CS/MATH 111, Discrete Structures - Fall 2018. Discussion 6 - Non-homogeneous Recurrences, Divide and Conquer & Inclusion - Exclusion

Andres, Sara, Elena

University of California, Riverside

November 1, 2018

Outline

Non-homogeneous recurrence

Theorem

[1] *Proof available at [Rosen, 2015. pg 515].*

$$a_n = a_n^h + a_n^p$$

[1] *

Non-homogeneous recurrence

- Find a particular solution for recurrence relation $C_n = 3C_{n-1} + Cn - 2 + 6$ show your work

Non-homogeneous recurrence

$C_n^n = \beta$, for some constant β. we plug it in: $\beta = 3\beta + \beta + 6$ which gives $\beta = -4$, So $C_n^n = -2$

Non-homogeneous recurrence

- Find a particular solution for recurrence relation $C_n = 3C_{n-1} + Cn - 2 + 3.2^n$ show your work

Non-homogeneous recurrence

$C_n^n = \beta.2^n$, for some constant β. we plug it in:
$\beta.2^n = 3\beta.2^{n-1} + \beta.2^{n-2} + 3.2^n$
after dividing by 2^{n-2}, this reduces to $\beta.4 = 3\beta.2 + \beta + 3.4$
we solve it for β, which gives $\beta = -4$. So, $C_n^n = -4.2^n$

Non-homogeneous recurrence

Solve the following non-homogeneous recurrence:
$A_n = 4A_{n-1} - 4A_{n-2} + 2 * 5^n$, $A_0 = 1, A_1 = 2;$

Non-homogeneous recurrence

Solve the following non-homogeneous recurrence:

$A_n = 4A_{n-1} - 4A_{n-2} + 2 * 5^n$, $A_0 = 1, A_1 = 2;$

$A'_{nc} = \alpha_1 2^n + \alpha_2 n 2^n$

$A''_n = n^m(p_t n^t + p_{t-1} n^{t-1} + ... + p_1 n + p_0)S^n$

S=5 is not a root of characteristic equation its multiplicity is m=0

$A''_n = (p_0)5^n$

$(p_0)5^n = 4(p_0)5^n - 4(p_0)5^n + 2 * 5^n$

$P_0 = 2$

$A_0 = \alpha_1 + 2 = 1$

$A_1 = 2\alpha_1 + 2\alpha_2 + 10 = 2$

$\alpha_1 = -1, \alpha_2 = -3$

Outline

Divide and Conquer

Problem 1: Give the asymptotic value (using the Θ-notation) for the number of letters that will be printed by the algorithms below. Your solution needs to consist of an appropriate recurrence equation and its solution, with a brief justification. (See the suggested format at the bottom of the assignment).

Divide and Conquer

(a) **Algorithm** PRINTXS (n : **integer**)
  if $n < 3$
    print("X")
  else
    PRINTXS($\lceil n/3 \rceil$)
    PRINTXS($\lceil n/3 \rceil$)
    PRINTXS($\lceil n/3 \rceil$)
    for $i \leftarrow 1$ **to** $2n$ **do** print("X")

(b) **Algorithm** PRINTYS (n : **integer**)
  if $n < 2$
    print("Y")
  else
    for $j \leftarrow 1$ **to** 16 **do** PRINTYS($\lfloor n/2 \rfloor$)
    for $i \leftarrow 1$ **to** n^3 **do** print("Y")

Divide and Conquer

(c) **Algorithm** PrintZs (n : **integer**)

```
if n < 3
    print("Z")
else
    PrintZs(⌈n/3⌉)
    PrintZs(⌈n/3⌉)
    for i ← 1 to 7n do print("Z")
```

(d) **Algorithm** PrintUs (n : **integer**)

```
if n < 4
    print("U")
else
    PrintUs(⌈n/4⌉)
    PrintUs(⌊n/4⌋)
    for i ← 1 to 11 do print("U")
```

Divide and Conquer

(e) **Algorithm** PRINTVS (n : **integer**)

  if $n < 3$

    print("V")

  else

    for $j \leftarrow 1$ **to** 9 **do** PRINTVS($\lfloor n/3 \rfloor$)

    for $i \leftarrow 1$ **to** $2n^3$ **do** print("V")

Divide and Conquer

(a)
There are 3 recursive calls, each with parameter $\lceil n/3 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$X(n) = 3X(n/3) + 2n.$$

We apply the Master Theorem with $a = 3$, $b = 3$, $c = 2$, $d = 1$. Here, we have $a = b^d$, so the solution is $\Theta(n \log n)$.

(b)
There are 16 recursive calls, each with parameter $\lfloor n/2 \rfloor$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$X(n) = 16X(n/2) + n^3.$$

We apply the Master Theorem with $a = 16$, $b = 2$, $c = 1$, $d = 3$. Here, we have $a > b^d$, so the solution is $\Theta(n^{\log_2 16})$.

(c)
There are 2 recursive calls, each with parameter $\lceil n/3 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$X(n) = 2X(n/3) + 7n.$$

We apply the Master Theorem with $a = 2$, $b = 3$, $c = 7$, $d = 1$. Here, we have $a < b^d$, so the solution is $\Theta(n)$.

Divide and Conquer

(d)
There are 2 recursive calls, each with parameter $\lceil n/4 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$X(n) = 2X(n/4) + 11.$$

We apply the Master Theorem with $a = 2$, $b = 4$, $c = 11$, $d = 0$. Here, we have $a > b^d$, so the solution is $\Theta(n^{\log_4 2})$.

(e)
There are 9 recursive calls, each with parameter $\lfloor n/3 \rfloor$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$X(n) = 9X(n/3) + 2n^3.$$

We apply the Master Theorem with $a = 9$, $b = 3$, $c = 2$, $d = 3$. Here, we have $a < b^d$, so the solution is $\Theta(n^3)$.

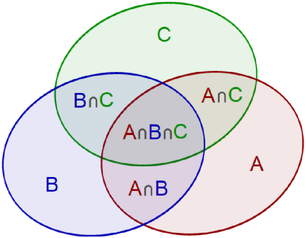
Outline

Inclusion-Exclusion

Problem 2: We have a group of people, each of which is a citizen of either US or Mexico or Canada. Half of the people in this group are US citizens, 10 are Mexican citizens, 17 are Canadian citizens, 4 people have dual US-Mexican citizenship, 5 have US-Canadian citizenship, 6 have Canadian-Mexican, and 2 are citizens of all three countries. How many people are in this group? Show your work.

Inclusion-Exclusion

$$
\begin{aligned}
|A \cup B \cup C| = {} & |A| + |B| + |C| \\
& - |A \cap B| - |A \cap C| - |B \cap C| \\
& + |A \cap B \cap C|
\end{aligned}
$$



Inclusion-Exclusion

$$
\begin{aligned}
\text{Us citizens:} \quad & \mid A \mid & = \tfrac{X}{2} \\
\text{Mexican citizen:} \quad & \mid B \mid & = 10 \\
\text{Canadian citizen:} \quad & \mid C \mid & = 17 \\
\text{US-Mexican citizen:} \quad & \mid A \cap B \mid & = 4 \\
\text{US-Canadian citizen:} \quad & \mid A \cap C \mid & = 5 \\
\text{Canadian-Mexican:} \quad & \mid B \cap C \mid & = 6 \\
\text{Citizens of all countries:} \quad & \mid A \cap B \cap C \mid & = 2
\end{aligned}
$$

$$X = \frac{X}{2} + 10 + 17 - 4 - 5 - 6 + 2$$

$$X = \frac{X}{2} + 14$$

$$X = 28$$