CS172 Tutorial Sheet for Tutorials in Week 6

1. (a) Using Lewis Carroll's method, decide whether or not the following (non-standard) syllogistic inferences is valid.

   All Athenians are Greeks
   All philosophers are Athenian
   All Spartans are Greeks
   No Spartans are Athenian

   No Spartans are philosophers

   This inference is non-standard because it uses four premises and (more importantly) because it uses four predicates. I've drawn out the diagram for you overleaf.

2. Suppose we define two relations as $R \triangleq \{0,1\} \times \{0,1\}$ and $S \triangleq \{0\} \times \{0,1\}$.
   Write each of the following binary relations as sets *in extension* (that is, as a list of elements between braces), and categorise the relation is reflexive, symmetric or transitive, as appropriate:

   (a) R
   (b) S
   (c) $R \cap S$
   (d) $R \setminus S$
   (e) $R \mathbin{;} S$

3. Suppose we have defined some set $Person$ along with a binary relation *childOf* $\subseteq (Person \times Person)$ that maps an individual person to each of their children.

   Define the following binary relations in terms of *childOf*, using the appropriate relational and set-theoretic operators (e.g. union, intersection, composition, inverse etc.). Categorise each relation as reflexive, symmetric or transitive, as appropriate.

   (a) the relation *parentOf*, which holds between two people exactly when the first is the parent of the second.
   (b) the relation *grandchildOf*, which holds between two people exactly when the first is a grandchild of the second.
   (c) the relation *siblingOf*, which holds between two people exactly when they have the same parent.
   (d) the relation *cousinOf*, which holds between two people exactly when a parent of one is a sibling of the parent of the other.

4. Suppose two binary relations R and S over $(\mathbb{N} \times \mathbb{N})$ have been defined as:
   $R \triangleq \{(1,4),(2,3),(3,2),(3,3),(4,1)\}$
   $S \triangleq \{(1,2),(2,3),(3,4)\}$

   Write each of the following binary relations in extension and then categorise the relation as reflexive, symmetric or transitive, as appropriate:

   (a) $R \mathbin{;} S$
   (b) $S \mathbin{;} R$
   (c) $R \mathbin{;} R$
   (d) $R \mathbin{;} R \mathbin{;} R$

5. Let the set B be defined as $B \triangleq \{0,1\}$, and define the identity relation over this set as $I_B \triangleq \{(0,0),(1,1)\}$.

   How many different binary relations are there in total over $(B \times B)$?

   Find binary relations R, S, and T that are subsets of $(B \times B)$ such that:

   (a) $R \mathbin{;} R = I_B$ but $R \neq I_B$.
   (b) $S \mathbin{;} S^{-1} \mathbin{;} S = S$, but $S \neq S^{-1}$.
   (c) $T \mathbin{;} T = \{\}$, but $T \neq \{\}$.

Acknowledgement: these exercises are based on some of those at http://www.usingz.com

Lewis Carroll's diagram

All Athenians are Greeks
All philosophers are Athenian
All Spartans are Greeks
No Spartans are Athenian

No Spartans are philosophers

First we label the four predicates:

a	=	Athenians
g	=	Greeks
p	=	philosophers
s	=	Spartans

Then, following Lewis Carroll's directions (*Symbolic Logic*, Appendix addressed to teachers, §7, pg 177):

> we assign the North Half to a (and of course the *rest* of the Diagram to a'), the West Half to g, the Horizontal Oblong to p and the Upright Oblong to s. We have now got 16 Cells.

I've written these in on the cells below to give you some help when putting in the counters: