CS172: Computer Systems II

Lecture 12

Predicate Logic (syntax)

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The predicate language: more formally

The formal language of predicate logic is built in two steps:

1. A term t is a variable $(x, y, z, \ldots)$ or a constant $(a, b, c, \ldots)$.

2. A formula is built via the following rules:
   (a) If $t_1, \ldots, t_n$ are terms and P is a predicate symbol,
   then $Pt_1 \ldots t_n$ is a formula.
   (b) If ϕ and ψ are formulas, then so are each of:
   $\neg\phi$, $\phi \vee \psi$, $\phi \wedge \psi$, $\phi \rightarrow \psi$, $\phi \leftrightarrow \psi$.
   (c) If ϕ is a formula and x is a variable,
   then both $(\forall\, x \cdot \phi)$ and $(\exists\, x \cdot \phi)$ are formulas.

A formula built according to these rules if called a **well-formed formula**.

Examples of well-formed formulas

- $Lxx \wedge \neg Lmx$
- $(\exists\, x \cdot Ljx)$
- $(\forall\, x \cdot Ljx)$
- $(\exists\, y \cdot (\forall\, x \cdot Lxy))$
- $(\forall\, x \cdot Bx \rightarrow (\exists\, y \cdot Gy \wedge Lxy))$
- $\neg(\exists\, x \cdot Gx \wedge (\forall\, y \cdot \neg Lxy))$
- $(\forall\, x \cdot (\exists\, y \cdot Lxy \rightarrow (\exists\, z \cdot Lzx)))$

Translating English sentences into predicate logic.

Example 1: "Every boy loves Mary."

Translating English sentences into predicate logic.

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- **Step 1:** identify the constants and properties/relations in the sentence, and define symbols to represent them.

Constants/Predicates:	Mary	x is a boy	x loves y
Symbols:			

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Example 1: "Every boy loves Mary."

- **Step 1:** identify the constants and properties/relations in the sentence, and define symbols to represent them.

Constants/Predicates:	Mary	x is a boy	x loves y
Symbols:			

- **Step 2:** break down the English sentence, and build it up again in predicate logic.

Every boy	...
x	loves ...
x	loves Mary
Every boy	loves Mary.

Translating English sentences into predicate logic.

Example 2: "Every boy loves a girl."

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Example 2: "Every boy loves a girl."

- **Step 1:** identify the constants and properties/relations in the sentence, and define symbols to represent them.

Translating English sentences into predicate logic.

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Constants/Predicates:	x is a boy	x is a girl	x loves y
Symbols:			

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Example 2: "Every boy loves a girl."

- **Step 1:** identify the constants and properties/relations in the sentence, and define symbols to represent them.

Constants/Predicates:	x is a boy	x is a girl	x loves y
Symbols:			

- **Step 2:** break down the English sentence, and built it up again in predicate logic.

Every boy	...
x	loves ...
x	loves *a girl*
Every boy	loves a girl.

Translating English sentences into predicate logic.

Example 3: "Every girl who loves all boys does not love every girl"

Translating English sentences into predicate logic.

Example 3: "Every girl who loves all boys does not love every girl"

- **Step 1:** same predicates as before: Bx, Gx, Lxy
- **Step 2:**

Every girl ...

x loves y

x loves all boys

x loves every girl

x does not love every girl

- Result:

Translation: options.

- We translated "x loves every girl" as: $(\forall z \cdot Gz \rightarrow Lxz)$
- So, do we translate: "x *does not* love every girl" ...
  - as: $\neg(\forall z \cdot Gz \rightarrow Lxz)$
  - or: $(\forall z \cdot Gz \rightarrow \neg Lxz)$
- Can you see the difference in meaning?

Famous translation example: donkey sentences

Try translating the following into predicate logic:

"Every farmer who owns a donkey beats it."

Famous translation example: donkey sentences

Try translating the following into predicate logic:

"Every farmer who owns a donkey beats it."

- Getting the quantifiers and scope right involves quite a bit of rearranging of the sentence.
- Make sure your farmer doesn't beat *every* donkey, and that the donkey is in scope when it gets beaten...
- Does your farmer beat every donkey he owns, or just one of them?

This famous example is from "Reference and Generality" by Peter Geach. Cornell University Press, 1962.

SEP: Discourse Representation Theory