CS172 Tutorial Sheet for Tutorials in Week 4

1. First translate the following arguments into the formal language of propositional logic. Then decide (informally) whether or not if the arguments are valid: if in doubt, check the truth tables.

   (a) If she is guilty, she will go to jail
She will not go to jail
―――――――――――
She is not guilty

   (b) It's not the case that it's not raining
―――――――――――
It's raining

   (c) I have ordered either fish or meat
I have not ordered fish
―――――――――――
I have ordered meat

   (d) If I prove that there is life in space, then it is true
There is life in space
―――――――――――
I have a proof of it

   (e) If I can prove the statement, the statement is true
The statement is true
―――――――――――
I can prove the statement

   (f) I am lying
―――――――――――
I am not telling the truth

2. A deduction that is not valid is also known as a (formal) logical *fallacy*.

   For each of the following two invalid deductions first express the premises and conclusion in propositional logic, and then use truth tables to show that the deductions are fallacious.

   (a) The following invalid deduction is an instance of a fallacy called *affirming the consequent*:

   If John is a student, then he is a human
John is a human
―――――――――――
John is a student

   (b) The following invalid deduction is an instance of a fallacy called *improper transposition*:

   If the earth is flat, I am an elephant
The earth is not flat
―――――――――――
I am not an elephant

   Technically these are called *formal* fallacies to distinguish them from the informal fallacies such as *begging the question*, *ad hominem* or *faulty analogy* that are not based on formal logical inconsistencies. For example, the following argument is an instance of the (informal) *fallacy of equivocation*, but is not directly refutable using truth tables:

   The end of life is death.
Happiness is the end of life.
―――――――――――
So, death is happiness.

   More examples of informal fallacies can be found in the entry on "Fallacies" in the Stanford Encyclopedia of Philosophy.

3. Use truth tables to prove the following *logical equivalences*:

   (a) $p \to q$ is equivalent to $\neg(p \wedge \neg q)$

   (b) $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$

   (c) $(p \vee \neg p)$ is equivalent to $((p \wedge q) \to p)$

4. The equivalence in 3(a) above can be used to express conjunction in terms of implication. Starting with this, work out how to express each of the following formula as an *equivalent* formula that uses only negation and implication:

   (a) $(p \wedge q)$

   (b) $(p \vee q)$

   (c) $(p \leftrightarrow q)$