CS3010 Spring 2024 Homework 1

Total points: 100

Due date: Monday, March 11, 2024

Purposes:

1. Review concepts: vectors, vector notations, and vector operations.
2. Review concepts: matrices, matrix notations, and matrix operations.
3. Review concepts: matrix trace, matrix determinant, and matrix rank.
4. Get familiar with Gram-Schmidt process to orthonormalize a set of vectors.
5. Get familiar with the naïve Gaussian Elimination process to solve a system of linear equations.

"Please start working on this assignment as early as possible!"

Task Description:

- **(5 pts) Task 1:** Compute **the cosine of the angle** between the following two vectors. Please **provide** the steps on how to get the answers.

$$x = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 3 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

- **(5 pts) Task 2:** Compute the **magnitude of the cross product** of the following two vectors. Please **provide** the steps on how to get the answers.

$$x = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

- **(25 pts) Task 3:** apply **the Gram-Schmidt process** to orthonormalize the following set of vectors. Please **provide** the steps on how to get the answers, and **justify** that the resulting vectors are orthonormal.

$$v_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix},\ v_2 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}, \text{ and } v_3 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

- **(5 pts) Task 4:** Given the following two matrices A and B, **compute both the matrix-matrix multiplications *AB*** and ***BA***.

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 8 \\ 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 6 & -2 \\ -1 & 4 & 3 & 1 \end{bmatrix}$$

Please **use the 1st approach** (as described in Pages 3 and 4 of this assignment) and **provide** the steps on how to get the answers. **Note:** $AB \neq BA$ in general.

- **(10 pts) Task 5:** Given the following two matrices A and B, **compute both the matrix-matrix multiplications *AB*** and ***BA***.

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 8 \\ 4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 6 & -2 \\ -1 & 4 & 3 & 1 \end{bmatrix}$$

Please **use the 2nd approach** (as described in Page 4 of this assignment) and **provide** the steps on how to get the answers. **Note:** $AB \neq BA$ in general.

- **(5 pts) Task 6: Compute** the **matrix inverse A^{-1}** of the following 2×2 matrix. Please **justify** your answers based on the definition of matrix inverse.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

- **(25 pts) Task 7: Use** the **naïve Gaussian Elimination method** to find the solution to the following general system of linear equations. Please **provide** the steps on how to get the answers.

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 5 & 8 & 6 & 3 \\ 4 & 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

- **(20 pts) Task 8: Use** the **naïve Gaussian Elimination method** to find the solution to the following **tridiagonal** system of linear equations. Please **provide** the steps on how to get the answers.

$$\begin{bmatrix} 5 & -1 & & & \\ -1 & 5 & -1 & & \\ & -1 & 5 & -1 & \\ & & -1 & 5 & -1 \\ & & & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

What to Submit?

1. **One pdf file** which includes the answers with clear steps to Tasks 1 to 8.
2. Please **name your pdf file in the format "yourname_p1.pdf"** and submit it in Canvas.

Matrix-Matrix Multiplication

Recall that in class we studied the following **two different approaches** to computing the Matrix-Matrix Multiplication AB of two matrices $A = (a_{ij})_{r\times s}$ and $B = (b_{ij})_{s\times t}$,

- The **1st approach** is to use the row vectors of the matrix A and the column vectors of the matrix B, such that

$$AB = \begin{bmatrix} A^{(1)} \\ A^{(2)} \\ \vdots \\ A^{(r)} \end{bmatrix} \begin{bmatrix} b^{(1)} & b^{(2)} & \cdots & b^{(t)} \end{bmatrix} = \begin{bmatrix} A^{(1)}b^{(1)} & A^{(1)}b^{(2)} & \cdots & A^{(1)}b^{(t)} \\ A^{(2)}b^{(1)} & A^{(2)}b^{(2)} & \cdots & A^{(2)}b^{(t)} \\ \vdots & \vdots & \ddots & \vdots \\ A^{(r)}b^{(1)} & A^{(r)}b^{(2)} & \cdots & A^{(r)}b^{(t)} \end{bmatrix}_{r\times t}$$

where $A^{(i)}$ denotes the i-th row vector of the matrix A and $b^{(j)}$ the j-th column vector of the matrix B. Note that each $A^{(i)}b^{(j)}$ is a scalar.

- The **2nd approach** is to use the column vectors of the matrix A and the row vectors of the matrix B, such that

$$AB = \begin{bmatrix} a^{(1)} & a^{(2)} & \cdots & a^{(s)} \end{bmatrix} \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ \vdots \\ B^{(s)} \end{bmatrix} = a^{(1)}B^{(1)} + a^{(2)}B^{(2)} + \cdots + a^{(s)}B^{(s)} = \sum_{i=1}^{s} a^{(i)}B^{(i)}$$

where $a^{(i)}$ denotes the i-th column vector of the matrix A and $B^{(j)}$ the j-th row vector of the matrix B. Note that each $a^{(i)}B^{(i)}$ is a matrix of size $r \times t$.