Midterm 2

1. a)

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_G^* - V_{Tn}\right)^2 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{DD} - V_G^* - V_{Tp}\right)^2$$

$$V_G^* - 0.3 = 0.9 - V_G^* - 0.4$$

$$\boxed{V_G^* = 0.4V}$$

b)

$$g_{m_n} = \mu_n C_{ox}\frac{W}{L}\left(V_G^* - V_{Tn}\right)$$

$$= 0.5 \times 200 \times 0.1 \text{ mS}$$

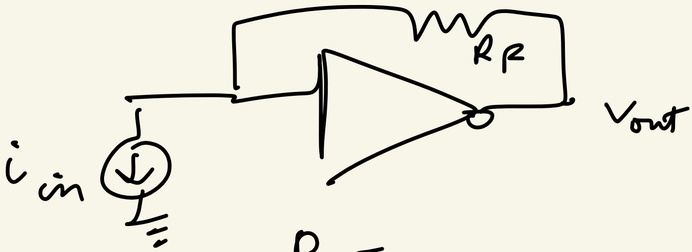
$$= 10 \text{mS}$$

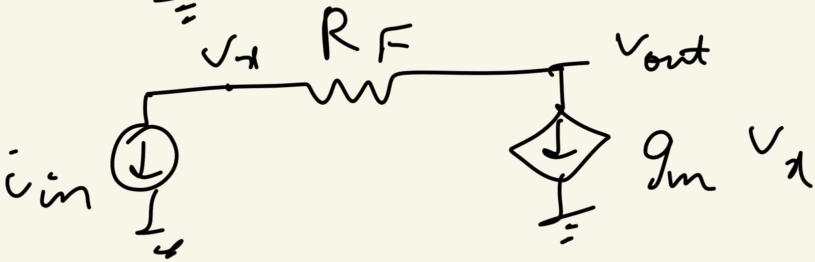
$$g_{m_p} = \mu_p C_{ox}\frac{W}{L}\left(V_{DD} - V_G^* - V_{Tp}\right)$$

$$= 0.4 \times 250 \times 0.1 \text{mS}$$

$$= 10 \text{mS}$$

$$g_m = g_{m_n} + g_{m_p} = 20 \text{mS}$$





$$v_x = -\frac{i_{in}}{g_m}$$

$$V_{out} = v_x + i_{in} R_F$$

$$= -\frac{i_{in}}{g_m} + i_{in} R_F$$

$$\frac{V_{out}}{i_{in}} = R_F - \frac{1}{g_m}$$

$R_F = 1.05 k\Omega$

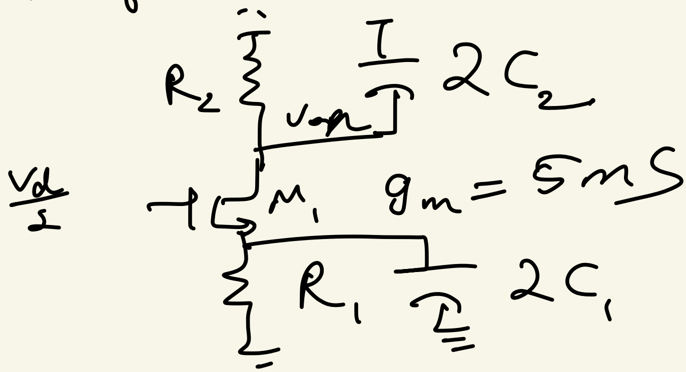
$g_m = 20mS \qquad \frac{1}{g_m} = 50\Omega$

$$\boxed{\frac{V_{out}}{i_{in}} = 1k\Omega}$$

c) $\frac{v_{out}}{i_{in}} = 0 \Rightarrow R_F = \frac{1}{g_m}$

$= \boxed{50\Omega}$

2. a) DM Half-ckt.



