



## **GRASP Seminar Series**

**Wu & Chen**

**Fridays 11am – 12pm**

Sep 14	GRASP Research Overview
Sep 21	Raquel Urtasun (Uber)
Sep 28	Deepak Pathak (UC Berkeley)
Oct 5	Lex Fridman (MIT)
Oct 12	Naomi Leonard (Princeton)
Oct 19	Yaser Sheikh (CMU/Oculus-Facebook)
Oct 26	Serge Belongie (Cornell)
Nov 2	Mark Cutkosky (Stanford)
Nov 9	Kevin Lynch (Northwestern)
Nov 16	Elliot Hawkes (UC Santa Barbara)
Nov 30	Kristen Graumen (UT Austin)

**Tentative schedule.  
Check GRASP  
website for updates.**

# **MEAM 520**

## **Lecture 6: DH Parameters**

Cynthia Sung, Ph.D.

Mechanical Engineering & Applied Mechanics

University of Pennsylvania

# Last Time

The **Denavit-Hartenberg transform** results from successive rotations and translations via the four DH parameters

*a parameterization for homogeneous transformations*

The transform from i to i-1 is

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Three DH parameters will be **constant** for each joint's transformation, and one will **vary**.

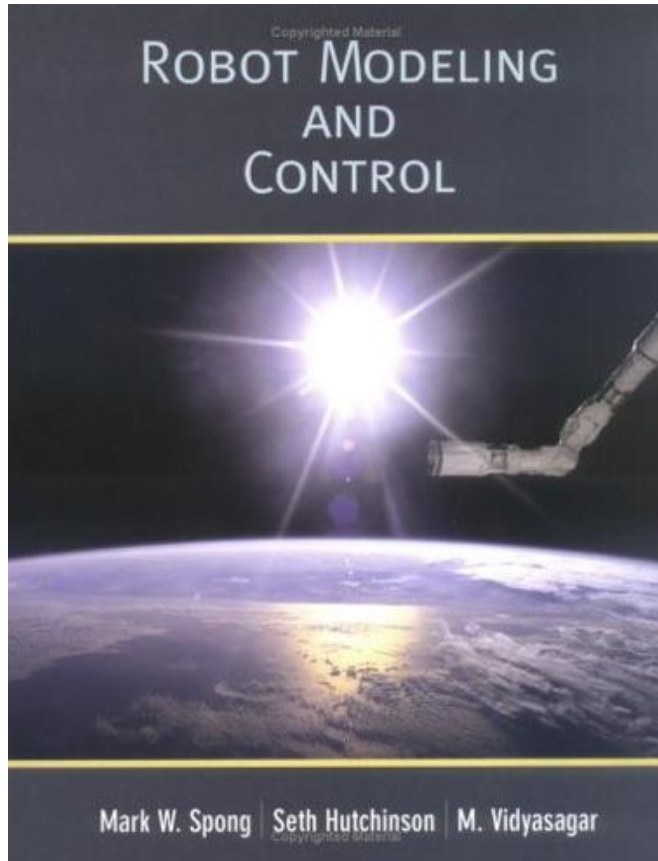
Plug DH parameters into the above formula to find each joint's transformation matrix.

The final transformation matrix from tip to base is

$$\mathbf{T}_n^0 = A_1(q_1) \cdots A_n(q_n)$$



# Today: More DH Parameters



## Chapter 3: Forward and Inverse Kinematics

- Read Sec. 3.2

### Lab 1: Kinematic Characterization of the Lynx

MEAM 520, University of Pennsylvania

September 5, 2018

This lab consists of two portions, with a pre-lab due on Wednesday, September 12, by midnight (11:59 p.m.) and a lab report due on Wednesday, September 19, by midnight (11:59 p.m.). Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

#### Individual vs. Pair Programming

The pre-lab component of this lab must be completed and submitted individually on Canvas. For the remainder of the lab, you may work either individually or with a partner. If you do this lab with a partner, you may work with anyone you choose, but you must work with them for all parts of this assignment.

If you are in a pair, you will both turn in the same report and code (see Submission Instructions below), for which you are jointly responsible and you will both receive the same grade. Work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Williams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Supplemental Material.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, even if one partner is much more experienced than the other. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

## Lab 1 due 9/19, 11:59 p.m.

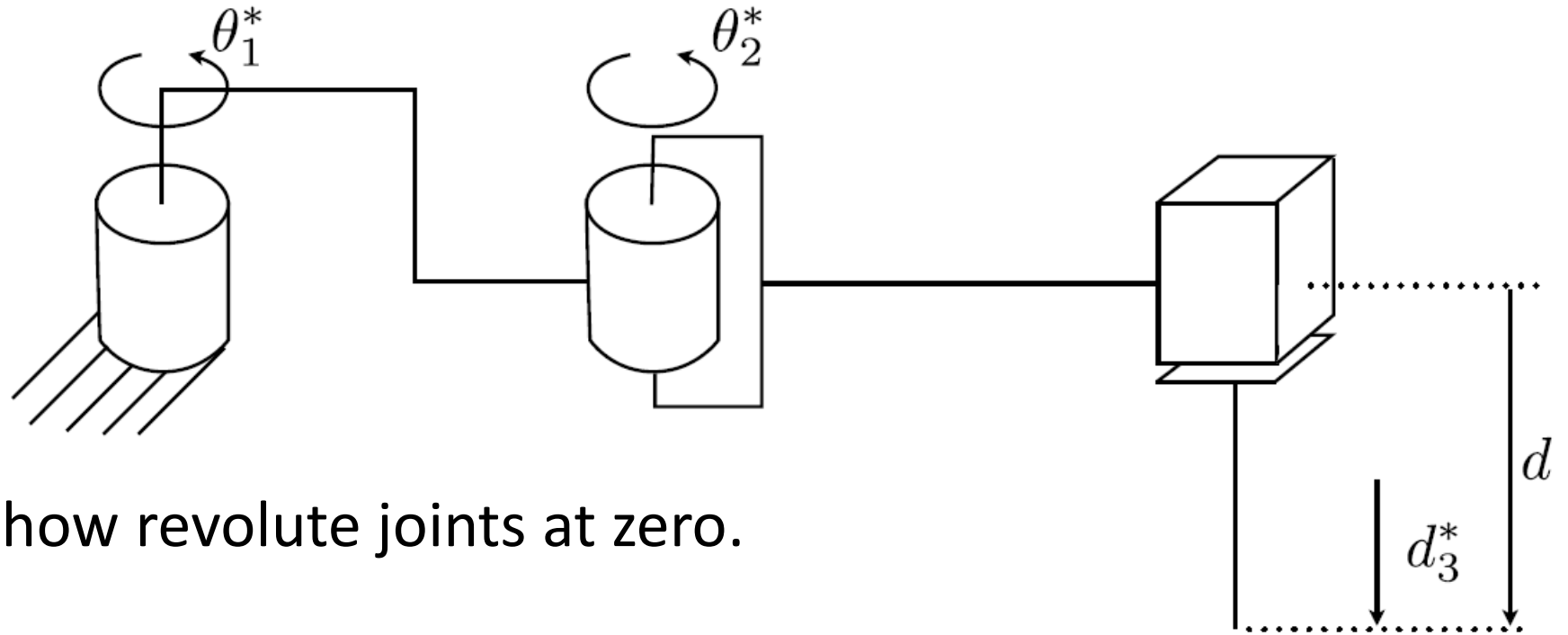
# Today: More DH Parameters

- Do one example
- Check one example
- You do an example



SCARA

Shown at  $\theta_1^* = 0, \theta_2^* = 0$ , and  $d_3^* = d$



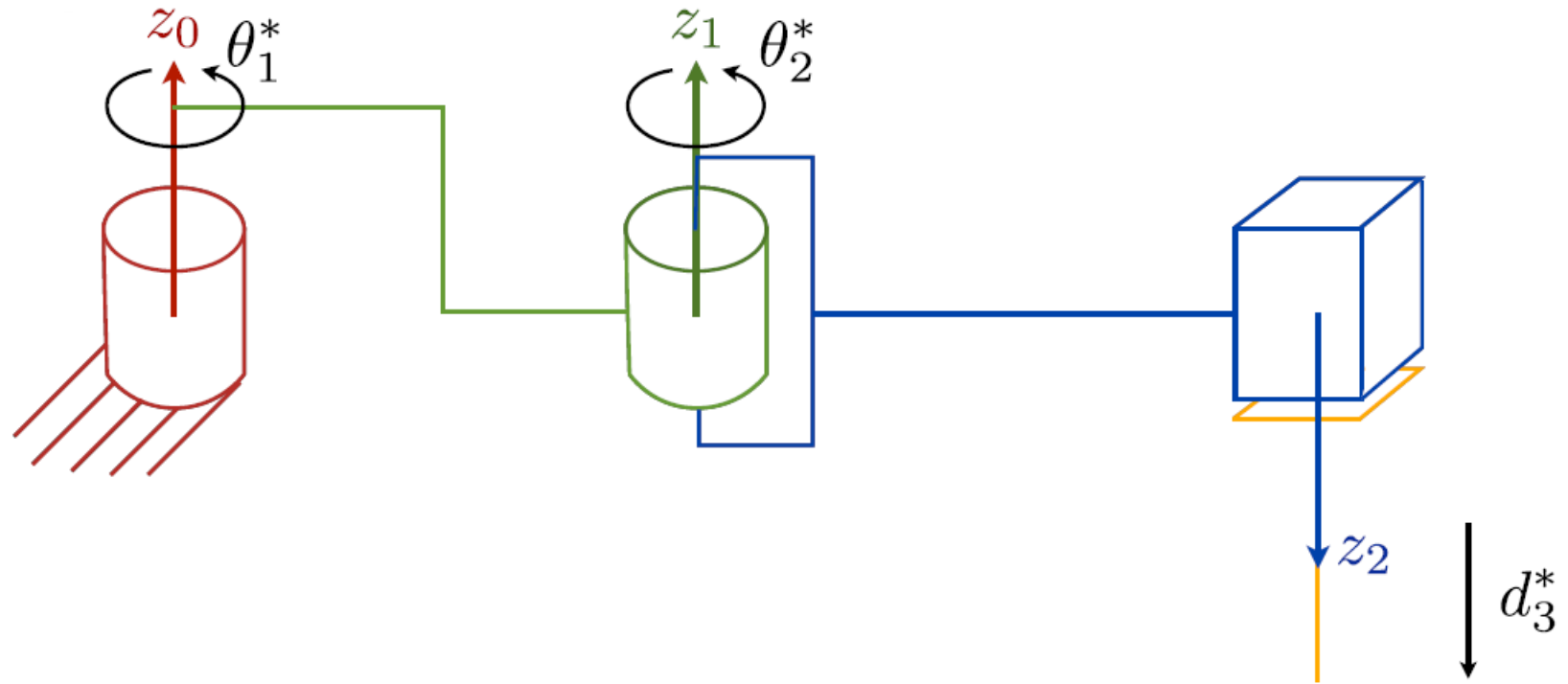
The diagram should show revolute joints at zero.

Prismatic joints are usually shown at a positive displacement instead of zero.



SCARA

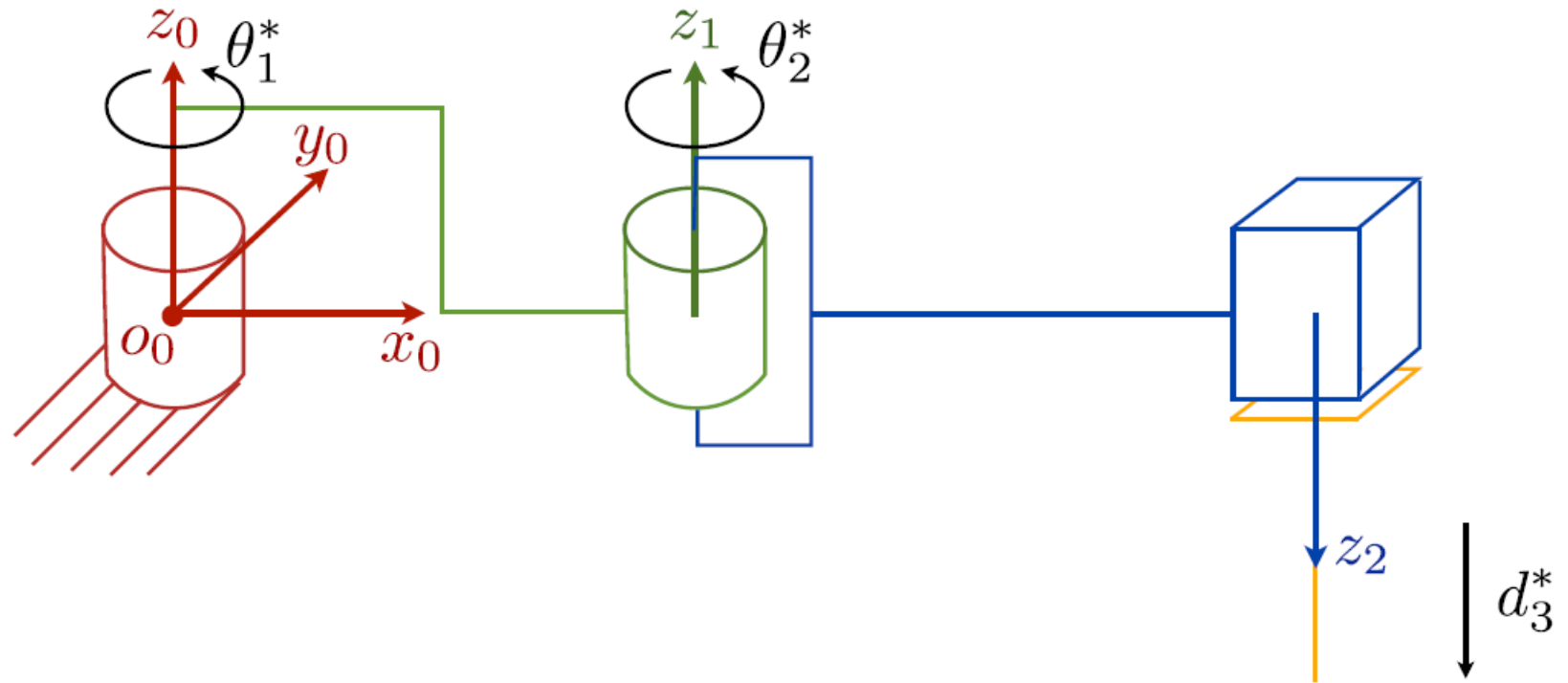
Step 1: Locate and label the joint axes  $z_0, \dots, z_{n-1}$ .





SCARA

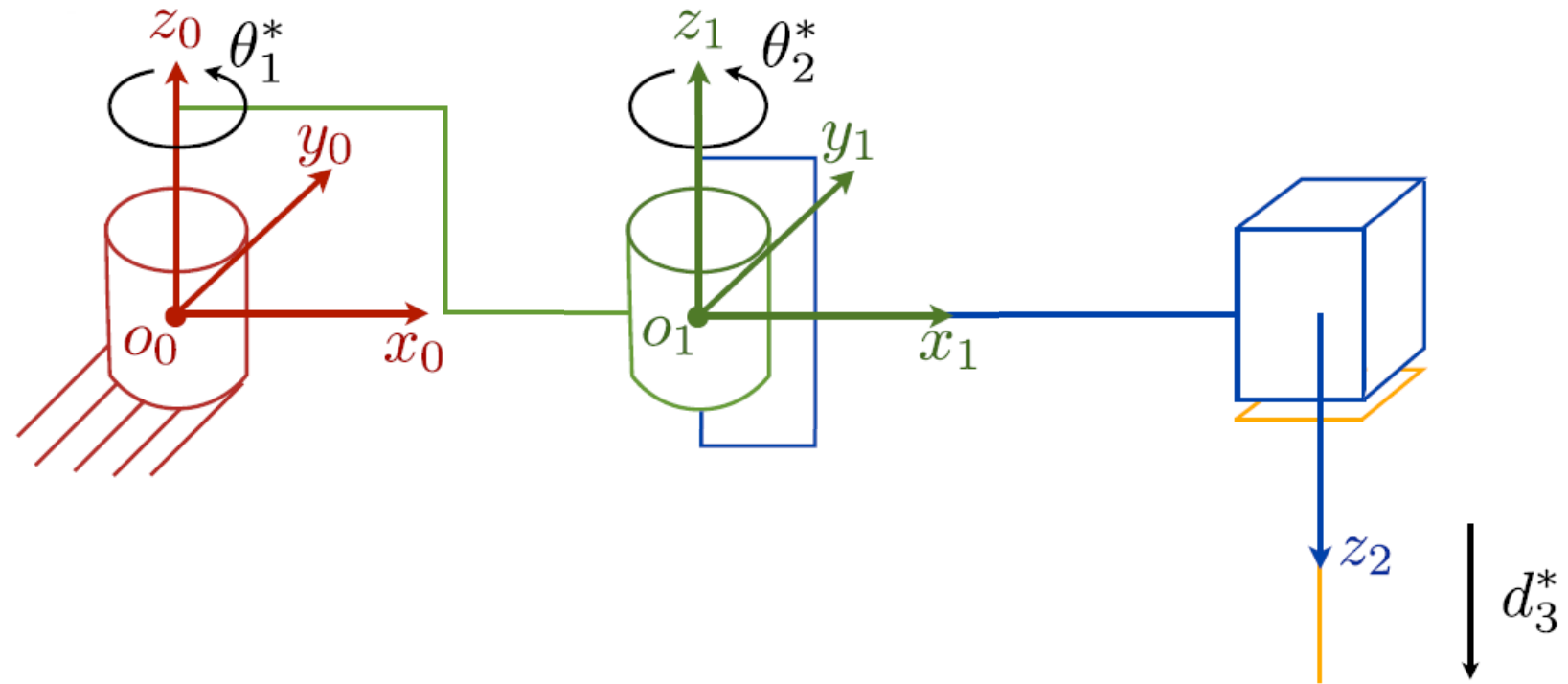
**Step 2:** Establish the base frame. Set the origin anywhere on the  $z_0$ -axis. The  $x_0$  and  $y_0$  axes are chosen conveniently to form a right-handed frame.







