# CS/MATH 111, Discrete Structures - Fall 2018. Discussion 8 - Graphs

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# Outline

# Euler path and tour

## Definition 1.1

An *Euler tour* in a graph $G$ is a simple circuit containing **every edge** of $G$. An *Euler path* in $G$ is a simple path containing every edge of $G$.

# Euler tour

- An Euler tour (or Eulerian tour, Euler circuit) traverses each edge of the graph **exactly once**.
- Graphs that have an Euler tour are called Eulerian.

# Euler tour

**Theorem 1**

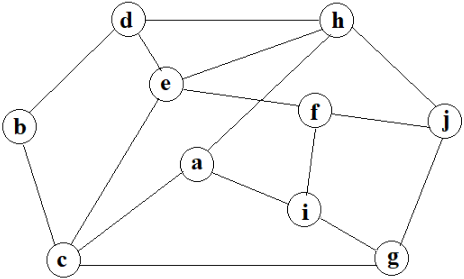
*An undirected graph has a closed Euler tour iff it is connected and each vertex has an even degree.*

**Theorem 2**

*An undirected graph has an Euler path but not an Euler tour iff it has exactly two vertices of odd degree.*

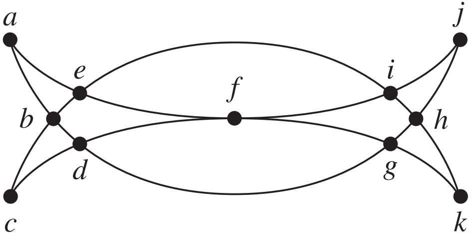
# Euler tour

- So this graph is not Eulerian:



# Euler tour

- Mohammed's Scimitars:



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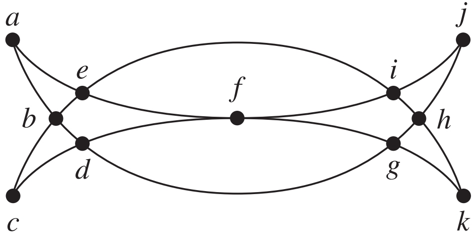
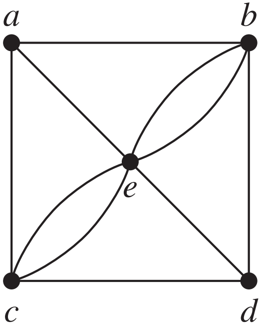



a, b, d, g, h, j, i, h, k, g, f, d, c, b, e, i, f, e, a

# Euler tour

- Determine weather the given graph has an Euler circuit:



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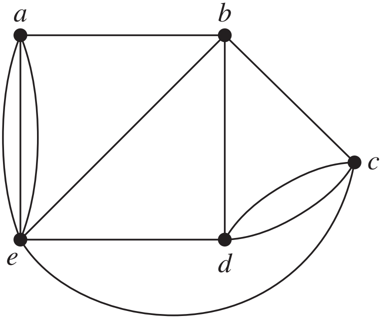


# Outline

Euler tour

Hamiltonian Cycle

Vertex Coloring

# Hamiltonian Cycle

- Hamiltonian Cycle (or Hamilton circuit) is a graph cycle (i.e., closed loop) through a graph that visits each node exactly once

# Hamiltonian Cycle

## Theorem 3 (Dirac's Theorem)

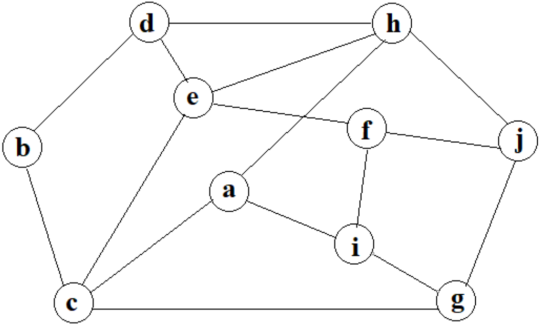
*If $G$ is a simple graph with $n$ vertices with $n \geq 3$ such that the degree of every vertex in $G$ is at least $\frac{n}{2}$, then $G$ has a Hamilton cycle.*

## Theorem 4 (Ore's Theorem)

*If $G$ is a simple graph on $n$ vertices, $n \geq 3$, and $d(v) + d(w) \geq n$ whenever $v$ and $w$ are not adjacent, then $G$ has a Hamilton cycle.*

# Hamiltonian Cycle

- The graph does not have Hamiltonian cycle.