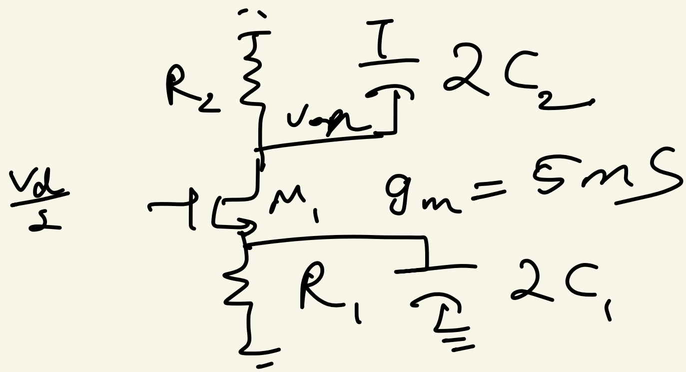
$$v_{on} = -\frac{g_m R_2}{1+g_m R_1} \cdot \frac{V_d}{2}$$

$$v_{op} = \frac{+g_m R_2}{1+g_m R_1} \cdot \frac{V_d}{2}$$

$$\frac{v_{op} - v_{on}}{V_d} = \frac{g_m R_2}{1+g_m R_1}$$

$$= \frac{5 \times 0.5}{5} = \boxed{0.5}$$

b)



$$\frac{v_{on}}{v_d/2} = -\frac{g_m R_2}{1+g_m R_1}\cdot\frac{1+s(2R_1C_1)}{1+s\left(\frac{R_1C_1}{1+g_mR_1}\right)}\cdot\frac{1}{1+s(2C_2R_2)}$$

$$\frac{v_{op}}{v_d/2} = +\frac{g_m R_2}{1+g_m R_1}\cdot\frac{1+s(2R_1C_1)}{1+s\left(\frac{R_1C_1}{1+g_mR_1}\right)}\cdot\frac{1}{1+s(2C_2R_2)}$$

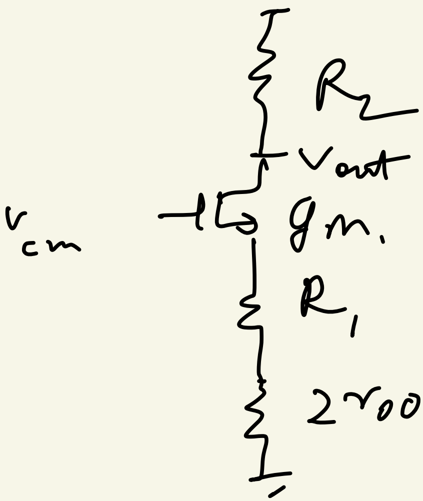
$$\frac{v_{op}-v_{on}}{v_d} = \frac{g_m R_2}{1+g_m R_1}\cdot\frac{1+s(2R_1C_1)}{\left(1+\frac{s2R_1C_1}{1+g_mR_1}\right)\left(\frac{1}{1+s(2C_2R_2)}\right)}$$

2 poles, 1 zero

Zero at 1.25GHz

Poles at 6.25GHz, 50GHz

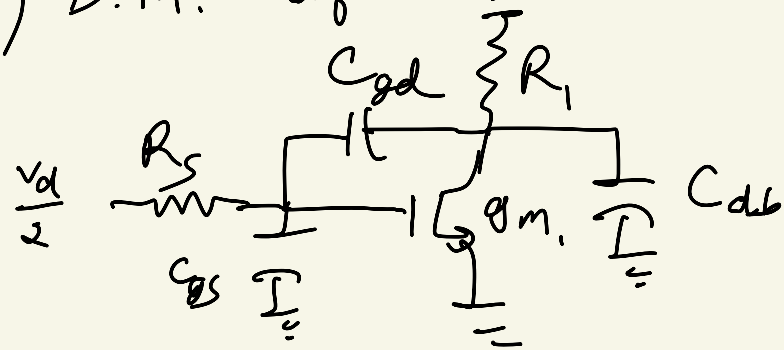
c) CM. half-ckt,



$$\frac{v_{op}}{v_{cm}} = \frac{v_{on}}{v_{cm}}$$

$$= -\frac{g_m R_2}{1 + g_m(R_1 + 2r_{oo})}$$

3. a) D.M. half-ckt.



