

Homework 2

John Carroll

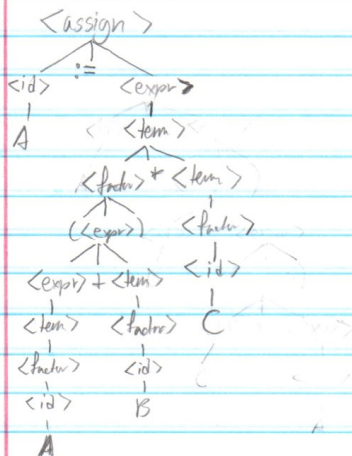
(1. 20pts) Question 3.7 (page 163) from the textbook. Scanned images:

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\langle \text{id} \rangle \rightarrow A | B | C$
 $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle | \langle \text{term} \rangle$
 $\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle | \langle \text{factor} \rangle$
 $\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle) | \langle \text{id} \rangle$

John Carroll
September 5th, 2014

1.) [3.7] a) Using Example
 $A = (A + B) * C$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{term} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{factor} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{id} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{factor} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{id} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{factor} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{id} \rangle$
 $\Rightarrow A = (A + B) * C$



John Canolle

[3.7] b) $A = B + C + A$

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{expr} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{expr} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{term} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{factor} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{id} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = B + \langle \text{term} \rangle + \langle \text{term} \rangle$

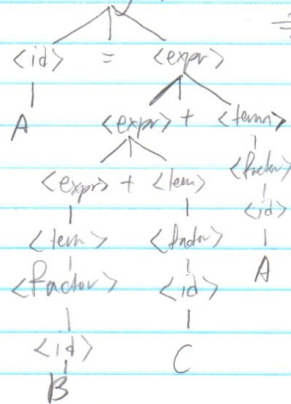
$\Rightarrow A = B + \langle \text{factor} \rangle + \langle \text{term} \rangle$

$\langle \text{assign} \rangle \Rightarrow A = B + \langle \text{id} \rangle + \langle \text{term} \rangle$

$\Rightarrow A = B + C + \langle \text{factor} \rangle$

$\Rightarrow A = B + C + \langle \text{id} \rangle$

$\Rightarrow A = B + C + A$



[3.7] c) $A = A * (B + C)$

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = \langle \text{factor} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = \langle \text{id} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = A * \langle \text{factor} \rangle$

$\Rightarrow A = A * \langle \text{expr} \rangle$

$\Rightarrow A = A * (\langle \text{expr} \rangle + \langle \text{term} \rangle)$

$\Rightarrow A = A * (\langle \text{term} \rangle + \langle \text{term} \rangle)$

$\Rightarrow A = A * (\langle \text{factor} \rangle + \langle \text{term} \rangle)$

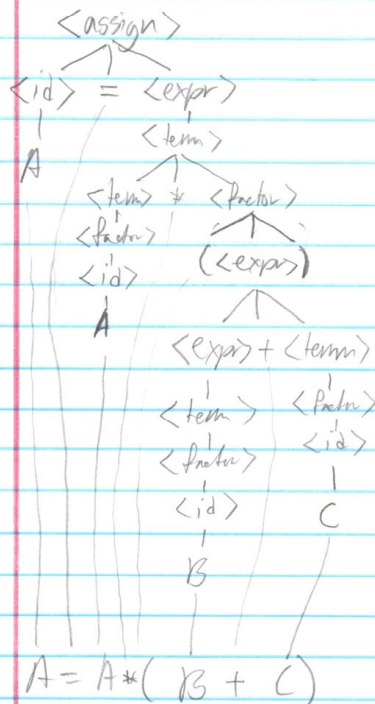
$\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{term} \rangle)$

$\Rightarrow A = A * (B + \langle \text{term} \rangle)$

$\Rightarrow A = A * (B + \langle \text{factor} \rangle)$

$\Rightarrow A = A * (B + \langle \text{id} \rangle)$

$\Rightarrow A = A * (B + C)$



[3.7] d.) $A = B * (C * (A + B))$

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{expr} \rangle$

$\Rightarrow A = \langle \text{term} \rangle$

$\Rightarrow A = \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = \langle \text{factor} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = \langle \text{id} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = B * \langle \text{factor} \rangle$

$\Rightarrow A = B * \langle \text{expr} \rangle$

$\Rightarrow A = B * \langle \text{term} \rangle$

$\Rightarrow A = B * (\langle \text{term} \rangle * \langle \text{factor} \rangle)$

$\Rightarrow A = B * (\langle \text{factor} \rangle * \langle \text{factor} \rangle)$

$\Rightarrow A = B * (\langle \text{id} \rangle * \langle \text{factor} \rangle)$

$\Rightarrow A = B * (C * \langle \text{factor} \rangle)$

$\Rightarrow A = B * (C * \langle \text{expr} \rangle)$

$\Rightarrow A = B * (C * (\langle \text{expr} \rangle + \langle \text{term} \rangle))$

$\Rightarrow A = B * (C * (\langle \text{term} \rangle + \langle \text{term} \rangle))$

$\Rightarrow A = B * (C * (\langle \text{factor} \rangle + \langle \text{term} \rangle))$

