

1.1:

a, * Logic:

$$((P \rightarrow Q) \wedge P) \rightarrow Q \quad (*)$$

Ta có:

$$\begin{aligned} \text{VT: } ((P \rightarrow Q) \wedge P) &\equiv (\bar{P} \vee Q) \wedge P \quad (\text{Luật kéo theo}) \\ &\equiv (P \wedge \bar{P}) \vee (P \wedge Q) \quad (\text{Luật phân phối}) \\ &\equiv 0 \vee (P \wedge Q) \quad (\text{Rút gọn}) \\ &\equiv P \wedge Q \end{aligned}$$

Xét (*) kết hợp VT:

$$\begin{aligned} \Rightarrow (P \wedge Q) \rightarrow Q &\equiv (\overline{P \wedge Q}) \vee Q \quad (\text{Luật kéo theo}) \\ &\equiv (\bar{P} \vee \bar{Q}) \vee Q \quad (\text{Luật De Morgan}) \\ &\equiv \bar{P} \vee (\bar{Q} \vee Q) \quad (\text{Luật kết hợp}) \\ &\equiv \bar{P} \vee 1 \\ &\equiv 1 \quad (\text{luôn đúng}) \end{aligned}$$

b, * Logic:

$$P \wedge Q \rightarrow P$$

$$\begin{aligned} \text{Ta có: } P \wedge Q \rightarrow P &\equiv (\overline{P \wedge Q}) \vee P \quad (\text{Luật kéo theo}) \\ &\equiv (\bar{P} \vee \bar{Q}) \vee P \quad (\text{De Morgan}) \\ &\equiv (\bar{P} \vee P) \vee \bar{Q} \quad (\text{kết hợp}) \\ &\equiv 1 \vee \bar{Q} \quad (\text{Rút gọn}) \\ &\equiv 1 \end{aligned}$$

c, * Logic:

$$\neg(P \wedge Q) \wedge P \rightarrow \bar{Q} \quad (*)$$

$$\begin{aligned} \text{Ta có: VT: } \neg(P \wedge Q) \wedge P &\equiv (\bar{P} \vee \bar{Q}) \wedge P \quad (\text{De Morgan}) \\ &\equiv (P \wedge \bar{P}) \vee (P \wedge \bar{Q}) \quad (\text{phân phối}) \\ &\equiv 0 \vee (P \wedge \bar{Q}) \\ &\equiv P \wedge \bar{Q} \end{aligned}$$

$$\begin{aligned} \text{Từ (*) kết hợp VT: } \Rightarrow (P \wedge \bar{Q}) \rightarrow \bar{Q} &\equiv (\overline{P \wedge \bar{Q}}) \vee \bar{Q} \\ &\equiv (\bar{P} \vee Q) \vee \bar{Q} \equiv \bar{P} \vee (Q \vee \bar{Q}) \\ &\equiv \bar{P} \vee 1 \equiv 1 \end{aligned}$$

d, logic:

$$(*) ((P \wedge Q) \leftrightarrow P) \rightarrow (P \rightarrow Q)$$

Proof:

$$\begin{aligned} \text{VT: } ((P \wedge Q) \leftrightarrow P) &\equiv (P \wedge Q \rightarrow P) \wedge (P \rightarrow (P \wedge Q)) \\ &\equiv (\neg(P \wedge Q) \vee P) \wedge (\bar{P} \vee (P \wedge Q)) \\ &\equiv (\bar{P} \vee \bar{Q}) \vee P \wedge ((\bar{P} \vee P) \wedge (\bar{P} \vee Q)) \\ &\equiv (\bar{P} \vee P) \vee \bar{Q} \wedge (1 \wedge (\bar{P} \vee Q)) \\ &\equiv (1 \vee \bar{Q}) \wedge (\bar{P} \vee Q) \\ &\equiv 1 \wedge (\bar{P} \vee Q) \\ &\equiv \bar{P} \vee Q \\ &\equiv P \rightarrow Q \quad (VP) \end{aligned}$$

1.7:

a,

$$\begin{array}{lcl}
 (\overline{x_1} \vee x_2) \rightarrow x_3 & & (\overline{x_1} \vee x_2) \rightarrow (x_4 \vee x_5) \\
 x_3 \rightarrow (x_4 \vee x_5) & & \overline{x_5} \\
 \overline{x_4} \wedge \overline{x_5} & & \downarrow x_6 \rightarrow x_5 \\
 \overline{x_6} \rightarrow \overline{x_5} & \equiv & \overline{x_6} \\
 \hline
 \therefore x_1 & & \therefore x_1
 \end{array}$$

$$\begin{array}{l}
 (\overline{x_1} \vee x_2) \rightarrow (x_4 \vee x_5) \\
 \equiv \overline{x_4} \wedge \overline{x_5} \equiv \overline{x_4 \vee x_5} \\
 \therefore x_1
 \end{array}$$

$$\begin{array}{l}
 \equiv \overline{\overline{x_1} \vee x_2} \equiv \overline{x_1 \wedge \overline{x_2}} \equiv x_1 \\
 \therefore x_1
 \end{array}$$

b,

$$\begin{array}{l}
 p \rightarrow ((q \vee r) \wedge s) \\
 p \\
 \hline
 (q \vee r) \wedge s
 \end{array}$$