Exact transfer fn.

$$\frac{v_{on}}{\frac{v_d}{2}} = \frac{-g_{m_1} R \left(1 - \frac{s\,C_{gd}}{g_{m_1}}\right)}{1 + s\left(R_S\left[C_{gs} + C_{gd}(1+g_{m_1}R_1)\right] + R_1\left[C_{db} + C_{gd}\right]\right) + s^2 R_S R_1\left[C_{gs}C_{gd} + C_{gd}C_{db} + C_{gs}C_{db}\right]}$$

$$\approx - \frac{-g_{m_1} R \left(1 - \frac{s\,C_{gd}}{g_{m_1}}\right)}{1 + s\left(R_S\left[C_{gs} + C_{gd}(1+g_{m_1}R_1)\right] + R_1\left[C_{db} + C_{gd}\right]\right)}$$

3-dB BW $\qquad g_{m1} R_1 = 9$

$$\approx \frac{1}{2\pi \cdot R_s\left[C_{gs} + C_{gd}(1 + g_m R_1)\right] + R_1\left[C_{gd} + C_{db}\right]}$$

$$= \frac{1}{\frac{2\pi}{2\pi} \times 2\,k\Omega\left[400\,fF + 200\,fF \times 10\right] + 200 \times \left[200\,fF + 200\,fF\right]}$$

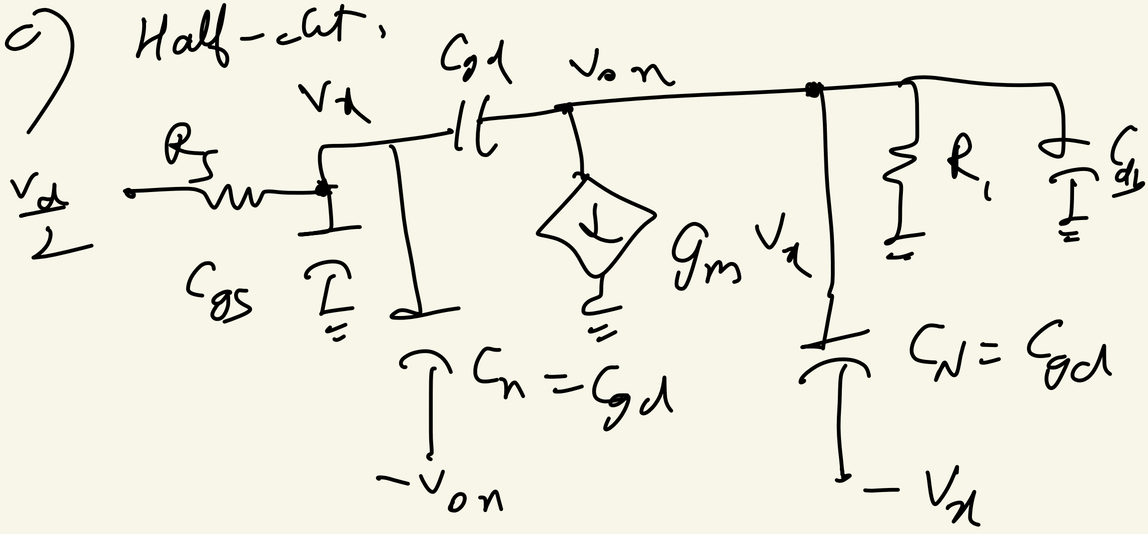
$$= \underline{205\,MHz}$$

b) Zero frequency $= \dfrac{g_m}{2\pi \cdot C_{gd}}$

$$= \frac{45\,mS}{200\,fF}$$

$$= \underline{225\,GHz}$$

c) Half-ckt,



To find zero freq.,
consider current from V_{on}
which depends on V_x.

$$g_m V_x + (V_{on} - V_x) \cdot sC_{gd} + (V_{on} + V_x) \cdot sC_N$$

$$C_N = C_{gd}$$

$$= g_m V_x + 2V_{on} \cdot sC_{gd}$$

$\Rightarrow$ No freq. at which current $= 0$

$\Rightarrow$ No zero

d) Time constant for node v_1

$$R_s\left[C_{gs} + C_{gd}(1 + g_m R_L) + C_N(1 - g_m R_L)\right]$$

$$= R_s\left[C_{gs} + C_{gd} + C_{gd}\right]$$

$$= R_s\left[C_{gs} + 2C_{gd}\right]$$

Time constant for node V_{on}

$$= R_1\left[C_{db} + C_{gd}\left(1 + \frac{1}{g_m R_L}\right) + C_N\left(1 - \frac{1}{g_m R_L}\right)\right]$$

$$= R_1\left[C_{db} + 2C_{gd}\right]$$

3-dB BW

$$\simeq \frac{1}{2\pi\left[R_s(C_{gs} + 2C_{gd}) + R_1(C_{db} + 2C_{gd})\right]}$$

