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AMATH 495 - 001
Final Project
Winter 2023 - June 11, 2024
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Recap: Project Outline (from Lab write-up)

I am going to create a toy model for an economy-climate coupled system as follows:

The model has two countries, one developed economy and one developing one. The developed economy grows it's GDP linearly over time and the developing economy grows exponentially. The developed economy starts with a GDP much larger than the developing economy. Both countries emit CO2 proportional to their current GDP.

Once a year, a global natural disaster happens and it reduces the GDP of both countries. The impact to GDP is inversely propotional to the current GDP of each country, and proportional to the current total amount of CO2 in the atmosphere.

The model will produce plots on how CO2 levels change over time as well as how the GDP of each country changes over time.

Note: I decided to tweak the model a bit since the last lab session. The GDP of both economies will now be modelled by a logistic curve, where the developed economy will have an initial condition closer to it's carrying capacity, and the developing economy will have an initial condition significantly less than it's carrying capacity. I will also add a few variables to represent "innovation" which will affect the carrying capacities of both economies and how much CO_2 they emit.

The Model

There are 5 variables that the model will keep track of:

$r =$ The GDP of the **r**icher, developed country
$p =$ The GDP of the **p**oorer developing country
$I_r =$ The "**I**nnovation" of the developed country. The higher the innovation, the higher the carrying capacity of the country, and the lower the CO_2 it emits.
$I_p =$ The "innovation" of the developing country.
$c =$ The **c**oncentration of CO_2 in the atmosphere

Under normal circumstances, the following system of ODEs represents how this system interacts:

$$
\begin{aligned}
\frac{dr}{dt} &= G_r\left(1-\frac{r}{K_r I_r}\right) r \\
\frac{dp}{dt} &= G_p\left(1-\frac{p}{K_p I_p}\right) p \\
\frac{dI_r}{dt} &= I_n \frac{dr}{dt}\left(1-M_p\right) \\
\frac{dI_p}{dt} &= I_n\left(\frac{dp}{dt}+\frac{dr}{dt} M_p\right) \\
\frac{dc}{dt} &= \gamma\left(\frac{r}{\alpha I_r}+\frac{p}{\beta I_p}\right)
\end{aligned}
$$

where I define the model adjustable parameters as follows:

G_r : Growth rate of the richer country's GDP
G_p : Growth rate of the poorer country's GDP
K_r : Initial carrying capacity of the richer country's GDP
K_p : Initial carrying capacity of the poorer country's GDP
γ : Emission rate of CO_2 with respect to GDP
α : CO_2 emission efficiency from GDP due to innovation of the richer country
β : CO_2 emission efficiency from GDP due to innovation of the poorer country
I_n : Innovation growth rate
M_p : Modernization program: The amount of innovation that the richer country donates to the poorer country to help them moderize.

The GDP of both countries is modelled by a modified logistics curve, where both countries can increase their carrying capacities by producing "innovation".

I wanted innovation to be proportional to current GDP, this is why the first term in both expressions includes the differential of their GDPs. The second term represents the richer country donating some of their innovation to help out the smaller country.

Both countries contribute an increase to the concentration of CO_2, but how much they contribute per GDP can be decreased by increasing their innovation. This is intended to simulate investments into green energy solutions and technologies.

However, once every f years, a big, global natural disaster happens and it will negatively impact the GDP of both countries. This is represented as follows:

$$\begin{aligned}
&\text{Every } f \text{ years:}\\
&g(t_f) = 10\% \ * \ g(t)\left(1 - \exp\left(-D * c(t)/\left[g(t)\right]^2\right)\right)\\
&\qquad\text{where:}\\
&\qquad\qquad g: \text{The GDP of a country}\\
&\qquad\qquad t_f: \text{A time immediately after the disaster}\\
&\qquad\qquad t: \text{A time immediately before the disaster}\\
&\qquad\qquad D: \text{Disaster intensity}\\
&\qquad\qquad f: \text{A disaster happens every } f \text{ years}
\end{aligned}$$

Originally, I had a much simpler expression for the disaster function, but it ended up causing GDP to go negative, so I decided to make it a bit more complicated by adding an exponential decay term to ensure that the disaster never destroys more than 90% of either economy.

