The Midterm

STAT 464/864 $\sim$ **Fall 2024**

Discrete Time Series Analysis | *Skye P. Griffith* ~ Queen's University

Monday, October 21st $\sim$ 10:30-11:20 (in class)

Learning Outcomes

If you successfully complete the midterm, you will have demonstrated an understanding of most of the following themes (depending on which stars you choose to complete).

Room 1: The Terrace | $Classical\ Time\ Series\ Modelling$

1. Trends
2. Seasonality
3. Autocorrelation & Stationarity
4. White Noise hypothesis testing

Room 2: The Fountain | $Filters,\ MA(q)\ and\ AR(p)$

1. Linear Filters & Proposition 2.2.1
2. Causality
3. MA(q) processes
4. AR(p) processes

Galaxy Grading

The Terrace contains 3 Stars. So does the Fountain. Each Star is worth 5 points.

Undergraduate students: Complete any 4 Stars.

Graduate students: Complete any 5 stars.

There are no alternate stars or anything because, like... this is a test.

50min. Calculators permitted. By that I mean like a Casio 991, not a computer.
No R, no cheetsheet.

The Terrace | Star 1 $\star$

True or False Mini-game

Each $\boxed{?}$ floating next to a TRUE statement contains a coin.

Each $\boxed{?}$ floating next to a FALSE statement contains a poison mushroom!

Break (colour-in, cross out, etc.) each box containing a coin, avoid the poison mushrooms.

You have an observed time series $\{x_t\}_{t=1}^{500}$.

You wish to model this data according to the classical decomposition

$$X_t = m_t + s_t + Y_t \qquad (\star),$$

where Y_t is assumed to be zero-mean, Gaussian (normally distributed $\forall t$) white noise.

Consider the operation $H_q(\cdot)$, defined by $H(x_t) \stackrel{def}{=} \dfrac{1}{2q+1} \sum_{j=-q}^{q} x_{t-j}$

$\boxed{?}$ $H_q(\cdot)$ outputs a polynomial regression line of degree q

$\boxed{?}$ $H_q(\cdot)$ is a moving average smoother of order q

$\boxed{?}$ $H_q(\cdot)$ is an exponential smoother with parameter q

Assuming x_t does NOT have a significant s_t component with period 10,

$\boxed{?}$ $H_{q:=10}(\cdot)$ will dampen high-frequency fluctuations (low-pass filter)

$\boxed{?}$ $H_{q:=10}(\cdot)$ eliminates gradual/polynomial trends over large time-scales

Assuming x_t DOES have a significant s_t component with period 10,

$\boxed{?}$ $H_{q:=10}(\cdot)$ will model s_t by fitting one or more sinusoids

$\boxed{?}$ $H_{q:=10}(\cdot)$ estimates s_t without assuming its behaviour within a given cycle of period 10.

The Terrace | Star 2 ⋆

Drawing Mini-game

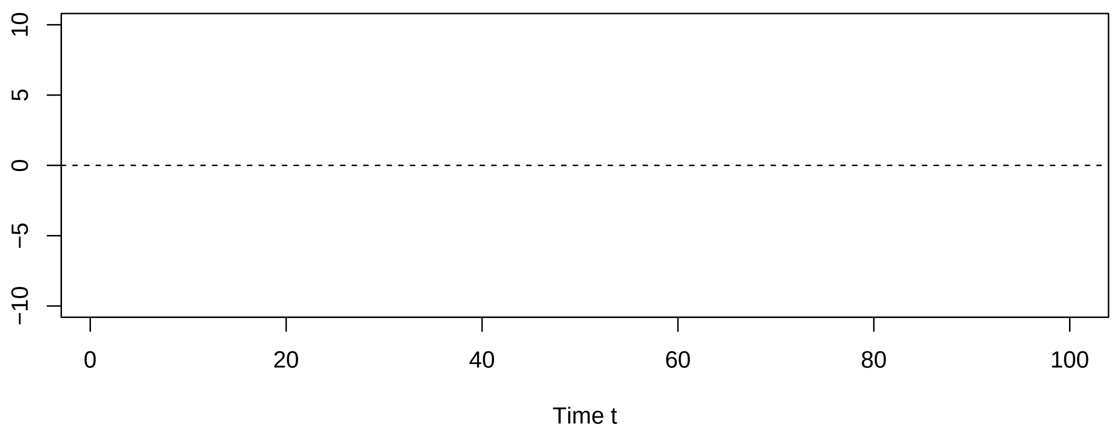
Name <u>one</u> of the three requirements for weak stationarity
(reminder: this is what we've simply been calling "stationarity" in class, as is conventional).

