Math 1551 Version A **Name (Print Clearly):** _____
Spring 2024
Exam 3 **GT ID Number:** _____
Time Limit: 75 Minutes

By signing here, you agree to abide by the **Georgia Tech Honor Code**:
I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.

Sign Your Name: _____

This exam contains 10 problems, some multiple choice and some free response, for a total of 50 points. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You may not receive any assistance with the exam or communicate with others (other than your professor) about the problems until it has been graded and returned to you on Gradescope.

Please read the following carefully:

- **This exam is double-sided.**
- **Do not cut apart, tear apart, or in any other way remove any pages from this exam packet.**
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit. Be sure to use correct notation throughout. ***Do NOT*** *forget the units in your answer, if applicable.*
- **You are required to show your work for free response problems.** Please circle or box in your final free response answer. **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- For multiple choice questions, **carefully fill in the bubble** corresponding to your final answer. Do not select multiple answers. Do not use X's, circles, slashes, or any other marks in/around the choice bubbles to indicate that you do or do not wish to choose a particular answer. **No partial credit will be given for these problems.**
- If you need extra space, you may use the back side of this cover page.

Version A Name: _____

SCRATCH WORK

If you want work on this page to be graded, you **must** make a note of this on the corresponding problem and clearly label your work.

Version A Name: _____

1. (2.5 points) Let $f(x) = \frac{1}{3}x^3 + 2x^2$. We want to maximize $f(x)$ on the interval $[-8, -2]$. At which of the following values of x does the closed interval method require us to evaluate $f(x)$?

   ◯ $x = 0$ only

   ◯ $x = -4$ only

   ◯ $x = -8$, $x = -4$, and $x = -2$ only

   ◯ $x = -8$, $x = -4$, $x = -2$, and $x = 0$.

2. For each of the following, fill in the bubble to indicate which of the options in the derivative bank is the correct derivative of the function. Each option will be used at most once.

Derivative Bank:	
A. $\frac{-1}{x(x+1)}$	C. $\frac{1}{x(x+1)}$
B. $-6\cot^2(2x)\csc^2(2x)$	D. $3\cot^2(2x)\sec^2(2x)$

   (a) (2 points) $\frac{d}{dx}\cot^3(2x)$ ◯ A. ◯ B. ◯ C. ◯ D.

   (b) (2 points) $\frac{d}{dx}\ln(\frac{2x}{x+1})$ ◯ A. ◯ B. ◯ C. ◯ D.

3. If a ball is given a push so that it has an initial velocity of 5 meters per second down a certain inclined plane, then the distance it has rolled after t seconds is $s(t) = 5t + 3t^2$.

   (a) (1.5 points) Find the velocity after 2 seconds. ***Don't*** *forget the units in your answer.*

   (b) (1.5 points) Find the acceleration after 2 seconds. ***Don't*** *forget the units in your answer.*

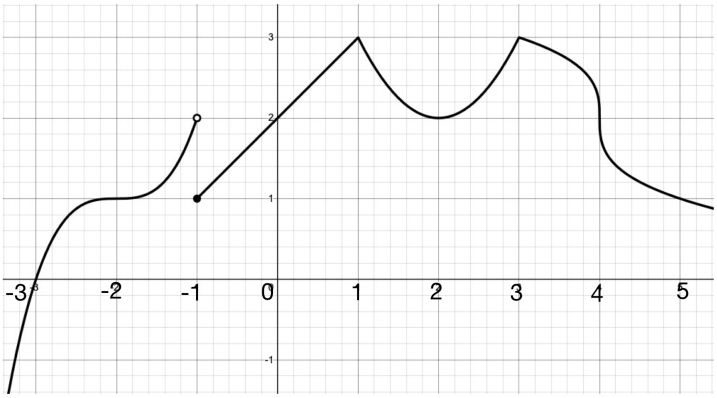
   (c) (1.5 point) How long does it take for the velocity to reach 35 meters per second? ***Don't*** *forget the units in your answer.*

4. Find the derivative of the following functions. You can use any of the derivative rules introduced in class, but be sure to show your work. **DO NOT** simplify your answers.