Initial Conditions

To simplify things, I will work with values normalized to the current status quo in this imaginary Earth. So we have that:

$$\begin{aligned}
r_0 &= 1\\
p_0 &= 0.01\\
I_{r0} &= r_0\\
I_{p0} &= p_0\\
c &= 1
\end{aligned}$$

Real World Data

For comparison purposes, here's some real world data of some of the quantities my model is simulating.

Below is real GDP data for the USA (a rich country) since 1960. It appears to be trending with a slow exponential growth.

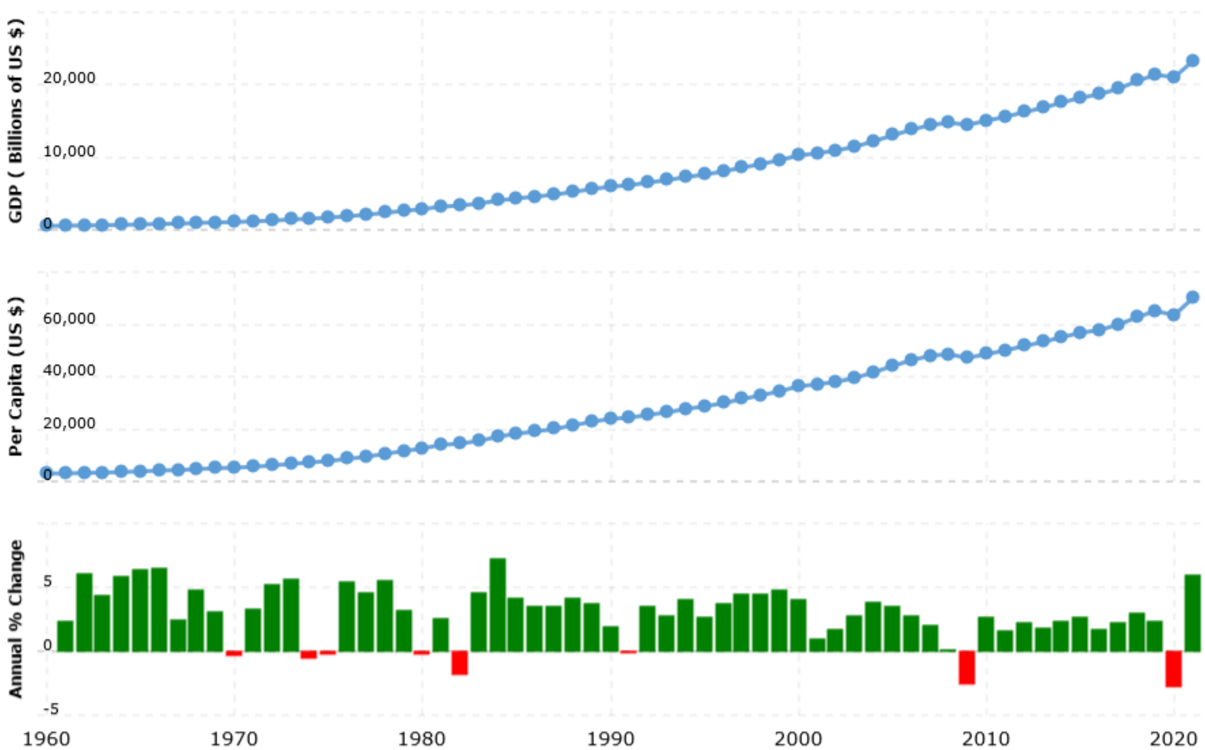

Figure 1: Real GDP data for the USA since 1960. Retrived from [4]

An example of a poorer country that's been heavily impacted by climate change in recent years is Chad. It's been experiencing intense droughts that get worse year by year. [1]. Below is a graph of their GDP since 1960, notice that it's much more volatile.

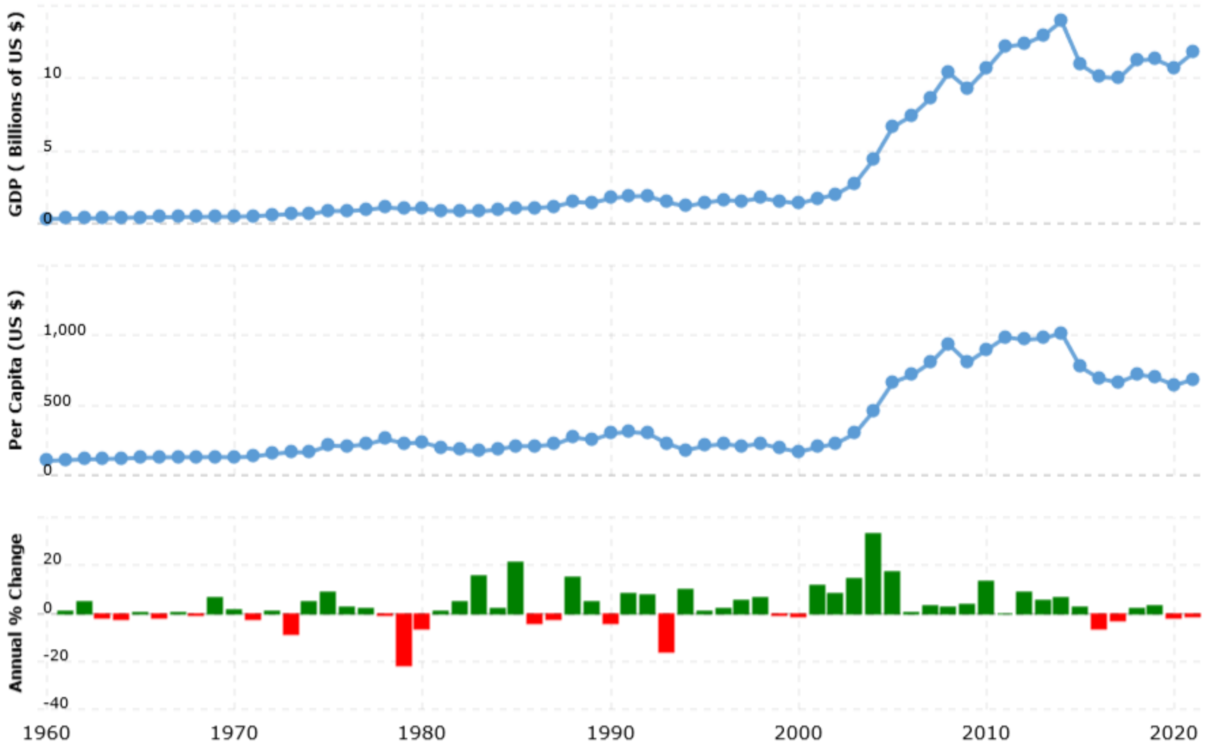

Figure 2: Real GDP data for Chad since 1960. Retrieved from [2]

And here's a graph of the concentration of CO_2 in the atmosphere since the industrial revolution. Notice how it's exponentially increasing as more and more countries started to industrialize and caused their own GDP's to increase exponentially.

