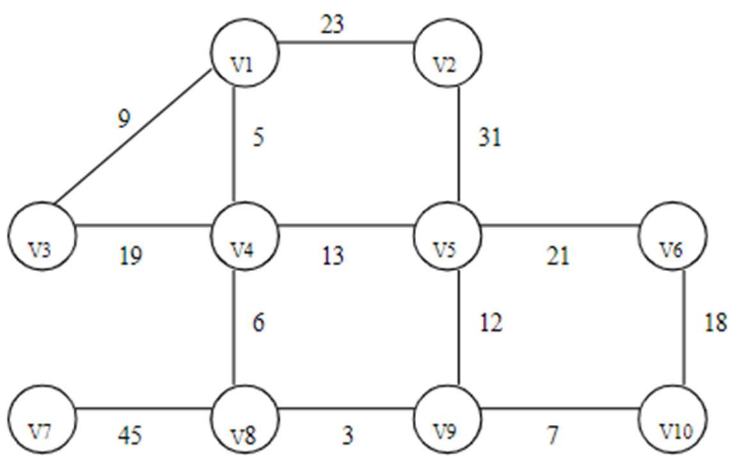
CS.3310 Homework 3

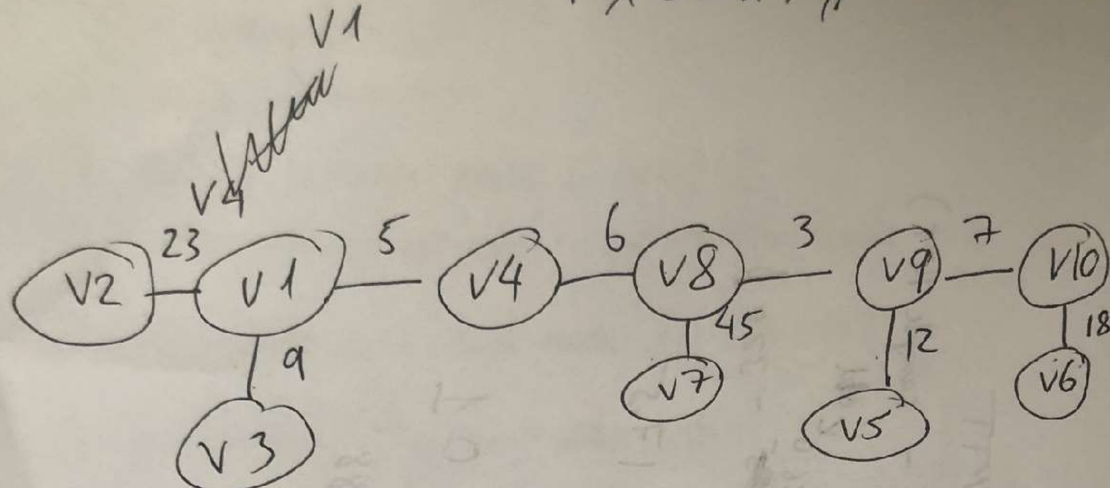
1. Construct Minimum Spanning Tree (MST) using Prim's algorithm and find the cost of that MST