   (a) (2.5 points) $(2x)^{\cos(x)}$

   (b) (2.5 points) $\arcsin\left(\frac{3}{x^2}\right)$

5. Use the graph below to answer the following questions.



(a) (2 points) At which one of the following intervals is $f'(x) < 0$?

◯ $(-2, -1)$ ◯ $(1, 2)$ ◯ $(-1, 0)$ ◯ $(2, 3)$

(b) (2 points) At which one of the following intervals is $f''(x) > 0$?

◯ $(-3, -2)$ ◯ $(4, 5)$ ◯ $(0, 1)$ ◯ $(3, 4)$

6. (2.5 points) The folium of Descartes curve is given by

$$x^3 + y^3 - 3xy = 0$$

Which of the following is true?

◯ The folium has a horizontal tangent line at the point $(\sqrt[3]{2}, \sqrt[3]{4})$

◯ The folium has a vertical tangent line at the point $(\sqrt[3]{2}, \sqrt[3]{4})$

◯ There is no point at which the folium has a horizontal tangent line.

◯ There is no point at which the folium has a vertical tangent line.

7. Consider the curve $2x^2 - xy = \sin(y)$.

a) (2.5 points) Find $\dfrac{dy}{dx}$. Show your work. Your answer may be expressed in terms of both x and y.

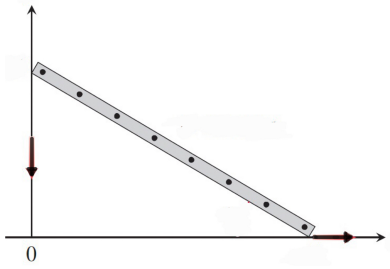
b) (2.5 points) Find the equation of the line tangent to the curve at the point $(0, \pi)$. Show your work.

8. (4 points) Consider

$$f(x) = \frac{1}{5}x^5 - x^4 + 1$$

Find all the value(s) of x at which the graph of $f(x)$ has an inflection point. Justify your answer.

9. A 5-ft ladder is leaning against a house when its base starts to slide away (see accompanying figure). By the time the base is 3 ft from the house, the base is moving at the rate of $\frac{1}{2}$ ft/sec.



(a) (2 points) What is the equation that relates x and y, where x represents the distance from the house to the base of the ladder, and y represents the distance from the ground to the top of the ladder?

- ◯ $x^2 + y^2 = 3$
- ◯ $x^2 + y^2 = 9$
- ◯ $x^2 + y^2 = 5$
- ◯ $x^2 + y^2 = 25$

(b) (4.5 points) How fast is the top of the ladder sliding down the wall at the moment when the base is 3 ft from the house? Show your work. ***Don't*** *forget the units in your answer.*

(c) (2 points) What is the equation that relates x and θ, where x represents the distance from the house to the base of the ladder, and θ represents the angle between the ladder and the ground?

- ◯ $\cos(\theta) = \frac{x}{5}$
- ◯ $\sin(\theta) = \frac{x}{5}$
- ◯ $\tan(\theta) = \frac{x}{5}$
- ◯ $\cos(\theta) = \frac{x}{25}$

10. We want to create a rectangular dog park. The top side must use a wood fence, but the other three sides can use chain-link fence. The wood fence costs \$30 per foot, and the chain-link fence costs \$10 per foot. We will spend \$1200, and we want to maximize the area of the dog park.

a) (1 point) State the goal of this problem in a few words.

b) (2 points) Give an equation that ***solely*** relates x and y, where x represents the length of the top and bottom sides of the park, and y represents the length of the left and right sides of the park. No need to simplify your answer.

c) (2 points) Which of the following functions $f(x)$ (i.e., a function ***just*** in terms of the variable x) do we want to maximize or minimize in order to solve this problem?

- ◯ $f(x) = x(60 - 2x)$
- ◯ $f(x) = x(60 - 2y)$
- ◯ $f(x) = x(60 + 2x)$
- ◯ $f(x) = x(60 + 2y)$

d) (1 point) Which of the following is the domain of $f(x)$?

- ◯ $(0, \infty)$
- ◯ $(0, 15)$
- ◯ $(0, 30)$
- ◯ $(10, 60)$

e) (4 points) Find the value of x that will lead to the extremum you are trying to find in this problem. Be sure to show your work and justify that your final answer is a global extremum. ***Don't*** *forget the units in your answer.*