Prob. & Stats. - Week 6 Outline

Yi (Alice) Yin

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1 Expectation

1.1 Definition

X is a random variable. The expectation of X is denoted as $E(X)$.

- Discrete Case
$$E(X) = \sum_x x\, p_X(x),$$
where $p_X(x)$ is the *probability mass function* of X.

- Continuous Case
$$E(X) = \int x\, f_X(x) dx,$$
where $f_X(x)$ is the *probability density function* of X.

1.2 Expectations for some special distributions

- $X \sim \text{Ber}(p)$
$$E(X) = p$$

- $X \sim \text{Geo}(r)$
$$E(X) = \frac{1}{r}$$
Recall:
  - Geometric Series:
  $$\sum_{n \geq 0} a^n = \frac{1}{1-a},\ |a| < 1$$
  - Taylor Expansion:
  $$\sum_{n \geq 0} \frac{a^n}{n!} = e^a$$

- $X \sim \text{Poi}(\lambda)$
$$E(X) = \lambda$$

- $X \sim \text{Exp}(\lambda)$
$$E(X) = \frac{1}{\lambda}$$
Recall:
  - Integration by Parts
$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$
Remark: For an exponential random variable
$$\text{Rate } (= \lambda) \neq \text{ Mean } (= E(X))$$
Look carefully how the variable is defined in each case.
- $X \sim \text{N}(\mu, \sigma^2)$
$$E(X) = \mu$$
Recall:
  - Standardization: $X \sim \text{N}(\mu, \sigma^2) \Longleftrightarrow Z \sim \text{N}(0, 1)$ with
$$Z = \frac{X - \mu}{\sigma} \qquad , \qquad X = \sigma\, X + \mu \; .$$
  - Gaussian Integral:
$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \qquad , \qquad \int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$$
- $X \sim \text{Bi}(n, p)$
$$E(X) = np$$
Recall:
  - Binomial Theorem:
$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$
  - Another way to approach:
    Use Bernoulli Distribution: $Y_i \sim Ber(p)$, where $i \in \{1, 2, ..., n\}$
$$X = Y_1 + Y_2 + \cdots + Y_n$$
$$E(X) = \sum_{i=1}^{n} E(Y_i) = np$$

1.3 Measures of dispersion

- P1
$$E[X - E(X)] = E(X) - E(X) = 0$$
- P2 (next week)
$$E[(X - E(X))^2] = Var(X)$$