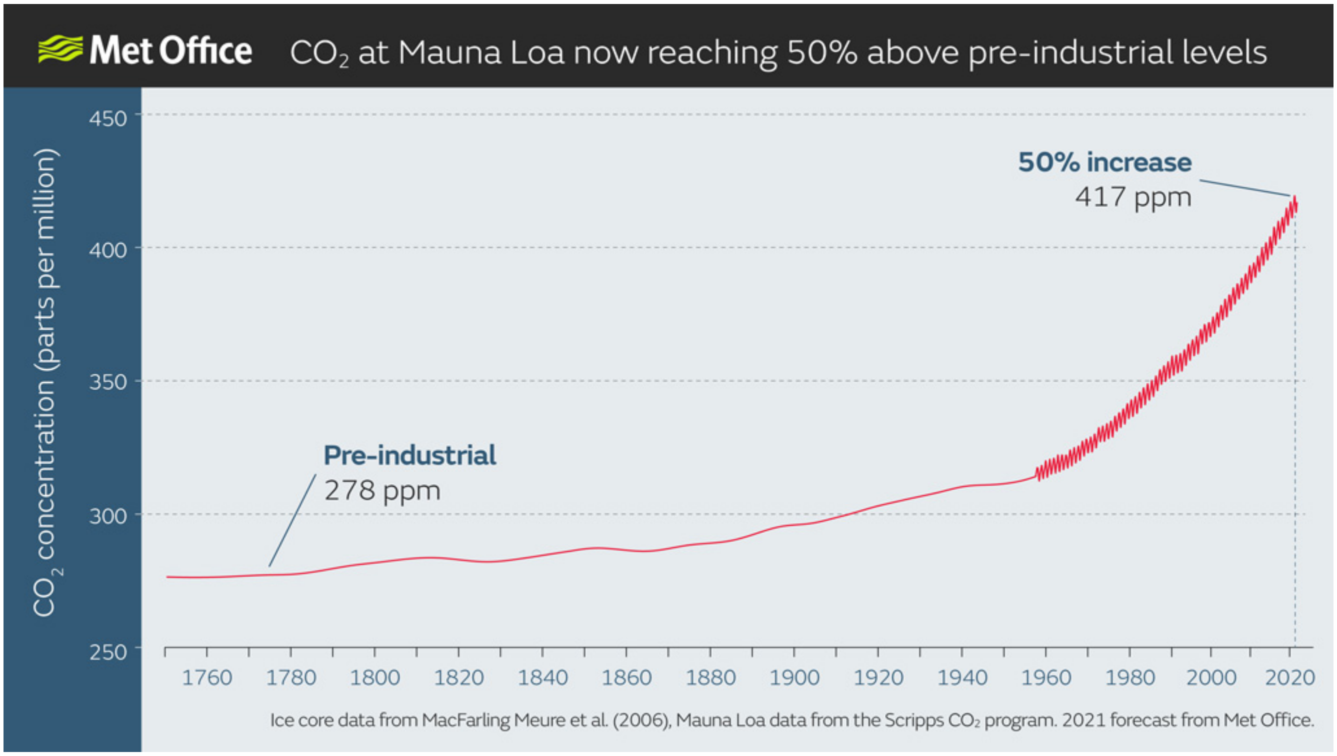

Figure 3: Concentration of CO_2 since the industrial revolution [3]

Results

Here is an attempt at replicating our own present day conditions for the climate and the economy. I set my ODE system parameters as follows:

$$
\begin{aligned}
G_r &= 4 \times 10^{-3} \\
G_p &= 2 \times 10^{-3} \\
K_r &= 1.01 \\
K_p &= 2.02 \\
\gamma &= 2 \\
\alpha &= 1 \\
\beta &= 1 \\
D &= 1 \times 10^{-5} \\
f &= 6 \\
I_n &= 2 \times 10^{-1} \\
M_p &= 0.01
\end{aligned}
$$

And then ran the simulation for 50 years:

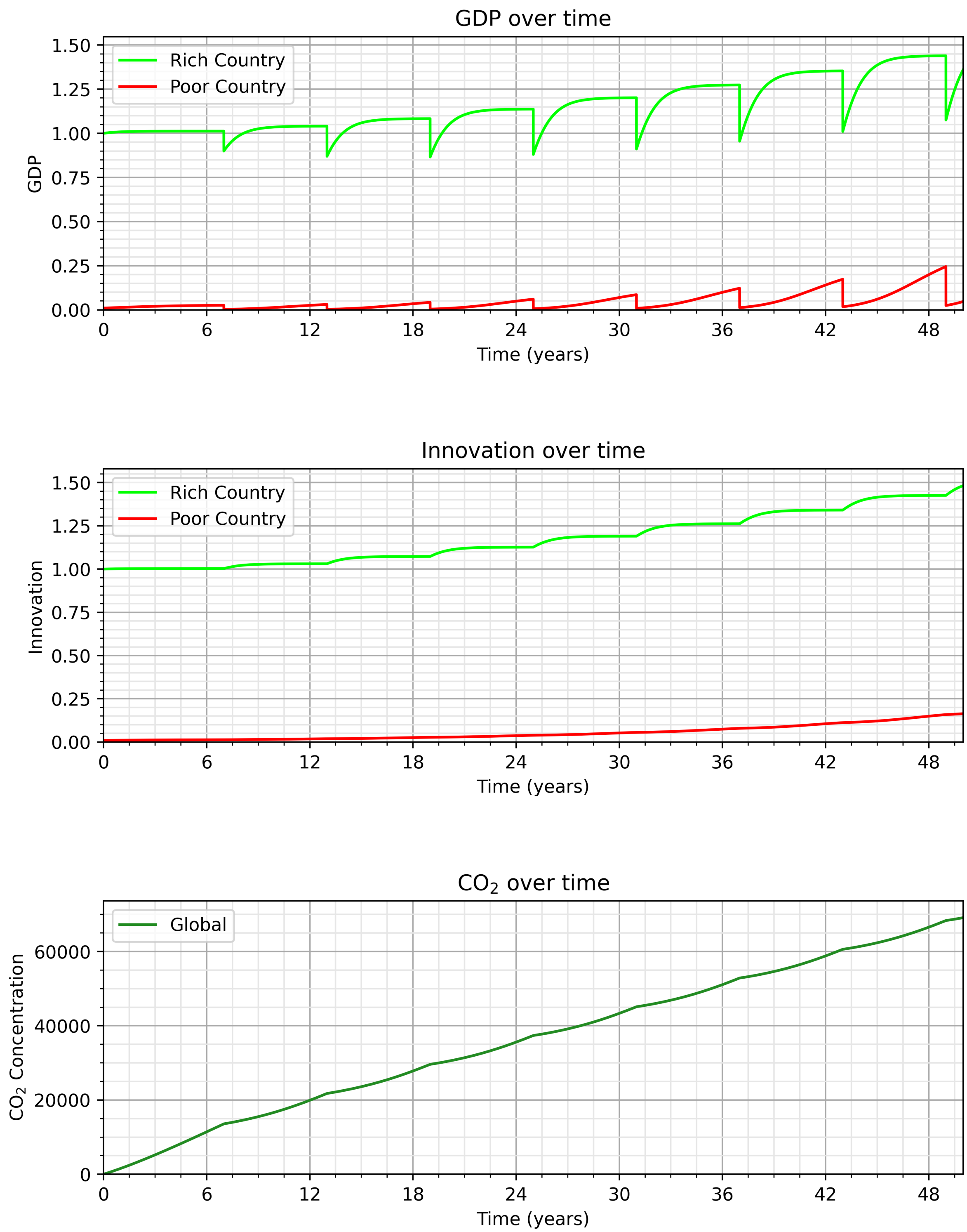

Figure 4: Control simulation for the Economy-Climate system.

Notice how the rich country isn't as affected by the disaster and quickly rebounds. Whereas the disaster is more devasting for the poorer country and takes them more time to recover:

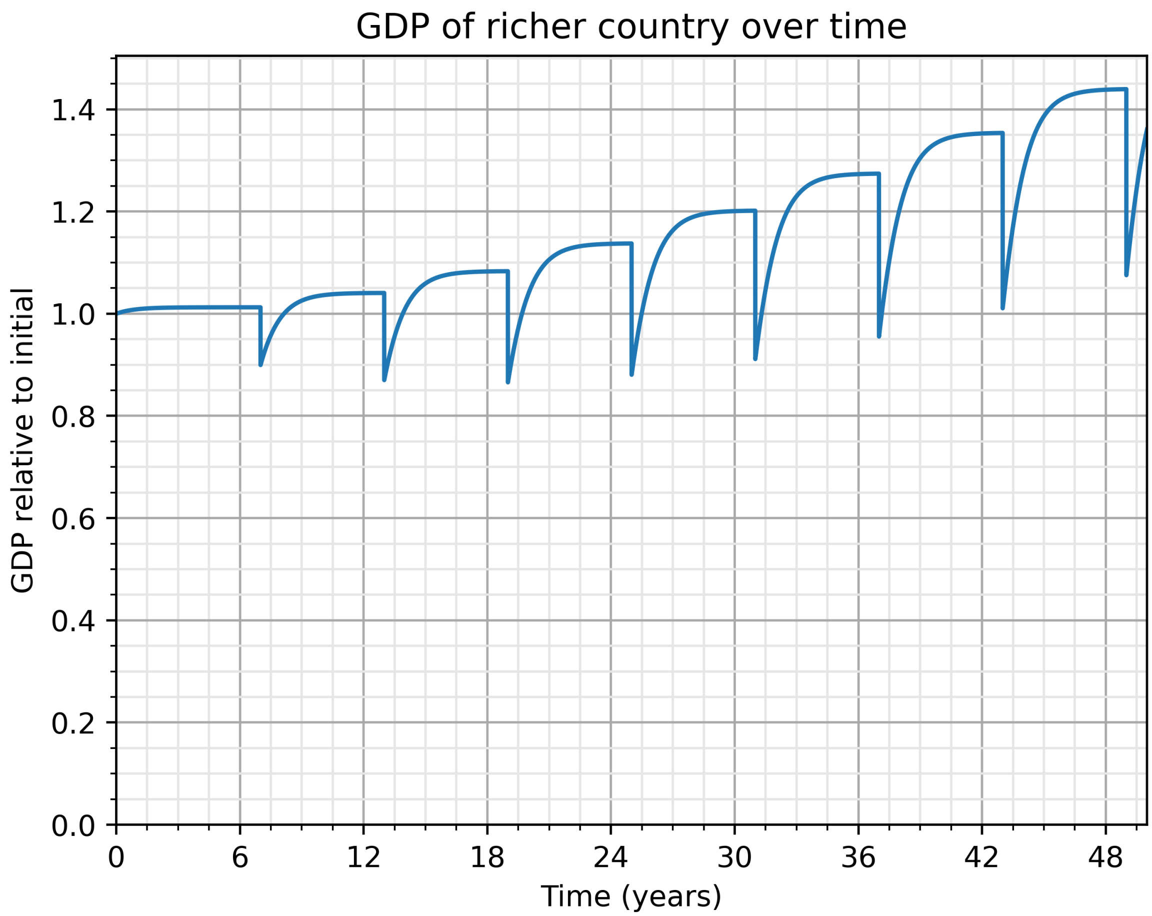

Figure 5: GDP for the rich country in control simulation.

