynamic portfolio and 0.29 for the ASXAO. This shows that even with the seemingly higher volatility, the GARCH(1,1) Kalman filter dynamic portfolios not only significantly improved average returns, but also reduced the volatility of the returns as well. In contrast the DCC-GARCH based Kalman filter dynamic portfolio had a Sharpe ratio of 0.58, which outperformed the index but not the Markowitz portfolio's 0.65.

5.2 20-year Period Performance

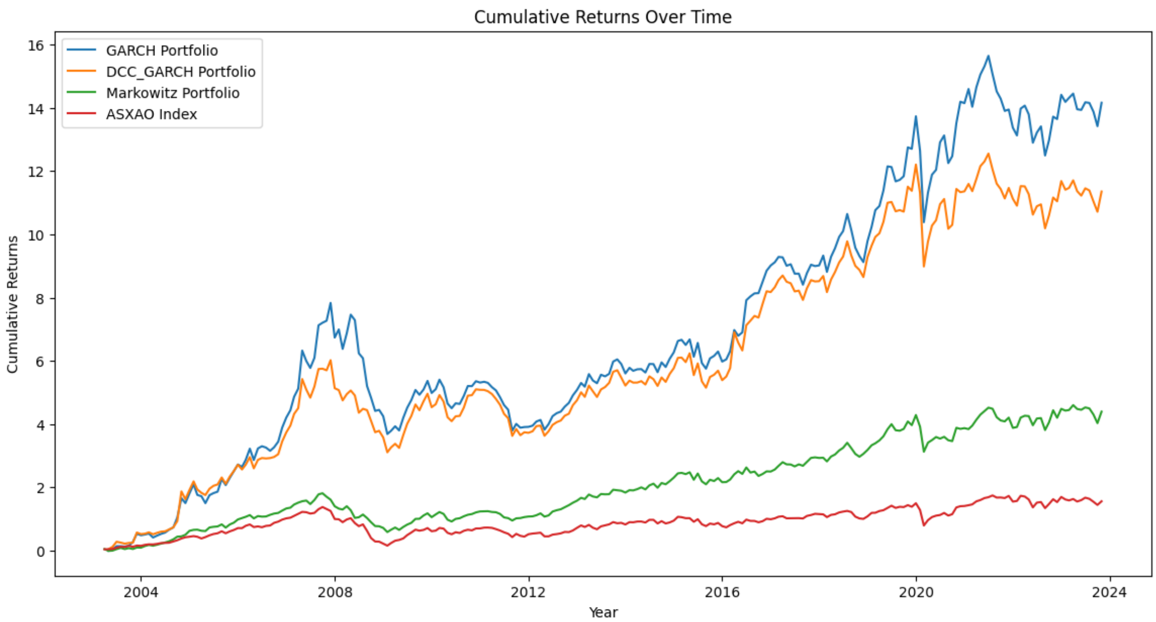

Figure 2 Comparison of portfolios and index November 2003-November 2023

Table 4 Performance comparison of portfolios and index November 2003 - November 2023

Portfolio	Cumulative Returns	Annualized Returns	Sharpe Ratio	Volatility
DCC-GARCH	11.3534	0.1299	0.6386	0.0589
GARCH	14.1614	0.1412	0.7202	0.0558
Markowitz	4.3954	0.0850	0.6106	0.0372
ASXAO	1.5619	0.0466	0.3170	0.0401

In the 20 year period between 2003 and 2023, the GARCH(1,1) Kalman filter dynamic portfolio model still performs the best, but with a smaller margin compared to its DCC-GARCH variant and the Markowitz portfolio. Cumulative return for the GARCH variant is

1416.14% compared to the DCC-GARCH variant's 1135.34% and the ASXAO's 156.19%. These results are much closer than the full sample period's metrics.

The Sharpe ratio for this period is better across the board for all portfolios and index. The GARCH variant retains the highest Sharpe ratio at 0.72 in this period, but the DCC-GARCH variant now outperforms the static Markowitz at 0.64 to 0.61. All the actively managed portfolios still have better Sharpe ratios than the index.

