

## **Master of Science Thesis**

### **Program: Financial Markets & Investments**

## **Estimation of Value at Risk for CSI 300 Index via Nonlinear GARCH Model**

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## 1. Abstract and Keywords

Since the collapse of China's stock index in summer 2015, financial risks inherent are gradually being everyone's attention. How to build an effective risk measurement system has become the focus of research circles. VaR (Value at Risk) as a general method for measuring the current financial risks has become an effective way to measure risk in financial markets for all financial institutions. A comprehensive analysis of the situation of China's main stock index CSI 300, is given out on the basis of in reference to the outstanding domestic and foreign literature and a detailed explanation of the implementation process VaR method based on GARCH Model is added. In the demonstration, I select the most recent several years' CSI 300 index for basic analysis, and then by comparing the various GARCH model, I choose the best model to calculate VaR and use the failure frequency testing method to test the calculation results. The empirical results show that the VaR method based on GARCH model can effectively measure the market risk in the China's stock index sector.

**Keywords:** CSI 300 Index, GARCH, VaR, Financial Risk

## 2. Preface

April 2015, China's stock market has entered a new round of outbreaks. From the data showed in May, the CSI 300 index has exceeded 4,000 points, the Chinese stock market entered into a new bull market. While the stock market ran in high level, China's financial sector also showed a very good situation, millions of people devoted their asset to stock market and the security companies and benefited a lot from this bullish trend. On June, CSI 300 index had already broken the 5000 points and about to reach 6000, a historical level in China's financial history.

In fact, the carnival in stock market was owed to the 'two sessions' and the 'internet plus' conception issued by Keqiang Li, the premier of China. By using the power of internet to drive entire financial industry and active the energy of innovative business, the entire stock market in China wiped out the haze of subprime crisis and started rising. However, no one found a serious fact that in the first quarter of 2015, China's economic growth was only 7%, the whole real economy was suffered from the recession. Without the support from the real economy the whole stock market will certainly became a weak structure. Most Chinese people enter the stock market just by following the trend blindly while ignoring the basic conception of risk. Especially when people under the lure of huge profits, they will totally believe that whole market will continue rising. However, most of investors in China rely too much on the analysis of macroeconomic policies and lacking of empirical measurement. However, in some mature market like U.S and EU countries, a large number of mathematical model have already been applied to the analysis of the stock market. The special circumstances in China, making everyone believe that all kinds of econometrics theories are never worked in china's stock market and government will do all they can do to maintain the stable of the market economy. However, with the development of the capital market in China, the financial system in China is becoming more and more complex, sometimes the administrative mean will lose its power and irreparable harm will just happen. Only after some costly lesson will people in China be aware of the importance of risk management. The prediction of the stock movements requires precise empirical analysis as a basis, by making every step of the investment process measurable and predictable investors can avoid unnecessary losses. Therefore, the introduction of advanced risk management system is very urgent in this country.

This article expounds the particular process of the econometric analysis for CSI 300 index and from the theory to practice, each step is described in detail. During the research, my supervisor Gros Lambert Bertrand offered my great help and my colleagues also give me much support on empirical model construction

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## 1. Introduction

Risk is a volatility of unexpected income, most firms are exposed to various kinds of risk, which can be a potential threat to their development. Since the Global Financial Crisis in 2008, which has been considered as the worst financial crisis since the Great Depression in 1930s, many financial institutions suffer from great losses due to a lack of awareness of risk measurement. CSI 300 Index, as the most important benchmark of China's stock market, is an important reference for related Index Fund. As the improvement of risk awareness in the field of index fund, most fund managers in China find it an urgent mission to build an efficient risk measurement system for all main stock indexes both in the Shanghai Stock Exchange Market and Shenzhen Stock Exchange Market in case of the sudden slump in the stock market.

Although Value at Risk is a common risk measuring method in developed countries, it's still a new thing in China. However, few people use it for measuring the risk for the index of China's stock because most fund managers thought that the China's government is omnipotent in solving all kinds of systematic and non-systematic risk in the domestic financial market. As a result, stock index fund managers in China suffered a great loss in China's stock crash in the summer of 2015. My question is exactly a deep exploration to this absurd phenomenon, because most fund managers in China still analysis the risk only by using macro information combined with simple technical analysis. So I put the emphasis on improving methods they use to measure the risk, by introducing Value at Risk, a risk measurement method that has been widely used among the financial system in most developed countries I hope I can find an answer to it.

## 2. Literature Review

### 2.1. Introduction to CSI 300 Index & the general situation of the stock market in China

The CSI 300 is a stock market index designed to replicate the performance of 300 stocks traded in the Shanghai and Shenzhen stock Market. As we can see from the chart, the CSI 300 Index covers all kinds of sectors in China's economic market, it reflects the main trend of major listed companies in the stock market. To have a comprehensive understanding in this CSI 300 Index, it's unavoidable to consider the current financial environment in China. Before the launching of CSI 300 stock index futures, investors can only trade in Hong Kong index futures market to reduce their risk. It took about four years for China to promote the CSI300 Stock Index Futures and the performance of Index in hedging the risk is satisfied.

*Stock Index Futures and Stock Index Correlation Analysis—An Empirical Analysis Based on the CSI-300 Stock Market* written by Lu Zhang (2014) give a quantitative and qualitative analysis on the hedging performance of CSI300 stock index futures by using the futures trading data in almost four years. In this research, two commonly used models (OLS and GARCH) are introduced to estimate the optimal hedge ratio. From the result of his simulation, we can find it satisfied that CSI 300 index futures have a great hedge effect and using CSI 300 index futures can effectively reduce risks and the stability of returns can be ensured. This thesis is an excellent summarize of the performance of CSI 300 Index and fully proved that the CSI 300 Index has an irreplaceable position in hedge fund. His excellent analysis let me have an unprecedented understanding in the CSI 300 Index. However, the limitation in the paper is obviously that lacking of sufficient data collected from the history quote of CSI 300 Index. Drawbacks of insufficient sample data and short time period may not make the result of empirical results generally accepted by everyone.

Another 2 papers are mainly about the analysis of the macro environment in China's stock market. *Chinese stocks end brutal August with another daily loss* reported that the slump of factory price and the slowdown in China's economic has a negative impact on the China's stock price. The author Kyoundwha Kim (2015) pointed that since the big decline in stock, the CSI

300 Index fell 1.2 percent <sup>1</sup> and the trading in Shanghai was <sup>2</sup>33 percent below the 30-day average, which has been a significant warning to the investors in this market. From his analysis, we can find that the future of China's stock market will not be bright. Although he didn't give accurate correlation function in his report but the description of the actual situation in China's stock still have meaningful reference to my research.

Unlike the previous one, another page emphasis on analyzing the influence of policy environment to the stock market. Chao Deng (2015) issued in his paper indicating that the China's central government have no idea how to solving volatility of China's stock market. One thing he mentioned in his article is that the crackdown follows efforts earlier this summer by Chinese authorities to curb what they called "malicious short selling". From this paper, we can see the whole China's financial system is lacking of effective risk control measurement when facing such kind of unpredictable decline in stock market. This is an important motivation of my research, because we can't always count on the authority of government departments when facing the collapse in the stock market.

## **2.2. The potential crisis in China's stock market**

According the report from *Takeshi, Jingu. Internet finance growing rapidly in China[R]. Tokyo: Nomura Research Institute, Ltd., 2014. 1-5* issued in May 2015. Takeshi and Jingu found that just 2 weeks ago before February 2014, property developer Shanghai Duolun Industry had seemed to hit a rough patch. It had just issued its annual report, which said it had experienced a 90% drop <sup>3</sup> in revenue. By a few days later, though, its fortunes seemed to have gotten a boost. On Sunday, the company issued a statement announcing its decision to change its name. Its new moniker P2P Financial Information Service. Co. Ltd.

The move puzzled some observers, as the property developer hadn't shown any indication previously that it was involved in financial services.

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<sup>1</sup> Kyoundwha Kim, 2015, *Chinese stocks end brutal August with another daily, marketwatch* p2

<sup>2</sup> Kyoundwha Kim, 2015, *Chinese stocks end brutal August with another daily, marketwatch* p4

<sup>3</sup> The estimation comes from annual financial report written by Takeshi, Jingu

Nevertheless, when the market opened Monday and Tuesday morning, Duolun's stock jumped to the upper trading limit of 10%. Duolun didn't immediately reply to a request for comment.

China's red-hot stock market has also rewarded other companies for changing their names in recent years. Particularly popular are buzzy names that include terms such as 'environmental protection,' 'technology,' 'entertainment' and 'finance.' According to one tally by the Beijing News, more than 70 public companies <sup>4</sup>have changed their names so far this year.

But some of the early adopters of this tactic have already learned that making a corporate change is more complicated than adopting a new name. A spicy Hunan-style restaurant chain renamed itself Cloud Live Technology last July and pledged to focus on cloud computing software. But to date, 90% of its revenues are still collected from diners, and the cloud computing concept hasn't managed to save its troubled dining business – last month, the company became the first Chinese company to default on the principal of locally issued bonds. If Duolun is truly delving into the P2P lending business, it will find an already crowded, poorly regulated scene.

In this report, they pointed out that in China, P2P is often used for lending activities taking place on private online platforms, without going through a traditional financial intermediary, according to their calculations from leading trade website Wangdai Zhijia, there were more than 2,000 P2P platforms by the end of April. P2P platforms in China play a larger role than they do in the U.S.: they are financial service providers who pick projects, provide guarantees by themselves or via a chose third party, as well as actively recruit investors.

Lax regulation makes entering the P2P field very easy, meaning a mixed picture of operators, including both reputable P2P platforms and underground loan sharks who pack up fast profits and disappear. Data from Wangdai Zhijia shows that in 2014 alone, more than 200 P2P firms — or 10% of the market total — saw their founders disappear and the company default on payments to lenders. The Chinese Banking Regulatory Commission, which is charged with regulating P2P lending, hasn't unveiled any formal rules on the sector yet.

"The sector is still small, only about 100 billion in total market size, and regulators are reluctant to step in," said Takeshi. "Also the reaction to any new regulation can be nasty. As soon as rules

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<sup>4</sup> Data comes from China Company Registration official website

are enacted, one can expect a wave of close-down of P2P platforms, and the following massive defaults might pose a challenge to social stability, which regulators hate to see.”

So far, they found that the only apparent sign of Duolun’s entrance into the wild industry of P2P lending is the fact that its controlling shareholder, Duolun Hong Kong, owns the domain name [www.p2p.com](http://www.p2p.com). On Sunday, Duolun Hong Kong stated publicly that its listed arm can use the domain name for free for a year. That means that such companies who run P2P business in China are shell companies or backdoor listing cooperation.

So at the end of the report Takeshi and Jingu pointed that not only Duolun, too many companies in China are involved in the P2P industry and even many quoted companies have nothing to do with the finance also invested large amount of money into P2P business, this action will surely improve the whole risk level all over the stock market in China. I totally agree with their point, because just 2 months after the collapse of CSI 300 Index, most institution investors said that China’s bull market is just ruined by overfull P2P business and lack of basic risk regulation in financial sectors.