SCARA



**For  $i = 1, \dots, n - 1$ ,**

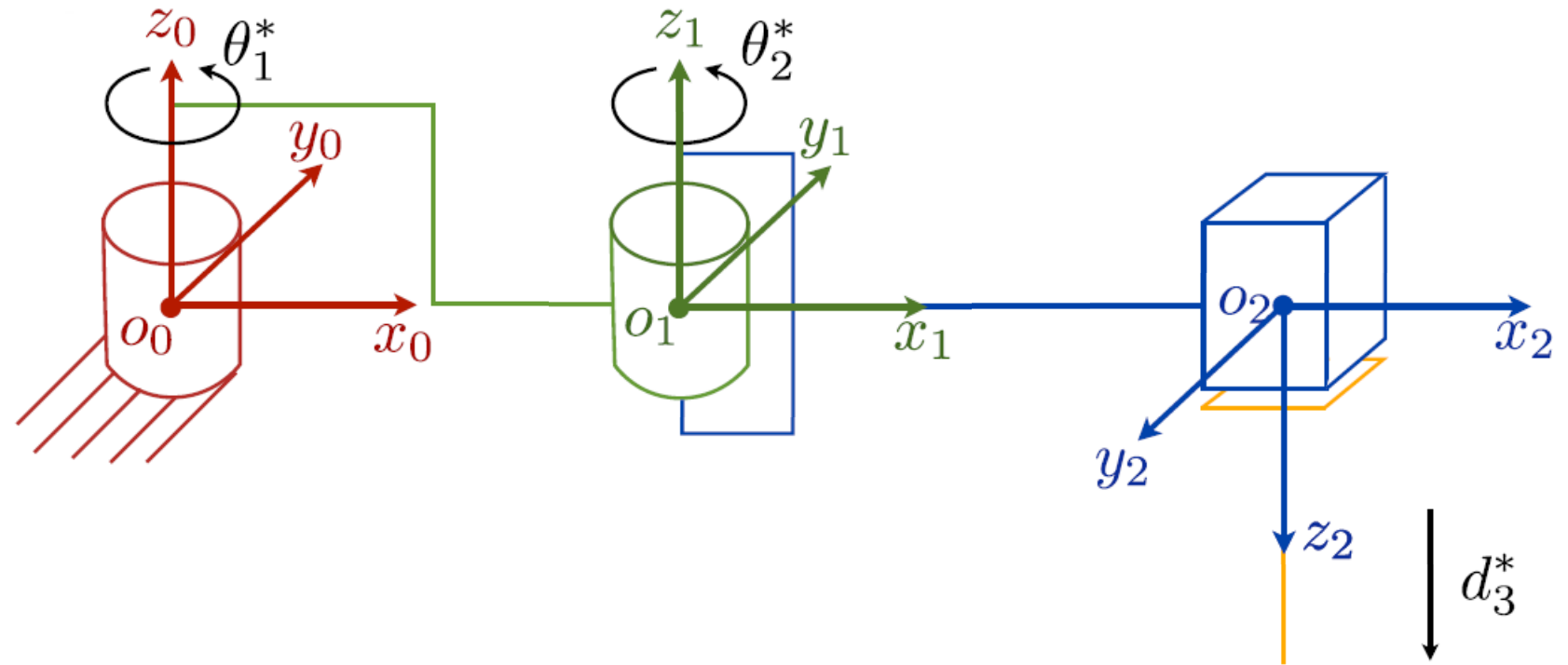
**Step 3:** Locate the origin  $o_i$  where the common normal to  $z_i$  and  $z_{i-1}$  intersects  $z_i$ . If  $z_i$  intersects  $z_{i-1}$  locate  $o_i$  at this intersection. If  $z_i$  and  $z_{i-1}$  are parallel, locate  $o_i$  in any convenient position along  $z_i$ .

**Step 4:** Establish  $x_i$  along the common normal between  $z_{i-1}$  and  $z_i$  through  $o_i$ , or in the direction normal to the  $z_{i-1} - z_i$  plane if  $z_{i-1}$  and  $z_i$  intersect.

**Step 5:** Establish  $y_i$  to complete a right-handed frame.



SCARA



**For  $i = 1, \dots, n - 1$ ,**

**Step 3:** Locate the origin  $o_i$  where the common normal to  $z_i$  and  $z_{i-1}$  intersects  $z_i$ . If  $z_i$  intersects  $z_{i-1}$  locate  $o_i$  at this intersection. If  $z_i$  and  $z_{i-1}$  are parallel, locate  $o_i$  in any convenient position along  $z_i$ .

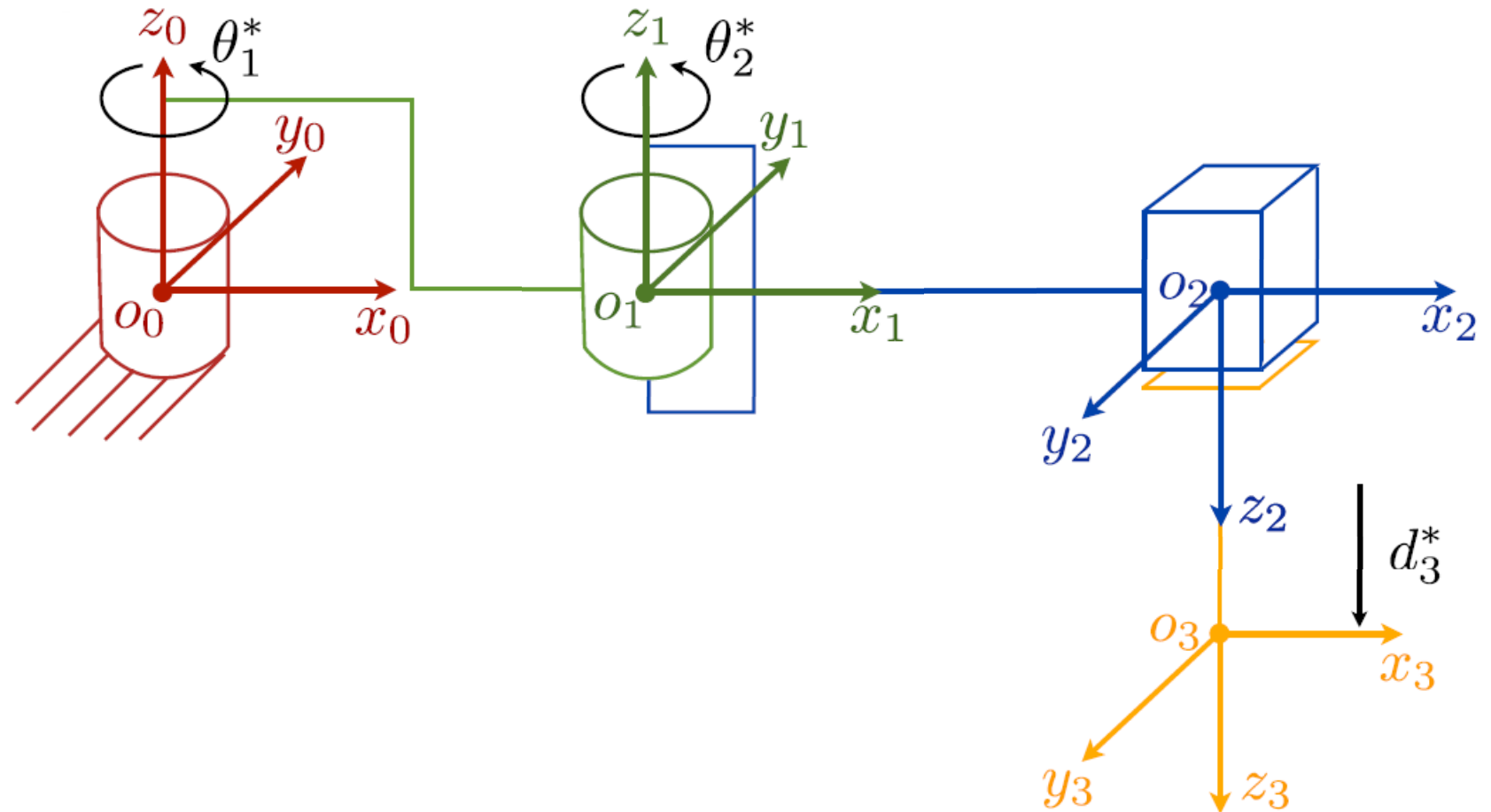
**Step 4:** Establish  $x_i$  along the common normal between  $z_{i-1}$  and  $z_i$  through  $o_i$ , or in the direction normal to the  $z_{i-1} - z_i$  plane if  $z_{i-1}$  and  $z_i$  intersect.