5.3 10-year Period Performance

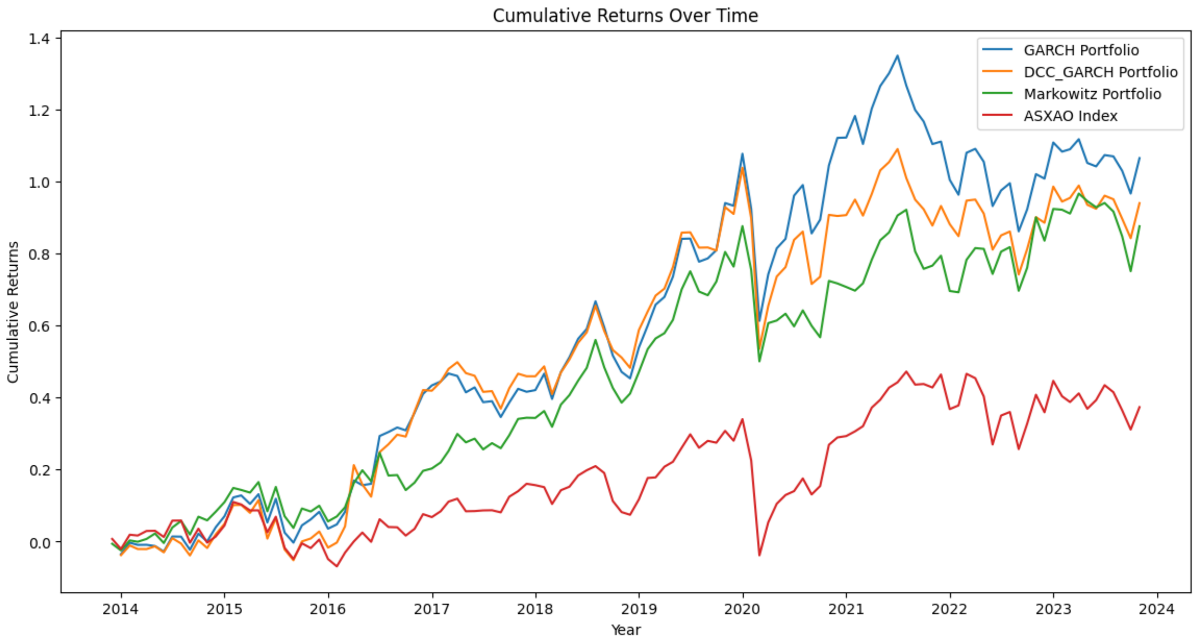

Figure 3 Comparison of portfolios and index November 2013- November 2023

Table 5 Performance comparison of portfolios and index November 2013 - November 2023

Portfolio	Cumulative Returns	Annualized Returns	Sharpe Ratio	Volatility
DCC-GARCH	0.9396	0.0691	0.4809	0.0433
GARCH	1.0645	0.0758	0.5529	0.0401
Markowitz	0.8754	0.0649	0.5103	0.0368
ASXAO	0.3732	0.0322	0.2549	0.0410

In the 10-year period, the gap between the portfolio optimisation strategies and the index diminished even further. The GARCH variant remains the best performing model, but only by just. It's cumulative return in the 10-year period is 106.45%, while its DCC-GARCH counterpart's is 93.96%. The gap is even smaller for their annualised returns, with the gap between the GARCH variant and the DCC-GARCH variant shrinking to 0.67% from the 2.07% in the full 27-year period.

In terms of order, the Sharpe ratios of the portfolio returned to how it was in the full sample period. The GARCH Kalman filter dynamic portfolio has the highest Sharpe ratio, followed by the Markowitz portfolio then the DCC-GARCH Kalman filter dynamic portfolio. Unlike in the previous sample period, there are points in periods of lower volatility where the DCC-GARCH model managed to outperform its GARCH counterpart.

Chapter 6

Discussion

The performance of the GARCH and DCC-GARCH Kalman filter dynamic portfolios aligns with expectations for a Kalman filter-based model. The performance of the models in the full sample period in comparison to the ASXAO index is reminiscent of another Kalman filter portfolio management model done by Alwyn J. Hoffman (2024), where the Kalman filter model quickly and sustainedly outpaced the index performance by a very significant margin over a period of circa 20 years. What is a surprise though, is how the better performing model between the two is the GARCH(1,1) based model. Theoretically, the DCC-GARCH Kalman filter model should have an advantage through its capabilities in capturing asset correlations, which would better reflect the true market conditions and its dynamics.

