Going over sample mid-term

1)

Recurence relation: $T(n) = 6T(n/2) + 20n^2$
$= 6T(n/2) + O(n^2)$

Go to the Master Theorem

$A = 6,\ b = 2,\ f(n) = n^2$

Steps:

1) $\log_b A = n^\wedge(\log_2 6) = n^\wedge(2.58)$

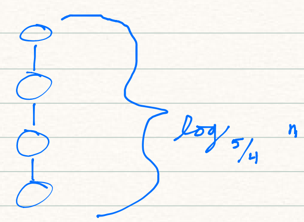
2) Case 1 applies

$n^2 = O(n^\wedge(2.58 - \varepsilon))$ as long as $\varepsilon \underset{>0}{<} 0.58$

$T(n) = \Theta(n^\wedge(\log_b A)) = \Theta(n^\wedge(\log_2 6)) = \Theta(n^\wedge(2.58))$

2) A) $T(n) = T(4/5\ n) + O(1)$

$A = 1,\ b = 5/4,\ f(n) = 1$



$n^\wedge(\log_{5/4} 1) = n^0 = 1$

$f(n) = 1 = \Theta(n^0)$

$T(n) = \Theta(n^0 \lg(n)) = \Theta(\lg(n))$

b) $T(n) = 2T(n/2) + O(n)$

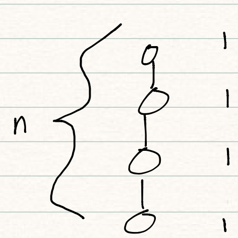
$A = 2,\ b = 2,\ f(n) = n$

$n^\wedge(\log_2 2) = n^1$

$f(n) = \Theta(n^\wedge \log_b A)$

$\Rightarrow T(n) = \Theta(n^\wedge(\log_2 2) \lg(n)) = \Theta(\lg(n))$

c) $T(n) = T(n-1) + 1$

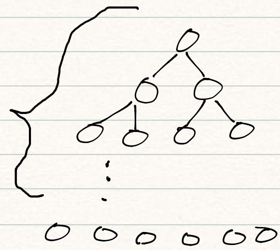


Total Work $= n \cdot 1$
$= \Theta(n)$

3) $T(n) = 2T(n/2) + \theta(n)$

n

$\log_2 2$



Total Work = ?

4)

each inner loop = $\sum_{i=0}^{n} i \cdot \theta(1)$

inside each inner loop $\theta(1)$

2 inner loops

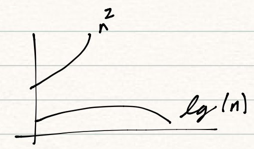
Total Work = $2\sum_{i=1}^{n} i \cdot \theta(1)$

$$2 \cdot \frac{n(n+1)}{2} \cdot \theta(1) = \theta(n^2)$$

Where does the outer loops come in to place?

5)

6) $n^2 = O(n^2 lg(n))$



for $n > 1$
for $c = 1$

$n^2 \leq cn^2 lg(n)$ divide by n^2
$1 \leq lg(n)$

7)

$n^2 \leq cn^2 lg(n)\, n^\wedge(-\varepsilon)$

$$\lim_{n \to \infty} \frac{lg\, n}{n^{\varepsilon}} = \infty$$

8)

let $f(n) = n^\wedge(2.5)$

for $n > 1$
let $c = 1$

$$0 \leq \underbrace{n^2 < n^{2.5}}_{\omega} \underbrace{< n^3}_{o}$$

$n^2 \overset{?}{<} n^{2.5}$
$1 < n^{0.5}$
$1 < \sqrt{n}$ ✓

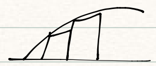
$n^{2.5} \overset{?}{<} n^3$
$1 < n^{0.5}$ ✓

HW

3.2-3

$lg(n!) = lg(1 \cdot 1 \cdot 2 \cdot \ldots (n-1)n)$
$= \sum_{i=1}^{n} lg(i) \leq \sum_{i=1}^{n} lg(n) = lg(n)$

(to show upper bound)



3-1) (a-c)

a)

Worst case

$A_0 n^0 + A_1 n^1 + A_2 n^2 + \dots + A_k n^k < A_0 n^k + A_1 n^k + A_2 n^k + \dots + A_k n^k$

b) $-|A_0| n^{k-1} - |A_1| n^{k-1} - |A_2| n^{k-1} - \dots - |A_{k-1}| n^{k-1} + A_k n^k$

$S_1 = \text{STRASSEN}(\quad , \quad)$

$\vdots$

let C be a new matric

$C_{11} =$

4 $\quad \dfrac{\ln(k)}{\ln(3)} \leq \underline{\ln(7)}$

4

a)

$n^\wedge(\log_2 2) = n$

$f(n) = \Omega(n^\wedge(\log_b a + \varepsilon)$

$a(f(n/b)) = 2(n/2)^4 = \dfrac{n^4}{8}$ let $c = 1/8$ 2_0

Case 3 applies

b)

$a f(n/b) = 7/10 \ n$ let $7/10 = c$

c)

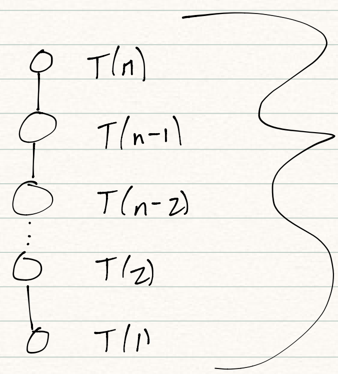
$f(n) = n^2 = \Omega(n^\wedge(\log_b a + 0.2))$

let $c = 7/9$

Questions

$T(n) = T(n-1) + O(1)$

substitution is just good for checking a soln we already have
let's go w/
$\Rightarrow$ recurrence



n calls $\quad n \times 1 = n$

$T(n) = \Theta(n)$

$f(n) = O(g(n))$ if $\exists\ c > 0,\ n_0 > 0 \ni$ for $n > n_0$ $\quad f(n) \leq c\,g(n)$

$f(n) = o(g(n))$ if $\exists\ c > 0,\ n_0 > 0 \ni$ for $n > n_0$ $\quad f(n) < c\,g(n)$