Homework 0

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	$X=1$	$X=2$	$X=3$	$X=4$
$Y=0$	0	1/9	1/6	1/18
$Y=1$	1/12	1/12	1/12	1/12
$Y=2$	1/6	1/30	1/30	1/10

1) 1. Marginal Distribution

$$\left.\begin{aligned} P(X=1) &= 1/12 + 1/6 = 1/4 \\ P(X=2) &= 1/9 + 1/12 + 1/30 = 41/180 \\ P(X=3) &= 1/6 + 1/12 + 1/30 = 17/60 \\ P(X=4) &= 1/18 + 1/12 + 1/10 = 43/180 \end{aligned}\right\} = 1$$

$$\left.\begin{aligned} P(Y=0) &= 1/9 + 1/6 + 1/18 = 1/3 \\ P(Y=1) &= 4(1/12) = 1/3 \\ P(Y=2) &= 1/6 + 1/30 + 1/30 + 1/10 = 1/3 \end{aligned}\right\} = 1$$

2. $P(X_1 \cap Y_1) = P(X)P(Y)$
If independent they will be the same and dependent if not for all possible events.

$P(X_2 \cap Y_1) = P(X_2)P(Y_1) \qquad 1/12 = (41/180)(1/3)$

$1/12 \neq 41/540$ so X and Y are dependent.

3. Conditional Distribution

$$\left.\begin{aligned} x=1 \quad & 1/6 \,/\, 1/3 = 1/2 \\ x=2 \quad & 1/30 \,/\, 1/3 = 1/10 \\ x=3 \quad & 1/30 \,/\, 1/3 = 1/10 \\ x=4 \quad & 1/10 \,/\, 1/3 = 3/10 \end{aligned}\right\} = 1$$

$E[X \mid Y=2] = 1(1/2) + 2(1/10) + 3(1/10) + 4(3/10)$

$= 22/10 = \boxed{2.2}$

4. $E[X] = 1(1/4) + 2(41/180) + 3(17/60) + 4(43/180)$
$= 254/180 + 153/180 + 45/180 = 452/180 = \boxed{2.51}$
$E[Y] = 0(1/3) + 1(1/3) + 2(1/3) = \boxed{1}$
$E[XY] = E[X]\,E[Y] = (2.51)(1) = \boxed{2.51}$

2) 1. Independent - $P(X \text{ and } Y) = P(X)P(Y)$
$\left(\frac{1}{2} \cdot \frac{1}{2}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $\quad 1/4 = 1/4$ ✓

2. Dependent - $P(X \cap Y) = P(X)P(Y)$
$\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{8}\right)$ $\quad \frac{1}{8} \neq \frac{1}{16}$

3. Dependent - $P(X \cap Y) = P(X)P(Y)$
$\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)$ $\quad \frac{1}{8} \neq \frac{1}{16}$

4. Dependent - $P(X \cap Y) = P(X)P(Y)$
$0 = \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)$ $\quad 0 \neq 1/64$

3) 1. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $x^T = \begin{bmatrix} 10 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10+20+30 \\ 4+5+6 \\ 7+8+9 \end{bmatrix} = \begin{bmatrix} 60 \\ 15 \\ 24 \end{bmatrix}$

$x^T [10, 1, 1]$ $B^T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = [2, 11, 11]$

2. $A = \begin{array}{ccccc} & & 105 & 48 & 72 \\ 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \\ & & 45 & 84 & 96 \end{array}$

$45 + 84 + 96 - 105 - 48 - 72 = 0$
$\text{Det}(A) = 0$

$B = \begin{array}{ccccc} & & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ & & 0 & & \end{array}$

$1 + 1 + 0 - 0 - 0 - 0 = 2$
$\text{Det}(B) = 2$

3. No $AB = \begin{pmatrix} 5 & 4 & 3 \\ 11 & 10 & 9 \\ 17 & 16 & 15 \end{pmatrix}$ $BA = \begin{pmatrix} 11 & 13 & 15 \\ 8 & 10 & 12 \\ 5 & 7 & 9 \end{pmatrix}$

4)

1.
$$\begin{array}{ccccccc} & & & 0 & 3 & 2 & 0 \\ 1 & -1 & 2 & 0 & 1 & -1 & 2 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & -2 & 3 & -1 & 1 & -2 & 3 \\ 3 & 1 & 2 & 4 & 3 & 1 & 2 \\ & & & 0 & 3 & 2 & 0 \end{array}$$

$3+2-3-2=0$
Determinant $=0$ so the vectors $v_1 \dots v_4$ are linearly dependent.

2.
$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & -2 & 3 & -1 \\ 3 & 1 & 2 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & -2 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \boxed{\text{Rank}(U) = 2}$$

3. Basis for range: 3rd colum $v_3 = -1v_2 + v_1$
4th column $v_4 = v_2 + v_1$

So Basis is just $v_1 + v_2$
$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

Basis for null space:
$x_1 = -x_3 - x_4 \qquad x_2 = x_3 - x_4$
$$\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

5) 1. $E[X_1] = 1(1/52) + 2(1/52) + 3(1/52) \dots + 52(1/52)$
$$= \frac{n(n+1)}{2} = \frac{52(53)}{2} / 52 = \boxed{26.5}$$

2. $E[Z] = E[X_1 - 2X_2 + 3X_3] = E[X_1] - 2E[X_2] + 3E[X_3]$
$$= 26.5 - 2(26.5) + 3(26.5) = \boxed{53}$$

3. $E[Y] = E[X_1 - X_2 + X_3 \dots X_{50}] = \boxed{0}$

6) 1. $f_1'(x) = 10e^{10x+2}$ monotonically increasing

$f_2'(x) = 60x^{11}$ monotonically increasing

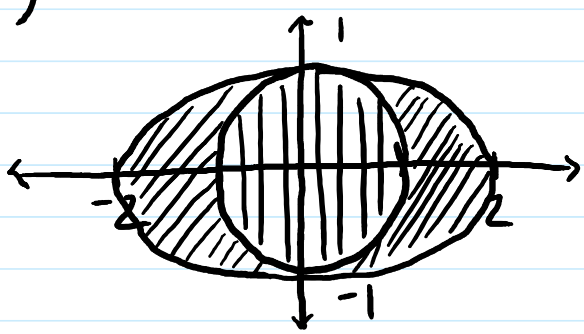
$f_3'(x) = (1-x)^{-1} = 1/(1-x)^2$ monotonically decreasing

2. $\int f_1(x)\,dx = \frac{1}{10} e^{10x+2}$

$\int f_2(x)\,dx = \frac{5}{13} x^{13} + 2x$

$\int f_3(x)\,dx = -\log(1-x)$

3.



$\left.\begin{matrix} x=1 & y=1 \\ x=-1 & y=-1 \end{matrix}\right\} > 4$ $\quad$ $x=1\ y=0$, $x=0\ y=0$

//// > 4

|||| < 4