**Step 5:** Establish  $y_i$  to complete a right-handed frame.



SCARA

**Step 6:** Establish the end-effector frame  $o_n x_n y_n z_n$ . Assuming the  $n$ -th joint is revolute, set  $z_n = a$  along the direction  $z_{n-1}$ . Establish the origin  $o_n$  conveniently along  $z_n$ , preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set  $y_n = s$  in the direction of the gripper closure and set  $x_n = n$  as  $s \times a$ . If the tool is not a simple gripper set  $x_n$  and  $y_n$  conveniently to form a right-handed frame.





SCARA

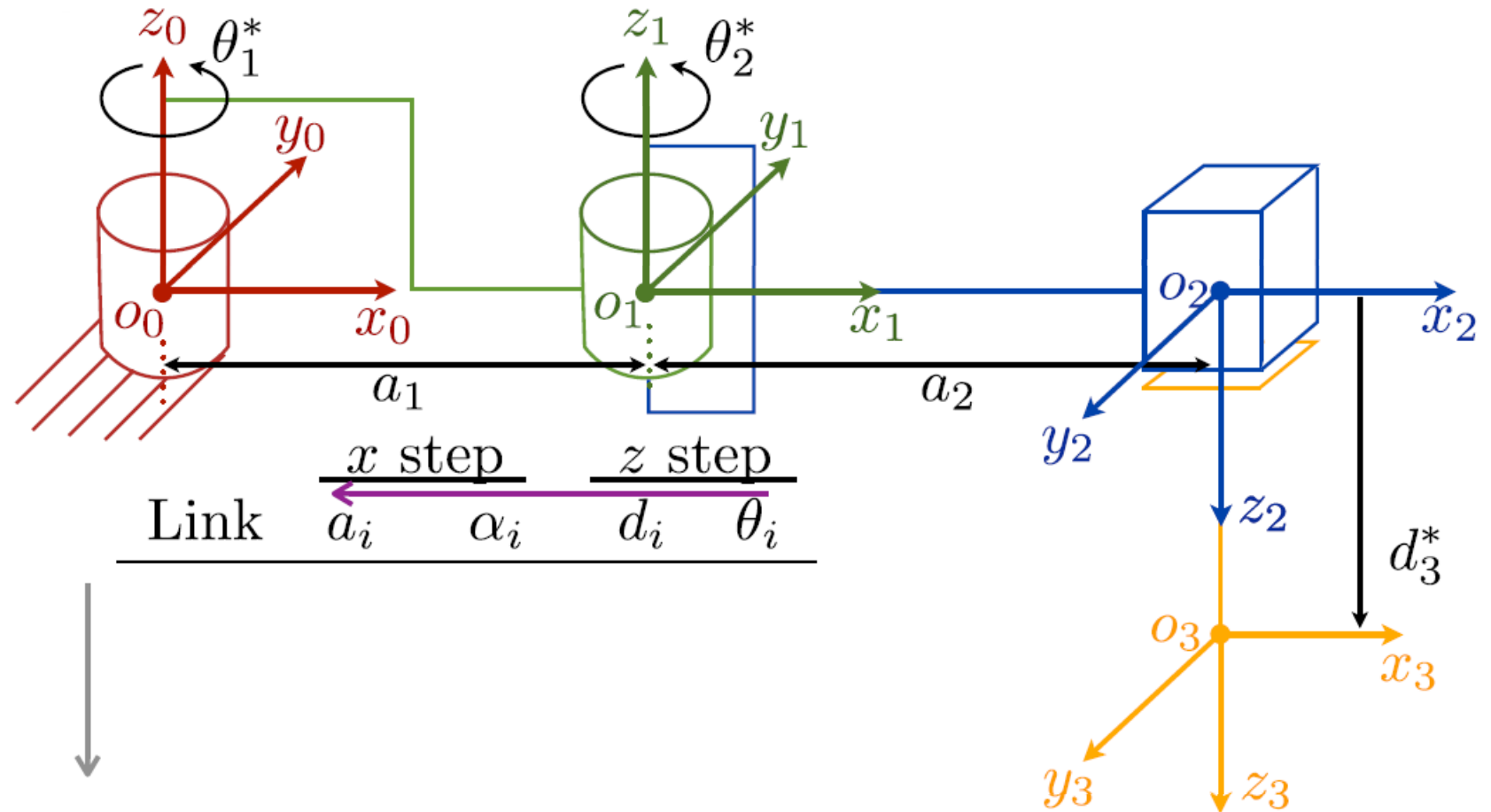
**Step 7:** Create a table of link parameters  $a_i$ ,  $d_i$ ,  $\alpha_i$ ,  $\theta_i$ .

$a_i$  = distance along  $x_i$  from the intersection of the  $x_i$  and  $z_{i-1}$  axes to  $o_i$ .

$d_i$  = distance along  $z_{i-1}$  from  $o_{i-1}$  to the intersection of the  $x_i$  and  $z_{i-1}$  axes.  $d_i$  is variable if joint  $i$  is prismatic.

$\alpha_i$  = the angle between  $z_{i-1}$  and  $z_i$  measured about  $x_i$ .

$\theta_i$  = the angle between  $x_{i-1}$  and  $x_i$  measured about  $z_{i-1}$ .  $\theta_i$  is variable if joint  $i$  is revolute.





SCARA

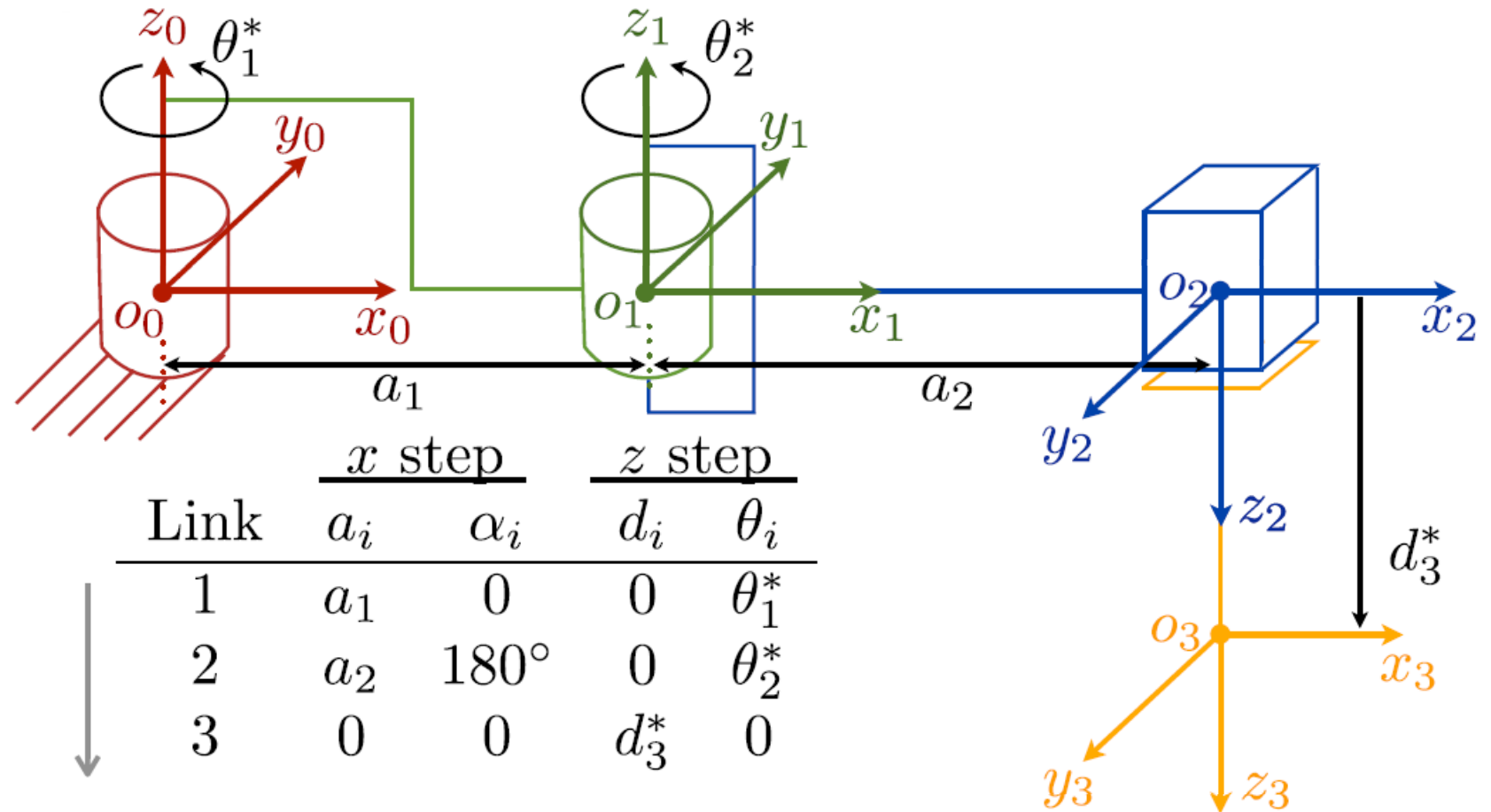
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$a_i$  = distance along  $x_i$  from the intersection of the  $x_i$  and  $z_{i-1}$  axes to  $o_i$ .