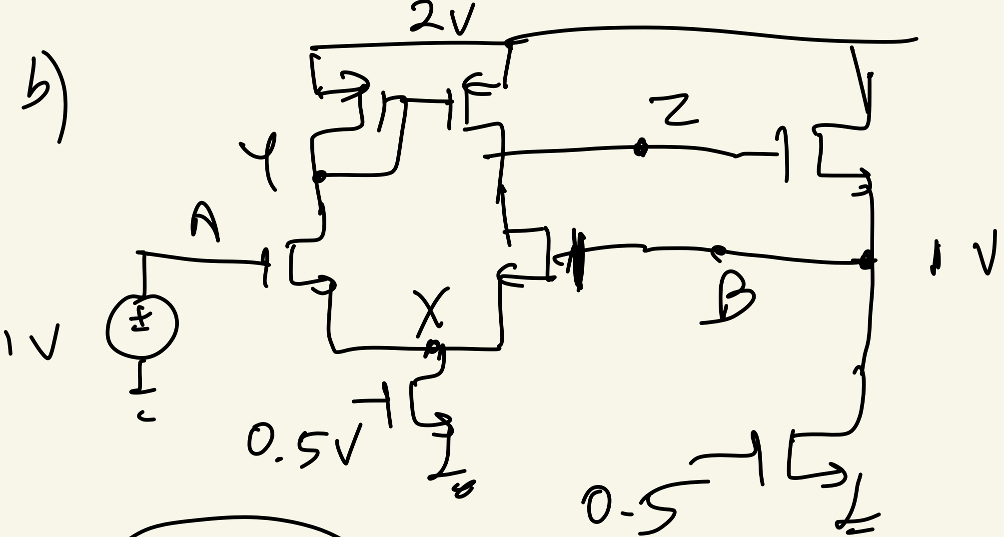
$$= \boxed{581\,MHz}$$

You can actually show that full transfer function

$$\frac{v_{op} - v_{on}}{v_d}(s) = \frac{-g_{m_1} R_1}{\left(1 + sR_S\left[C_{gs} + 2C_{gd}\right]\right)\left(1 + sR_1\left[C_{db} + 2C_{gd}\right]\right)}$$

4. a) Terminal B ~~if~~ for negative feedback

b)



$\boxed{\begin{aligned} A &= 1V \\ B &= 1V \end{aligned}}$ (by negative feedback)

$$Y = V_{DD} - V_{T_p} - \Delta V$$

$$I_0 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_B - V_{T_n}\right)^2$$

