Math 111 Shirazi S01

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8/29 thru 9/2

Topics covered:

By the end of calculus (Math 111) I should have the ability to analyze any $f(x)$ function.

- Derivatives
- Integrals
- Limits

Pre-requisite knowledge to Calc:

- Functions ie. $f(x) = y$
- Fractions
- Basic geometry
- Domain and Range of Functions

Homework will either be written in LaTeX or through the WebWork application. WebWork will be typical math homework.

Communication should be through:

- Email (Absences, Medical, Extensions)
- Office Hours a Math Help
- Slack (ask my fellow students questions)

Disclaimer

As of right now, the following is only my notes. I may include homework assignments after they are due.

Lecture One (8/29)

What is calculus?

The long answer to this will be after 13 weeks

The short answer is that calculus was created to answer two questions

1. How do you find the tangent a given function?
2. How do you find the area under a curve?

Tangents

“What is the slope of a line tangent to a function?”

Tangets are imporant when analyzing the rate of change at a point in time. This is incredibly common in hte real world, especially when tracking how certain things change over periods of time. This could be a simple question of how many students graduate every single year from a highschool, but could be a far more complicated and important question of how fast are COVID cases are increasesing at any given point in time.

Lines

Lines can be descibed in two different ways:

$Ax + By = C$

which can also be further re-written to

$y = ax + b$ (more commonly known to me as $y = mx + b$)

Rate of change can be described by the equation:

$$\frac{f(t_0) - f(t_1)}{t_0 - t_1}$$

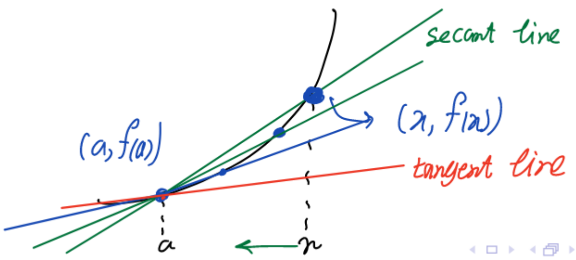
This equation can eventually be simplified to just the slope. Through substitution:

$$\frac{A * t_0 + B - A * t_1 + B}{t_0 - t_1}$$

You can factor out the A from the equation and the two Bs cancel each other out, leaving you with

$$A * \frac{t_0 - t_1}{t_0 - t_1}$$

Which, after the fraction cancels out to 1, you are just left with A or the slope. Therefore, the slope of A is just the rate of change.



The rate of change at a is the tangent line at a. This can be best approximated by picking a random point at n. The closer n gets to a the more accurate, but still non-exact, the tangent line gets. This can best be described as:

$$\text{Rate of Change at A} \approx \frac{f(t_0) - f(t_1)}{t_0 - t_1}$$

The estimated rate of change, or the fraction component, can also be described as n_{sec}. The secant line is the approximation of the tangent, so therefore it is annotated as n_{sec}.

Velocity

Another way to find the rate of change at a given point can best be described by velocity problems. Given the velocity of a car is described by $S(t) = -5t^2 + 35$, between point A (0 hrs, 0km) and B (1.5 hrs, 60km) how do you find the $\approx$ rate of change at 0.4 hrs. We will call the point at 0.4 hrs X. We should also create two points surrounding X, those being Y which will be 0.3 hrs, and Z which will be 0.5 hrs.

The way you approximate the rate of change at point X by bounding the secant lines or the estimated rates of changing between $Y \to X$ and $X \to Z$. This way, the answer to the $\approx$ would look like:

$$Y_{sec} < \text{Rate of Change at } X < Z_{sec}$$

Y_{sec} can be found with:

$$Y_{sec} = \frac{S(Y) - S(X)}{Y - X}$$

Which can then be substituted to:

$$Y_{sec} = \frac{-5 * 0.3^2 + 35 - -5 * 0.4^2 + 35}{0.3 - 0.4}$$

Both constants of 35 can be canceled out, and everything else simplified to:

$$Y_{sec} = \frac{0.45 - 0.8}{-0.1}$$

$$Y_{sec} = \frac{-0.35}{-0.1}$$

$$Y_{sec} = 3.5$$

And the same thing can be done again w/ Z_{sec}

$$Y_{sec} = \frac{S(Z) - S(X)}{Z - X}$$

$$Y_{sec} = \frac{-5 * 0.5^2 + 35 - -5 * 0.4^2 + 35}{0.5 - 0.4}$$

$$Y_{sec} = \frac{1.25 - 0.8}{0.5 - 0.4}$$

$$Y_{sec} = \frac{0.45}{0.1}$$

$$Y_{sec} = 4.5$$

Therefore, if we plug it into the original prompt, we can determine that X_{sec} is between Y_{sec} and Z_{sec}.