$d_i$  = distance along  $z_{i-1}$  from  $o_{i-1}$  to the intersection of the  $x_i$  and  $z_{i-1}$  axes.  $d_i$  is variable if joint  $i$  is prismatic.

$\alpha_i$  = the angle between  $z_{i-1}$  and  $z_i$  measured about  $x_i$ .

$\theta_i$  = the angle between  $x_{i-1}$  and  $x_i$  measured about  $z_{i-1}$ .  $\theta_i$  is variable if joint  $i$  is revolute.





SCARA

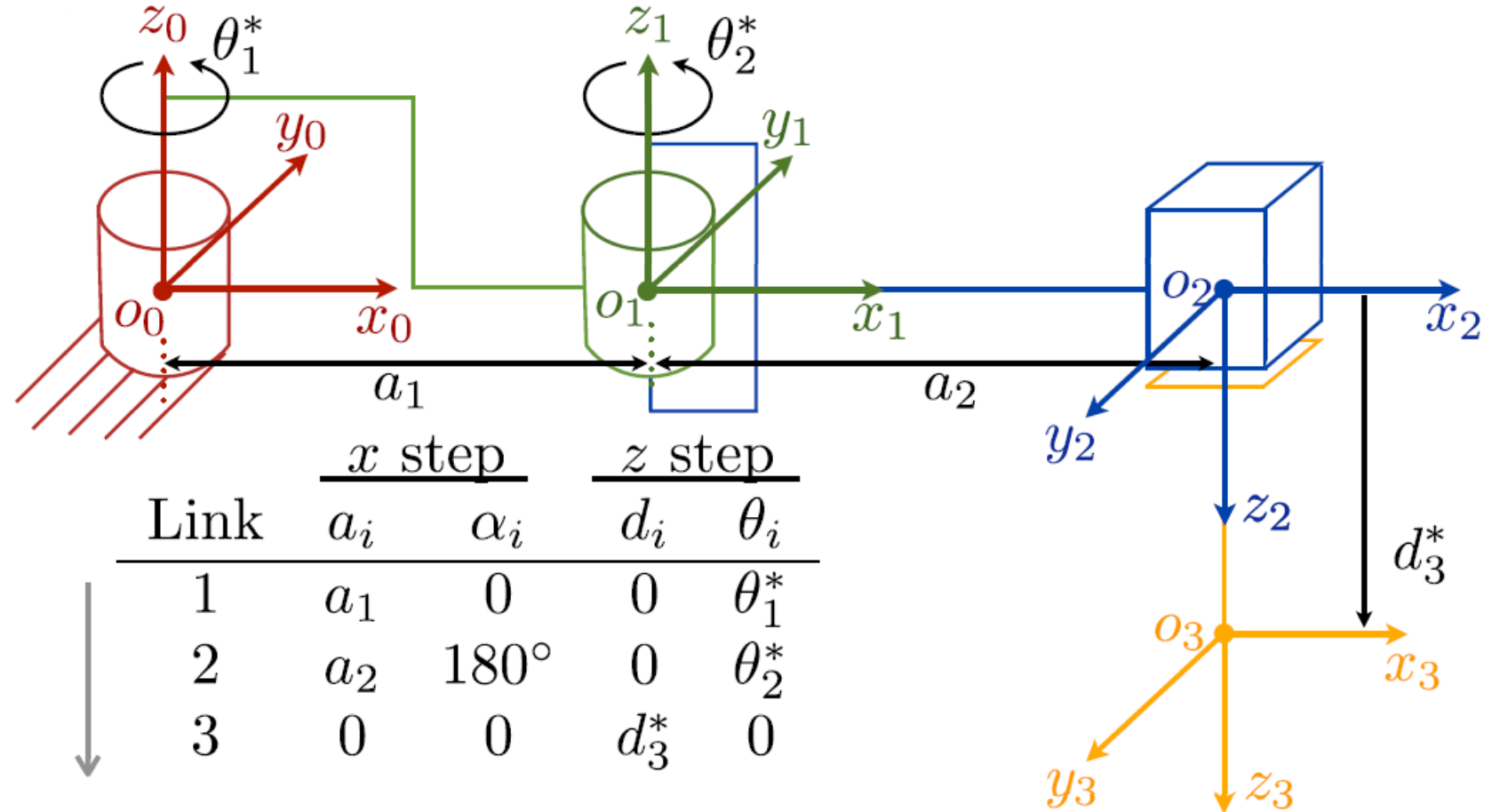
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Step 8:** Form the homogeneous transformation matrices  $A_i$  by substituting the above parameters into (3.10).

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





SCARA

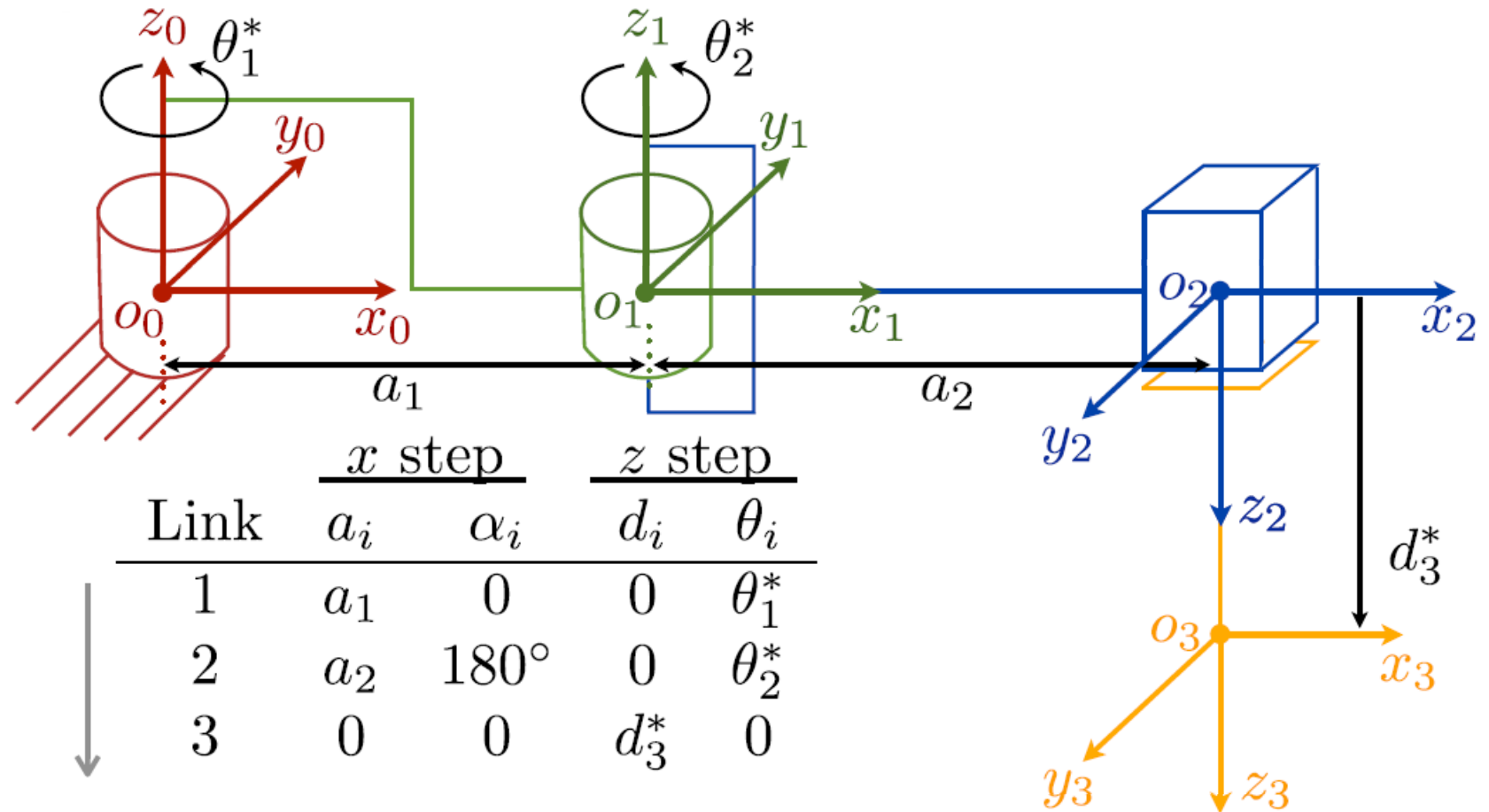
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