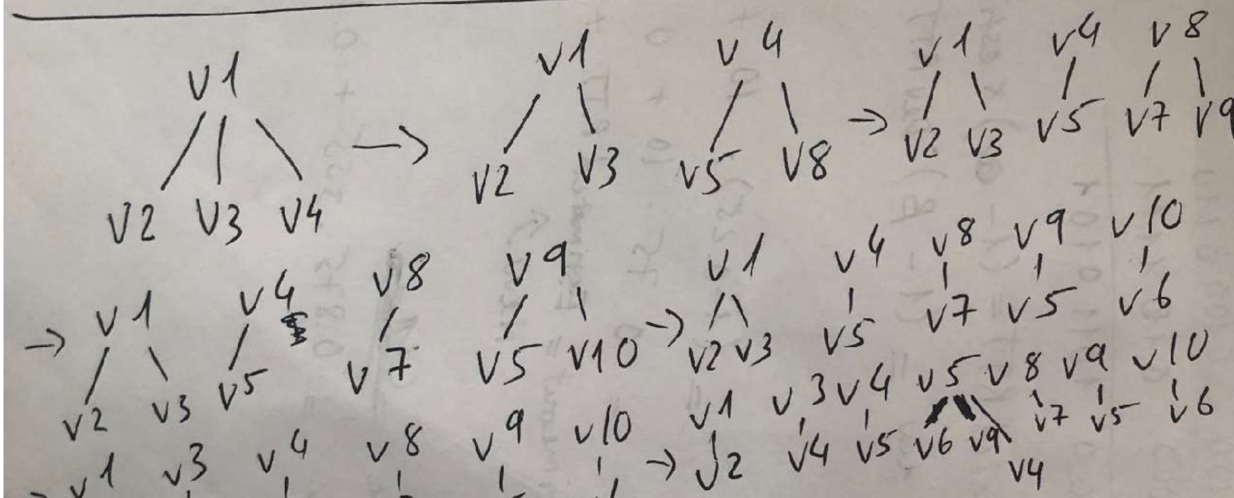



***Notation – v12 = 23 means edge weight from v1 to v2 is 23

Step 1 – Start from v1, add it to the SetMST and compute adjacent edges weight.
V12 = 23; v13 = 9; v14 = 5. Connect v1 and v4
Step 2 – Travel to v4, add it to SetMST, and compute adjacent edges weight.
V12 = 23; v13 = 9; v45 = 13; v48 = 6. Connect v4 and v8
Step 3 – Travel to v8, add it to SetMST and compute adjacent edges weight.
V12 = 23; v13 = 9; v45 = 13; v87 = 45; v89 = 3. Connect v8 and v9
Step 4 – Travel to v9, add it to SetMST and compute adjacent edges weight.
V12 = 23; v13 = 9; v45 = 13; v87 = 45; v95 = 12, v910 = 7. Connect v9 and v10
Step 5 – Travel to v10, add it to SetMST and compute adjacent edges weight.
V12 = 23; v13 = 9; v45 = 13; v87 = 45; v95 = 12, v106 = 18. Connect v1 and v3
Step 6 – Travel to v3, add it to SetMST and compute relevant adjacent edges weight.
V12 = 23; v45 = 13; v87 = 45; v95 = 12, v106 = 18 Connect v9 and v5
Step 7 – Travel to v5, add it to SetMST and compute relevant adjacent edges weight.
V12 = 23; v87 = 45; v106 = 18; v56 = 21. Connect v10 to v6
Step 8 – Add v6 to SetMST and check remaining edges weight.
V12 = 23; v87 = 45. Connect v1 to v2
Step 9 – Add v2 to SetMST and check remaining edges weight.
V87 = 45. Connect v8 to v7. **Completed MST as shown below with 9 edges**
**Total cost/weight** = 23+5+9+6+45+3+12+7+18 = **128**