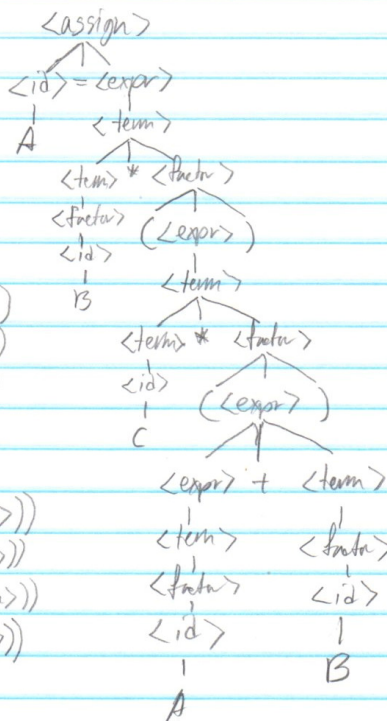
$\Rightarrow A = B * (C * (\langle \text{id} \rangle + \langle \text{term} \rangle))$

$\Rightarrow A = B * (C * (A + \langle \text{term} \rangle))$

$\Rightarrow A = B * (C * (A + \langle \text{factor} \rangle))$

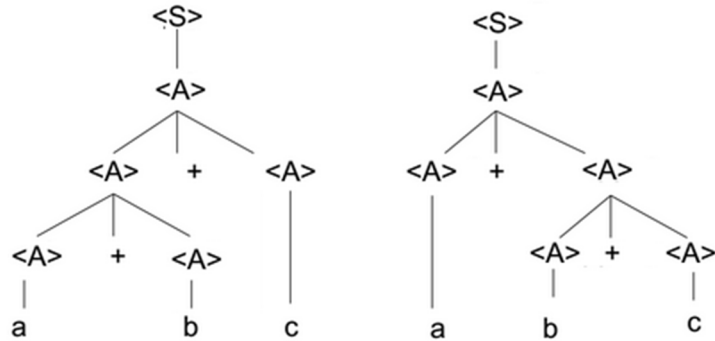
$\Rightarrow A = B * (C * (A + \langle \text{id} \rangle))$

$\Rightarrow A = B * (C * (A + B))$



(2. 20pts) Question 3.8 (page 164) from the textbook.

The following two distinct parse trees for the same string prove that the grammar is ambiguous.



(3. 20pts) Question 3.11 (page 164) from the textbook.

$\langle S \rangle \rightarrow \langle A \rangle a \langle B \rangle b$

$\langle A \rangle \rightarrow \langle A \rangle b \mid b$

$\langle B \rangle \rightarrow a \langle B \rangle \mid a$

Which of the following sentences are in the language generated by this grammar?

- a. baab - Yes
- b. bbbab - No
- c. bbaaaaa - No
- d. bbaab - Yes

(4. 10pts) Question 3.13 (page 164) from the textbook.

Write a grammar for the language consisting of strings that have n copies of the letter a followed by the same number of copies of the letter b , where $n > 0$. For example, the strings ab , $aaaabbbb$, and $aaaaaaabbbbbbb$ are in the language but a , abb , ba , and $aaabb$ are not.

Ans: $S \rightarrow a S b \mid a b$

(5. 20pts) Question 3.23 (page 165) from the textbook.

- a) $a = 2 * (b - 1) - 1 \{a > 0\}$
- $2 * (b - 1) - 1 > 0$
- $2 * b - 2 - 1 > 0$
- $2 * b > 3$
- $b > 3 / 2$

$$\begin{aligned} \text{b) } & b = (c + 10) / 3 \{b > 6\} \\ & (c + 10) / 3 > 6 \\ & c + 10 > 18 \\ & c > 8 \end{aligned}$$

$$\begin{aligned} \text{c) } & a = a + 2 * b - 1 \{a > 1\} \\ & a + 2 * b - 1 > 1 \\ & 2 * b > 2 - a \\ & b > 1 - a / 2 \end{aligned}$$

$$\begin{aligned} \text{d) } & x = 2 * y + x - 1 \{x > 11\} \\ & 2 * y + x - 1 > 11 \\ & 2 * y + x > 12 \end{aligned}$$

(6. 10pts) Question 3.24(page 165-166) from the textbook.

$$\begin{aligned} \text{a) } & a = 2 * b + 1 \\ & b = a - 3 \{b < 0\} \\ & a - 3 < 0 \\ & a < 3 \end{aligned}$$

Now, we have:

$$\begin{aligned} & a = 2 * b + 1 \{a < 3\} \\ & 2 * b + 1 < 3 \\ & 2 * b < 2 \\ & b < 1 \end{aligned}$$

$$\begin{aligned} \text{b) } & a = 3 * (2 * b + a); \\ & b = 2 * a - 1 \{b > 5\} \\ & 2 * a - 1 > 5 \\ & 2 * a > 6 \\ & a > 3 \end{aligned}$$

Now we have:

$$\begin{aligned} & a = 3 * (2 * b + a) \{a > 3\} \\ & 3 * (2 * b + a) > 3 \\ & 6 * b + 3 * a > 3 \\ & 2 * b + a > 1 \\ & b > (1 - a) / 2 \end{aligned}$$