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MAT 350 Module Five: Equilibrium Temperature Distribution

**Problems**

Consider the discretized square plate in Figure 1.

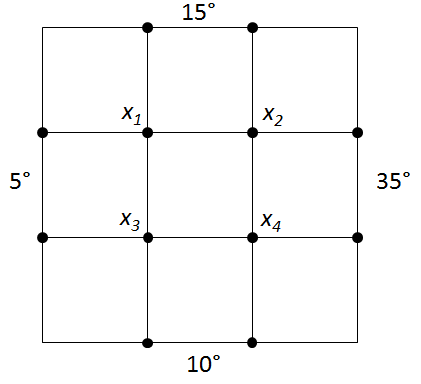


Figure 1: Discretized Square Plate

1. Write a system of linear equations for this system and clearly identify the matrix *A* in the matrix equation *A****x****=****b***.

, , ,

x = , A = , b =

1. Compute the determinant of *A*.

**Matrix A:**

A =

**Add ¼ \* Row 1 to Row2, removing the -1 from Row 2 Col 1:**

A =

**Add ¼ \* Row 1 to Row3, removing the -1 from Row 3, Col 1:**

A =

**Add 1/15 \* Row 2 to Row 3 to remove the -1/4:**

A =

**Remove the leading -1 from Row 4 by adding 4/15 \* Row 2:**

A =

**Remove the from Row 4 by adding 2/7 \* Row 3:**

A =

**Multiply the diagonals of the matrix to find the determinant:**

4 \* 15/4 \* 56/15 \* 24/7 = 192

1. Compute the classical adjoint matrix (i.e., the adjugate matrix) of *A*.

**Matrix A:**

A =

**First find the Matrix of Minors before computing the Adjoint Matrix. This can be done by computing the determinants of the smaller square matrices formed by removing row i and column j from Matrix A for each determinant calculation:**

**= det**  , Determinant = 4 \*det - 0 \* det -1 \* det = 4 \* 15 – 0 \* -1 – 1 \*4 = 60 – 4 = = 56

**Follow the same process for the rest of the elements of the Matrix of Minors:**

**= det**  , Determinant = -16

**= det**  , Determinant = 16

**= det**  , Determinant = -8

**= det**  , Determinant = -16

**= det**  , Determinant = 56

**= det**  , Determinant = -8

**= det**  , Determinant = 16

**= det**  , Determinant = 16

**= det**  , Determinant = -8

**= det**  , Determinant = 56

**= det**  , Determinant = -16

**= det**  , Determinant = -8

**= det**  , Determinant = 16

**= det**  , Determinant = -16

**= det**  , Determinant = 56

**Combining these terms into a matrix, we get the Matrix of Minors:**

**The Matrix of cofactors is determined by applying a checkerboard of minus signs (alternating minus signs) to the Matrix of Minors. Because the negative numbers are alternating in our Matrix of Minors, we get a positive cofactor matrix:**

Cof A:

**Finally, to calculate the Adjugate Matrix, we transpose the elements of the cofactor matrix (swapping over the diagonal). In this case, because of the similarity of the terms in the cofactor matrix, the Adjugate Matrix is identical:**

Adjugate Matrix of A:

1. Using your answers from problems 2 and 3, solve for *A-1* and ***x***.

Adjugate Matrix of A:

**To find *A-1,* multiply the Adjugate matrix by 1/determinant(A):**

***A-1* =**

**Simplify:**

***A-1* =**

***X* = *A-1*b:**

***A-1* =** \* b = =

**X =**  =

1. Use Cramer’s rule to check your answer by solving for just *x1* and *x4*.

**Cramer’s Rule requires that we swap the b vector for a column in Matrix A and then solve for the determinant of the new matrix. Once we have the determinant of the new matrix, we can find *x1* and *x4* by dividing the determinant of the new matrix by the determinant of the old matrix:**

A = b =

**Solve for *x1*:**

**Substitute b for Column 1:**

=

**Convert the matrix into RREF:**

=

**Swap Row 1 and Row 2:**

=

**Reduce the 20 in Row 2 by subtracting 2/5 Row 1 from Row 2:**

=

**Remove the 15 from Row 3 by subtracting 3/10 \* Row 1 from Row 3:**

=

**Remove the 45 from Row 4 by subtracting 9/10 \* Row 1 from Row 4:**

=

**Swap rows 2 and 4:**

=

**Remove the -6/5 from row 3 by subtracting 6/23 \* row 2:**

=

**Remove the -13/5 from Row 4 by subtracting 13/23 \* row 2:**

=

**Add 5/49 \* Row 3 to Row 4 to remove the -10/23 from Row 4:**

=

**Multiply the diagonal to calculate the determinant:**

50 \* \* \* = 2520

b = 2520

***x1*** = 2520/192

***x1*** = 13.125

A = b =

**Solve for *x4*:**

**Substitute b for Column 4:**

=

**Convert the matrix into RREF:**

**Remove the -1 from Row 2 by adding ¼ \* Row 1:**

=

**Remove the -1 from Row 3 by adding ¼ \* Row 1:**

=

**Remove the -1/4 from Row 3 by adding 1/15 \* Row 2:**

=

**Remove the -1 from Row 4 by adding 4/15 \* Row 2:**

=

**Remove the -16/15 by adding 2/7 \* Row 3:**

=

**Multiply the diagonal to calculate the determinant:**

4 \* 15/4 \* 56/15 \* 465/7 = 3720

b = 3720

***x4*** = 3720/192

***x4*** = 19.375