### **2.3. The exploration of VaR (Value at Risk) method.**

Traditional ALM (Asset-Liability Management, asset and liability management) in *Fama, Eugene Efficient Capital Markets: A Review of Theory and Empirical Work*[J]. *Journal of Finance* 1970 (2): P383–417 is dependent on the report analysis and lack of effectiveness. And using  $\beta$  coefficient of variance to measure the risk is too abstract, intuitive, only reflecting the market (or assets) volatility; The CAPM (capital Asset Pricing Model) is unable to be compatible with financial derivatives. All these traditional methods above cannot accurately define and measure financial risk, G30 Research Group entitled "practices and rules derived products," in 1993. The report proposed a new method to measure market risk called VaR (Value at Risk: value at Risk). Now it has become the mainstream method of measuring the financial market risk. Later, JPMorgan issued Risk Metrics mode to calculate the VaR, which has been widely used by many financial institutions.

In order to have a better understand of Value at Risk method, I highly recommended Value at Risk: *A new Benchmark for Managing Financial Risk (3rd Edition)* written by Philippe Jorion. This is a classical work for people who want to learn Value at Risk from the scratch. Also the

demo of VaR computing by using Monte Carlo Method is very detailed. In this book we can have a brief learning about VaR measurement and its application in risk managements. However, it still doesn't have a comparison of all the computing methods for Value at Risk.

Luckily, Mária Bohdalová (2007) made a full comparison of comparison of Value-at-Risk methods for measurement of the financial risk in *A comparison of Value-at-Risk methods for measurement of the financial risk*. In this paper, he divided the Estimating methods of VaR into two categories. One is parametric method (Variance-Covariance method and Monte Carlo Simulation), another is nonparametric method (History Simulation). The author compares the main approaches of calculating VaR and implements Variance-Covariance, Historical and Bootstrapping approach on stock portfolio then comparing the empirical part by using histogram. The final result is that there is no easy answer which method is the best. Investor or risk manager have to choose appropriate method according different kinds of portfolio. Obviously that's not the exactly answer I need for my thesis for the lack of use of advanced valuation model like ARCH or GARCH, but it still gives out a hint to let us choose an appropriate way to computing the VaR by ourselves.

Jui-Cheng Hung (2008) issued a paper about the estimation of value at risk for energy commodities. This paper adopts the GARCH model with the heavy-tailed (HT) distribution proposed by Politis (2004) to estimate one-day-ahead VaR for several energy companies including West Texas Intermediate crude oil (WTI), Brent crude oil, heating oil, propane and New York conventional gasoline regular (NYHCGR). As we can see from the paper, the fat tails in return innovation process indeed play an important role in VaR estimates and should be considered in risk management, also GARCH-type models can provide more accurate VaR estimates when assessing the market risk of energy commodities. It also provides all steps needed for dealing with the VaR measurement by using GARCH Model. So it can be the most valuable thesis I found since it gives out a specific way to measure the VaR and that's exactly what I want in my thesis.

#### **2.4. The Use of GARCH Models in VaR Estimation**

In this sector, I refer a classical thesis *The Use of GARCH Models in VaR Estimation*[M]. Greece: University of Piraeus, 2003. P14-15. In this paper, the author explain the usage of

GARCH model in VaR step by step. Following the extensive and detailed investigation of a plethora of volatility modeling techniques, briefly presented in the preceding sections, a number of comments are of order, aiming to summarize our results and give both to the researcher and the practitioner, some fundamental guidelines with which to proceed in VaR estimation. There are strong indications that the mean process specification plays no important role here. He tries to extract autoregressive phenomena from the returns such that only the underlying volatility is left in the residuals, Timotheos and Angelidis experimented with a number of AR processes. The results show that such a methodology does not add anything significant to the VaR framework other than complexity in the estimation procedure. Moreover, using only an ARCH term (without any lagged conditional variances) sometimes yields acceptable results only when residuals are modeled under either the Student's-t distribution or the GED; it is never the case for a Normal distribution. Generally speaking, in the VaR framework the leptokurtic distributions and especially the Student's-t, are more appropriate than the Normal assumption, as they generate more accurate conditional or unconditional forecasts, while there is no volatility model which is clearly superior than the others. Furthermore, the size of the rolling sample used in estimation turns out to be rather important: in simpler models and low confidence levels a sample size smaller than 2000 improves probability values. In more complex models, where leptokurtic distributions are used or where the confidence level chosen is high, a small sample size may lead to lack of convergence in the estimation algorithms. Finally, there is no consistent relation between the sample sizes and the optimal models, as we observe significant differences in the VaR forecasts for the same model under the four sample sizes.

### **2.5. The usage of VaR Model in current China's financial market**

Currently there are many scholars use VaR estimation to study stock indexes in China. I have seen many literatures focused on studying the VaR of Shanghai and Shenzhen traditional stock index, but few of them can provide me with a complete guide for estimating the VaR of my portfolio by using GARCH Model. Although most of them even don't give out the source code of their empirical models. But these papers still give me a clear mind in planning the steps of my project. Therefore, in the selection of the literature, I focus on two aspects of the absorption. On the one hand, it is necessary to refer to the excellent literature on VaR risk

measurement of securities index in order to have a deep learning on VaR analysis using GARCH model, on the other hand, I need to find a better way to realize the whole empirical experiment with the statistics software.

In the analysis of the VaR using the GARCH model, my analysis steps come from the *YAN Zheng-chun. Study on Dynamic VaR Based on GARCH Model - A Case Study of Shanghai Stock Index [J]. Economic Research Review, 2011 (28): P80-82*. In this paper, the GARCH model is used to forecast the VaR of the Shanghai Composite Index. Yan used GARCH (1,1) model to calculate the VaR value of the Shanghai Composite Index and also imported failure test for checking the reliability of the model. The results show that the estimated the failure time is only 54 times<sup>5</sup> and the error rate is 0.54% at 95% confidence level, which means that GARCH model can accurately reflect the risk of Shanghai stock market and it's suitable for risk management in Shanghai Stock Market. However, Yan didn't give out the detail of the empirical experiment, making it hard for me to realize the whole experiment. Meanwhile, in aspect of the mathematical derivation of the VaR and GARCH models, I refer to the *Song Bo. VaR Calculation Based on GARCH Model [D]. Nanjing: Nanjing Polytechnic University, 2004. 3-66*. This article talks about the mathematical mechanism of the GARCH model and VaR estimation. This paper describes the process of GARCH model building and VaR mathematical estimation in detail, and compares the different kinds of calculation methods of VaR. Finally, Song chose GARCH (1,1) model to simulate and use Microsoft stock as sample to prove the effective of the model. Although Song didn't use China's stock index as example, but the whole process is given out in the form of Python code. It's a good complement for Yan's article.

To find a better way to realize the whole empirical experiment with the statistics software, I read *Verzani, John. Getting Started with Rstudio. O'Reilly Media, Inc. p. 4. ISBN 9781449309039*. It's a book covers all aspects of Rstudio usage, including how to estimate VaR by using Monte Carlo simulation. Obviously, none of these papers gives out any off-the-shelf solution for my empirical experiment, but they gave me a clear mind to build my own model step by step. Also an Eviews Manual *Anders Thomsen, Rune Sandager, Andreas Vig Logerman, Jannick Severin*

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<sup>5</sup> The data comes from the appendix in *Zheng-chun. Study on Dynamic VaR Based on GARCH Model - A Case Study of Shanghai Stock Index [J]. Economic Research Review, 2011 (28): P80-82*



*Johanson, Steffen Haldrup Andersen. Introduction to EViews 6.0/7.0.* gives me much help in building GARCH Model, it provides a real VaR estimation model programmed by Eviews language.

## **2.6. Extension Research for GARCH Model.**

Before the extension research I read a paper about ARCH Model to have a better understand in advanced GARCH Model, it's *Box, G. E. P. and Pierce, D. A. Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models[J], Journal of the American Statistical Association, 65: 1509–1526. JSTOR 1970: P1-47*

Then the most difficult part of my thesis should be the extension of Generalized Autoregressive Conditional Heteroscedasticity model, short for GARCH model. The whole GARCH family models is enormous, even the using of the simplest GARCH Model, GARCH (1,1), also has to consider difference of distribution like normal distribution, student t distribution, heavy-tailed distribution. However, for simple portfolio, we just need to choose Nonlinear GARCH Model (introduced by Engle and Ng in 1993).

For most beginner, it's necessary to begin from *Properties and Estimation of GARCH (1,1) Model*. Petra Posedel (2005) had a deep study the properties of the GARCH (1,1) model and the assumptions on the parameter space in the circumstance that the process is stationary. The whole thesis is boring because it just a Mathematical derivation to GARCH (1,1) Model, but it's still a necessary step since I'm still not familiar with such kind of empirical model.

However, univariate GARCH Model is far from enough for the further researching. In order to have a further research on GARCH Model, I read the *Calculating Value-at-Risk* written by William Fallon (1997). By using equity return data and a hypothetical portfolio of options, the author evaluated the performance of all six models. The result showing how accurately each model measures the Value at Risk on an out-of-sample basis. In this case, the author developed a hypothesis that multivariate GARCH (1,1) model is superior than other five models in computing the VAR and finally it proved to be true. However, EGRACH & IGARCH are not included in this paper, more research should be done in other GARCH family models.

When it comes to simulation period, I choose R Studio as my analysis tools. So I read *Nonlinear Time Series Analysis: Available R packages for GARCH modeling and a real data analysis*. It's

a guidebook developed by Mingzhong Zhang. It introduced the way to use GARCH model and the application of ‘fAGRCH’ package in, a package contains econometric functions for parameters estimation of GARCH Model, in R Studio. It contains all specific process of the application of GARCH Model in real dataset. The estimation progress of Value at Risk for CSI 300 Index will be based on the operation in it.

## **2.7. Conclusion of the literature review**

After reading all the reading material, I can totally compare these papers and extract essential information from them.

Lu Zhang’s research help me understand the significant position of CSI 300 Index in China’s stock market and Kyoundwha Kim and Chao Deng’s report give me much market information of China’s stock. But what confused me most is that all papers related to VaR have few real example for the estimation of VaR for stock index, especially the stock index in China. Petra Posedel’s introduction to GARCH model is good, but compared to other 2 papers, it lacks empirical process with real data. All paper can develop and prove their hypothesis correctly, I totally agree with the opinion in their papers except Lu Zhang’s *Stock Index Futures and Stock Index Correlation Analysis—An Empirical Analysis Based on the CSI-300 Stock Market*.

Because CSI 300 index is a young index without experiencing any financial crisis, the data he got is end in 2014, I don’t know whether the author will still have a positive attitude towards the hedge efficiency of CSI 300 Index when he saw the collapse of stock market in the August 2015. In general, all material is still very helpful to my research.

*Takeshi, Jingu. Internet finance growing rapidly in China[R]. Tokyo: Nomura Research Institute, Ltd., 2014. 1-5* uncovered an important reason for the collapse of China’s stock in summer 2015, that’s the lack of regulation in financial field leading to a huge internet finance bubble.

In terms of GARCH Model and VaR estimation, all papers give out detailed explanations, making it convenient for beginners like me to understand it. Especially, *Zheng-chun. Study on Dynamic VaR Based on GARCH Model - A Case Study of Shanghai Stock Index [J]. Economic Research Review, 2011 (28): P80-82* and *Song Bo. VaR Calculation Based on GARCH Model [D]. Nanjing: Nanjing Polytechnic University, 2004. 3-66*. These papers are the few paper who

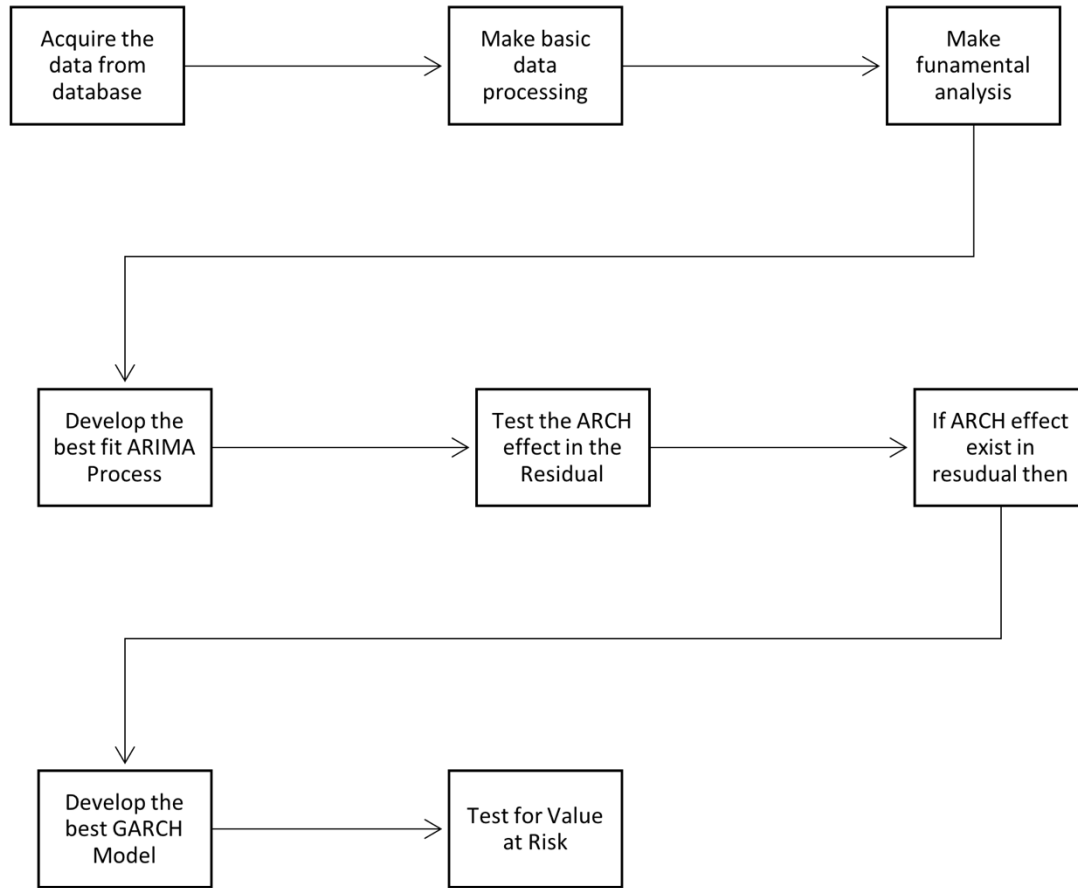
can give a real example for the application of VaR in China's stock and most important of all, they use GARCH model as the estimation method. I totally agree with the common ideas in these papers that empirical models will play an important role in China's stock and it's reasonable. In terms of software usage, Mingzhong Zhang's *Nonlinear Time Series Analysis: Available R packages for GARCH modeling and a real data analysis* and Verzani, John's *Getting Started with Rstudio. O'Reilly Media, Inc. p. 4. ISBN 9781449309039* are the best Rstudio guides I have ever seen, all of operations are shown in picture, which makes it easier for me to start from R. Overall, all these papers satisfy my need for the final experiment and I am grateful for the contributions they did for us.

### **3. Methodology**

#### **3.1. Mind Map**

Here is the mind map of the whole empirical experiment. Here we can see from Table 3-1, there will be 8 steps in our experiment. All of these steps will be done by an Rstudio Shiny.app automatically.

Table 3-1 Mind map for the experoment



## 2.1 Data Acquisition

In this step, I dropped the traditional data acquisition methods and use cloud data mining. I choose Quandl as our data source. The reasons are followed:

First, it's the only database that can support multi coding language and only few lines of commands, it can withdraw quotes of multi portfolios in different time. This will reduce large amount of work time when we want to apply our model into different markets.

Second, it can automatically drop the error and the abnormal returns in the series by using different API if you want. Especially when compare the return of two portfolios day by day, holidays, abnormal events and other blank records must be dropped.

Third, it's dynamic. Unlike traditional database, the link we embed into Rstudio Shiny.app can be dynamic and run as a cloud app, which means you can calculate the VaR of any portfolio at any time periods. That's very useful when comparing the VaR in different indexes.

The history quotes of CSI 300 Index can be get from Quandl, by using 'Quandl' package in Rstudio, we can get the history quota of CIS 300 Index from 2006-6-21 to 2016-6-21.

### **3.2. Data Processing**

In this progress, we will convert the close price of the CSI 300 into the daily return of the CSI 300 Index, so that we can take the log return of CSI 300 Index. Here we have to step, one step is using 'diff' function to get the log return of the CSI 300 Index, the other is using 'plot' function to plot the daily compound returns of the CSI 300 Index. By observing the trend of volatility in the chart, we can judge the basic trend of the index and make a brief judgement of the stability of the index we choose.

### **3.3. Fundamental analysis**

The first step should be the fundamental analysis to the original data, the results we wanted in this step are the mean, standard deviation, maximum return, minimum return, kurtosis and skewness. Then we can judge whether the sequence satisfied the normal distribution and have the feature of fat tail. A perfect series we want should be normal distribution and with no fat tail. However, it's almost impossible in the real market. That's why we must use GARCH model, because it can deal with the series with an obvious peak and fat tails.

### **3.4. ARIMA Model Test**

Before we build the GARCH Model, we have to build a ARIMA model first, because two models have a special relationship. Bollerslev (1986) said 'If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroscedasticity(GARCH)'. And the ARIMA is actual only an ARMA (p, q) process. Also, in the GARCH Model building process, you will find the GARCH model must be build based on a given ARIMA model got in the previous step.

### 3.5. Test for ARCH Effects

Fourth step is the ARCH test for the residuals. Before we go ahead to GARCH model developing, we must ensure the residual of our ARIMA must have ARCH effect. To test the ARCH effect, I choose the Ljung-Box test on the first 12 lags of the squared residuals of the best ARIMA model under the null hypothesis of no ARCH effects. So if the hypothesis is rejected by the test, the experiment can continue.

### 3.6. Developing GARCH Model

If all steps above passed, we can simulate the parameter in GARCH Model as followed. In the formulas below, we can see the GARCH Model is combined by 2 parts. One is the mean equation, and the other is the variance equation (standard regression equation).

$$\begin{cases} y_t = x^T \gamma + u_t \\ u_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \beta_0 + \beta_1 u_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \end{cases}$$

In the first two formulas,  $y_t = x^T \gamma + u_t$  and  $u_t = \sigma_t \varepsilon_t$  It is a simple regression equation, where  $x^T$  is the vector of explanatory variables,  $\gamma$  is the coefficient vector before the variable is explained, and  $u_t$  is the residual, which forms the mean equation in the GARCH model. In the last formula  $\sigma_t^2 = \beta_0 + \beta_1 u_{t-1}^2 + \beta_2 \sigma_{t-1}^2$  is used to reflect the fluctuation of residuals in the mean equation.  $\beta_0$  is a constant term. The changes of  $\beta_1$  and  $\beta_2$  have a great impact on the formula. The greater the value of  $\beta_1$ , the greater the effect of the conditional variance on the impact. The greater the value of  $\beta_2$ , the more time-dependent the conditional variance will be.

In this experiment because the limit of the time, I only compare GARCH family models, GARCH (1,1), GARCH (1,2) and GARCH (2,1) and find the best GARCH Model. My hypothesis is that GARCH (1,1) model is the most suitable model for the VaR measurement. If the final result proves it. Then I can answer my question: why it's essential to measure the risk level of CSI 300 Index by using Value at Risk method?

### 3.7. Kupiec's Unconditional Coverage and the Christoffersen Independence Test

To examine the back testing report using the 'report' function. The 'report' can automatically generate the report that executes the unconditional and conditional coverage tests for

exceedances. ‘VaR.alpha’ is the tail probability and ‘conf.level’ is the confidence level on which the conditional coverage hypothesis test will be based.

### 3.8. Plotting the VaR Limits

Finally, we can get the VaR chart of our portfolio. The methods for VaR calculation are divided into two kinds one is parametric method, another non-parametric method, parametric method is variance-covariance method and, non-parametric hair is a historical simulation method and Monte Carlo Simulation.

The main form of the formula for VaR is shown below, it includes 2 formulas, the first one is used to calculate the VaR of the underlying asset, the another one is used to set the probability that the asset value loss is less than the possible loss cap, the probability is the confidence level of our VaR model.

$$VaR = V \cdot [E(R) - R^*]$$

$V$  —The beginning value of the portfolio

$R$  —Rate of return

$R^*$  —The minimum rate of return for a certain confidence interval ( $c$ )

$$Prob(\Delta p \Delta t \geq VaR) = 1 - c$$

$Prob$  —The probability that the asset value loss is less than the possible loss cap

$c$  —The given confidence level

However, it’s too abstract. It’s only the way to help us understand the mathematical principle of the VaR. In the real empirical experiment, we use the formula as followed:

$$VaR = M \cdot (Z_\alpha \cdot \sigma_p - \mu)$$

$M$  —The beginning value of the portfolio

$Z_\alpha$  —The quantile corresponding to the confidence level under the standard normal

$\sigma_p$  — one-step forward prediction of the conditional standard deviation

$\mu$  —one-step forward prediction of the conditional mean

To calculate the VaR just fill it with all parameters get from the GARCH (1,1) above one by one. To make a plot of the back testing performance, we use the ‘plot’ function tin the R. Because the ‘ugarchfit’ function in ‘rugarch’ package has already equipped with automatic plot menu, we just need to input the command to get what we need.

By analyzing the VaR we get, we can know whether VaR is an effective model to fit our Index. If all of these above proved to be true, I can say that China’s financial institution need this kind of advanced empirical model.

### **3.9. Forecasting Risk and VaR**

After that, we have put our model into a real practice, by forecasting the VaR in the next few days, we can judge the validity of the model in the future. Only when the most daily return doesn’t hit the 1% VaR can we say that our model is effective.