Answer here: ____

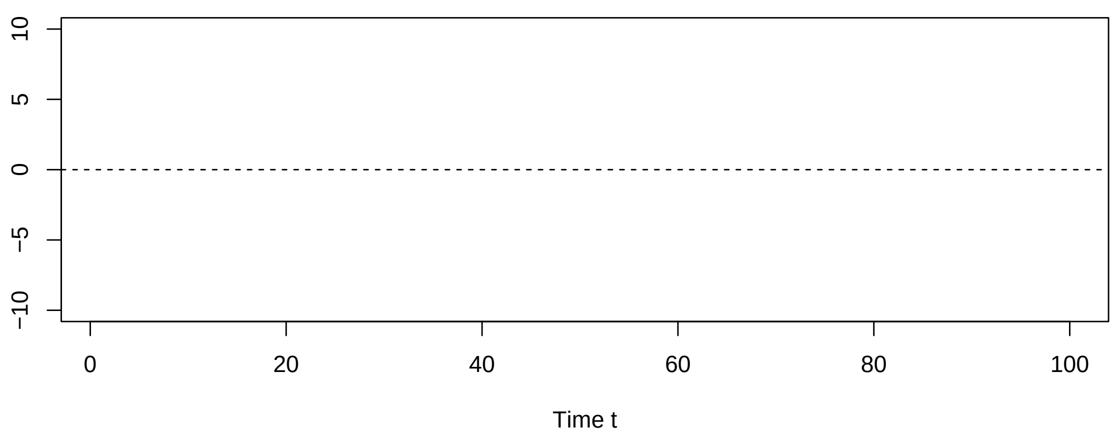
In plot 1: Draw a time series which satisfies the condition you chose, above.
It does not necessarily need to be stationary, it just needs to satisfy your chosen condition.

In plot 2: Draw a time series which does NOT satisfy your chosen condition.

Plot 1: Condition Satisfied



Plot 2: Condition NOT Satisfied

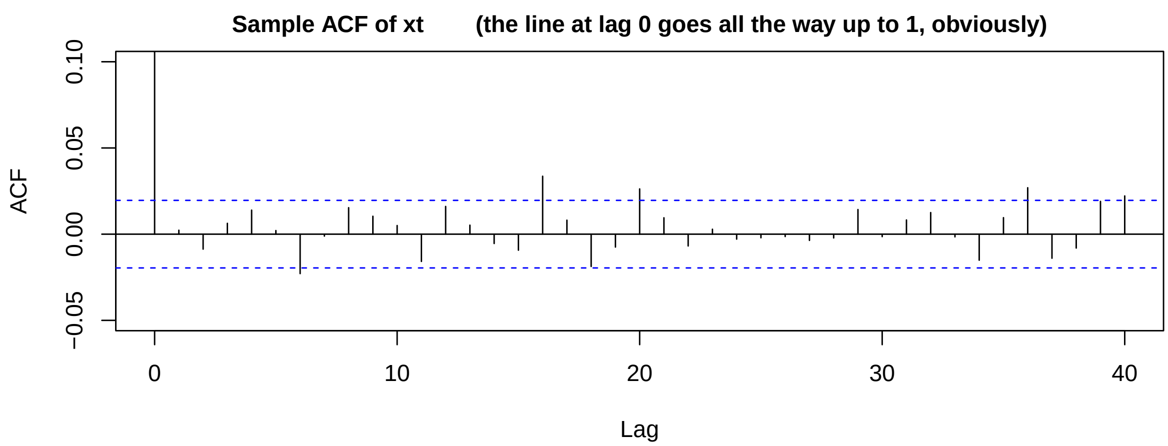


The Terrace | Star 3 ⋆

White Noise Hypothesis Mini-game

Consider the time series $\{x_t\}_{t=1}^{10\,000}$, whose sample ACF $\widehat{\rho}_x(h)$ is plotted, below.

We want to test whether this series is a white noise process at the $\alpha = 0.05$ level.



1. Write H_0 as a mathematical expression, in terms of $\rho(h)$.

2. Write H_A as a mathematical expression, in terms of $\rho(h)$.

3. The ACF plot shows $\widehat{\rho}_x(h)$ for $h = 0, \ldots, 40$. Recall, $\alpha = 0.05$.
   How many times is $\widehat{\rho}_x$ allowed to enter the plot's *rejection region* before we reject H_0?

4. What is the conclusion of your hypothesis test, according to the sample ACF plot? Give your answer in terms of the null hypothesis, H_0. Do you believe x_t is white noise? Why?

The Fountain | Star 1 ⋆

True or False Mini-game

Each ? floating next to a TRUE statement contains a coin.

Each ? floating next to a FALSE statement contains a poison mushroom!

Break (colour-in, cross out, etc.) each box containing a coin, avoid the poison mushrooms.