# Hamiltonian Cycle

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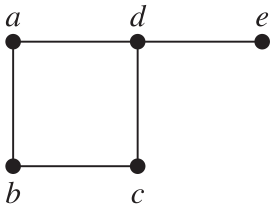


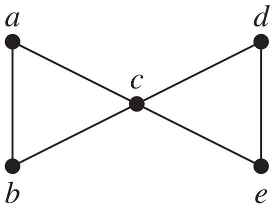

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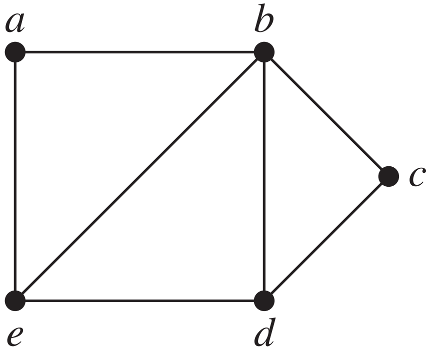

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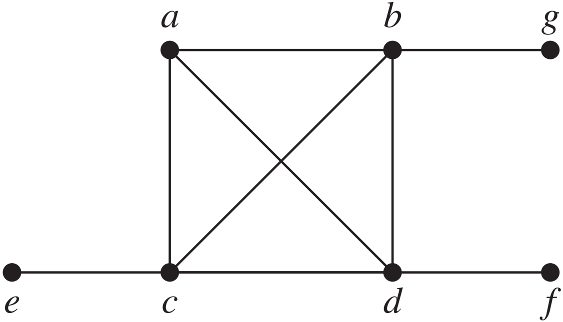

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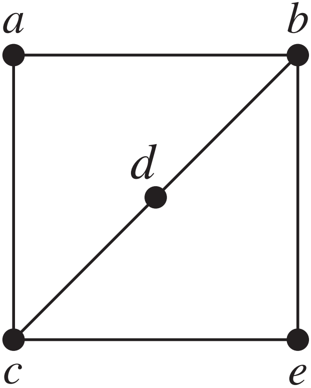
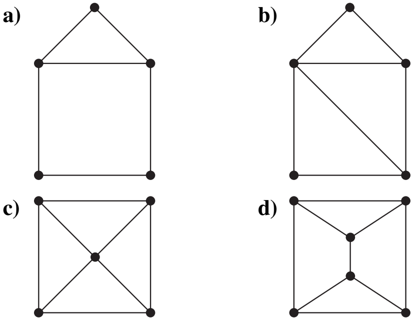


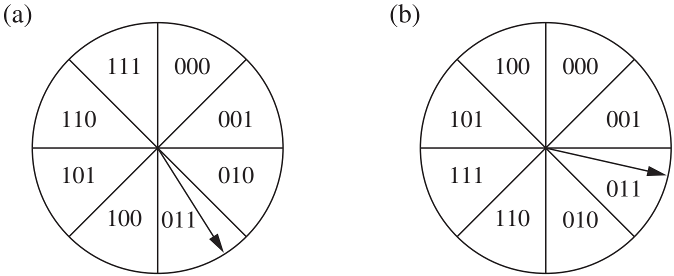
# Hamiltonian Cycle

- For each of these graphs, determine:
  - (i) whether Dirac's theorem can be used to show that the graph has a Hamilton circuit
  - (ii) whether Ore's theorem can be used to show that the graph has a Hamilton circuit
  - (iii) whether the graph has a Hamilton circuit



# Gray codes [1]

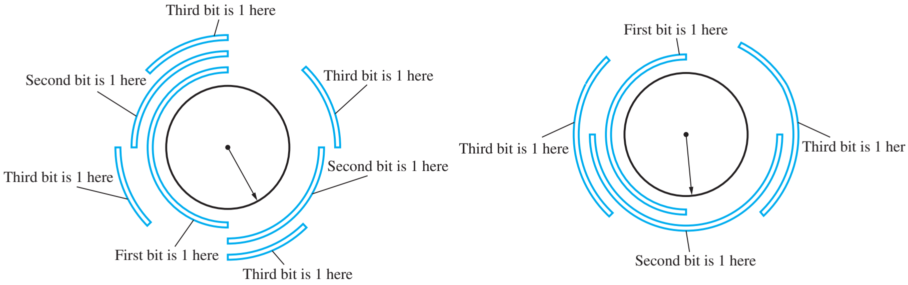
- Converting the position of a pointer into digital form:




[1] https://en.wikipedia.org/wiki/Gray_code

# Gray codes

- The digital representation of the position of the pointer:



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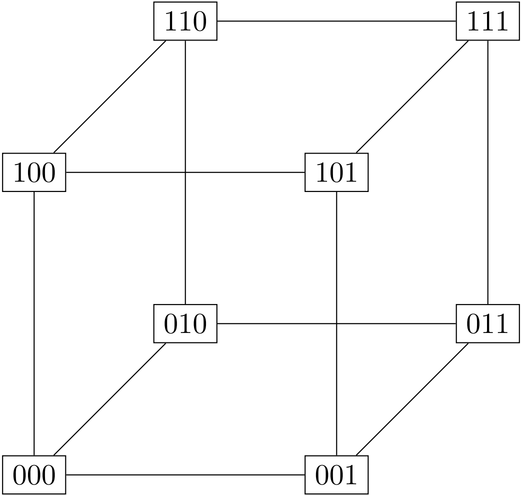

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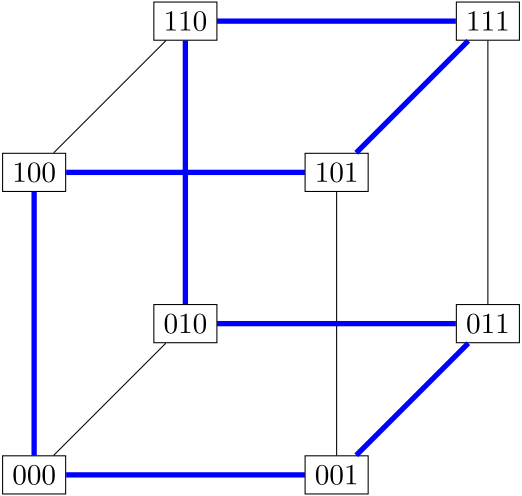
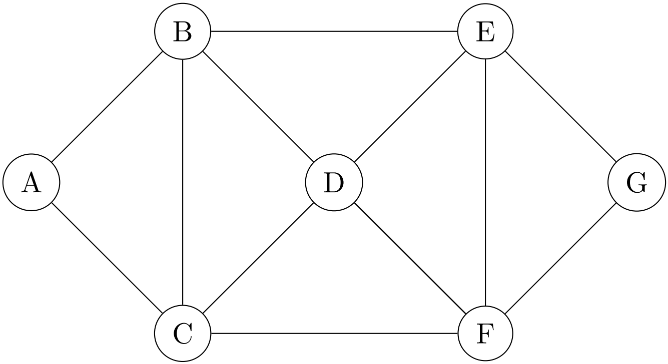


# Outline

# Vertex Coloring

- The chromatic number of a graph is the smallest number of colors needed to color the vertices so that no two adjacent vertices share the same color.
- Hardness: A very hard problem(an NP-Complete problem).

# Vertex Coloring



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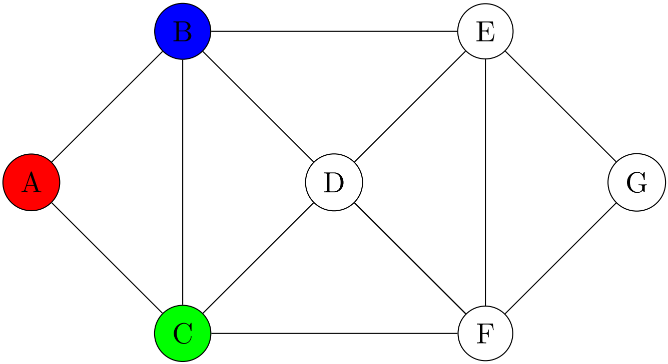