$$= 200\mu A$$

$$I_3 = \frac{1}{2} \times I_0 = 100\mu A$$

$$\Rightarrow \Delta V_3 = 0.2V$$

$$Y = 2 - 0.4 - 0.2$$

$$Y = \boxed{1.4V}$$

$$V_{GS_1} = V_{GS_2} = V_1 + \Delta V \Big|_{100\mu A}$$

$$= 0.3 + 0.2$$

$$= 0.5V$$

$$A = B = 1V$$

$$X = 1V - 0.5V$$

$$\boxed{X = 0.5V}$$

(2) $$V_{GS_6} = \Delta V \Big|_{100\mu A, \left(\frac{40}{1}\right)} + V_{Tn}$$

$$= 0.1V + 0.3V$$

$$= 0.4V$$

$$V_{S_6} = V_B = 1V$$

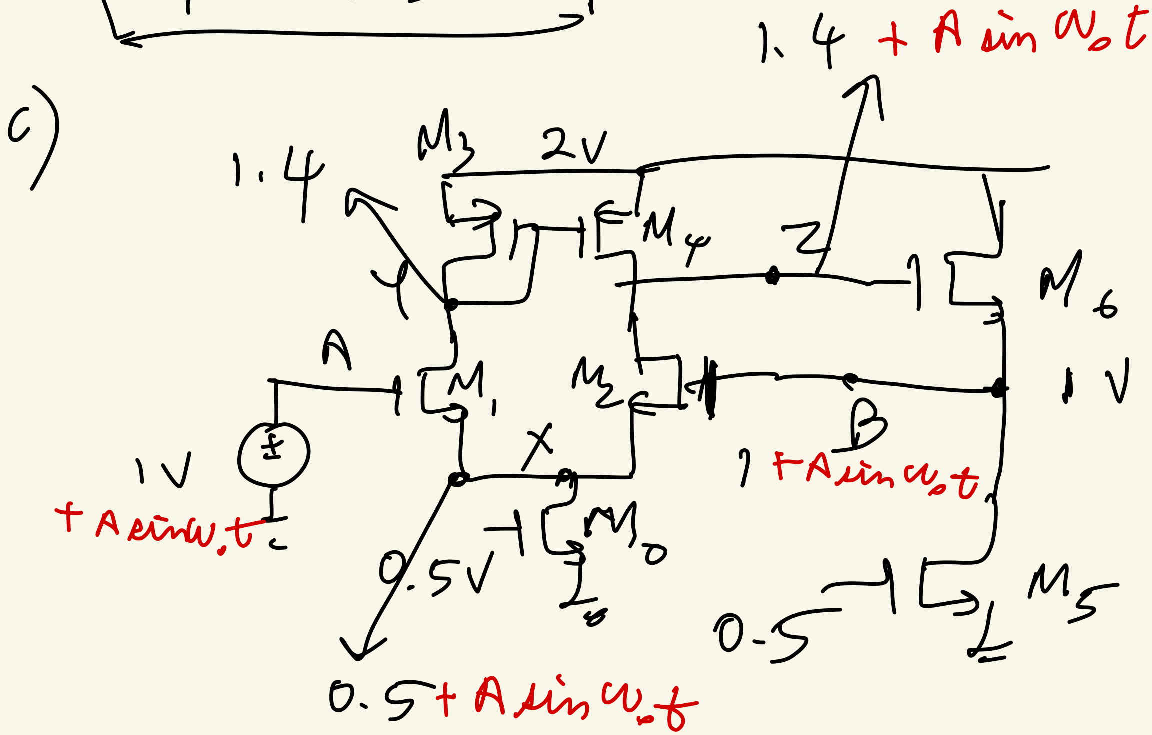
$$V_{G_6} = 1.4V$$

$$\boxed{Z = 1.4V}$$

$A, B = 1V$

$Y, Z = 1.4V$

$X = 0.5V$



M_2 can't get into triode
(drain & gate swing by same amount)

$\underline{M_1}$ $\quad V_{G,max} = 1 + A$

$V_D = 1.4$

$V_D = V_{G,max} - V_T$

$1.4 = 1 + A - 0.3$

$A = 0.7V$

$\underline{M_2}$ $V_{D,min} = 0.5 - A$

$V_G = 0.5$

$V_{D,min} = V_G - V_T$

$0.5 - A = 0.5 - 0.3$

$A_{max} = 0.3V$

$\underline{M_3}$ never gets to triode

$\underline{M_4}$ $V_G = 1.4V$

$V_{D_{max}} = 1.4 + A$

$V_{D,max} = V_G + V_{tn}$

$1.4 + A = 1.4 + 0.4$

$A_{max} = 0.4V$

$\underline{M_5}$ $V_{D_{min}} = 1 - A$ $V_G = 0.5V$

$V_{D,min} = V_G - V_T$ $1 - A = 0.5 - 0.3$

$A = 0.8V$

$\underline{M_6}$ $V_{G,max} = 1.4 + A$ $V_D = 2V$

$\boxed{A_{max} = 0.9V}$

$$\boxed{A_{max} = 0.3V \quad M_o \text{ in cut-off}}$$

d)

$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_o}}$$

$$A_o = g_{m_1}(r_{o2} \| r_{o4})$$

$$\omega_o = \frac{1}{(r_{o2} \| r_{o4}) \cdot C_L}$$

In unity gain feedback, the CLG has a 3-dB BW of $A_o \omega_o$

$$= \frac{1}{2\pi} \frac{g_{m_1}}{C_L} = \frac{1}{2\pi} \frac{1 mS}{\frac{1}{2\pi} pF}$$

$$= 1 GHz$$