

Sets

First let us define the set F to be the first 20 natural numbers: $F \triangleq \{n \in \mathbb{N} \mid n < 20\}$.
Then we can define three subsets of F :

- $X \triangleq \{n \in F \mid (n+1)/2 \in \mathbb{N}\}$
- $Y \triangleq \{n \in F \mid \sqrt{n} \in \mathbb{N}\}$
- $Z \triangleq \{n \in F \mid n^2 \in F\}$

1. Based on these definitions, list the elements of the following sets

- (a) $X = \{ \quad \quad \quad \}$
- (b) $Y = \{ \quad \quad \quad \}$
- (c) $Z = \{ \quad \quad \quad \}$
- (d) $X \cup Y = \{ \quad \quad \quad \}$
- (e) $X \cap Z = \{ \quad \quad \quad \}$
- (f) $X \setminus Z = \{ \quad \quad \quad \}$
- (g) $X \cap Y \cap Z = \{ \quad \quad \quad \}$

2. Let us define the *complement* of a subset of F as being those elements in F that are not in that set. Specifically, we define $\bar{X} = F \setminus X$ and $\bar{Y} = F \setminus Y$. We can then formulate a version of **de Morgan's laws** for our sets. To show this, list the elements of the following sets:

- (a) $\overline{X \cup Y} = \{ \quad \quad \quad \}$
- (b) $\overline{X \cap Y} = \{ \quad \quad \quad \}$
- (c) $\overline{X \cap Y} = \{ \quad \quad \quad \}$
- (d) $\overline{X \cup Y} = \{ \quad \quad \quad \}$

Syllogistic Reasoning

1. Using Venn diagrams, decide whether or not each of the following syllogistic inferences are valid.

- (a)
$$\begin{array}{l} \text{No animals are plants} \\ \text{No sheep are plants} \\ \hline \text{All animals are sheep} \end{array}$$
- (b)
$$\begin{array}{l} \text{No student is clever} \\ \text{Some clever people are not rich} \\ \hline \text{Some rich person is not a student} \end{array}$$
- (c)
$$\begin{array}{l} \text{No human is a monster} \\ \text{All animals are monsters} \\ \hline \text{No human is an animal} \end{array}$$
- (d)
$$\begin{array}{l} \text{All numbers are green} \\ \text{Some green things are not salty} \\ \hline \text{Some numbers are not salty} \end{array}$$
- (e)
$$\begin{array}{l} \text{Some shoes are not socks} \\ \text{All shoes are not brown} \\ \hline \text{No brown thing is a sock} \end{array}$$
- (f)
$$\begin{array}{l} \text{Some Bandersnatch are not manxome} \\ \text{All manxome things are uffish} \\ \hline \text{Some Bandersnatch are not uffish} \end{array}$$



Neither Bandersnatch nor uffish