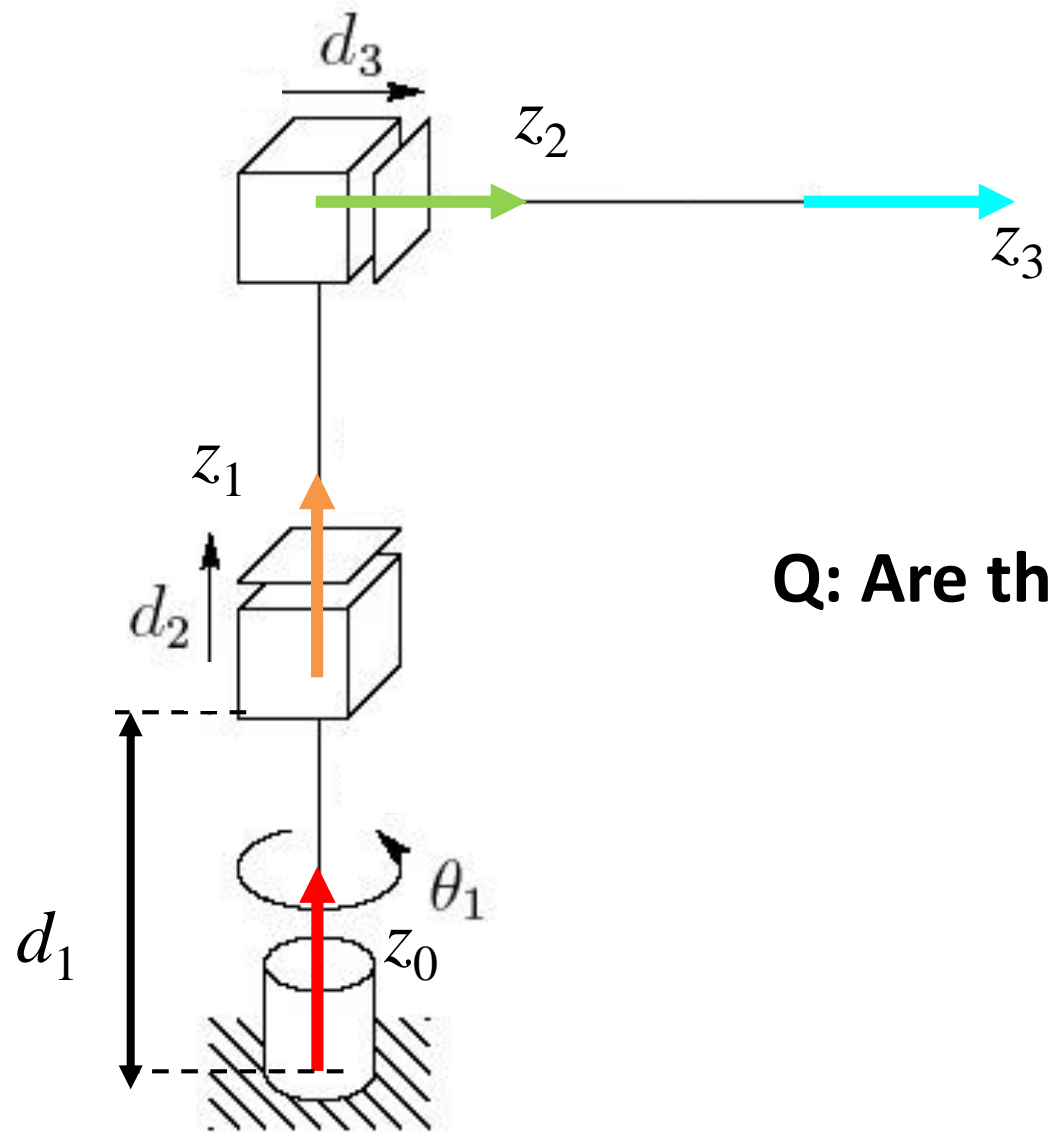
$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Step 9:** Form  $T_n^0 = A_1 \cdots A_n$ . This then gives the position and orientation of the tool frame expressed in base coordinates.

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_{12}^* & s_{12}^* & 0 & a_1 c_1^* + a_2 c_{12}^* \\ s_{12}^* & -c_{12}^* & 0 & a_1 s_1^* + a_2 s_{12}^* \\ 0 & 0 & -1 & -d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