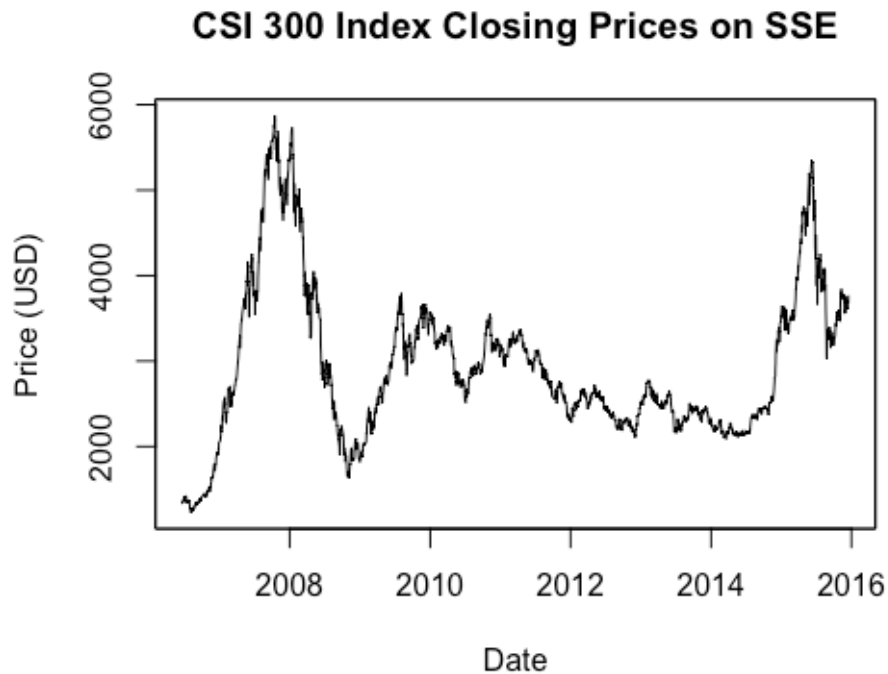


## 4. Results and Analysis

### 4.1. Result for Data Acquisition

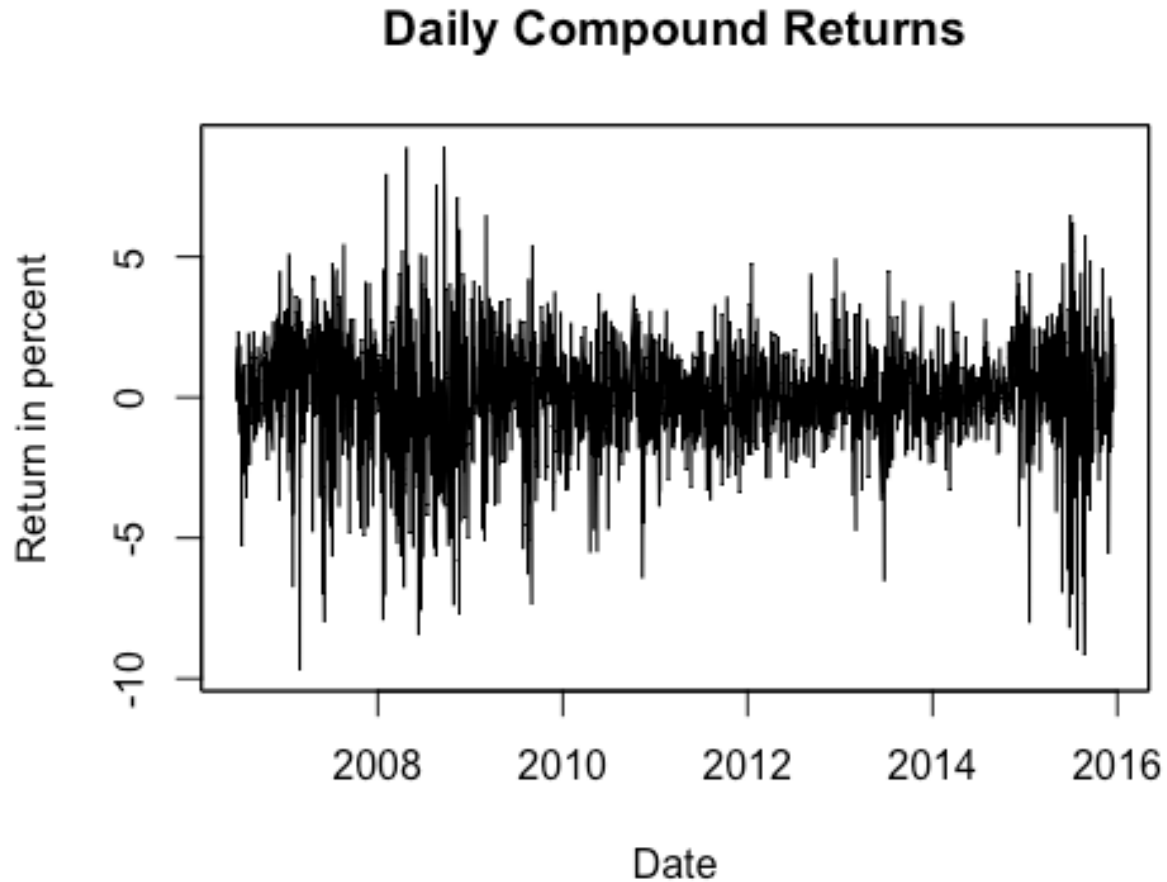
By using the ‘Quandl’ Database, we successfully achieve the closing price of the CSI 300 Index. From the plot of the trend we can see that the CSI 300 Index reached the highest level in October, 2007 at 5737.22 and then dropped the lowest level in October, 2008 at 1667.83. However, in the last year, it reached another highest point at 5335.12 in June, 2015, but fell into 2948.03 in 6 months.

*Figure 4-1 CSI 300 Index History Price*



### 4.2. Result for data processing

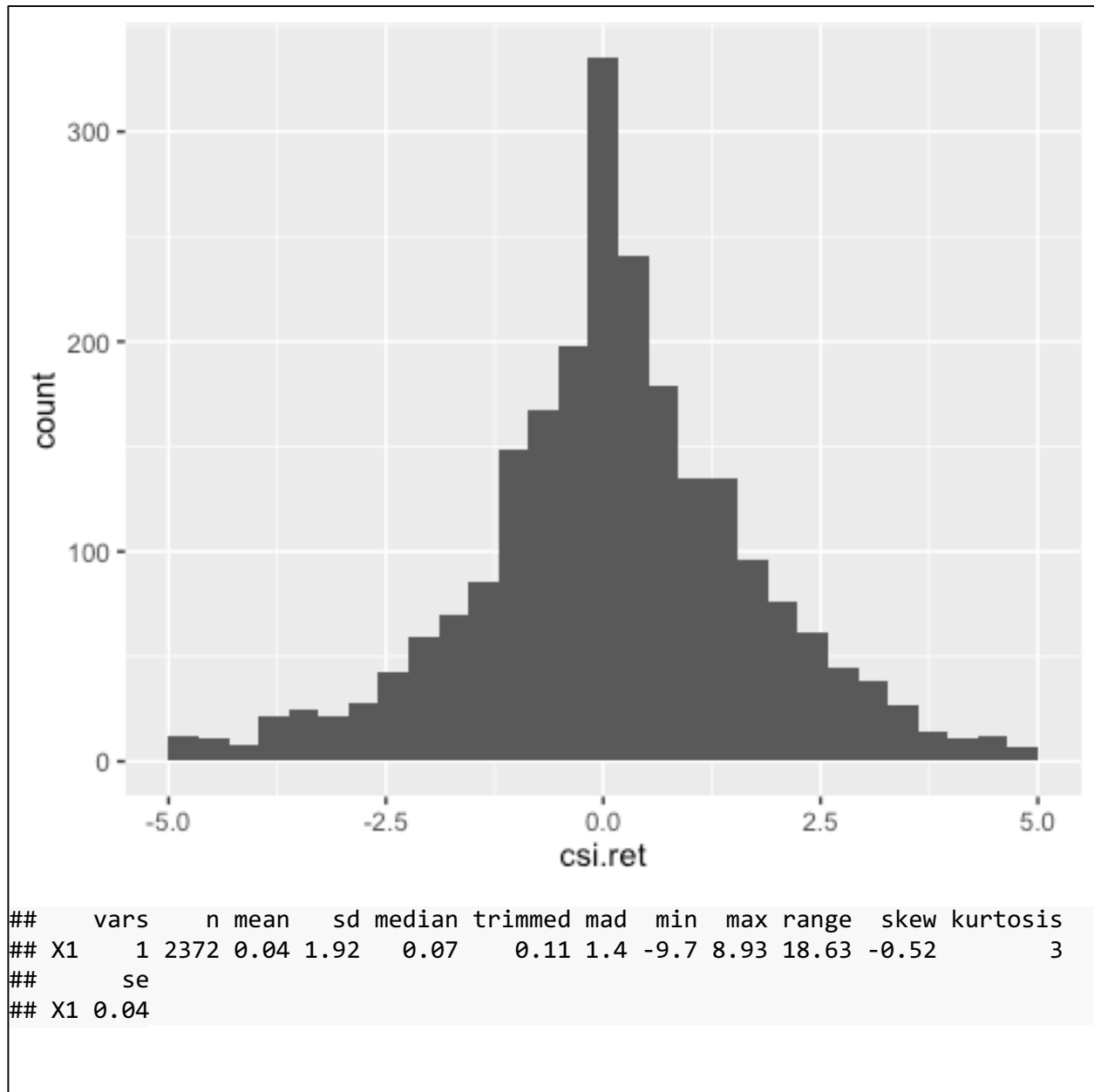
After acquiring the data from database, I converted it to daily compound returns in percentage to fit the next few steps of the analysis. From the chart we can see the return of CSI 300 Index is form 10% to -10%, that's because there is a limit move regulation in China's stock market.

*Figure 4-2 Daily Compounded Returns for CSI 300 Index*

#### 4.3. Result for Fundamental Analysis

The results for fundamental analysis is as followed, as we can see from the exported data, the total amount of the sequence is 2372, the mean of the return is 4%, the median is 7%, the max return of the whole market during the observation period is 800.93%% and the minimum is -90.7% the Kurtosis is 3, which means that the convex degree of the distribution in our data is much higher than the normal distribution. The Skew is -0.52 (Below 0), which means that the right tail of the distribution is longer. It's sure that our data is not a normal distribution, but we still need further test for ARCH effect to see whether GARCH model is suitable for our empirical analysis.

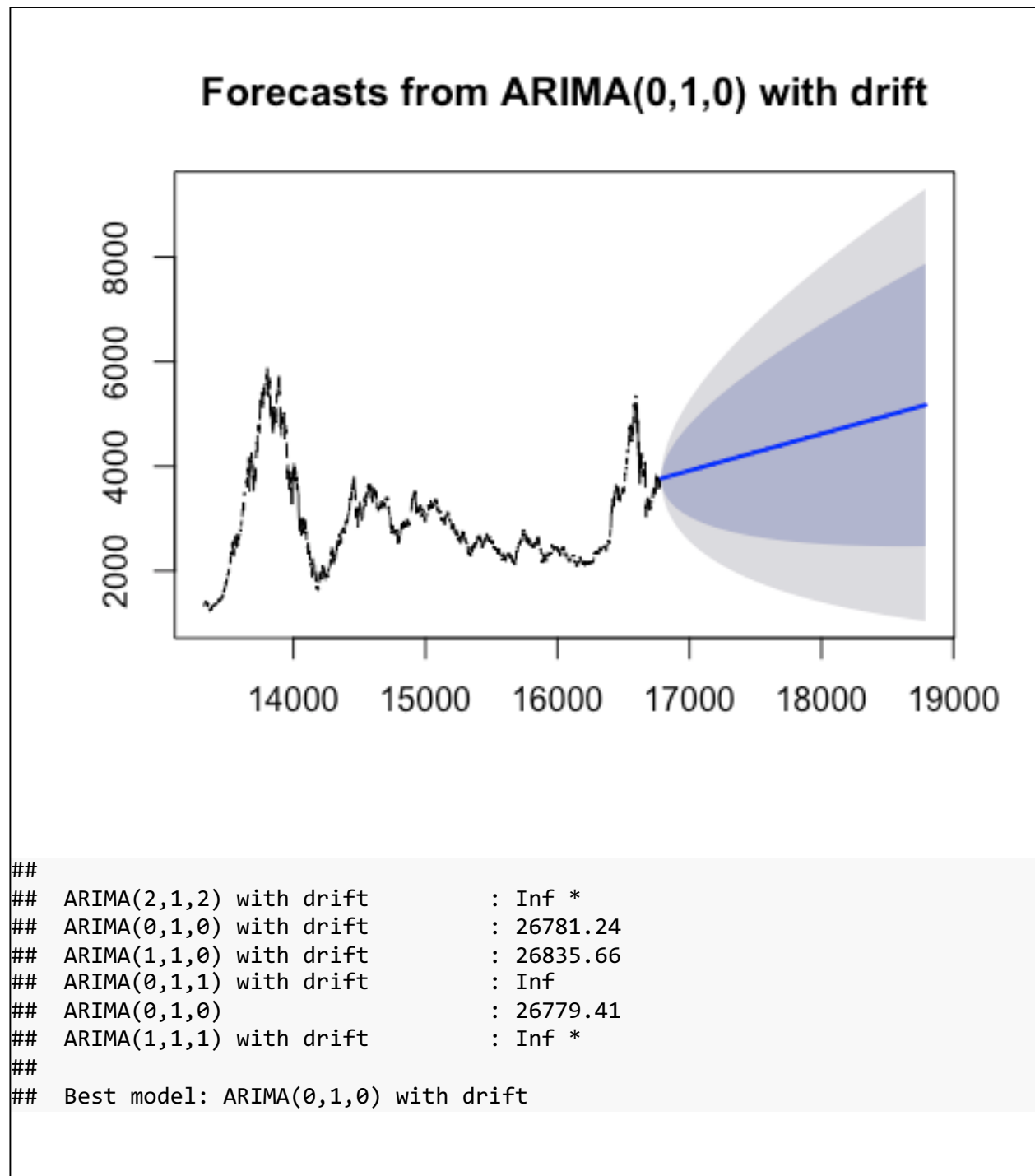
Figure 4-3 Histogram and Descriptive data for CSI 300 Index



#### 4.4. The result of ARMA Model Test

We tested 6 kinds of ARIMA model, the best model is ARIMA (0,1,0) with drift. We will build GARCH Model based on this model.

Figure 4-4 Best ARIMA Model



#### 4.5. Result of the test for ARCH Effect

The following exported data is the test of ARCH effect by using “Ljung-Box” test, the p-value is only 2.2e-16, far from 5% level of significance then we can see there exist ARCH effect, we can continue to the GARCH Model building.

*Figure 4-5*

```
##  
## Box-Ljung test  
##  
## data: fit1$residuals^2  
## X-squared = 1241.2, df = 12, p-value < 2.2e-16
```

#### 4.6. Result for Developing GARCH Model

Because of our data is only a simple time series data, I only use GARCH (p, q) model for the final calculation of VaR.

To build a GARCH Model based on ARIMA (0,1,0), we use the ‘ugarchroll’ function in R Studio, it can automatically build a GARCH Model and gives out all kinds of choices you need for next steps. Here I compared 3 common GARCH model, including GARCH (1,1), GARCH (1,2), GARCH (2,1) and got the results as followed.

Table 4-1 GARCH Models Comparison

Model	Variable	Value	Standard Deviation	t Statistic	Prob
GARCH(1,1)	RESID(-1)^2	0.055652	0.007707	7.2210	0.000000
	GARCH (-1)	0.937452	0.008585	109.2011	0.000000
GARCH9(1,2)	RESID(-1)^2	0.055766	0.009878	5.645510	0.000000
	GARCH (-1)	0.937309	0.004435	211.350984	0.000000
	GARCH (-2)	0.000007	0.006797	0.001094	0.999127
GARCH(2,1)	RESID(-1)^2	0.041065	0.028148	1.45893	0.144585
	RESID(-2)^2	0.017126	0.031040	0.55173	0.581134
	GARCH(-1)	0.934779	0.014560	64.20192	0.000000

As we can see in Table, the P values of all the parameters in GARCH (1,1) reject the null hypothesis, so the parameters are valid. At the same time  $\text{RESID}(-1)^2 + \text{GARCH}(-1) < 1$ , which satisfy the constraints of parameters. The parameter GARCH (-2) in GARCH (1, 2) is 0.000007, the standard deviation is 0.006797, the t statistic is 0.001094 and the P value is 0.999127, accepted the null hypothesis. The parameter is invalid. Since one of the parameters does not satisfy the condition, this model is not applicable to empirical research.

The parameter  $\text{RESID}(-1)^2$  in GARCH (2,1) is 0.041065, standard deviation is 0.028148, t statistic is 1.45893, P value is 0.144585, accept the original hypothesis, so the parameter is invalid. The parameter  $\text{RESID}(-2)^2$  in GARCH (2,1) is 0.017126, standard deviation is 0.031040, t statistic is 0.55173, P value is 0.581134, accept the original hypothesis, so the parameter is invalid. Since there are two parameters that do not meet the conditions, this model is not applicable to empirical research.

In summary, only the estimated parameters of the GARCH (1,1) model reject the null hypothesis and pass the test. In GARCH (1, 2) and GARCH (2,1), the probability of some parameters,  $P > 0.05$ , accepts the null hypothesis and cannot pass the validity test. So GARCH (1,1) is the best choice for VaR estimation.

#### 4.7. Result for Kupiec's Unconditional Coverage and the Christoffersen Independence Test

Before plotting the VaR chart, we have to test the effective of our VaR Model, In Table 4-2, you can the test for the VaR of CSI 300 Index. The Expected Exceed I set is 23 times, but the Actual VaR Exceed is 55 times. The Null-Hypothesizes of conditional coverage (Christoffersen) and unconditional converge (Kupiec) are all rejected, we cant's passed the test. So there is still a chance of 2.4% our model will be out of function, but considering there are 2252 trading series in our test and our confidence interval is set to 1%, it's a strict testing condition. We can also test our model under 95% of confidence interval and watch the difference.

*Table 4-2 Report for the Back-test under 99% confidence interval*

VaR Backtest Report	
Model:	sGARCH-norm
Backtest Length:	2252
Data:	
alpha:	1%
Expected Exceed:	23
Actual VaR Exceed:	55
Actual %:	2.40%
Unconditional Coverage (Kupiec)	
Null-Hypothesis:	Correct Exceedances
LR.uc Statistic:	33.738
LR.uc Critical:	6.635
LR.uc p-value:	0
Reject Null:	YES
Conditional Coverage (Christoffersen)	
Null-Hypothesis:	Correct Exceedances and Independence of Failures
LR.cc Statistic:	34.032
LR.cc Critical:	9.21
LR.cc p-value:	0
Reject Null:	YES

Here is the Table when we set Alpha at 5%, we can see that in Table 4-3, when the confidence interval is set to 95%, the Null-Hypothesizes of conditional coverage (Christoffersen) and unconditional converge (Kupiec) are all accepted, it means our model can only passed the test under 95% of confidence interval.

Table 4-3 Report for the Back-test under 95% confidence interval

VaR Backtest Report	
Model:	sGARCH-norm
Backtest Length:	2252
Data:	
alpha:	5%
Expected Exceed:	112
Actual VaR Exceed:	55
Actual %:	2.40%
Unconditional Coverage (Kupiec)	
Null-Hypothesis:	Correct Exceedances
LR.uc Statistic:	33.738
LR.uc Critical:	6.635
LR.uc p-value:	0
Reject Null:	NO
Conditional Coverage (Christoffersen)	
Null-Hypothesis:	Correct Exceedances and Independence of Failures
LR.cc Statistic:	34.032
LR.cc Critical:	9.21
LR.cc p-value:	0
Reject Null:	NO

To find out whether it's a common phenomenon among all markets, I also apply my model in other indexes, including DAX Performance Index, Dow Jones Industrial Average Index and CAC 40 Index. Soon, we can see from Table 4-4, it's a common thing that under 99% confidence interval, our model will not work, but under 95% confidence interval, it works.

Table 4-4

Index	VaR Exceed	Test under 99% confidence Interval	Test Under 95% confidence Interval
CSI 300 Index	23	Reject	Accept
DJIA Index	83	Reject	Accept
DAX Index	18	Reject	Accept
CAC 40 Index	18	Reject	Accept

#### 4.8. Result for VaR Limits Plotting



After the precious step, we will plot the chart like this. From the previous step we know that the daily return of the data (blue) hits the 1% VaR (Red) 55 times, it means that there is still some possibility that we will suffer the loss above the limitation we set. However, we still don't know when such kind of accident happened. After observing the plot, I found two interesting things: First, during the subprime mortgage crisis, the limitation of VaR of DAX Performance Index, Dow Jones Industrial Average Index and CAC 40 Index all break the 10%, while China's CSI 300 Index didn't. Even in the 2015–16 Chinese stock market crash, it never over 10%. That's because the Chinese government interrupt the financial market and set Limit-up action and market-saving events in China's stock market, which has a great impact to my final result. So the fact is that my model is running under a non-market economy environment, it works in current situation but I don't know whether it will work in the next few years when China's financial market environment is totally turned into a market economy environment.

Second, the situation that the daily return of the data (blue) hits the 1% VaR (Red) always happened in the extreme disaster in financial area, like subprime mortgage crisis. It means that VaR has its limitation when facing extreme disasters.

