CS172 Tutorial Sheet for Tutorials in Week 3

1. **Translating sentences into propositional logic**
   Translate the following natural language sentences into the formal language of propositional logic. In each case, remember to explicitly give the meaning of any atomic propositions you use, e.g.

   M = Mary likes Chinese food   B = Bill likes Chinese food

   (a) Mary and Bill like Chinese food.
   (b) Mary likes Chinese food, but Bill doesn't.
   (c) This engine is not noisy, but it does use a lot of energy.
   (d) If you do not know how to spend money, you are not rich.
   (e) Life is a misery if you do not get more than you deserve.
   (f) It is not the case that Cain is guilty and Abel is not.
   (g) If Elizabeth did not sign this letter, then her assistant did.
   (h) John is not only stupid but nasty too.
   (i) Johnny wants both a train and a bicycle from Santa Claus, but he will get neither.
   (j) It is not the case that Guy comes, if Peter or Harry comes.

2. **Valuations in propositional logic**
   Let $P = \{p, q, r\}$ be a set of atomic propositions, and let V_1 and V_2 be two valuations defined over P, as follows:

   - $V_1(p) = 1, \quad V_1(q) = 0, \quad V_1(r) = 1$
   - $V_2(p) = 0, \quad V_2(q) = 1, \quad V_2(r) = 0$

   Given these definitions, draw out the syntax tree for the formulas in each of the following judgements, and use these trees to decide if the judgement is correct.

   (a) $V_1 \models p \rightarrow (q \vee r)$
   (b) $V_2 \models p \rightarrow (q \vee r)$
   (c) $V_1 \not\models p \wedge (q \vee r)$
   (d) $V_2 \not\models p \wedge (q \vee r)$

3. **Classifying formulas in propositional logic**
   Using truth tables, classify each of the following propositions (exclusively) as one of:
   - a contradiction
   - a tautology
   - satisfiable (but not a tautology)

   (a) $(p \rightarrow q) \rightarrow p$
   (b) $p \wedge (\neg p)$
   (c) $\neg(p \vee \neg q) \leftrightarrow (\neg p \wedge q)$

4. **Axiom systems in propositional logic**
   One way of dealing with proofs in logic is to try and identify core axioms, and then to use these to prove all other statements. The most famous axiom system for propositional logic uses these three axioms (see textbook, definition 2.23):

   (a) $(\phi \rightarrow (\psi \rightarrow \phi))$
   (b) $((\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi)))$
   (c) $((\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi))$

   These were first discovered by Jan Łukasiewicz, who was part of a notable group of Polish logicians in the 1920s that also included Stanisław Leśniewski and Alfred Tarski. The first two axioms are also the types of the S and K combinators that can be used to develop a Turing-complete programming language.

   Use truth tables to show that each of these three axioms is a tautology.