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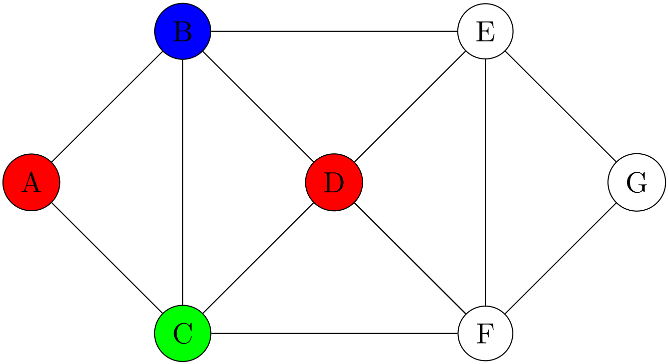

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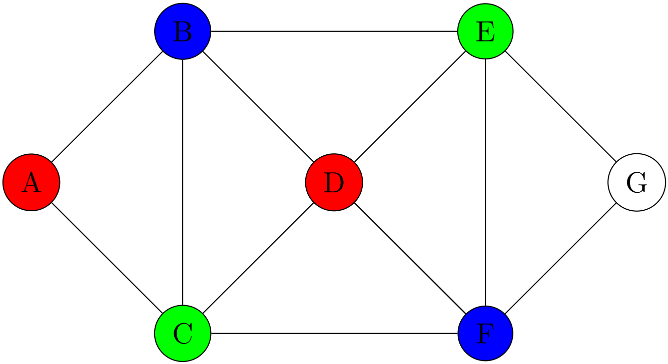

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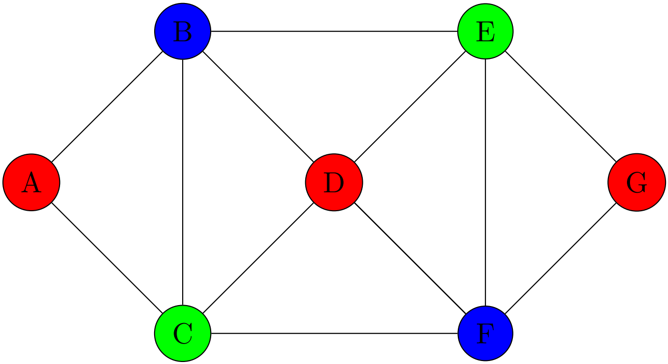

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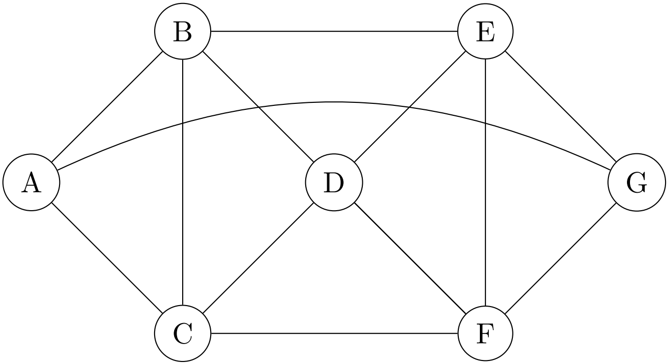

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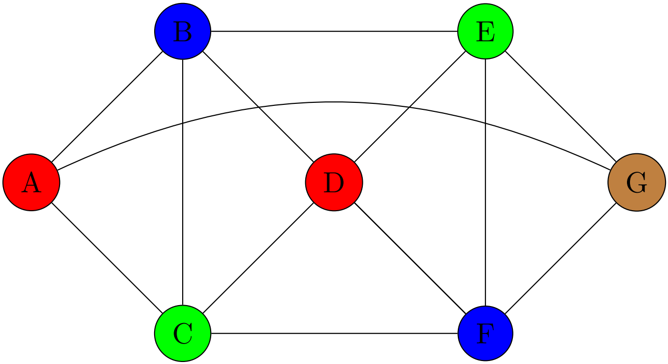

