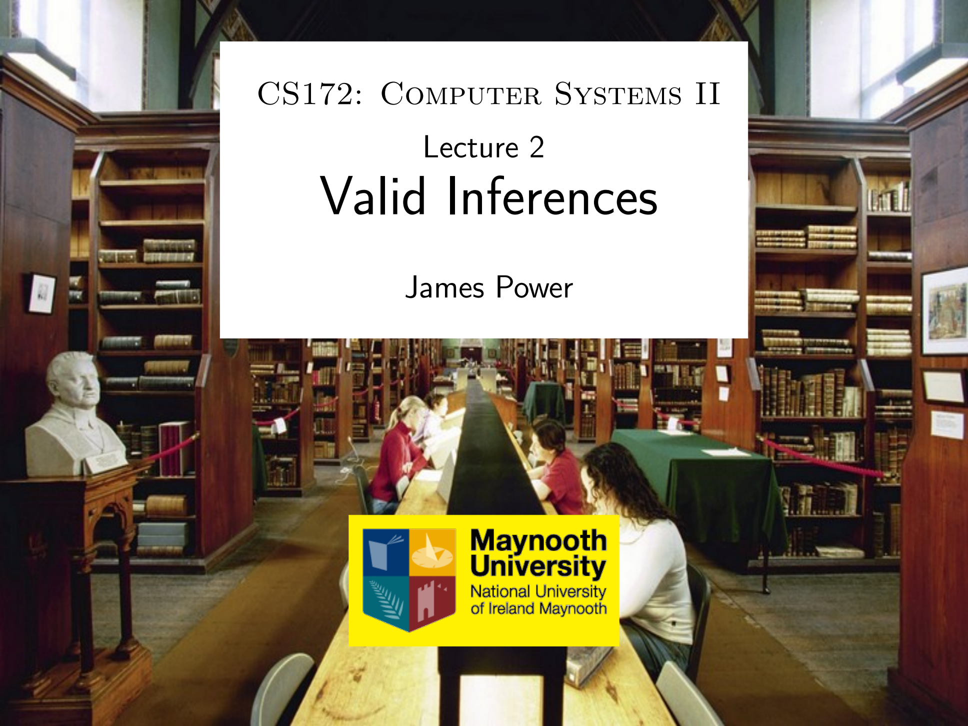

CS172: COMPUTER SYSTEMS II

Lecture 2

Valid Inferences

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Inferences

LiA §2.2

This is an example of an **inference**:

If you take my medication, you will get better.
You are taking my medication.

So, you will get better.

Q: is this inference *valid*?

Valid Inferences

The validity of an inference can be stated in one of two ways:

An inference is valid if:

- whenever *all* premises are true, then the conclusion is true.

An inference is valid if:

- whenever the conclusion is false, then *at least one* premise is false.

Valid Inferences?

1. If you take my medication, you will get better.
   You are taking my medication.
   ___
   So, you will get better.

2. If you take my medication, you will get better.
   You are getting better.
   ___
   So, you are taking my medication.

3. If you take my medication, you will get better.
   But you are not taking my medication.
   ___
   So, you will not get better.

4. If you take my medication, you will get better.
   But you are not getting better.
   ___
   So, you have not taken my medication

What are the inference *forms*?

Let M = "you take my medication", B = "you will get better"

1. If you take my medication, you will get better.
   You are taking my medication.
   ___
   So, you will get better.

2. If you take my medication, you will get better.
   You are getting better.
   ___
   So, you are taking my medication.

3. If you take my medication, you will get better.
   But you are not taking my medication.
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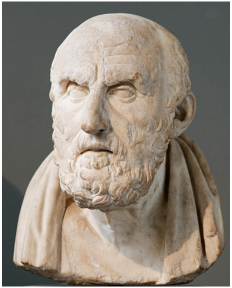
   So, you will not get better.

4. If you take my medication, you will get better.
   But you are not getting better.
   ___
   So, you have not taken my medication

Stoic Logic

LiA §2.3



Chrysippus of Soli, c.280-c.207 BC

Five *indemonstrable* statement forms:

1. If the first, [then] the second. The first. Therefore, the second.
2. If the first, [then] the second. Not the second. Therefore, not the first.
3. Not both the first and the second. The first. Therefore, not the second.
4. Either the first or the second. The first. Therefore, not the second.
5. Either the first or the second. Not the first. Therefore, the second.

Another example

An integer x is even or odd.
If x is even, then x + x is even.
If x is odd, then x + x is even.