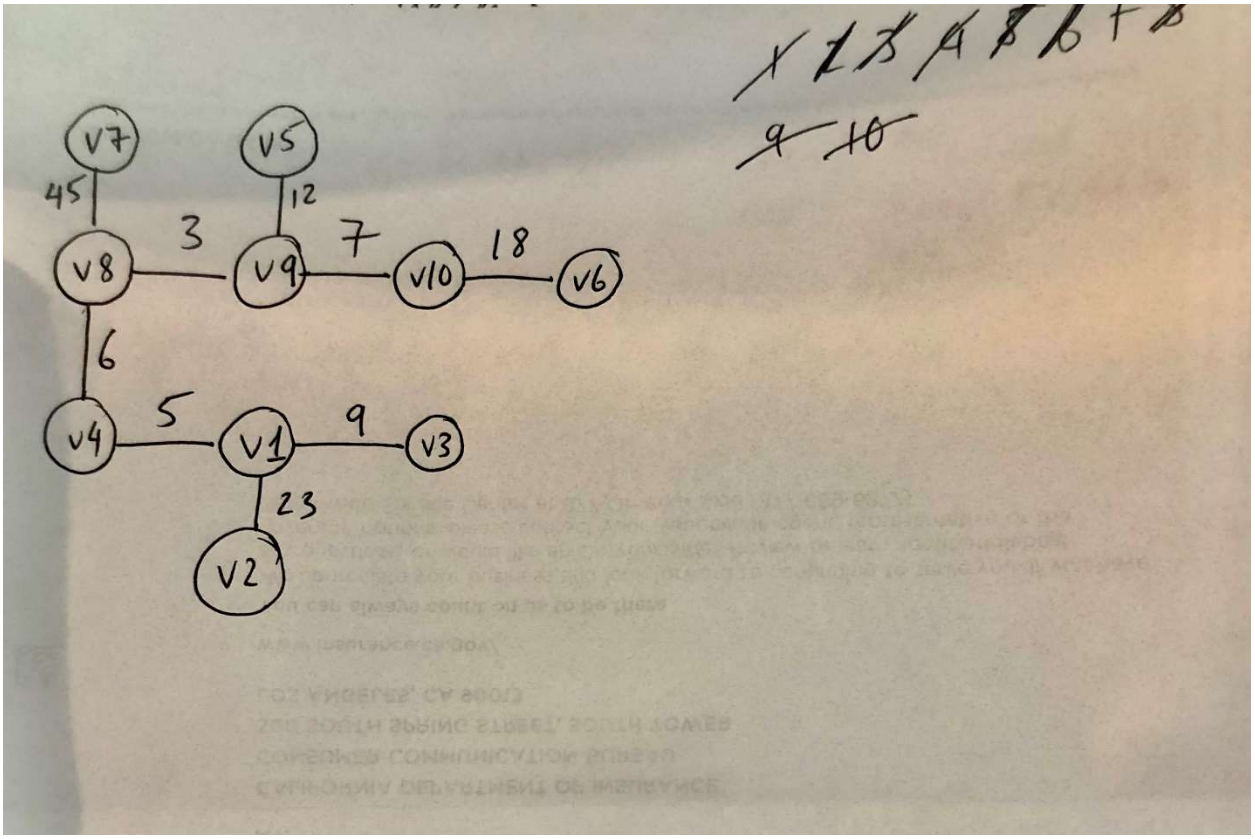
~~1 2 3 4 5 6 7 8 9 10~~



2. Construct MST using Kruskal's algorithm and cost of that MST
   Sort the edges
   (v8,v9) = 3
   (v1,v4) = 5
   (v4,v8) = 6
   (v9, v10) = 7
   (v1, v3) = 9
   (v5,v9) = 12
   (v4, v5) = 13 X
   (v6, v10) = 18
   (v3, v4) = 19 X
   (v5, v6) = 21 X
   (v1, v2) = 23
   (v2, v5) = 31 X
   (v7, v8) = 45

   Step 1 – connect v8 and v9. New set
   Step 2 – connect v1 and v4. New set
   Step 3 – connect v4 and v8. Since they came from different sets, there's no cycle created.
   Step 4 – connect v9 and v10. Since they came from different sets, there's no cycle created.
   Step 5 – connect v1 and v3. Since they came from different sets, there's no cycle created.
   Step 6 – connect v5 and v9. Since they came from different sets, there's no cycle created.
   Step 7 – discard (v4, v5) because a cycle would be created
   Step 8 – connect v6 and v10. Since they came from different sets, there's no cycle created.
   Step 9 – discard (v3, v4), (v5, v6) because a cycle would be created.
   Step 10 – connect v1 and v2. Since they came from different sets, there's no cycle created.
   Step 11 – discard (v2, v5) because a cycle would be created.
   Step 12 – connect v7 and v8. Since they came from different sets, there's no cycle created.
   MST completed and as shown below with 9 edges
   **Total cost =** 3+5+6+7+9+12+18+23+45 = **128**