$$3.5 < \text{Rate of Change at } X < 4.5$$

Lecture Two (8/31)

Calc History and Recap

The previous lecture was written in a notebook then transcribed to LaTeX. Notes from now on will be written purely in LaTeX.

All we know in mathematics is addition, subtraction, multiplication, and division (although multiplication and division are more short-hand forms of addition and subtraction respectively). The only reason we use simple calculators is to save time. Not because we cannot do it without a calculator. Historically, the creation of calculus would have been created without the use of calculators, etc. Computers would have been created this way, for example. Where the accumulator on a computer is simply just a mechanism that adds and subtracts.

Recap of the previous lecture:

- m_{sec} is a good $\approx$ of the tangent line at m (m_{tan})
- Rate of Change at $A \approx \frac{f(t_0)-f(t_1)}{t_0-t_1}$; the reason this is an approximation is because the rate of change at A is based on simply one point, meanwhile the fraction is based on an interval.
- As the estimation (A_{sec}) gets closer to A the closer an estimation of A_{tan} it is.

Based on the previous problem, based on a car moving at the velocity of $S(t) = -5t^2 + 35$ where S_{sec} is based on the rate of change between the interval $[0.3, 0.4]$ or $[Y, Z]$, the closer Y and Z get to 0.4 or X, the better the approximate is.

If Y is set to Z is set to 0.39 and 0.41 respectively, you end up with a better approximate of X_{tan} or:

$$4.05 < \text{Rate of Change at } X < 3.95$$

The second question that led to the creation of calculus, was that of "how do you find the area under a curve?" Why does this matter? Good question! Time for me to find out !

In physics, equations with Force or F are solved by finding the work done by an object on an interval from $[a, b]$. The issue becomes, what happens when the equation of Force is *not* constant? In the real world, Force equations are never really constant. If you push an object, you cannot indefinitely push. You eventually need to take a few deep breathes and slow down a bit. The way you find the work done is by finding the area under the curve of the non-constant Force equation.

[insert graph later]

In this wobbly looking equation, how the heck are you supposed to find the amount of work done to put Force on an object? Well, good question! This is why we have calculus . God my notes are getting gay now that I'm typing them in LaTeX lmao.

To recap, the two biggest questions in calculus, are:

1. How do you find the tangent on a given equation?
2. How do you find the area under a curve?

As a result, the biggest things we (I?) will learn in this course is:

- Derivatives
- Integrals

Before we get to those fun stuff, we have to look at limits or $\lim$!

Intervals

uHHH what the heck is going on with delta neighbourhoods.... look at / ask for help later ;-; but will try to work on now

Neighborhoods

Good to know notation: δ or delta. Delta is the difference between two points. So if you have a and $a + \delta$ and $\delta = 5$ you end up having the point $a + \delta$ be equal to $a + 5$. Likewise, $a - \delta$ would be $a - 5$. Both points are symmetrical based on a and are the same distance/difference away from each other; or the same δ away from each other.

In the velocity problem from earlier, you end up with 0.3 and 0.5 with the center point being 0.4. In this instance you end up with a center point a of 0.4 and a δ of 0.1. This is a neighborhood, and a symmetrical one at that, where the δ between a and the point on the right and the left are both the same δ difference away from each other.

A more mathematical way of expressing this problem would be:

$$\{x \in \mathbb{R} : a - \delta < x < a + \delta\}$$

This, in actual English, is that x on a line is an element ($\in$) of all real / natrual numbers ($\mathbb{R}$) where x is in-between $a - \delta$ and $a + \delta$. This is a very, very fancy and quite obtuse and non-intuitive way of just saying that "x is a point between the symmetrical difference a - number to a + number."