So, x + x is always even.

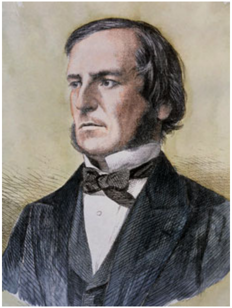
What can we tell about this inference?

$$\dfrac{\dfrac{A \qquad \dfrac{A \quad B \quad C}{D}}{E} \qquad \dfrac{E}{F} \qquad \dfrac{A}{B}}{G}$$

Suppose:

- Each inference in the tree is valid.
- We are told: A is true and G is false.
- What about the others?

Boole and logic



George Boole
1815-1864

- 1847: *Mathematical Analysis of Logic*
- 1849: First professor of mathematics at Queen's College, Cork
- 1854: *An Investigation of the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probabilities*

See also: Augustus De Morgan (1806-1871)

What was Boole doing?

"The design of the following treatise is

- to investigate the fundamental laws of those operations of the mind by which reasoning is performed;
- to give expression to them in the symbolical language of a calculus,
- and upon this foundation to establish the science of Logic and construct its method "

Chapter 1,
An Investigation of the Laws of Thought, 1854.

Boole's notation

Chapter 2, Proposition I.

All the operations of Language, as an instrument of reasoning, may be conducted by a system of signs composed of the following elements, viz:

1st. Literal symbols, as x, y etc., representing things as subjects of our conceptions.

2nd. Signs of operation, as $+, -, \times$, standing for those operations of the mind by which the conceptions of things are combined or resolved so as to form new conceptions involving the elements.

3rd. The sign of identity, $=$.

And these symbols of Logic are in their use subject to definite laws, partly agreeing with and partly differing from the laws of the corresponding symbols in the science of Algebra.

Logic notation

	Boole (1854)	*Digital Logic*	*C*, *C++*, *C#*, *Java*	*Math. Logic*
x and y	$x \times y$	$x.y$	x `&&` y	$x \wedge y$
x or y	$x + y$	$x + y$	x `||` y	$x \vee y$
not x	$1 - x$	$\overline{x}$	`!`x	$\neg x$

- There are many different kinds of logic
- The one closest to Boole's logic is called **propositional logic**.

Ingredients of the propositional language

LiA §2.4

1. Basic statements (atomic propositions): $p, q, r, \ldots$

2. Operators to build more statements:

"not ..."	becomes	$\neg \ldots$
"... and ..."	becomes	$\ldots \wedge \ldots$
"... or ..."	becomes	$\ldots \vee \ldots$
"if ... then ..."	becomes	$\ldots \to \ldots$
"... if and only if ..."	becomes	$\ldots \leftrightarrow \ldots$

Ingredients of the propositional language

LiA §2.4

1. Basic statements (atomic propositions): $p, q, r, \ldots$

2. Operators to build more statements:

negation	"not ..."	becomes	$\neg \ldots$
conjunction	"... and ..."	becomes	$\ldots \wedge \ldots$
disjunction	"... or ..."	becomes	$\ldots \vee \ldots$
implication	"if ... then ..."	becomes	$\ldots \rightarrow \ldots$
equivalence	"... if and only if ..."	becomes	$\ldots \leftrightarrow \ldots$

Examples

Write in propositional logic:

- John and Mary are running.
- I will go to school if I get a cookie now.
- I will go to school only if I get a cookie now.
- I will go to school, but I will do so reluctantly.
- He has an Ace if he does not have a King or a Spade
- A foreign national is entitled to social security if he has legal employment or if he has had such less than three years ago, unless he is currently also employed abroad.

The *formal language* of propositional logic

- Let P be a set of proposition letters and let $p \in P$.
- Then we define a formula in propositional logic ϕ as follows:

$$\begin{array}{lcl} \phi & ::= & p \\ & | & \neg\phi \\ & | & (\phi \wedge \phi) \\ & | & (\phi \vee \phi) \\ & | & (\phi \rightarrow \phi) \\ & | & (\phi \leftrightarrow \phi) \end{array}$$

This notation is called *Backus Naur Form*

Parsing a formula

Construct trees for the following formula:

- $(p \wedge q) \rightarrow r$
- $(\neg p) \wedge (\neg q)$
- $\neg(p \wedge \neg q)$
- $p \wedge q \wedge r$
- $(\neg p \vee q) \rightarrow r)$
- $p \wedge q \rightarrow r$