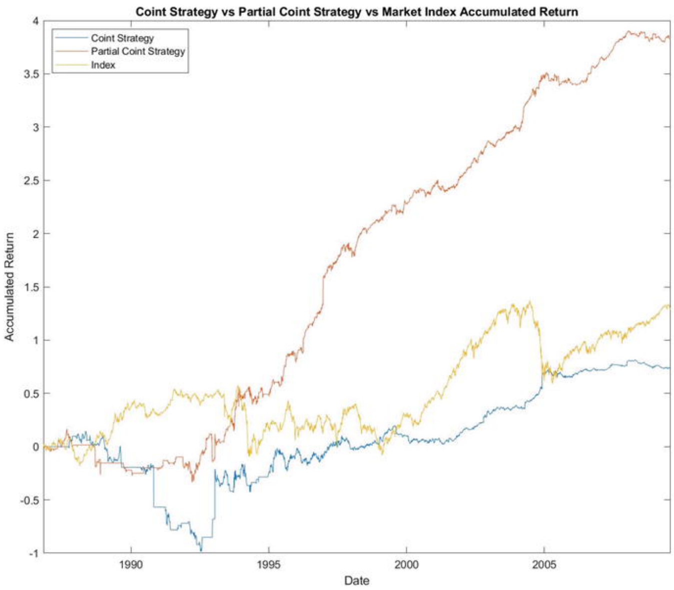

Figure 4 Hoffman's Kalman filter portfolio management model cumulative returns, from Hoffman, A. J. (2024).

It seems that the GARCH(1,1) model's simpler structure with fewer parameters makes it more robust under conditions where the data is not granular enough to capture the more complex dynamics that DCC-GARCH models are designed to address. In addition, by design, the GARCH(1,1) model adapts more quickly to changes in variance as it does not try to estimate multiple complex relationships as in MGARCH models (R. Engle, 2001). Another possibility on why the DCC-GARCH performed poorly in comparison to the GARCH(1,1) model is how the model is prone to overfitting to the idiosyncrasies of the sample data as a result of it having more parameters (Hansen & Lunde, 2005).

The monthly data granularity significantly impacts the performance of both dynamic portfolio models, as they are sensitive to the frequency of data points. Both models are designed to capture the volatility clustering in more frequent financial time series. Using monthly data greatly restricts the model's ability to leverage more frequent fluctuations and correlations present in daily or weekly data, which could enhance the model's accuracy and robustness in modelling volatility. The sensitivity of these models could be seen in the behaviour of cumulative returns during financial crises, where they experience sharp increases prior and steep declines afterwards. The accuracy of the Kalman filter's weight adjustments is dependent on the precision of the volatility and correlation estimates provided by the GARCH/DCC-GARCH models, which in turn is tied to data frequency. This is evident in the performance degradation observed during shorter estimation periods, where less data is available to inform the model's predictions, causing it to perform comparably to static portfolio optimisation models.

Evaluating the models' performance across various time periods–full sample, 20-year, and 10-year–illustrates that the models' advantages are more pronounced when larger datasets are available. This robustness over longer periods shows that larger datasets provide more stability in modelling and prediction. Such an evaluation confirms the hypothesis that increased data frequency improves model performance, providing a more reliable basis for dynamic portfolio optimisation using a tandem GARCH/DCC-GARCH Kalman filter model.

Chapter 7

Conclusion

In this paper, the application of a dynamic portfolio model using GARCH models and Kalman filters was explored on a portfolio composed of the 12 leading companies in the ASX. The study highlighted the importance and effectiveness of models such as GARCH(1,1) and DCC-GARCH in capturing time varying volatilities and correlations, and their use case with a Kalman filter. The study provides a compelling and responsive framework for managing financial assets.

The empirical results indicates that the GARCH(1,1) variant outperformed the DCC-GARCH variant and traditional static portfolio optimisation strategies, especially over extended periods. This suggests that the simpler structure of the GARCH(1,1) variant, with fewer parameters, offers greater robustness and adaptability to a financial market compared to the DCC-GARCH variant. However, this conclusion comes not without limitations. The use of monthly data potentially affects the responsiveness and accuracy of the models due to the reduced frequency of data points, which smooths out short term fluctuations.

Future research should utilise higher frequency (i.e. daily or weekly) data, integrate transaction costs, and explore the model's stability over different market conditions to refine their practical applicability. Additionally, examining the performance of these models in other markets or different economic settings could provide a better insight into their generalisability and effectiveness across diverse financial landscapes.

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