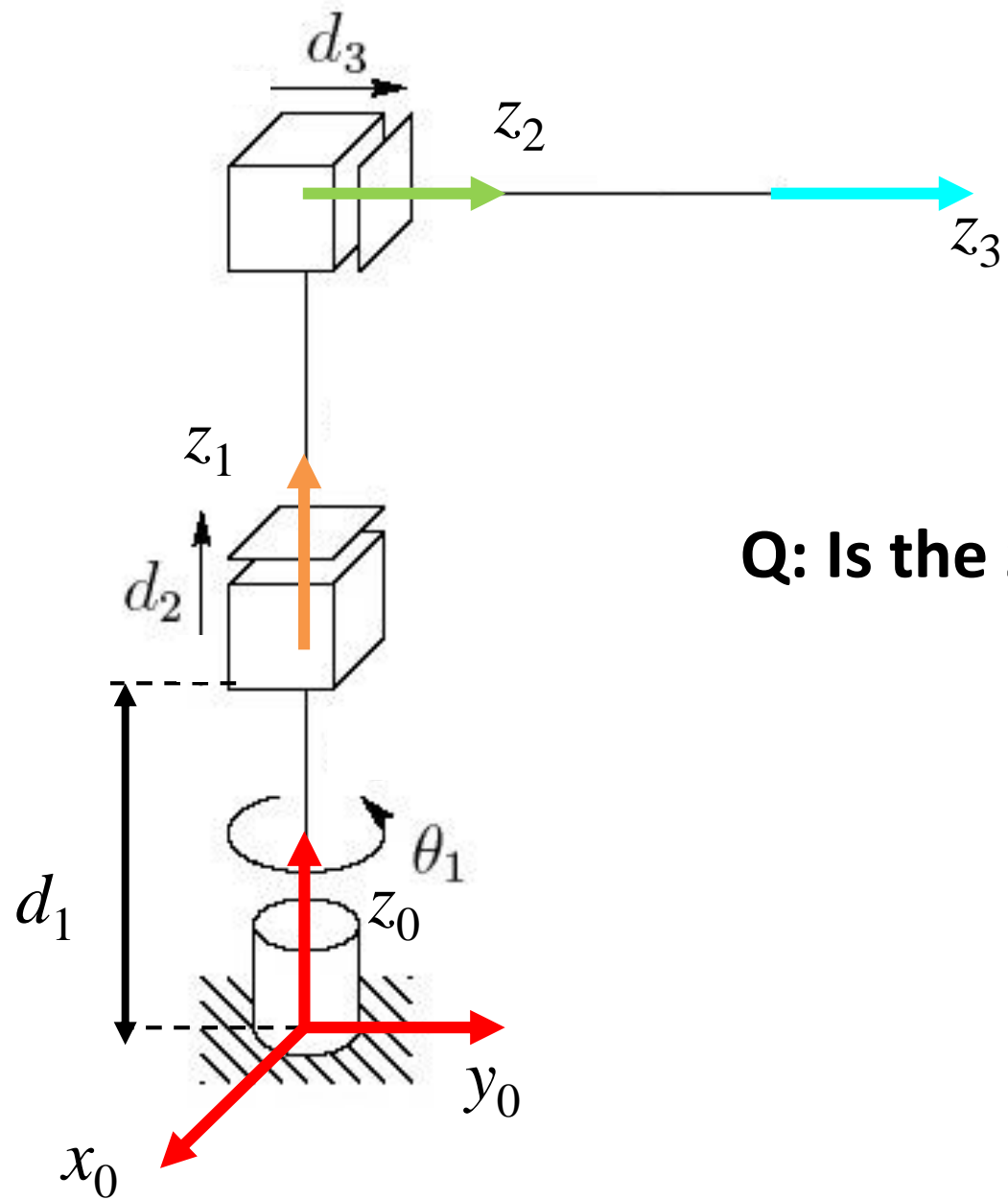


**Q: Are these z-axis locations correct?**



cylindrical

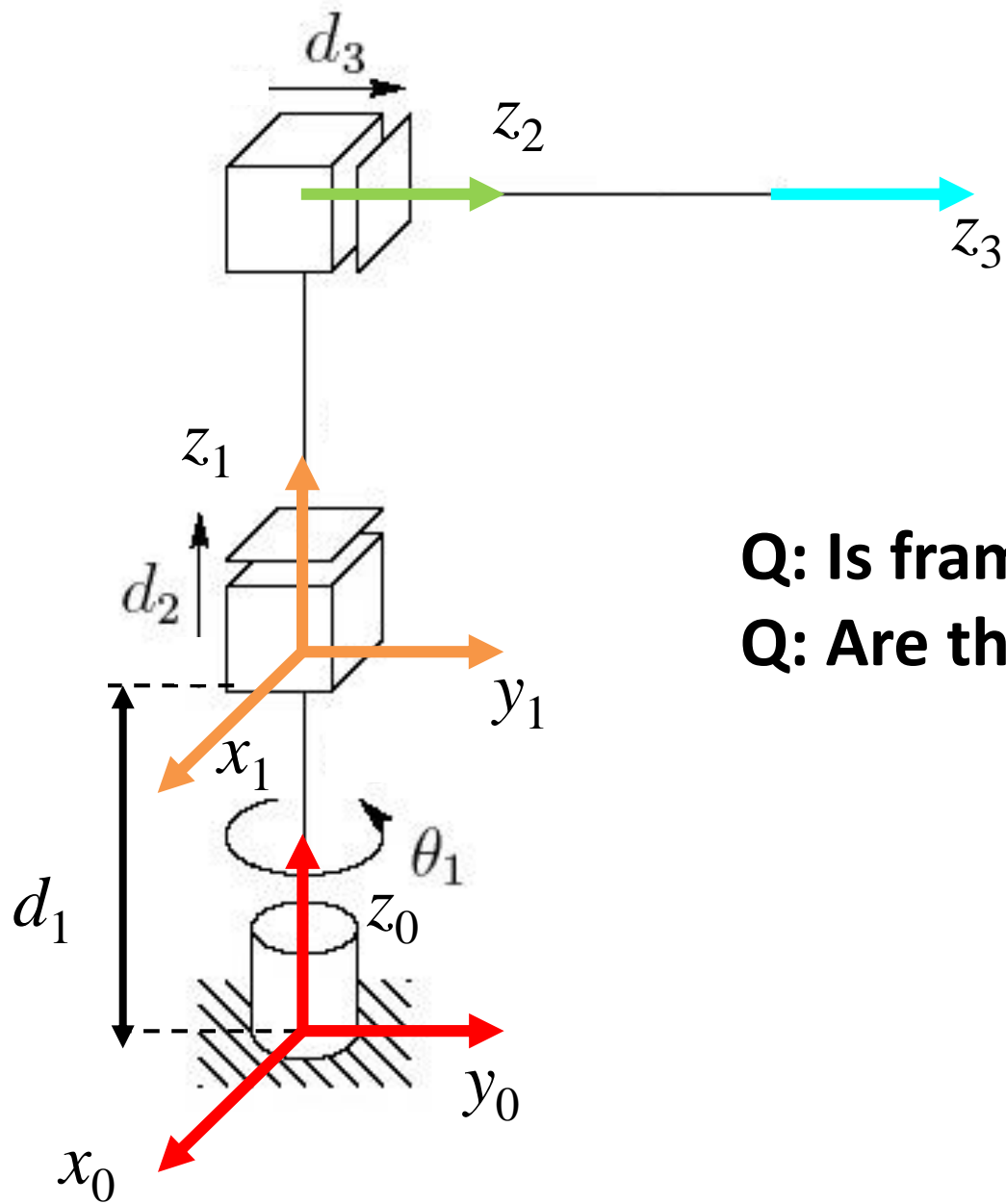




**Q: Is the zero frame correct?**



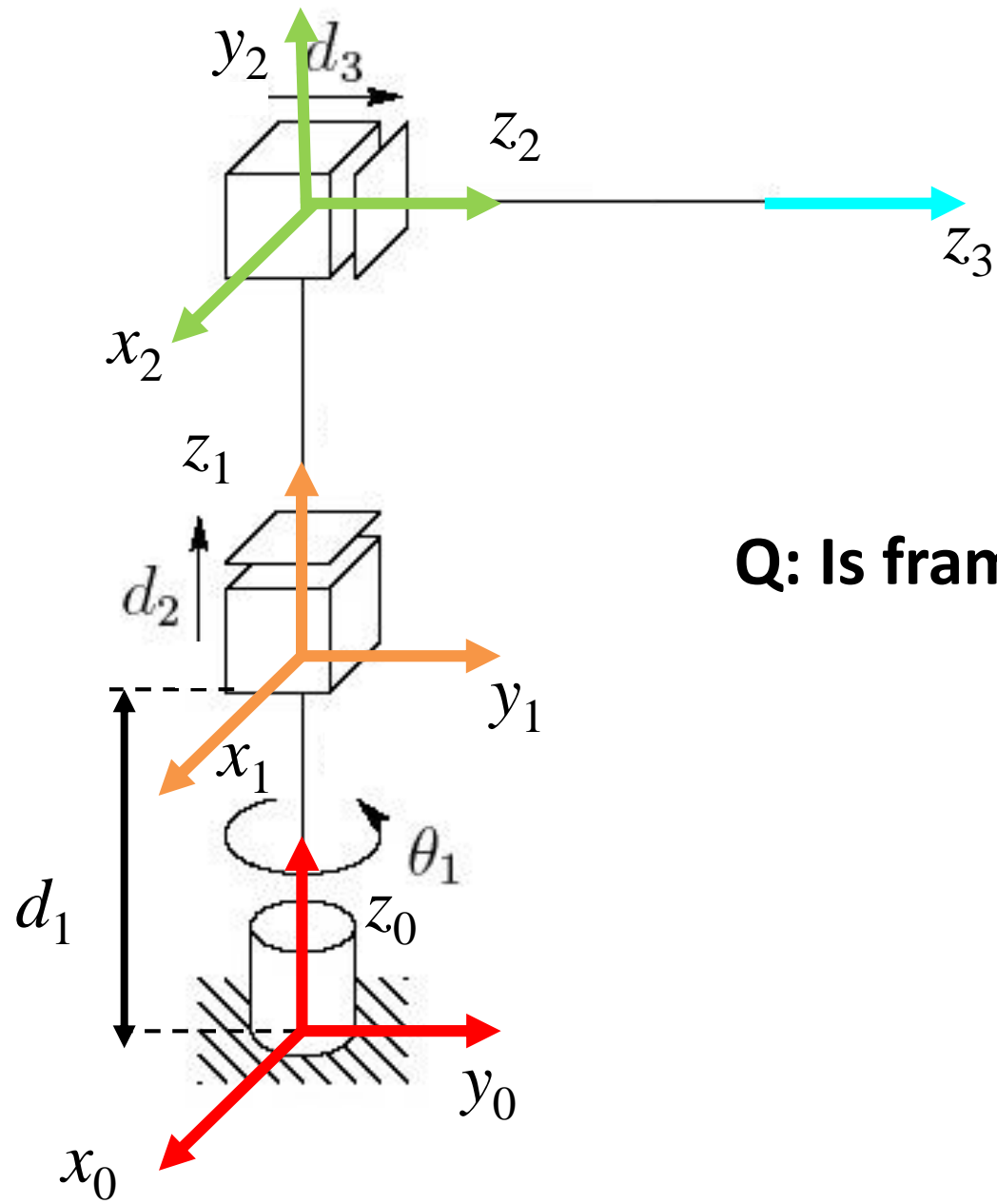
cylindrical



**Q: Is frame 1 correct?**  
**Q: Are there other options?