Math 111 Shirazi S01

Skye Kychenthal

8/29 thru 9/2

Topics covered:

By the end of calculus (Math 111) I should have the ability to analyze any $f(x)$ function.

- Derivatives
- Integrals
- Limits

Pre-requisite knowledge to Calc:

- Functions ie. $f(x) = y$
- Fractions
- Basic geometry
- Domain and Range of Functions

Homework will either be written in LaTeX or through the WebWork application. WebWork will be typical math homework.

Communication should be through:

- Email (Absences, Medical, Extensions)
- Office Hours a Math Help
- Slack (ask my fellow students questions)

Lecture One (8/29)

What is calculus?

The long answer to this will be after 13 weeks

The short answer is that calculus was created to answer two questions

1. How do you find the tangent a given function?
2. How do you find the area under a curve?

Tangents

"What is the slope of a line tangent to a function?"

Tangets are imporant when analyzing the rate of change at a point in time. This is incredibly common in hte real world, especially when tracking how certain things change over periods of time. This could be a simple question of how many students graduate every single year from a highschool, but could be a far more complicated and important question of how fast are COVID cases are increasesing at any given point in time.

Lines

Lines can be descibed in two different ways:

$Ax + By = C$

which can also be further re-written to

$y = ax + b$ (more commonly known to me as $y = mx + b$)

Rate of change can be described by the equation:

$$\frac{f(t_0) - f(t_1)}{t_0 - t_1}$$

This equation can eventually be simplified to just the slope. Through substitution:

$$\frac{A * t_0 + B - A * t_1 + B}{t_0 - t_1}$$

You can factor out the A from the equation and the two Bs cancel each other out, leaving you with

$$A * \frac{t_0 - t_1}{t_0 - t_1}$$

Which, after the fraction cancels out to 1, you are just left with A or the slope. Therefore, the slope of A is just the rate of change.

[insert graph later ;-;]

The rate of change at a is the tangent line at a. This can be best approximated by picking a random point at n. The closer n gets to a the more accurate, but still non-exact, the tangent line gets. This can best be described as:

$$\text{Rate of Change at A} \approx \frac{f(t_0) - f(t_1)}{t_0 - t_1}$$

The estimated rate of change, or the fraction component, can also be described as n_{sec}. The secant line is the approximation of the tangent, so therefore it is annotated as n_{sec}.

Velocity

Another way to find the rate of change at a given point can best be described by velocity problems. Given the velocity of a car is described by $S(t) = -5t^2 + 35$, between point A (0 hrs, 0km) and B (1.5 hrs, 60km) how do you find the $\approx$ rate of change at 0.4 hrs. We will call the point at 0.4 hrs X. We should also create two points surrounding X, those being Y which will be 0.3 hrs, and Z which will be 0.5 hrs.

The way you approximate the rate of change at point X by bounding the secant lines or the estimated rates of changing between $Y \rightarrow X$ and $X \rightarrow Z$. This way, the answer to the $\approx$ would look like:

$$Y_{sec} < \text{Rate of Change at } X < Z_{sec}$$

Y_{sec} can be found with:

$$Y_{sec} = \frac{S(Y) - S(X)}{Y - X}$$

Which can then be substituted to:

$$Y_{sec} = \frac{-5 * 0.3^2 + 35 - -5 * 0.4^2 + 35}{0.3 - 0.4}$$

Both constants of 35 can be canceled out, and everything else simplified to:

$$Y_{sec} = \frac{0.45 - 0.8}{-0.1}$$

$$Y_{sec} = \frac{-0.35}{-0.1}$$

$$Y_{sec} = 3.5$$

And the same thing can be done again w/ Z_{sec}

$$Y_{sec} = \frac{S(Z) - S(X)}{Z - X}$$

$$Y_{sec} = \frac{-5 * 0.5^2 + 35 - -5 * 0.4^2 + 35}{0.5 - 0.4}$$

$$Y_{sec} = \frac{1.25 - 0.8}{0.5 - 0.4}$$

$$Y_{sec} = \frac{0.45}{0.1}$$

$$Y_{sec} = 4.5$$

Therefore, if we plug it into the original prompt, we can determine that X_{sec} is between Y_{sec} and Z_{sec}.

$$3.5 < \text{Rate of Change at } X < 4.5$$

Lecture Two (8/31)