Homework 1

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P1 1. Maximum value = d
Minimum value = 0

$$z_i = \sum_{j=1}^{d} M_{ij} x_j = \langle M_i, x \rangle \leq \|M_i\| \|x\|$$

$$\|x\| \leq 1 \quad \text{so} \quad z_i = \|M_i\| = \sqrt{1^2+1^2+1^2} = \sqrt{d}$$

$$\|Mx\| = \|z\| = \sqrt{(\sqrt{d})^2+(\sqrt{d})^2+\ldots} = \sqrt{d+d+\ldots} = \sqrt{d(d)}$$

$$= \sqrt{d^2} = d$$

2. Maximum value = 1
Minimum value = 0

$\|Dx\| = 1$ since D = Identity matrix

$\|Dx\| = \|x\| \leq 1 = 1$ when $\|x\|$ is the max.

P2 1. $$x^T(x \circ y)y = x^T(xy^T)y = x^T x y^T y$$

$$x^T x = \sum_1^d x_i^2 = \|x\|^2 \quad y^T y = \sum_1^d y_i^2 = \|y\|^2$$

$$x^T x \, y^T y = \|x\|^2 \|y\|^2$$

This is equal to $x^T \langle x,y \rangle y$ when $x \circ y = x^T y = \sum_1^d x_i y_i = \langle x,y \rangle$

2. $x \circ y = xy^T$ if we multiply xy^T by a vector v
We have $xy^T v = x(y^T v)$ $y^T v$ is a scalar multiplied by a vector x Rank is = 1

$$\overset{x}{\begin{bmatrix} d\times 1 \end{bmatrix}} \overset{y^T}{\begin{bmatrix} 1\times d \end{bmatrix}} \overset{v}{\begin{bmatrix} d\times 1 \end{bmatrix}} = \overset{x}{\begin{bmatrix} d\times 1 \end{bmatrix}} \overset{y^T v}{\begin{bmatrix} 1\times 1 \end{bmatrix}} = \overset{\text{rank}=1}{\begin{bmatrix} d\times 1 \end{bmatrix}}$$

3. $$X^T X = \sum_{l=1}^{d} X_{il}^T X_{lj} = \langle X_i^T, X_j \rangle = \sum_{i,j=1}^{d} x_i^T x_j$$

$$= \sum_{i,j=1}^{d} X_j^T X_i = \sum_{i=1}^{d} X_i^T X_i = \sum_{i=1}^{d} X_i \circ X_i$$

P3 1. $x^T A x$ is PSD $\quad x^T 10 A x = 10(x^T A x)$

$x^T A x \geq 0$, so $10 x^T A x \geq 0$ Always PSD ✓

2. $x^T (A+B) x = (x^T A x) + (x^T B x)$ but $x^T A x \geq 0$

$x^T B x \geq 0$ so $(x^T A x) + (x^T B x) \geq 0$ Always PSD ✓

3 $x^T u u^T x = \langle x, u \rangle \langle u, x \rangle = \|x\|^2 \|u\|^2 =$ Always PSD ✓

4. $x^T (A-B) x = (x^T A x) - (x^T B x)$

Would only be PSD when A is bigger than B so it would not always be PSD.

5. $x^T (I - uu^T) x \qquad x^T I x - x^T u u^T x$

$= x^T x - \langle x, u \rangle \langle u, x \rangle = \|x\|^2 - \|x\| \|u\| \|u\| \|x\|$
$= \|x\|^2 - \|x\|^2 \|u\|^2 \qquad \|x\|^2 - \|x\|^2 = 0$
which is ≥ 0 PSD ✓

P4 1. $\|A_F\| = \sqrt{\sum_m \sum_n A_{ij}^2} = \sqrt{n}$

There are n ones. n ones squared = n

$$\|A\| = \max_x \frac{\|Ax\|}{\|x\|} = \max_x \frac{\|Ix\|}{\|x\|} = \max_x \frac{\|x\|}{\|x\|} = 1$$

2. $\|A_F\| = \sqrt{\sum_m \sum_n A_{ij}^2} = \sqrt{\sum_m \sum_n (uv^T)^2} = \sqrt{\sum_m \sum_n (\langle u,v \rangle)^2}$

$$= \sqrt{\sum_m \sum_n \|u\|^2 \|v\|^2}$$

3.
$$\|A\| = \max_x \frac{\|Ax\|}{\|x\|} = \max_x \frac{\|uv^Tx\|}{\|x\|}$$

$$= \max_x \frac{\|u\langle v,x \rangle\|}{\|x\|} = \max_x \frac{\|u \overbrace{\|v\| \|x\|}^{\text{scalar}} \|}{\|x\|}$$

$$= \max_x \frac{\|v\| \|x\| \|u\|}{\|x\|} = \max_x \|v\| \|u\|$$