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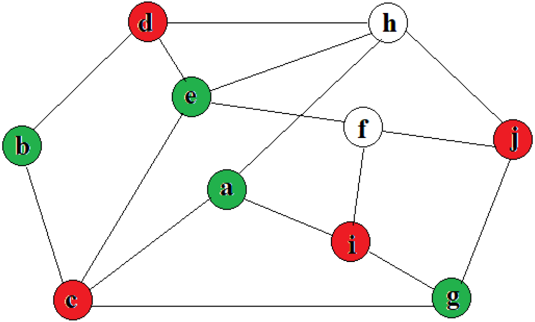

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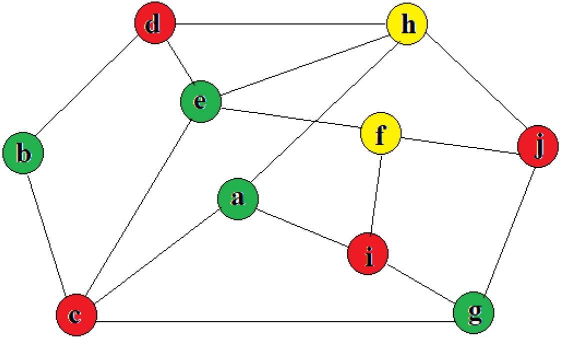
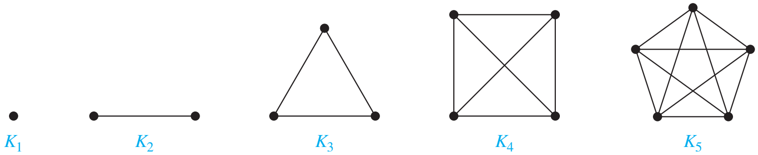


# Hamiltonian Cycle

- Complete graphs of $n$ vertices ($K_n$):



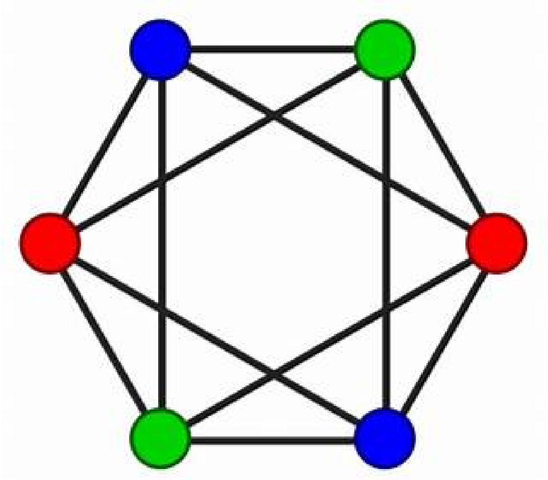
# Vertex Coloring

For certain classes of graphs, we can easily compute the chromatic number. For example, the chromatic number of $K_n$ is $n$, for any $n$. Notice that we have to argue two separate things to establish that this is its chromatic number:

- $K_n$ can be colored with $n$ colors.
- $K_n$ cannot be colored with less than $n$ colors.

For $K_n$, both of these facts are fairly obvious. Assigning a different color to each vertex will always result in a well-formed coloring (though it may be a waste of colors). Since each vertex in $K_n$ is adjacent to every other vertex, no two can share a color. So fewer than $n$ colors can't possibly work.

# Vertex Coloring



# Frequency Assignments

- Television channels 2 through 13 are assigned to stations in Colombia so that no two stations within 150 Km can operate on the same channel. How can the assignment of channels be modeled by graph coloring?

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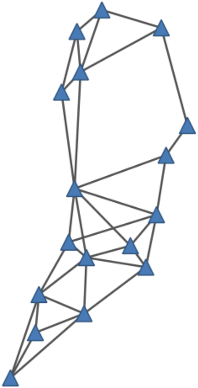

# Frequency Assignments

- Television channels 2 through 13 are assigned to stations in Colombia so that no two stations within 150 Km can operate on the same channel. How can the assignment of channels be modeled by graph coloring?
- Construct a graph by assigning a vertex to each station. Two vertices are connected by an edge if they are located within 150 Km of each other. An assignment of channels corresponds to a coloring of the graph, where each color represents a different channel.

# Reference


- Discrete Mathematics and Its Applications. Rosen, K.H. 2012. McGraw-Hill.
  Chapter 10: Graphs.
  Section 10.5: Euler and Hamilton Paths.
  Section 10.8: Graph Coloring.