Look more at open, closed, half-open, and similarly notation. Open notation for the interval between a and b would be (a, b) while the closed notation would be $[a, b]$. This is respectively equivilant to $\{x \in \mathbb{R} : a < x < b\}$ and $\{x \in \mathbb{R} : a \leq x \leq b\}$. $\{\}$ indicates a set in mathematics. Basically CS stuff but math stuff yknow???? Brain hurt.

Look at slideshow notes and go over what intersection and union means...

Intuitive Definition of a Limit

Add graphs from "the limit of a function" slide.

If a goes to x where a is equal to 5 and 6 is equal to 6. If you are going down from x to a, the number 6 is going down to eventually reaching 5. That means that $f(x)$ is getting closer to 5. This can be described as:

$$\begin{aligned}
\text{limit of } f \text{at } a &= 5 \\
f(a) &= 6 \\
\lim_{x \to a} f(x) &= L \\
L &= 5
\end{aligned}$$

L is usually assumed to be a real number. Help. Me.

Lecture Three (9/2)

The first homework will be assigned tonight (a written assignment), meanwhile the next homework assignment will be the web-work assignment due later.

How to calculate a limit?

Calculating limits using a table

Find the limit of:

$$f(x) = \frac{x^2 - 25}{x - 5} \text{at} a = 5$$

The best way to do this is to find the surrounding numbers using the equation of:

$$a - \delta < x < a + \delta$$

We will then find the values of $f(5.1)$ and $f(4.9)$ which are respectively equal to 10.1 and 9.9. It can then be assumed that $\lim_{x\to 5} f(x) = 10$ because it seems that x is approaching 10 from both sides. To be the most sure, you need to continuously compute the values in a way that δ gets closer and closer to 0.

We can re-write the process as:

$$\begin{aligned}
f(x) &= \frac{x^2 - 25}{x - 5} \text{ at } x = 5 \\
x - \delta < \lim_{x\to 5} f(x) &< x + \delta \\
\delta &= 0.1 \\
f(x - \delta) &= 9.9 \\
f(x + \delta) &= 10.1 \\
9.9 < \lim_{x\to 5} f(x) &< 10.1 \\
\delta &= 0.01 \\
f(x - \delta) &= 9.99 \\
f(x + \delta) &= 10.01 \\
9.99 < \lim_{x\to 5} f(x) &< 10.01
\end{aligned}$$

Programmatic way of solving limits

A more programmatic way of solving the problem could be:

```
    values = []

def compute (value, delta = 0.1, iter_divide = 2, iterations = 3):

    delta = delta / iter_divide
    iterations -= 1

    values.append (

        (
            ((value+delta)**2 - 25 ) / ( (value+delta) - 5 ) ,
            ((value-delta)**2 - 25 ) / ( (value-delta) - 5 )
        )

    )

    if (iterations <= 0):
        return
    return compute ( value, delta, iter_divide, iterations )

compute(5)

print (values)

>> (10.05, 9.95), (10.025, 9.975), (10.0125, 9.875)
```

Transcendental Functions

If a function is **algebraic**, it can be described by algebraic functions. Otherwise it is called **transcendental**. Transcendental functions may include: $\sin(x)$ or e^x.

Evaluate $\lim_{x\to 0}\sin(\frac{1}{x})$

$$
\begin{aligned}
f(x) &= \sin(\frac{1}{x}) \\
\lim_{x\to 0} f(x) &= ? \\
f(0) &= \text{undefined} \\
f(0.1) &= -0.544 \\
f(-0.1) &= 0.544 \\
f(0.01) &= -0.506 \\
f(-0.1) &= 0.506 \\
\lim_{x\to 0} f(x) &= \text{undefined}
\end{aligned}
$$

This limit is undefined, because as x approaches 0 from negative δ values, the $\lim$ approaches 0.5 meanwhile from positive δ values it approaches -0.5.

Epsilon shit at the end of the lecture is making no sense. Look moodle at notes later and ask for help ;-;

Precise Definition

Let $f(x)$ be defined for all $x\neg a$, over an interval containing a, and L is a real number, Then:

$$\lim_{x\to a} f(x) = L$$

if, for every (Epsilon???) ¿ 0, there exists a δ ¿ 0, such that for all x,

...lost the rest of the definition look at powerpoint notes.