Trees

COMP 2210 – Dr. Hendrix

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Trees

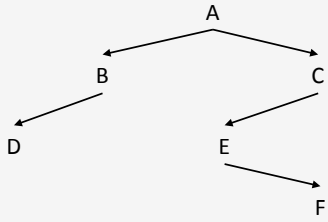
A tree is a collection in which the elements are arranged in a hierarchy.

A **list** is a *one dimensional* structure because it defines *linear relationships* between elements: **predecessor, successor**

A → B → C → D → E → F

successor(B) == C predecessor(C) == B

A **tree** is a *two dimensional* structure because it defines *hierarchical relationships* among elements: **parent, child**

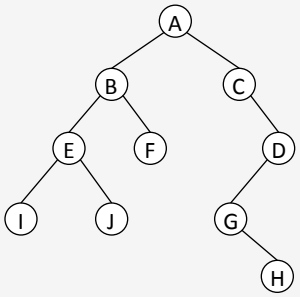


parent(B) == A

parent(C) == A

child(A) == B, C

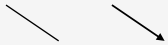
Tree terminology



A tree is composed of nodes and branches.

Node – places in the tree where the elements are stored

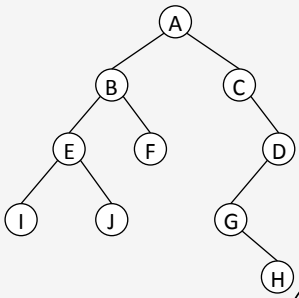
Branches – connections between nodes, from parent to child. Also called **edges**.



The terms "nodes" and "branches" are abstract and do not imply a particular implementation.

That is, we could implement a tree with either arrays or (physical) nodes and pointers.

Tree terminology



A **parent node** has one or more **children**.

A, B, C, E, D, and G are parents.

A **leaf node** has no children.

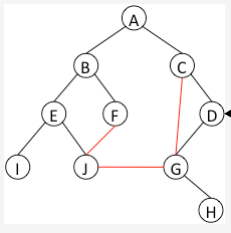
I, J, F, and H are leaves.

A **child node** has exactly one parent.

B, C, E, F, D, I, J, G, H are children.

The **root node** has no parent

A is the root.

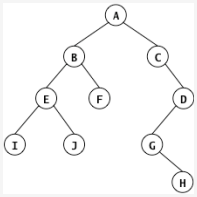


*Makes structures like this **not** a tree.*

Tree terminology

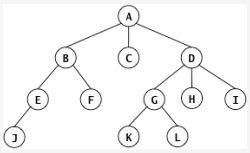
The **order** of a tree is an integer ≥ 2 that represents the upper limit on the number of children that any node can have.

Order = 2 Binary Tree



Each node can have at most 2 children.

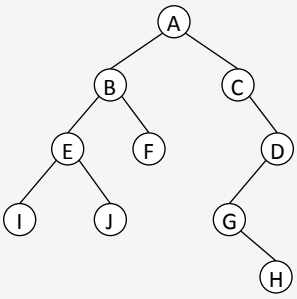
Order = 3 Ternary Tree



Each node can have at most 3 children.

General tree = a tree with no specified order.

Tree terminology



Path – a sequence of nodes from one node to another node, going from parent to child

Path from A to J = A-B-E-J

There is no path from J to A.

Path length – the number of nodes on the path

Path from A to J has length 4

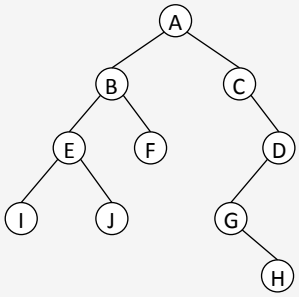
Ancestor – Node X is an ancestor of node Y iff there is a path from X to Y

Descendent– Node X is an descendent of node Y iff there is a path from Y to X.

A path is sometimes defined as a sequence of edges instead of nodes.

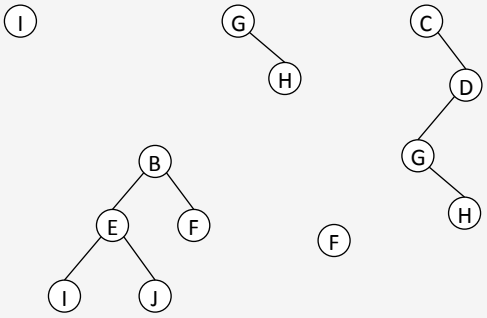
So, path length is sometimes counted differently.

Tree terminology



Subtree – A tree within a larger tree, rooted at a given node X. The subtree consists of X and all descendents of X.

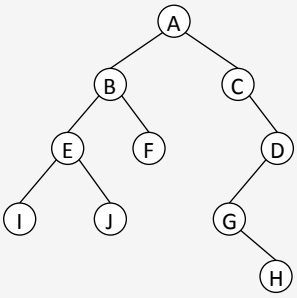
Example subtrees:



There are as many subtrees are there are nodes in the tree.

The tree itself is a subtree.

Tree terminology – height



Height is a metric that is defined in terms of a given node, but is typically used to describe a tree or subtree.

When height is applied a tree or subtree, it refers to the height of its root.

Height measures the distance of a given node from the "bottom" of the tree.

Height = length of the longest path from a given node to a descendent leaf

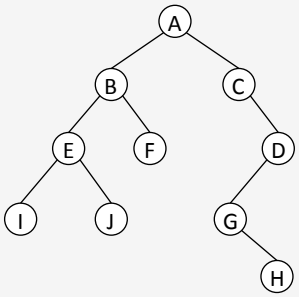
Height depends on how path and path length are defined.

You will be off by one from the text.

Height of A = 5 ← *height of the tree*

Height of B = 3 Height of J = 1 Height of H = 1

Tree terminology – depth



Depth measures the distance of a given node from the "top" of the tree.

Depth is the same concept as "level" in the text.

Depth = length of the path from the root of the tree to a given node.

Depth of A = 1

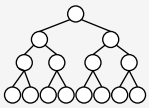
Depth of B = 2 Depth of J = 4 Depth of H = 5

Depth of a leaf on the lowest level is the same as the height of the tree.

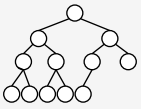
Depth depends on how path and path length are defined.

You will be off by one from the text.

Tree terminology

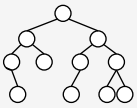


Full – A tree is full if all leaves have the same depth and every parent node has the maximum number of children.



Complete – A tree is complete if it is full to the next-to-last level, and the leaves on the lowest level are "left justified".

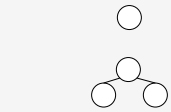
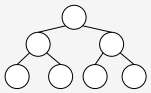
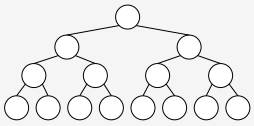
A full or complete tree is the shortest possible tree (minimum height) that could store N nodes.



Balanced – A tree is balanced if for each node, its subtrees have similar heights. The term "similar" is intentionally vague since different balancing schemes exist.

A balanced tree will have near-optimal height for storing N nodes.

Capacity v. Height

Full Binary Tree	#Nodes (n)	Tree Height (h)
	1	1
	3	2
	7	3
	15	4

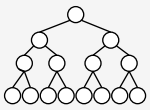
$$h = \lfloor \log_2 n \rfloor + 1 \qquad n = 2^h - 1$$

Shapes and height

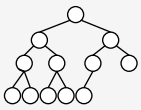
height 

Many tree algorithms are dependent to some extent on the tree's height.

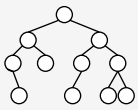
full



complete



balanced



$h = \log_2(n+1)$

Height is O(logn)