*Figure 4-6 CSI 300 Index*

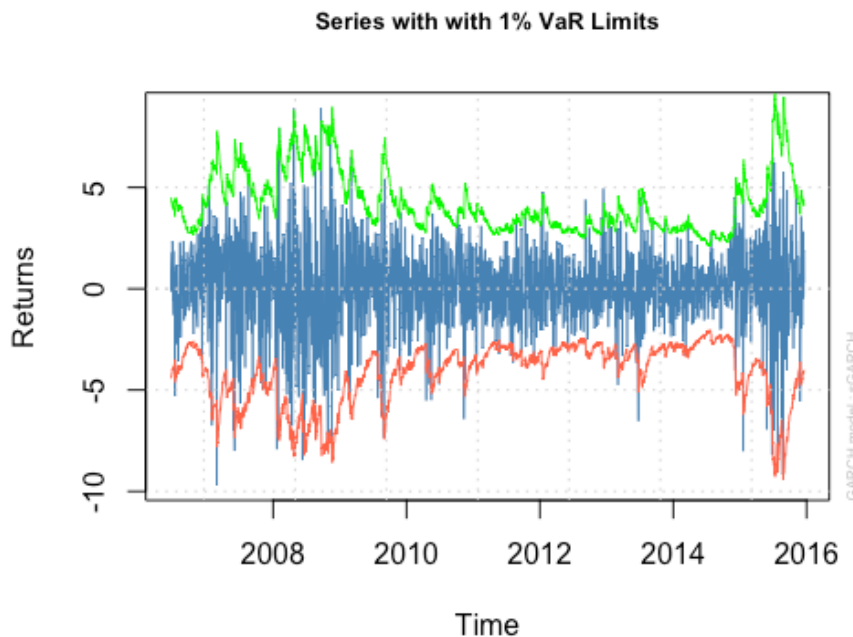


Figure 4-7 DJIA Index

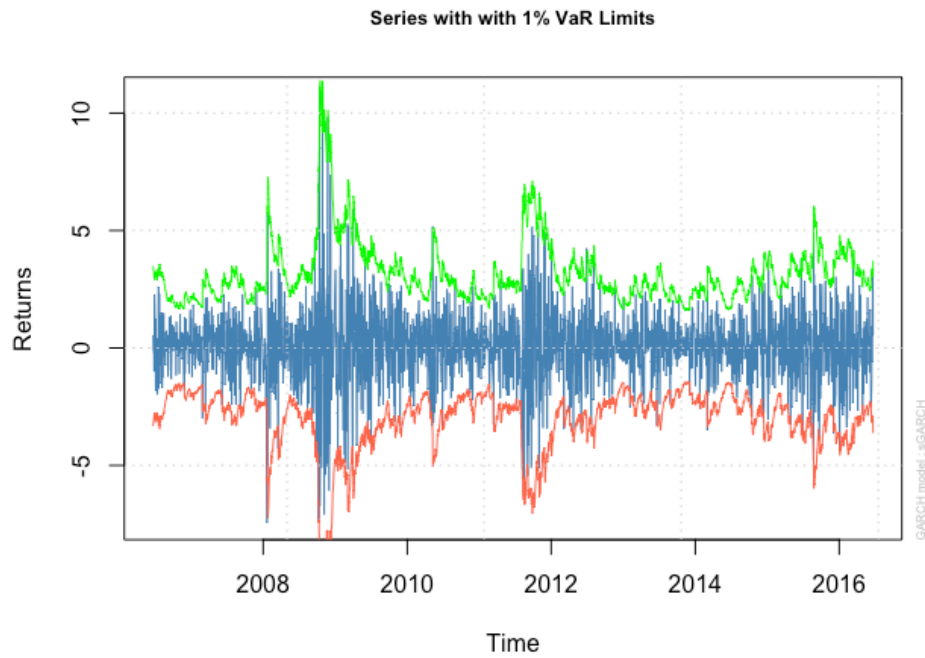


Figure 4-8 DAX Index

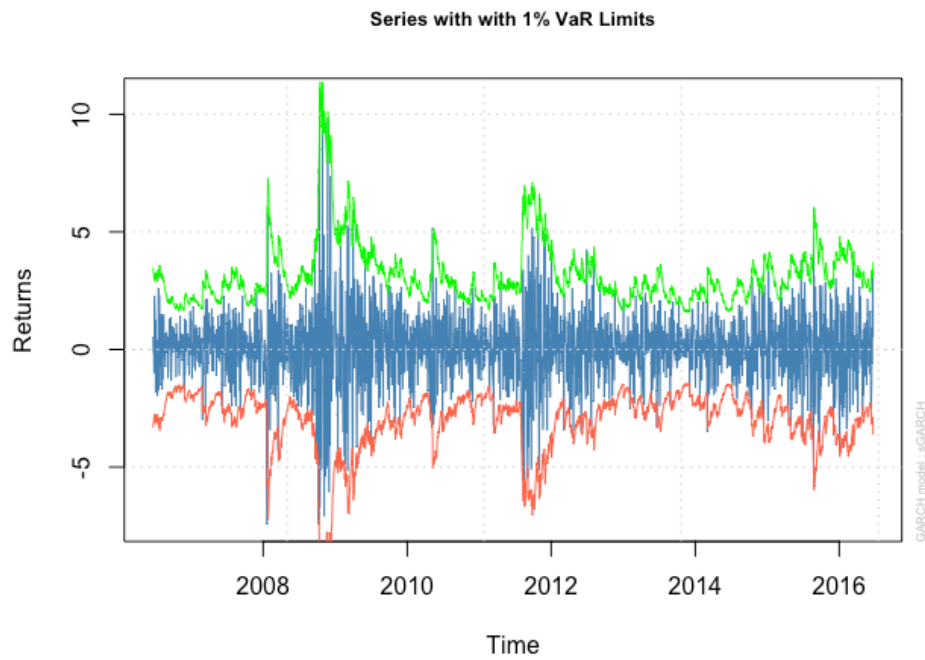
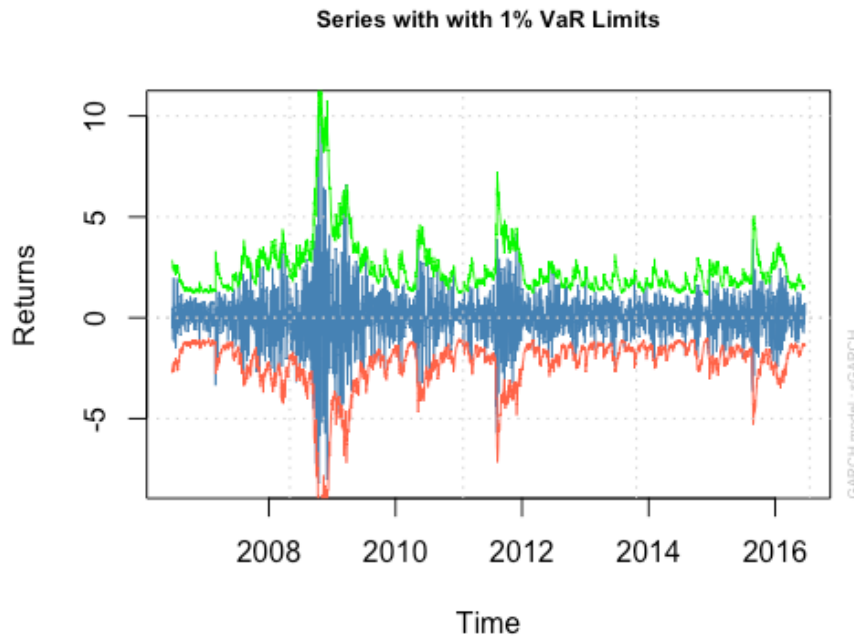


Figure 4-9 CAC 40 Index



#### 4.9. Prediction

Finally, I export the prediction of my VaR estimation for 90 days to test whether it can fit the future trend of CSI 300 Index, the result is satisfied, the model only failed once under the 99% confidence interval. But condemning there is no large impact on China's stock market, it still need further test.

Confidence Interval	Actual Days	Failed Times	Frequency
95%	100	0	0%
99%	100	1	1%

## 5. Conclusion

Finally, we have the conclusion as followed:

1. The China's stock market is suffering from downside risk because of slow in economy.
2. Compared to other developed countries, China's financial market is still far from mature and administrative intervention still plays an important role in it.
3. Under the current situation, the VaR method can fit the CSI 300 Index very well and it can be used for predicting the financial risk of the CSI 300 Index.
4. VaR method still has some limitation when facing extreme crisis, but works well in most of time.

At the same time, there are many shortcomings in this research:

1. The GARCH (1,1) model can also be compared with TGARCH and EGARCH in order to find the best model.
2. VaR does not meet the subadditivity, it cannot take the tail risk into account and can't control extreme events. Here worthy of further progress.
3. The Back-test of the VaR shows that our model can only work under 95% confidence interval so I need to improve it.

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## 7. Rstudio Code Explanation

The source code of my model is polished in <https://github.com/Skyepose/Estimation-of-Value-at-Risk-for-CSI-300-Index-via-Nonlinear-GARCH-Model> to comply with GPL policy. All people can use and modify it. You can also view the test environment visiting <http://www.deltapower.me:8787>

Here are the steps for all I did in Rstudio. But may be different if you want to use Rstudio Shiny.app to generate these code.

### 7.1. Environment Required and Packages

The first part is run the packages you needed for the experiment. Before the experiment, “tseries”, “forecast”, “FinTS”, “rugarch”, “ggplot2”, “psych” and “Quandl” must be installed. Then you can load these packages. For Mac and Linux users, XQuartz environment must be installed in your system.



```
library("tseries");
library("forecast");

library("FinTS");

##
## Attaching package: 'FinTS'

## The following object is masked from 'package:forecast':
##
##      Acf

library("rugarch");

## Loading required package: parallel

##
## Attaching package: 'rugarch'

## The following object is masked from 'package:stats':
##
##      sigma

library("ggplot2");
library("psych");

##
## Attaching package: 'psych'

## The following objects are masked from 'package:ggplot2':
##
##      %+%, alpha

library("Quandl");

## Loading required package: xts
```

## 7.2. Data Acquisition

After that, run the following command to fetch the data from Quandl and plot the picture.

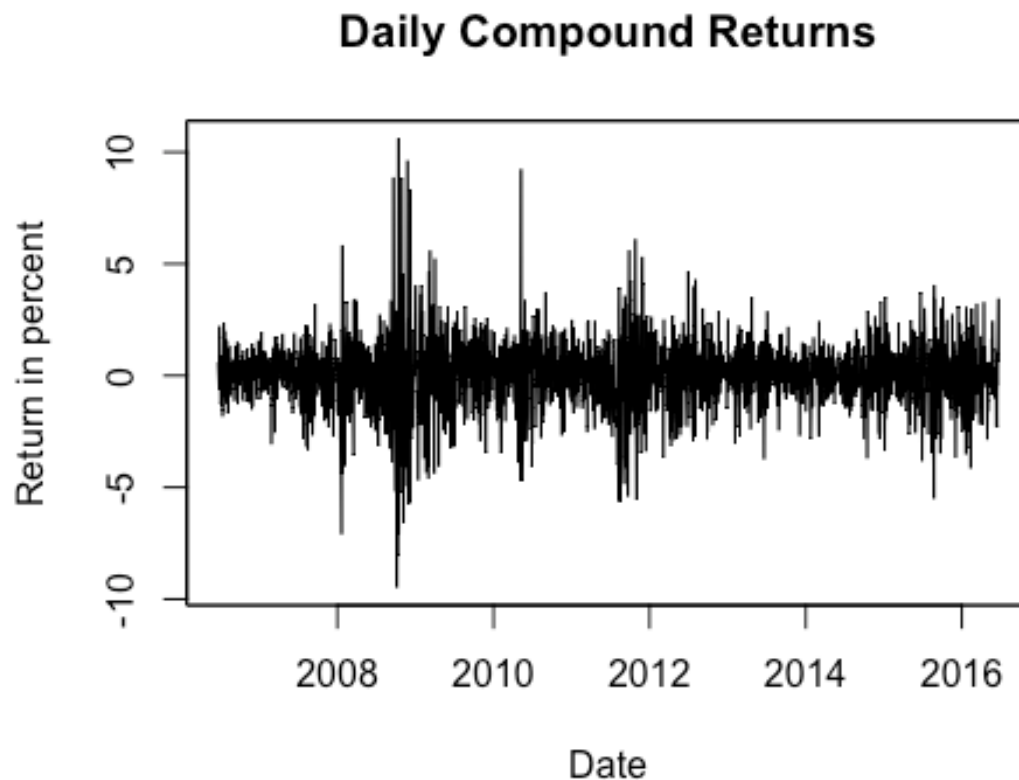
```
csi.data=Quandl(c("YAHOO/INDEX_FCHI.4"), api_key="sqUGVk1vqe37KNAYZ-1Z", start_date="2006-06-21", end_date="2016-06-21", type = "zoo");  
plot(csi.data, main = "CSI 300 Index Closing Prices on SSE", ylab = "Price (USD)", xlab = "Date");
```



### 7.3. Data Processing

Once get the data, you can process it and plot daily compounded plot as followed.