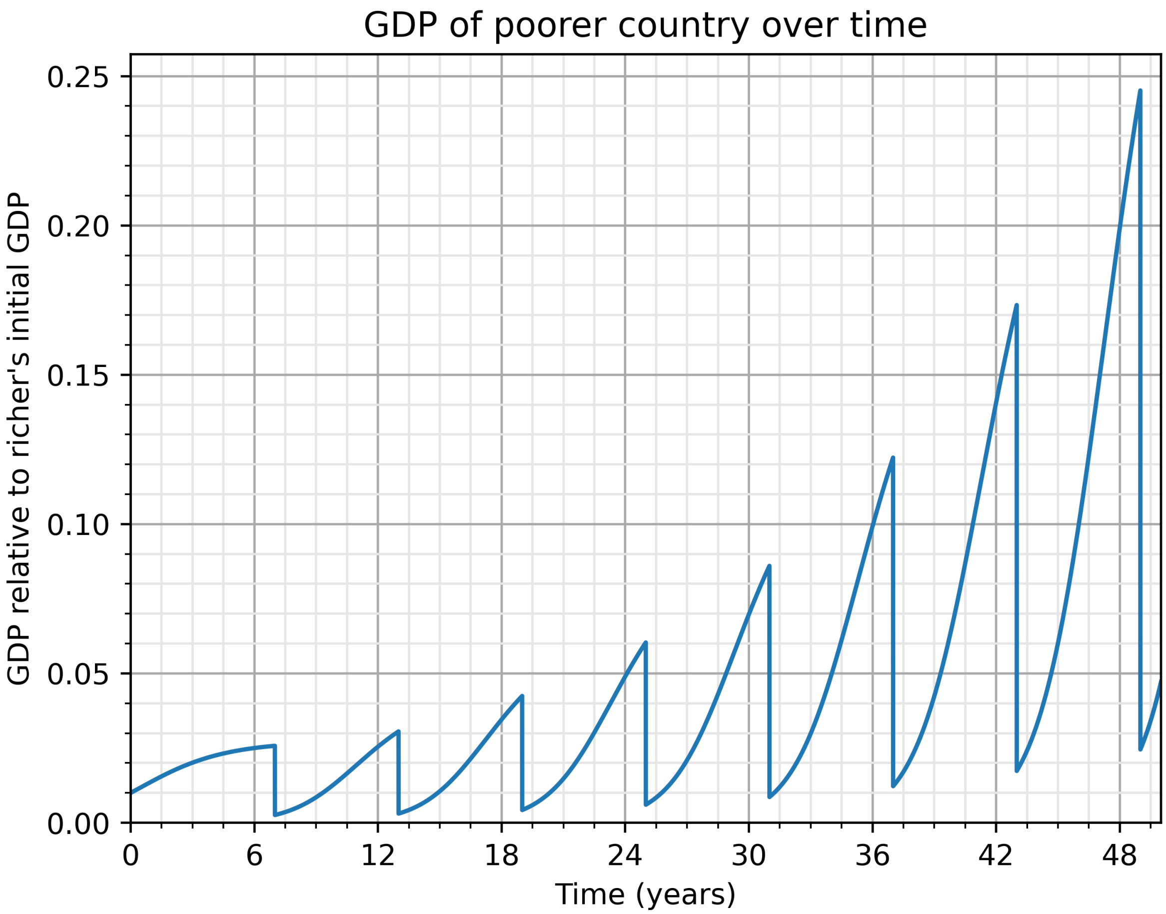

Figure 6: GDP for the poor country in control simulation.

Discussion

I wish I had more time to play around with my model. Because of it's complexity, it was kinda difficult finding the right parameters to replicate all the trends I was looking to replicate compared to real world data, so a lot of time was spent fine-tuning the parameters. But the model is also too simple to be used to make any meaningful predictions. So this was more useful as a lesson on how complicated the climate-economy system really is since it's a non-trivial task to use math to model it.

Availability of Code

The code used for this project can be found on my GitHub page at `https://github.com/SkyeChen-28/AMATH495_FinalProjectCodespace`

Acknowledgements

1. I used Python to code this project and GitHub to host all code files.
2. I used Overleaf to make this document.
3. See References for websites I referenced

References

[1] *10 of the countries most affected by climate change*. Aug. 2022. URL: https://www.concernusa.org/story/countries-most-affected-by-climate-change/.

[2] *Chad GDP 1960-2023*. URL: https://www.macrotrends.net/countries/TCD/chad/gdp-gross-domestic-product.

[3] Carbon Brief Staff. *Met office: Atmospheric CO2 now hitting 50% higher than pre-industrial levels*. Mar. 2021. URL: https://www.carbonbrief.org/met-office-atmospheric-co2-now-hitting-50-higher-than-pre-industrial-levels/.

[4] *U.S. GDP 1960-2023*. 2023. URL: https://www.macrotrends.net/countries/USA/united-states/gdp-gross-domestic-product.