V7
V5
45
12
3
7
18
V8
V9
V10
V6
6
5
9
V4
V1
V3
23
V2
1 2 3 4 5 6 7 8
9 10

3. Imagine that the objects and their weights are as below with knapsack M = 10

| Object | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|
| Weight | 5 | 6 | 7 |
| Profit | 11 | 18 | 20 |
| Profit/weight | 2.2 | 3 | 2.857 |

The order based on nonincreasing profit/weight is $I_2$, $I_3$, $I_1$.
Step 1 – We add $I_2$ to the knapsack. Then, the knapsack only have capacity of 4 left.

Step 2 – We can't add anymore items to the bag because their weights is greater than 3. This means that the knapsack utilization rate isn't 100% if it's a 0/1 Knapsack. In this case, **the profit from this 0/1 knapsack is 18.**

**Profit = 0*11 + 1*18 + 0*20 = 18**

However, the knapsack problem in class allow for partial/fractional inclusion of items so the knapsack utilization rate is always 100%.

The optimal solution is ordering by profit/weight of $I_2$, $I_3$, $I_1$ just as above.

Step 1 – We add $I_2$ to the knapsack. Then, the knapsack only have capacity of 4 left.

Step 2 – Since 4 is less than the weight of any object, we must add a fraction of an object. The next item is $I_3$ so we add 4/7 of $I_3$ to the knapsack. Then, the profit would be:

**Profit = 0*11 + 1*18 + 4/7*20 = 29.429**

By profit comparison, 29.429 > 18 so the proposed 0/1 knapsack with the proposed strategy is not the optimal solution. This will be the usually be the case because 0/1 knapsack doesn't always have 100% utilization rate whereas the knapsack strategy discussed in class will always have 100% utilization rate which always maximizes profit.

4. Let An = { a1, a2, ..., an } be a finite set of distinct coin types (e.g., a1= 50 cents, a2= 25 cents, a3= 10 cents etc.). We assume each ai is an integer and that a1 > a2 > ... > an. Each type is available in unlimited quantity. The coin changing problem is to make up an exact amount C using a minimum total number of coins. C is an integer > 0
    a. If $a_n \neq 1$, then for C value ending in 1's such as {1, 11, 21, 31,…}, there is no solution because $a_n \neq 1$
    b. The algorithm is as below
    #pass the amount to get changed for C, and A is the finite set of distinct coin type
    Def numCoin (C, A):
    ```
            Count = 0
            numOfCoin = 0
            n = 1
            while (C > 0):
            #find greatest amount of coin possible for this type of denomination
                    count = C // a_n
            #find remaining value after
                    C -= (count*a_n)
            #update values
                    numOfCoin += count
                    n += 1
    return numOfCoin
    ```
    c. A counter example would be to consider the amount of 55 and $A_n$ = {10,9.8.7,6,1} The algorithm in part (b) would yield the number of coins is 10 coins used consisting of five 10 coins and five 1 coins. However, there's another solution consisting of four 10 coins, one 9 coin, and one 6 coin, totaling six coins used instead of 10. The algorithm in part (b) uses more coins than the counterexample so part (b) algorithm isn't an optimal solution.
    d. If $A_n = \{K^{n-1}, K^{n-2}............K^0)$ where n > 1, we can substitute in some value and check. Suppose n = 5 and k = 2, we have $A_n$ = {16, 8, 4, 2, 1}. Then, suppose we have the amount C = 55, the algorithm would yield three 16 coins, one 4 coin, one 2 coin, and one 1 coin, totaling 6 coins.
    We know this is an optimal solution because the complexity when reducing is like a binary search tree where the amount of coins or steps require is approximately O(log n). The algorithm will always yield minimum number of coins.
    Lg(55) = 5.78 rounded up to 6