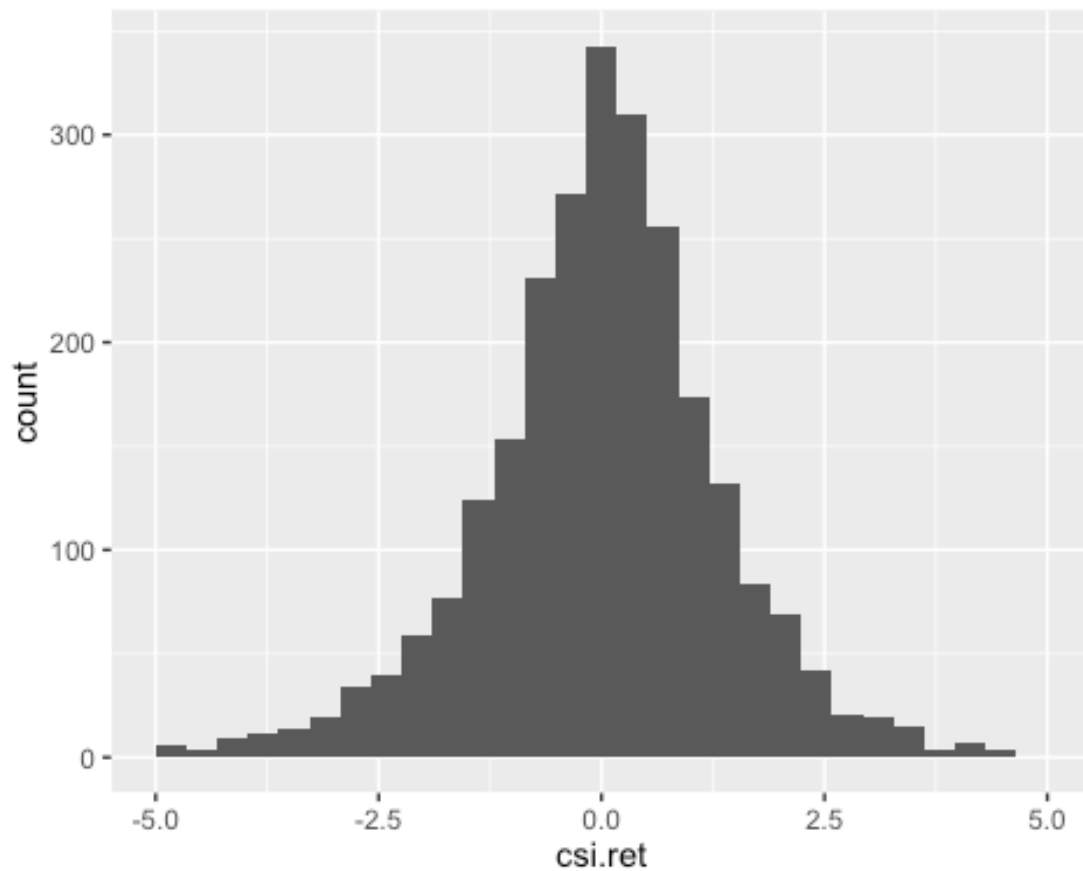
```
csi.ret<- diff(log(csi.data)) * 100;
plot(csi.ret, main = "Daily Compound Returns", xlab = "Date", ylab = "Return
in percent");
```



#### 7.4. Descriptive Analysis

In order to get the descriptive analysis, we have to get the histogram and descriptive data.

```
qplot(csi.ret,,xlim=c(-5,5), geom="histogram") ;
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## Warning: Removed 28 rows containing non-finite values (stat_bin).
```



```
describe(csi.ret);
```

```
##      vars    n mean   sd median trimmed mad   min   max range skew kurtosis
## X1      1 2559   0 1.52  0.04  0.02 1.1 -9.47 10.59 20.07 0.05      5.5
##      se
## X1 0.03
```

### 7.5. Find the best ARIMA Model

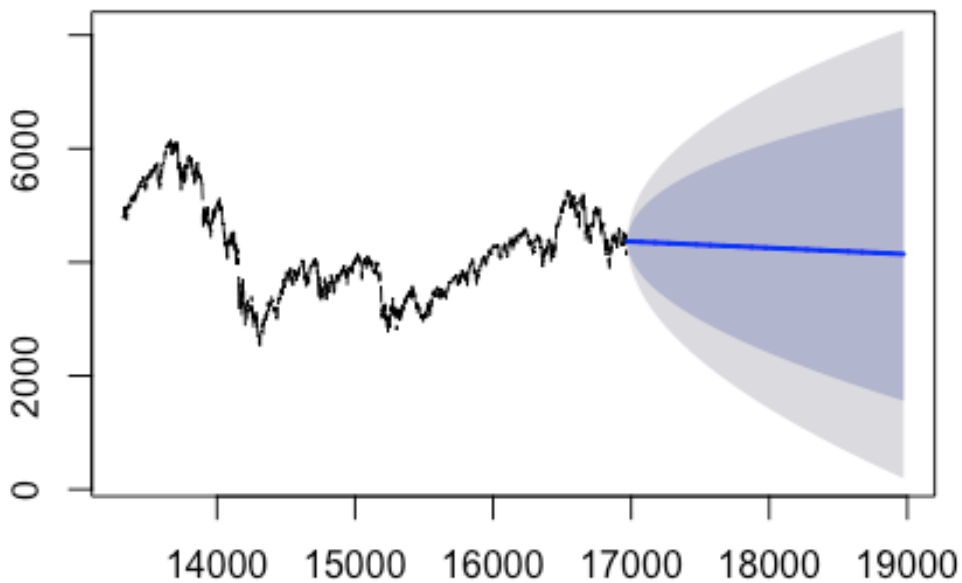
Then use `auto.arima` to find the best ARIMA model, here is ARIMA (0,1,0)

```
fit1 <- auto.arima(csi.data,max.p = 5,max.q = 5,max.P = 5,max.Q = 5,max.d =
3,seasonal = TRUE,ic = 'aicc',trace =TRUE );

##
## ARIMA(2,1,2) with drift      : Inf *
## ARIMA(0,1,0) with drift     : 28624.51
## ARIMA(1,1,0) with drift     : 28695.09
## ARIMA(0,1,1) with drift     : Inf
## ARIMA(0,1,0)                : 28622.51
## ARIMA(1,1,1) with drift     : Inf *
##
## Best model: ARIMA(0,1,0) with drift

plot(forecast(fit1,h=2000));
```

### Forecasts from ARIMA(0,1,0) with drift



```
str(fit1);

## List of 18
## $ coef      : Named num -0.112
## .. attr(*, "names")= chr "drift"
## $ sigma2    : num 2030
## $ var.coef  : num [1, 1] 0.793
## .. attr(*, "dimnames")=List of 2
## .. ..$ : chr "drift"
```

### 7.6. Ljung-Box test

Here is the Ljung-Box test. The p-value is  $2.2e-16$  less than 0.05, reject the null hypothesis.

```
Box.test(fit1$residuals^2, lag=12, type="Ljung-Box");

##
## Box-Ljung test
##
## data: fit1$residuals^2
## X-squared = 953.64, df = 12, p-value < 2.2e-16
```

### 7.7. Build GARCH Model and compare

When comes GARCH Model building, I test 3 GARCH Model and find the best model with all parameters passed the confidence test. Here the best model is GARCH (1,1). You can see the result in the appendix.

```
res_garch11_spec<- ugarchspec(variance.model = list(garchOrder = c(1, 1)), me
an.model = list(armaOrder = c(0, 1, 0)));
res_garch11_fit<- ugarchfit(spec = res_garch11_spec, data = csi.ret);
ctrl = list(tol = 1e-7, delta = 1e-9)

res_garch12_spec<- ugarchspec(variance.model = list(garchOrder = c(1, 2)), me
an.model = list(armaOrder = c(0, 1, 0)));
res_garch12_fit<- ugarchfit(spec = res_garch12_spec, data = csi.ret);
res_garch21_spec<- ugarchspec(variance.model = list(garchOrder = c(2, 1)), me
an.model = list(armaOrder = c(0, 1, 0)));
res_garch21_fit<- ugarchfit(spec = res_garch21_spec, data = csi.ret);
res_garch11_fit;
```

### 7.8. Back-Test for VaR

Then we can start the Back-test for our model, the Back-test condition is depending on your need, here only simulate under 99% confidence interval. The Back-test will take a long time about 25 minutes on PC, please be patient, so here is just an example.

```
res_garch11_rol1 <- ugarchroll(res_garch11_spec, csi.ret, n.start = 120, refi
t.every = 1, refit.window = "moving", solver = "hybrid", calculate.VaR = TRU
E, VaR.alpha = 0.01, keep.coef = TRUE, solver.control = ctrl, fit.control = l
ist(scale = 1))
report(res_garch11_rol1, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99);
```

We can see Kupiec and Christoffersen test is rejected. It's a common thing in all markets, if you reset the alpha to 5%, it will be different.

```
## VaR Backtest Report
## =====
## Model:                sGARCH-norm
## Backtest Length: 2252
## Data:
##
## =====
## alpha:                1%
## Expected Exceed: 23
## Actual VaR Exceed:   55
## Actual %:            2.40%
##
## Unconditional Coverage (Kupiec)
## Null-Hypothesis: Correct Exceedances
## LR.uc Statistic: 39.175
## LR.uc Critical:      6.635
## LR.uc p-value:      0
## Reject Null:        YES
##
## Conditional Coverage (Christoffersen)
## Null-Hypothesis: Correct Exceedances and
##                  Independence of Failures
## LR.cc Statistic: 40.357
## LR.cc Critical:     9.21
## LR.cc p-value:      0
## Reject Null:        YES
```

Here is the result for another situation. We can see that we passed the test.

```
## VaR Backtest Report
## =====
## Model:                sGARCH-norm
## Backtest Length: 2252
## Data:
##
## =====
## alpha:                 5%
## Expected Exceed: 23
## Actual VaR Exceed: 55
## Actual %:              2.5%
##
## Unconditional Coverage (Kupiec)
## Null-Hypothesis: Correct Exceedances
## LR.uc Statistic: 39.175
## LR.uc Critical:       6.635
## LR.uc p-value:       0
## Reject Null:         NO
##
## Conditional Coverage (Christoffersen)
## Null-Hypothesis: Correct Exceedances and
##                    Independence of Failures
## LR.cc Statistic: 40.357
## LR.cc Critical:      9.21
## LR.cc p-value:      0
## Reject Null:        NO
```

