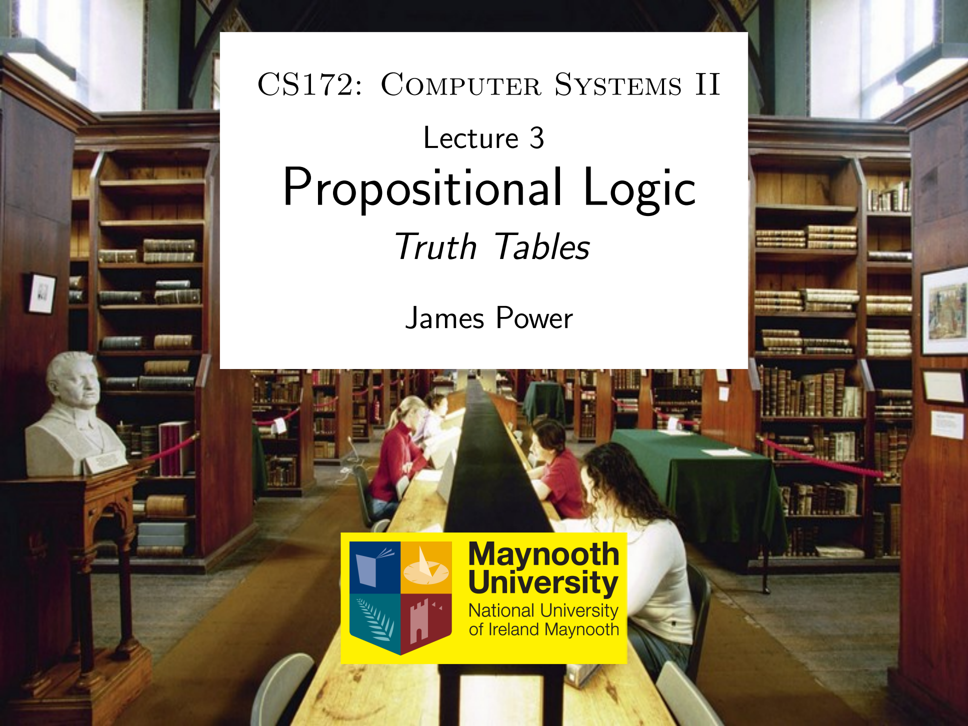

CS172: Computer Systems II

Lecture 3

Propositional Logic

Truth Tables

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Truth Tables

Topics on truth tables:

- Definition of the propositional operators
- Valuations, evaluating complex formulas
- Special kinds of formulas: satisfiable, unsatisfiable, tautology

- Using truth tables to decide if an inference is valid

Truth Tables: defining and, or, not

LiA §2.5

Truth tables define the semantics of the propositional operators

ϕ	$\neg\phi$
F	T
T	F

ϕ	ψ	$\phi \wedge \psi$
F	F	F
F	T	F
T	F	F
T	T	T

ϕ	ψ	$\phi \vee \psi$
F	F	F
F	T	T
T	F	T
T	T	T

- Invented by:

  C.S. Peirce (1893), Ludwig Wittgenstein (1912), Emil Post (1920)
- If you prefer, use 0 for "false" and 1 for "true".

Truth Tables: compound propositions

Truth tables can also be used to *calculate the values* for compound propositions.

Example: write out the truth table for $(p \vee q) \wedge \neg (p \wedge q)$

p	q	$(p$	$\vee$	$q)$	$\wedge$	$\neg$	$(p$	$\wedge$	$q)$
0	0								
0	1								
1	0								
1	1								

Truth Tables: implication and equivalence

You may not have seen these ones before:

ϕ	ψ	$\phi \to \psi$
F	F	T
F	T	T
T	F	F
T	T	T

ϕ	ψ	$\phi \leftrightarrow \psi$
F	F	T
F	T	F
T	F	F
T	T	T

Truth Tables: all five propositional operators

ϕ	$\neg\phi$
0	1
1	0

ϕ	ψ	$\phi \wedge \psi$	$\phi \vee \psi$	$\phi \rightarrow \psi$	$\phi \leftrightarrow \psi$
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	1	1

One way of remembering these is to treat ϕ and ψ as numbers:

$$\begin{array}{rcccccc} \textit{Logic}: & \neg\phi & \phi \wedge \psi & \phi \vee \psi & \phi \rightarrow \psi & \phi \leftrightarrow \psi \\ \textit{Arithmetic}: & 1-\phi & \textit{min}(\phi,\psi) & \textit{max}(\phi,\psi) & \phi \leq \psi & \phi = \psi \end{array}$$

Truth Tables: more complex propositions (again)

Example: write out the truth table for $(\neg(p \vee q) \rightarrow r)$

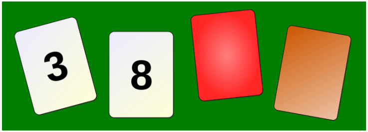
p	q	r	$(\neg$	$(p$	$\vee$	$q)$	$\rightarrow$	$r)$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Wason selection task

A test to see if you understand **implication**:

- Four cards
- Each card has a number on one side, and a patch of color on the other.

Which card(s) must be turned over to test the idea that if a card shows an even number on one face, then its opposite face is red?



Devised by Peter Cathcart Wason (1966).
See: Wikipedia, or textbook §2.12

Valuations

- In propositional logic, a *valuation* corresponds to a single row in the truth table.
- Notation:
  $V \models \varphi$ means "the proposition φ is *true* under the valuation V"
  $V \not\models \varphi$ means "the proposition φ is *false* under the valuation V"

Valuations

- In propositional logic, a *valuation* corresponds to a single row in the truth table.
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  $V \not\models \varphi$ means "the proposition φ is *false* under the valuation V"
- Example: Suppose we let $V(p) = 0$ and $V(q) = 1$
  Decide which of the following are correct **using their syntax trees**:
  - $V \models p \wedge q \vee p$
  - $V \models p \rightarrow (\neg p \rightarrow q)$
  - $V \models p \rightarrow (q \rightarrow p)$
  - $V \models p \vee \neg p$
  - $V \models p \leftrightarrow (p \vee q)$