Consider the following: $X_t = \sum_{j=-\infty}^{\infty} \psi_j X_{t-j}$ where $\psi_j = \begin{cases} (-1)^j & |j| < 10 \\ 0 & else \end{cases}$

? $\{\psi_j\}$ is a linear filter

? $\{\psi_j\}$ is a causal filter

? $\{\psi_j\}$ is a linear process

Now let Y_t be stationary and zero mean, but NOT white noise.
Let γ_Y and σ_Y^2 respectively denote the ACVF and variance of Y_t

Let $Z_t = \sum_{j=-\infty}^{\infty} \psi_j Y_{t-j}$

? $E[Z_t] = 0$

? $\text{Var}[Z_t] = \text{Var}[Y_t]$

? The ACVF $\gamma_Z(h)$ exists, and can be expressed in terms of γ_Y and σ_Y^2

? $\gamma_Z(h)$ exists, and can be expressed in terms of σ_Y^2 only

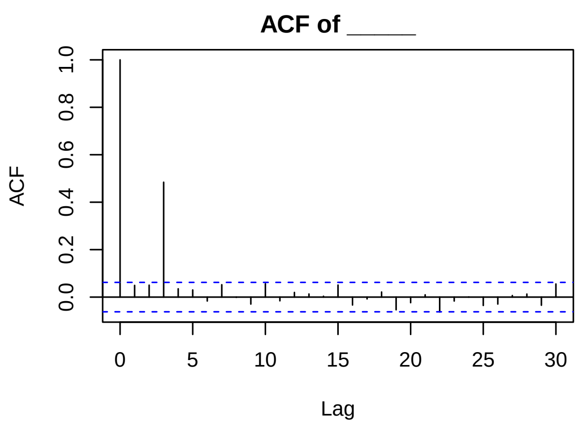
The Fountain | Star 2 ⋆

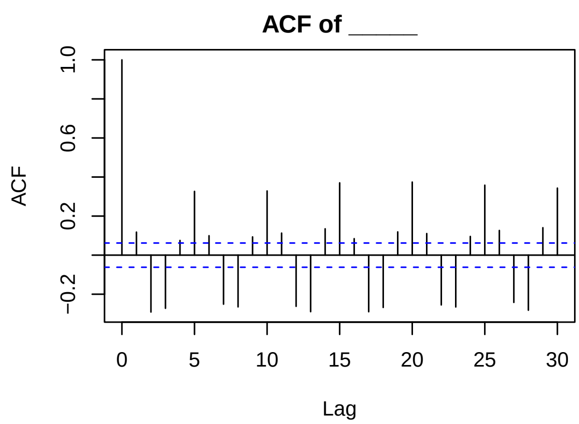
Matching Mini-game

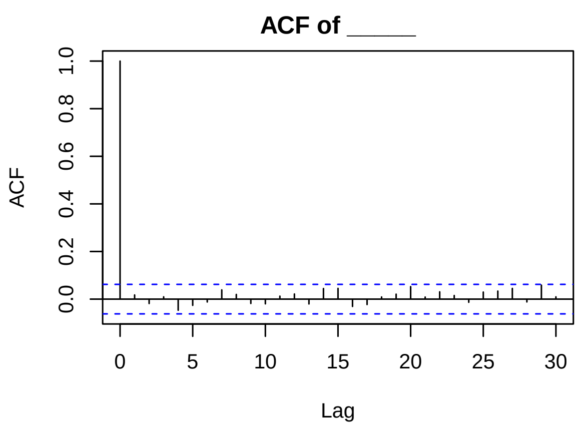
Fill in the plot titles based on which simulations (w,x,y,z) you believe produced each ACF.

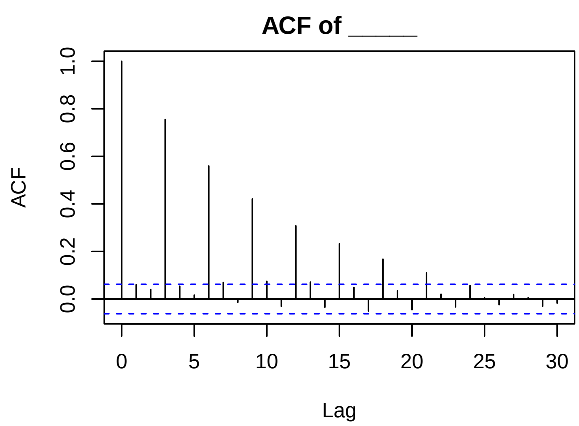
```r
N <- 1000
t <- 1:N
y.mod <- specify(ar = c(0, 0, 0.75))
z.mod <- specify(ma = c(0, 0, 0.75))

w <- rnorm(N)
x <- w + cos(2*pi*0.2*t)
y <- sim(y.mod, N)
z <- sim(z.mod, N)
```









The Fountain | Star 3 $\star$

Calculation Mini-game

Let $\{Z_t\}$ be zero mean and stationary, with ACVF $\gamma_Z(h)$.

Let $\{s_t\}$ be a seasonal component with period $d \geq 2$.

Let $Y_t = s_t + Z_t$

Let $X_t = Y_t - Y_{t-d}$, and suppose all this is defined for $t \in \mathbb{Z}$.

Questions:

Is $\{X_t\}$ stationary?

If yes, compute the ACVF $\gamma_X(h)$.

If no, does $\{X_t\}$ itself have a seasonal component? What is its period?