## 7.9. Plotting

After use plot command, you will see the menu as followed. Here is the most important step, you must input 2 to get the plot with 1% VaR Limits. Because the limitation of the package, there is no 5% confidence interval choice for your series, but you can replace the default confidence in the `res_garch11_fit`, you with 5% and get what you want, however, the title will not change.



```
plot(res_garch11_fit);
```

Make a plot selection (or 0 to exit):

- |   |  |
|---|--|
| 1: Series with 2 Conditional SD Superimposed limits | 2: Series with 1% VaR Limits                   |
| 3: Conditional SD (vs  returns )                    | 4: ACF of Observations                         |
| 5: ACF of Squared Observations                      | 6: ACF of Absolute Observations                |
| 7: Cross Correlation Standardized Residuals         | 8: Empirical Density of Standardized Residuals |
| 9: QQ-Plot of Standardized Residuals                | 10: ACF of Standardized Residuals              |
| 11: ACF of Squared Standardized Residuals           | 12: News-Impact Curve                          |

### **7.10. Prediction**

Finally, apply the model to the real CSI 300 Index prediction, the predication will take a long time, so I recommend you run this model in RServer, if possible, so I only give out 13 days' prediction. After 13 days, you can compare the real loss with these data.

```

res_garch11_fcst <- ugarchforecast(res_garch11_fit, n.ahead = 12);
res_garch11_fcst

##
## *-----*
## *      GARCH Model Forecast      *
## *-----*
## Model: sGARCH
## Horizon: 12
## Roll Steps: 0
## Out of Sample: 0
##
## 0-roll forecast [T0=2016-06-21]:
##      Series Sigma
## T+1  0.01683 1.592
## T+2  0.05038 1.591
## T+3  0.05038 1.590
## T+4  0.05038 1.589
## T+5  0.05038 1.588
## T+6  0.05038 1.587
## T+7  0.05038 1.587
## T+8  0.05038 1.586
## T+9  0.05038 1.585
## T+10 0.05038 1.584
## T+11 0.05038 1.583
## T+12 0.05038 1.583

```

## 8. Appendix

### 8.1. GARCH Model Test Report for GARCH (1,1), GARCH (1,2) and GARCH (2,1)

```
##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model   : sGARCH(1,1)
## Mean Model    : ARFIMA(0,0,1)
## Distribution   : norm
##
## Optimal Parameters
## -----
##           Estimate  Std. Error  t value  Pr(>|t|)
## mu         0.050378   0.021644   2.3275  0.019937
## ma1        -0.046701   0.021687  -2.1535  0.031282
## omega       0.038164   0.009365   4.0751  0.000046
## alpha1      0.097539   0.012760   7.6444  0.000000
## beta1       0.886308   0.014394  61.5729  0.000000
##
## Robust Standard Errors:
##           Estimate  Std. Error  t value  Pr(>|t|)
## mu         0.050378   0.021369   2.3575  0.018399
## ma1        -0.046701   0.020605  -2.2665  0.023421
## omega       0.038164   0.011832   3.2253  0.001258
## alpha1      0.097539   0.019634   4.9680  0.000001
## beta1       0.886308   0.020548  43.1344  0.000000
##
## LogLikelihood : -4318.039
##
## Information Criteria
## -----
##
## Akaike          3.3787
## Bayes           3.3901
## Shibata         3.3787
## Hannan-Quinn    3.3828
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----
##                               statistic  p-value
## Lag[1]                      0.3213  0.5708
## Lag[2*(p+q)+(p+q)-1][2]    0.7802  0.8612
```

```

## Lag[4*(p+q)+(p+q)-1][5]      2.2933  0.6260
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##                               statistic p-value
## Lag[1]                        2.229  0.1355
## Lag[2*(p+q)+(p+q)-1][5]      4.012  0.2526
## Lag[4*(p+q)+(p+q)-1][9]      5.178  0.4038
## d.o.f=2
##
## Weighted ARCH LM Tests
## -----
##           Statistic Shape Scale P-Value
## ARCH Lag[3]    0.3219 0.500 2.000  0.5705
## ARCH Lag[5]    1.8360 1.440 1.667  0.5086
## ARCH Lag[7]    2.5913 2.315 1.543  0.5938
##
## Nyblom stability test
## -----
## Joint Statistic:  0.5332
## Individual Statistics:
## mu      0.02641
## ma1     0.11764
## omega   0.12250
## alpha1  0.12843
## beta1   0.10701
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      1.28 1.47 1.88
## Individual Statistic:  0.35 0.47 0.75
##
## Sign Bias Test
## -----
##           t-value      prob sig
## Sign Bias      0.7074 4.794e-01
## Negative Sign Bias 1.2037 2.288e-01
## Positive Sign Bias 2.9167 3.568e-03 ***
## Joint Effect    24.7735 1.722e-05 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
##   group statistic p-value(g-1)
## 1    20      71.55  5.070e-08
## 2    30      75.31  5.497e-06
## 3    40      88.30  1.095e-05

```

```

## 4      50      95.30      8.323e-05
##
##
## Elapsed time : 0.188015

res_garch12_fit;

##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model   : sGARCH(1,2)
## Mean Model    : ARFIMA(0,0,1)
## Distribution   : norm
##
## Optimal Parameters
## -----
##      Estimate Std. Error  t value Pr(>|t|)
## mu      0.050306   0.021644  2.32425 0.020112
## ma1     -0.046741   0.021686 -2.15533 0.031136
## omega    0.038196   0.009987  3.82467 0.000131
## alpha1   0.097580   0.017403  5.60696 0.000000
## beta1    0.886244   0.181567  4.88110 0.000001
## beta2    0.000002   0.167776  0.00001 0.999992
##
## Robust Standard Errors:
##      Estimate Std. Error  t value Pr(>|t|)
## mu      0.050306   0.021376  2.353434 0.018601
## ma1     -0.046741   0.020599 -2.269143 0.023260
## omega    0.038196   0.011679  3.270596 0.001073
## alpha1   0.097580   0.018460  5.285941 0.000000
## beta1    0.886244   0.146546  6.047556 0.000000
## beta2    0.000002   0.140999  0.000012 0.999991
##
## LogLikelihood : -4318.169
##
## Information Criteria
## -----
##
## Akaike          3.3796
## Bayes           3.3933
## Shibata         3.3796
## Hannan-Quinn    3.3845
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----

```

```

##                                statistic p-value
## Lag[1]                        0.3204  0.5714
## Lag[2*(p+q)+(p+q)-1][2]      0.7798  0.8614
## Lag[4*(p+q)+(p+q)-1][5]      2.2925  0.6262
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##                                statistic p-value
## Lag[1]                        2.229  0.1354
## Lag[2*(p+q)+(p+q)-1][8]      4.960  0.3583
## Lag[4*(p+q)+(p+q)-1][14]     6.968  0.5100
## d.o.f=3
##
## Weighted ARCH LM Tests
## -----
##                Statistic Shape Scale P-Value
## ARCH Lag[4]      1.868 0.500 2.000  0.1717
## ARCH Lag[6]      1.937 1.461 1.711  0.5048
## ARCH Lag[8]      2.832 2.368 1.583  0.5754
##
## Nyblom stability test
## -----
## Joint Statistic:  2.7311
## Individual Statistics:
## mu      0.02612
## ma1     0.11804
## omega   0.12311
## alpha1  0.12848
## beta1   0.10753
## beta2   0.10615
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      1.49 1.68 2.12
## Individual Statistic:  0.35 0.47 0.75
##
## Sign Bias Test
## -----
##                t-value      prob sig
## Sign Bias      0.7051 4.808e-01
## Negative Sign Bias 1.2069 2.276e-01
## Positive Sign Bias 2.9174 3.561e-03 ***
## Joint Effect    24.7810 1.716e-05 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----

```

```

## group statistic p-value(g-1)
## 1 20 71.55 5.070e-08
## 2 30 75.62 4.974e-06
## 3 40 88.33 1.085e-05
## 4 50 95.65 7.589e-05
##
##
## Elapsed time : 0.2082851

res_garch21_fit;

##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model : sGARCH(2,1)
## Mean Model : ARFIMA(0,0,1)
## Distribution : norm
##
## Optimal Parameters
## -----
##      Estimate Std. Error t value Pr(>|t|)
## mu      0.048680 0.021566 2.2572 0.023993
## ma1     -0.046127 0.020703 -2.2280 0.025879
## omega    0.051592 0.012921 3.9930 0.000065
## alpha1   0.044707 0.021731 2.0573 0.039660
## alpha2   0.074440 0.028609 2.6020 0.009268
## beta1    0.858846 0.020493 41.9100 0.000000
##
## Robust Standard Errors:
##      Estimate Std. Error t value Pr(>|t|)
## mu      0.048680 0.021347 2.2804 0.022582
## ma1     -0.046127 0.020340 -2.2678 0.023342
## omega    0.051592 0.016980 3.0384 0.002378
## alpha1   0.044707 0.030801 1.4515 0.146647
## alpha2   0.074440 0.040415 1.8419 0.065488
## beta1    0.858846 0.029519 29.0946 0.000000
##
## LogLikelihood : -4315.095
##
## Information Criteria
## -----
##
## Akaike      3.3772
## Bayes       3.3909
## Shibata     3.3772

```

```

## Hannan-Quinn 3.3821
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----
##               statistic p-value
## Lag[1]         0.4732  0.4915
## Lag[2*(p+q)+(p+q)-1][2]  0.9603  0.7635
## Lag[4*(p+q)+(p+q)-1][5]  2.5217  0.5602
## d.o.f=1
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##               statistic p-value
## Lag[1]         0.0008106  0.9773
## Lag[2*(p+q)+(p+q)-1][8]  1.0919938  0.9673
## Lag[4*(p+q)+(p+q)-1][14] 2.7564992  0.9677
## d.o.f=3
##
## Weighted ARCH LM Tests
## -----
##               Statistic Shape Scale P-Value
## ARCH Lag[4]    0.6628 0.500 2.000  0.4156
## ARCH Lag[6]    0.9181 1.461 1.711  0.7714
## ARCH Lag[8]    1.9976 2.368 1.583  0.7421
##
## Nyblom stability test
## -----
## Joint Statistic:  0.8989
## Individual Statistics:
## mu      0.03113
## ma1     0.09852
## omega   0.14089
## alpha1  0.11355
## alpha2  0.15760
## beta1   0.11666
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      1.49 1.68 2.12
## Individual Statistic:  0.35 0.47 0.75
##
## Sign Bias Test
## -----
##               t-value      prob sig
## Sign Bias      0.6099 5.420e-01
## Negative Sign Bias 2.4376 1.485e-02 **
## Positive Sign Bias 2.4274 1.528e-02 **
## Joint Effect    29.7684 1.544e-06 ***

```



```
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
##  group statistic p-value(g-1)
## 1      20      71.79    4.632e-08
## 2      30      80.26    1.061e-06
## 3      40      93.83    2.041e-06
## 4      50     104.33    7.123e-06
##
##
## Elapsed time : 0.2201369
```