offer little help in overcoming the complexity of algorithms of exponential or factorial time complexity. Because of the increased speed of computation, increases in computer memory, and the use of algorithms that take advantage of parallel processing, many problems that were considered impossible to solve five years ago are now routinely solved, and certainly five years from now this statement will still be true. This is even true when the algorithms used are intractable.

Exercises

1. Give a big-O estimate for the number of operations (where an operation is an addition or a multiplication) used in this segment of an algorithm.

```
t := 0
for i := 1 to 3
  for j := 1 to 4
    t := t + ij
```

2. Give a big-O estimate for the number additions used in this segment of an algorithm.

```
t := 0
for i := 1 to n
  for j := 1 to n
    t := t + i + j
```

3. Give a big-O estimate for the number of operations, where an operation is a comparison or a multiplication, used in this segment of an algorithm (ignoring comparisons used to test the conditions in the **for** loops, where $a_1, a_2, ..., a_n$ are positive real numbers).

```
m := 0
for i := 1 to n
  for j := i + 1 to n
    m := max(a_i a_j, m)
```

4. Give a big-O estimate for the number of operations, where an operation is an addition or a multiplication, used in this segment of an algorithm (ignoring comparisons used to test the conditions in the **while** loop).

```
i := 1
t := 0
while i ≤ n
    t := t + i
    i := 2i
```

5. How many comparisons are used by the algorithm given in Exercise 16 of Section 3.1 to find the smallest natural number in a sequence of n natural numbers?

6. **a)** Use pseudocode to describe the algorithm that puts the first four terms of a list of real numbers of arbitrary length in increasing order using the insertion sort.

b) Show that this algorithm has time complexity $O(1)$ in terms of the number of comparisons used.

7. Suppose that an element is known to be among the first four elements in a list of 32 elements. Would a linear search or a binary search locate this element more rapidly?

8. Given a real number x and a positive integer k, determine the number of multiplications used to find x^{2^k} starting with x and successively squaring (to find x^2, x^4, and so on). Is this a more efficient way to find x^{2^k} than by multiplying x by itself the appropriate number of times?

9. Give a big-O estimate for the number of comparisons used by the algorithm that determines the number of 1s in a bit string by examining each bit of the string to determine whether it is a 1 bit (see Exercise 25 of Section 3.1).

***10.** **a)** Show that this algorithm determines the number of 1 bits in the bit string S:

```
procedure bit count(S: bit string)
count := 0
while S ≠ 0
  count := count + 1
  S := S ∧ (S − 1)
return count {count is the number of 1s in S}
```

Here $S - 1$ is the bit string obtained by changing the rightmost 1 bit of S to a 0 and all the 0 bits to the right of this to 1s. [Recall that $S \wedge (S - 1)$ is the bitwise *AND* of S and $S - 1$.]

b) How many bitwise *AND* operations are needed to find the number of 1 bits in a string S using the algorithm in part (a)?

11. **a)** Suppose we have n subsets $S_1, S_2, \ldots, S_n$ of the set $\{1, 2, \ldots, n\}$. Express a brute-force algorithm that determines whether there is a disjoint pair of these subsets. [*Hint:* The algorithm should loop through the subsets; for each subset S_i, it should then loop through all other subsets; and for each of these other subsets S_j, it should loop through all elements k in S_i to determine whether k also belongs to S_j.]

b) Give a big-O estimate for the number of times the algorithm needs to determine whether an integer is in one of the subsets.

12. Consider the following algorithm, which takes as input a sequence of n integers $a_1, a_2, \ldots, a_n$ and produces as output a matrix $\mathbf{M} = \{m_{ij}\}$ where m_{ij} is the minimum term in the sequence of integers $a_i, a_{i+1}, \ldots, a_j$ for $j \geq i$ and $m_{ij} = 0$ otherwise.

```
initialize M so that m_ij = a_i if j ≥ i and m_ij = 0
  otherwise
for i := 1 to n
  for j := i + 1 to n
    for k := i + 1 to j
      m_ij := min(m_ij, a_k)
return M= {m_ij} {m_ij is the minimum term of
  a_i, a_{i+1}, . . . , a_j}
```