

## 1

$$\begin{aligned}
 100n^2 &\leq 2^n \\
 \Rightarrow \log(100n^2) &\leq n \\
 \Rightarrow \log 100 + 2\log n &\leq n \\
 \Rightarrow n - 2\log n &\geq \log 100
 \end{aligned}$$

$$\begin{aligned}
 \text{令 } f(x) &= x - 2\log x \\
 f'(x) &= 1 - \frac{2}{\ln 2} * \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= 0 \\
 \Rightarrow x &= \frac{2}{\ln 2} \\
 \Rightarrow x &\simeq 2
 \end{aligned}$$

同时

$f(2) = f(4) = 0$  可知

$x > 4$ ,  $f(x)$  递增

由计算可知

$f(14) < \log 100 < f(15)$

结果为 15

## 2

为了方便起见, 不妨设  $f(n) = O(g(n))$

当  $n$  足够大时

$$\max(f(n), O(g(n))) = g(n)$$

$$\Theta(f(n) + g(n)) \leq \Theta(O(g(n)) + g(n)) = g(n)$$

上式取等号

所以

$$\max(f(n), g(n)) = \Theta(f(n) + g(n))$$

### 3

$$\begin{aligned}T(n) &= T(n-1) + n \\T(n-1) &= T(n-2) + n-1 \\&\dots \\T(2) &= T(1) + 2\end{aligned}$$

可以得到

$$T(n) = \frac{n(n+1)}{2} - 1 + T(1) = O(n^2)$$