Problem 1

$\nearrow I_{DS1} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(0.2V)^2 = 2mA$

a. $V_{b1} = 0.5V$, Find V_{b2} to ensure saturation

$V_{ov1} = 0.5 - 0.3 = 0.2V = V_{ov3} = V_{ov4}$

we want $I_{DS1} = I_{DS7}$

$$\cancel{\frac{1}{2}}\underbrace{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}_{=200\cdot 0.5 = 100\,mA/V^2}\underbrace{(V_{b1} - V_{tn})}_{0.2V}^2 = \cancel{\frac{1}{2}}\underbrace{\mu_p C_{ox}\left(\frac{W}{L}\right)_p}_{=0.4\cdot 1000 = 400}(V_{DD} - V_{b2} - V_{tp})^2$$

$$V_{DD} - V_{b2} - V_{tp} = \sqrt{\frac{100}{400}\cdot(0.2)^2} = 0.1V$$

$V_{ov7} = 0.1V$

$$V_{b2} = \underset{\substack{\uparrow\\ 2V}}{V_{DD}} - 0.1V - \underset{\substack{\uparrow\\ 0.4V}}{V_{tp}} = 1.5V$$

$\boxed{V_{b2} = 1.5V}$

b. CM Input range:

Keep M7 in sat

$$V_{cm}|_{max} + V_{GS5} = V_{DD} - V_{ov7}$$

$$\left(\frac{W}{L}\right)_5 = \frac{1}{4}\left(\frac{W}{L}\right)_7 \rightarrow V_{ov5} = 2V_{ov7} = 0.2V$$

$$V_{cm}\big|_{max} = V_{DD} - V_{ov7} - V_{GS5} = 1.3V$$

$$\uparrow \quad 2V \qquad \uparrow \quad 0.1V \qquad \uparrow \quad 0.6V$$

↑ constraining

keep m_1 in sat

$$V_{cm}\big|_{min} - V_{GS3} = V_{ov1}$$

$$V_{cm}\big|_{min} = V_{ov1} + V_{GS3} = 0.2V + 0.2V + 0.3V = 0.7V$$

Constraining →

keep m3 in sat

$$V_o - (V_{cm}\big|_{max} - V_{GS3}) = V_{ov3}$$

constraining

$$V_{cm}\big|_{max} = V_o + V_{GS3} - V_{ov3} = V_o + V_{tn}$$

$$= 1.3V$$

keep m5 in sat

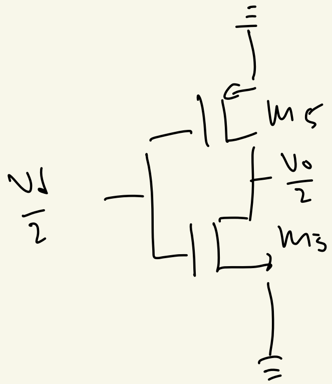
$$V_{cm}\big|_{min} + V_{SG5} - V_o = V_{ov5}$$

$$V_{cm}\big|_{min} = V_o + V_{ov5} - V_{SG5}$$

$$= V_o - V_{tp} = 1 - 0.4 = 0.6V$$

$$\boxed{0.7V < V_{cm} < 1.3V}$$

C. differential gain. Assume $\lambda_n = \lambda_p$



$I_{DS5} = I_{DS3} \rightarrow r_{o5} = r_{o3} = r_o$

$V_{ov3} = V_{ov5} \rightarrow g_{m3} = g_{m5} = g_m$

$$\boxed{A_d = 2g_m \cdot (r_o \| r_o) = g_m r_o}$$

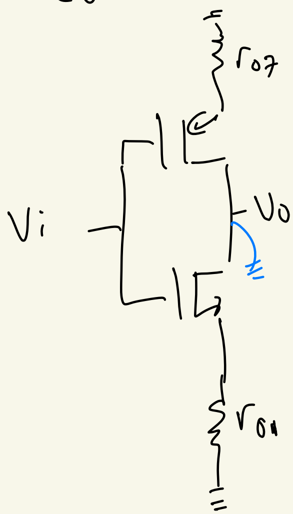
$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \cdot 1mA}{0.2} = 10mS$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \cdot 1mA} = 10k\Omega$$

$$\boxed{g_m r_o = 100}$$

d. Common-mode gain

$r_{o7} = \frac{1}{2} r_o \leftarrow$ 2x current same λ



$$i_{sc} \approx \frac{-g_m}{1+g_m r_{o1}} \cdot 2 = \frac{-2g_m}{1+g_m \frac{r_o}{2}}$$

$r_{o1} = \frac{1}{2} r_o \leftarrow$ 2x current, same λ

$$R_{out} \approx \left(g_m r_o \frac{r_o}{2}\right) \| \left(g_m r_o \frac{r_o}{2}\right) \approx \frac{g_m r_o^2}{4}$$

$$\boxed{A_{cm} = \frac{-2 g_m^2 r_o^2}{4\left(1+g_m \frac{r_o}{2}\right)} = \frac{-g_m^2 r_o^2}{2\left(1+g_m \frac{r_o}{2}\right)} \approx \frac{-g_m^2 r_o^2}{2\left(g_m \frac{r_o}{2}\right)} \approx -g_m r_o = -100}$$

e. $$\boxed{CMRR = \frac{|A_d|}{|A_{cm}|} \approx \frac{g_m r_o}{g_m r_o} \approx 1}$$

CMRR can be improved by increasing the tail source output impedances. This can be done by increasing L_7 and L_1 while keeping $\left(\frac{W}{L}\right)_7$ and $\left(\frac{W}{L}\right)_5$ constant.

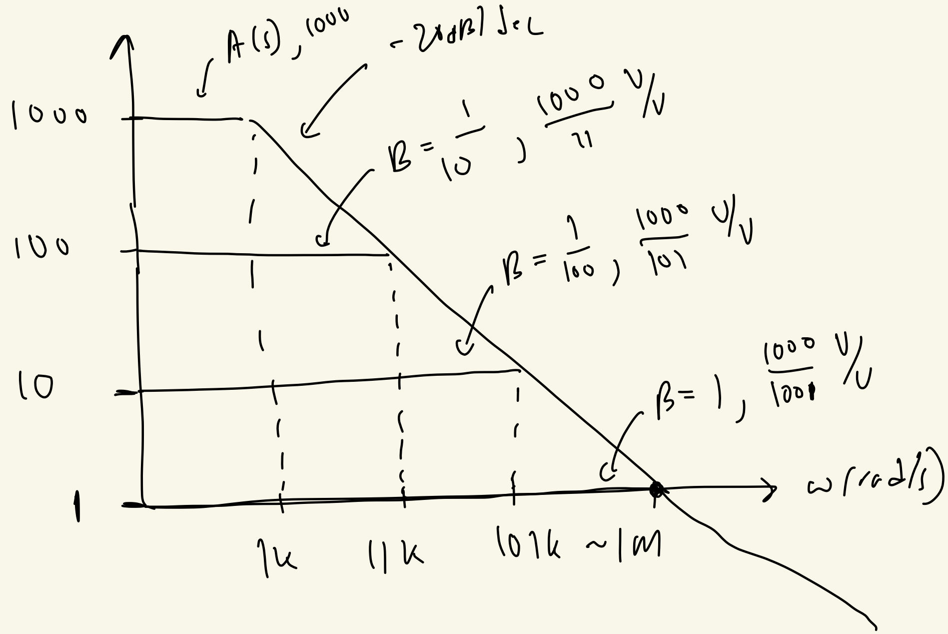
Problem 2

a. $$A_f(s) = \frac{A(s)}{1+BA(s)} = \frac{\frac{A_0}{1+s/\omega_p}}{1+\frac{BA_0}{1+s/\omega_p}}$$

$$= \frac{A_0}{1+s/\omega_p + BA_0} \cdot \frac{1/(BA_0+1)}{1/(BA_0+1)} = \boxed{\frac{A_0}{1+BA_0} \cdot \frac{1}{1+\frac{s}{\omega_p(1+A_0B)}}}$$

1 pole @ $\boxed{\omega_{pf} = \omega_p(1+A_0B)}$

b.



$\omega_p = 1k$

C. $V_e = V_i - B V_o$

$V_o = A_o V_e$

$V_e = V_i - B A_o V_e$

$$V_e = \frac{V_i}{1 + B A_o} \rightarrow \frac{V_e}{V_i} = \frac{1}{1 + A_o B}$$

$B = 1$

$$0.01 = \frac{1}{1 + A_o}$$

$$1 + A_o = 100 \rightarrow \boxed{A \geq 99}$$