**



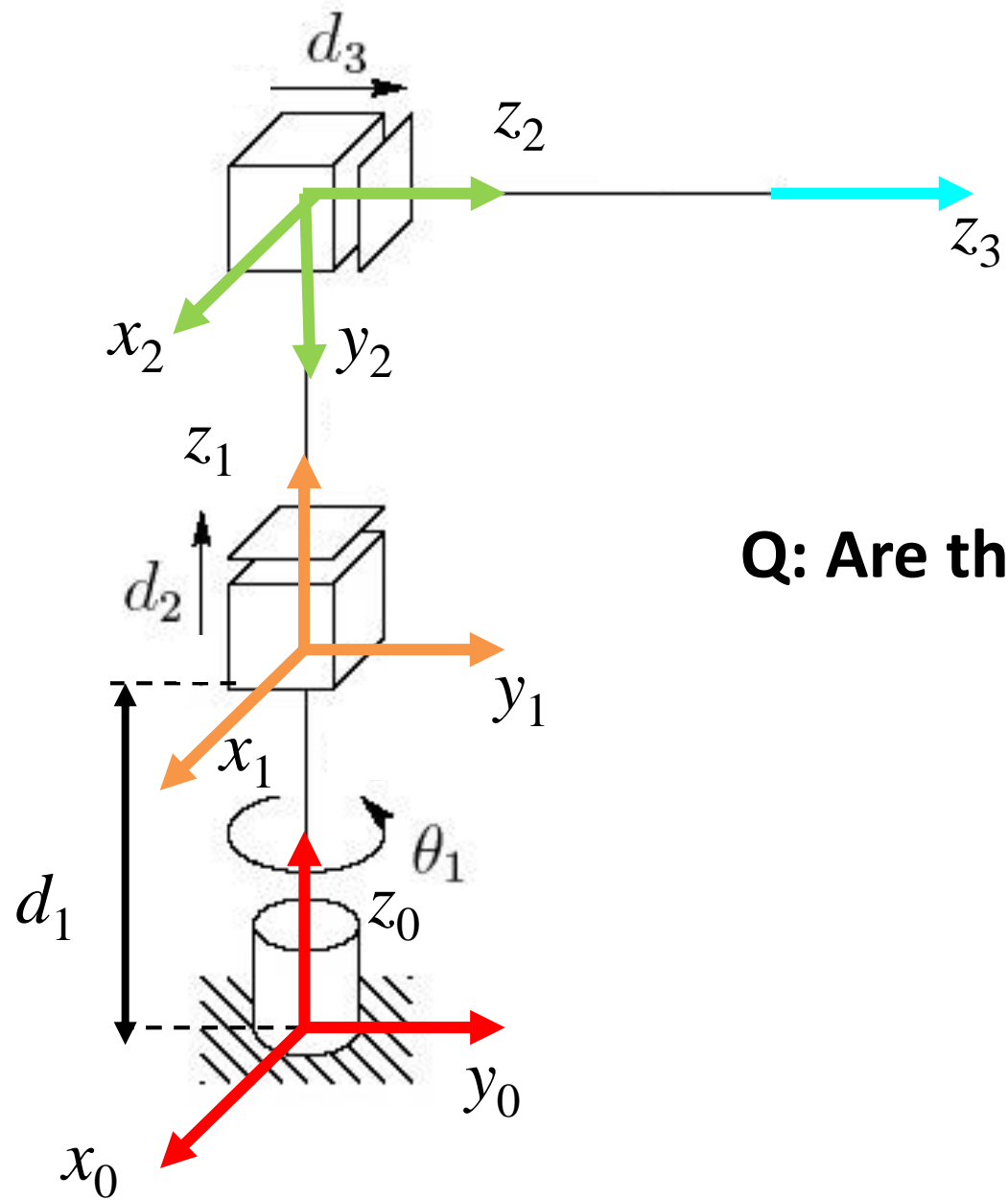
cylindrical



**Q: Is frame 2 correct?**



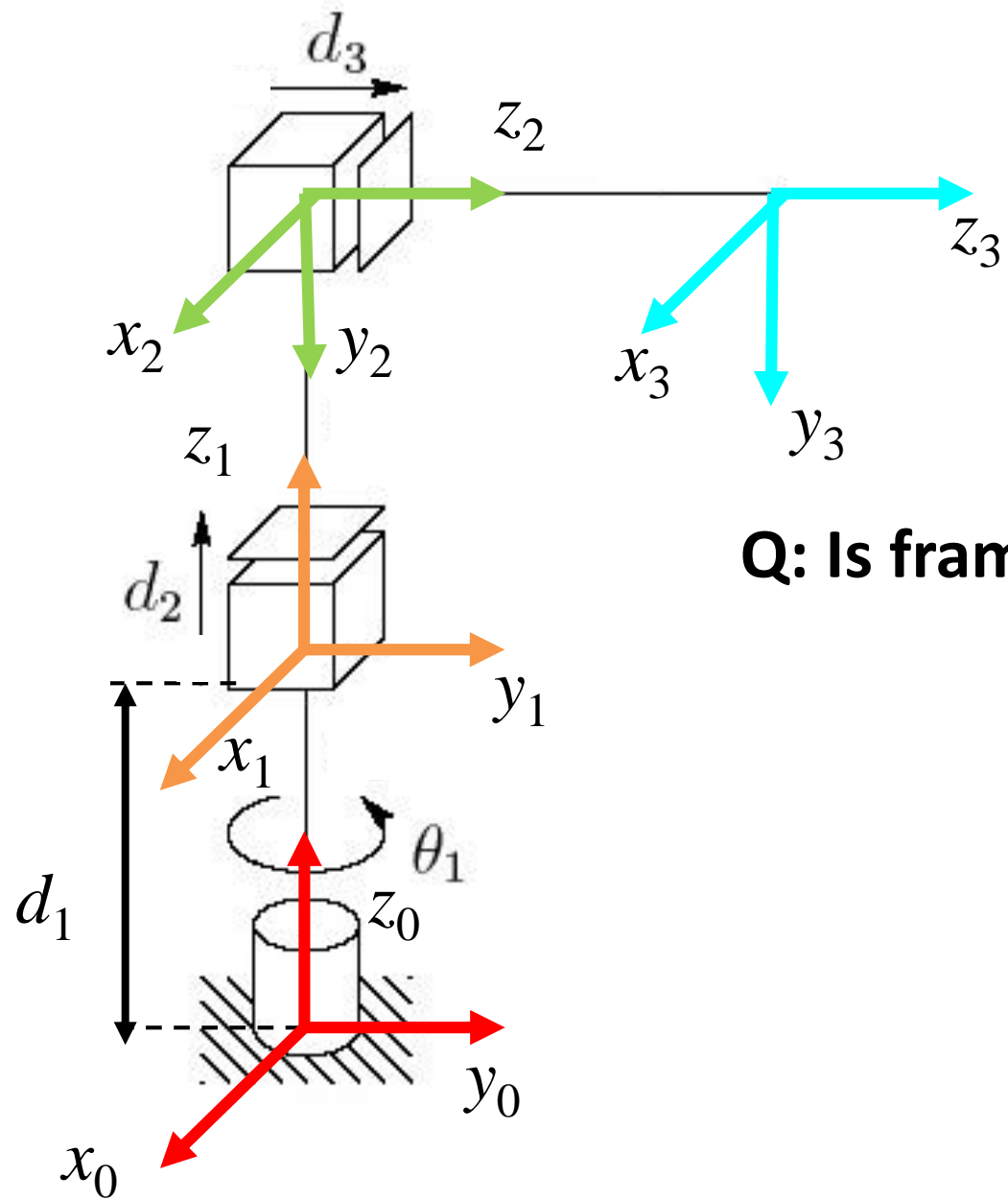
cylindrical



**Q: Are there other options?**



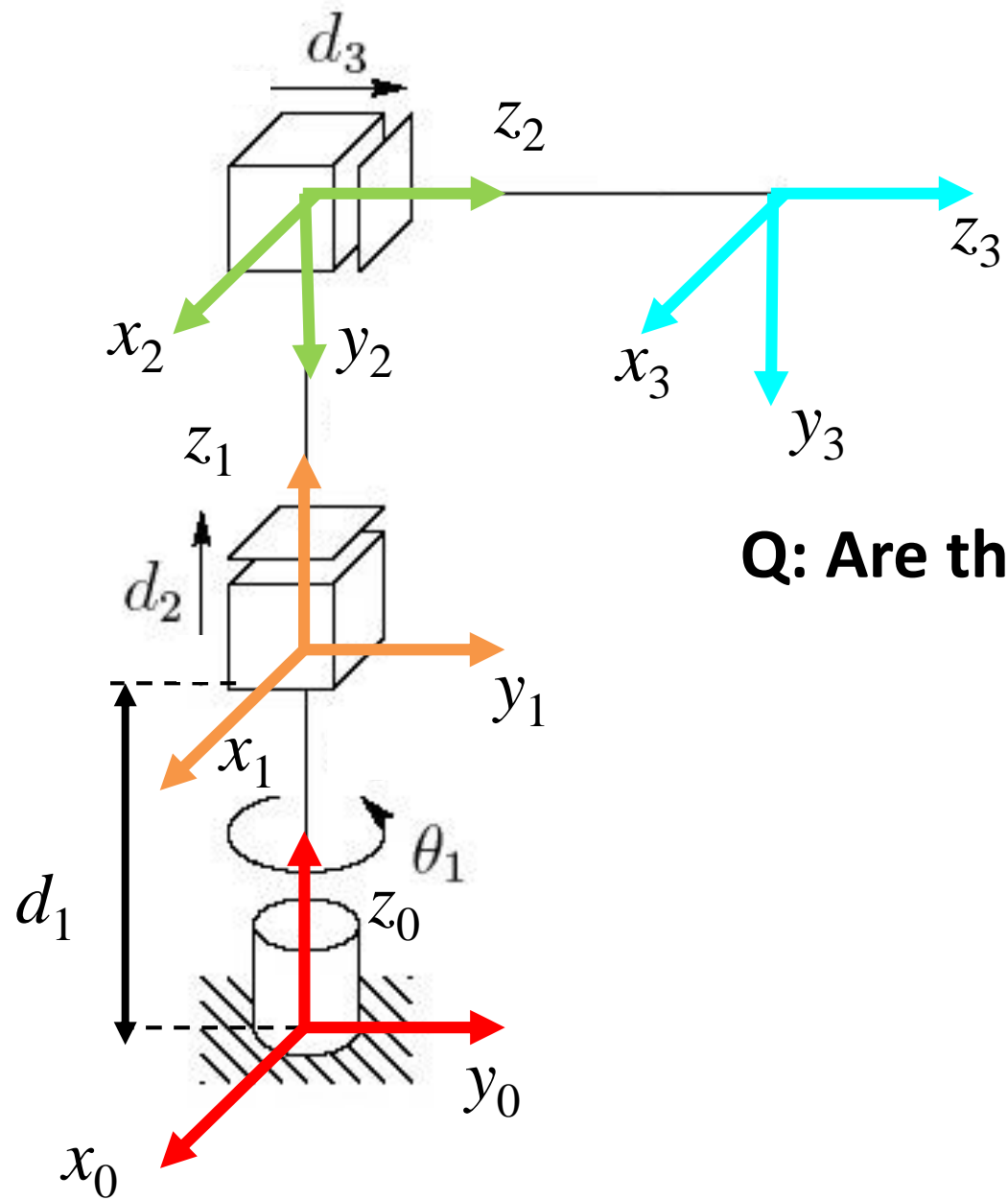
cylindrical



**Q: Is frame 3 correct?**



cylindrical

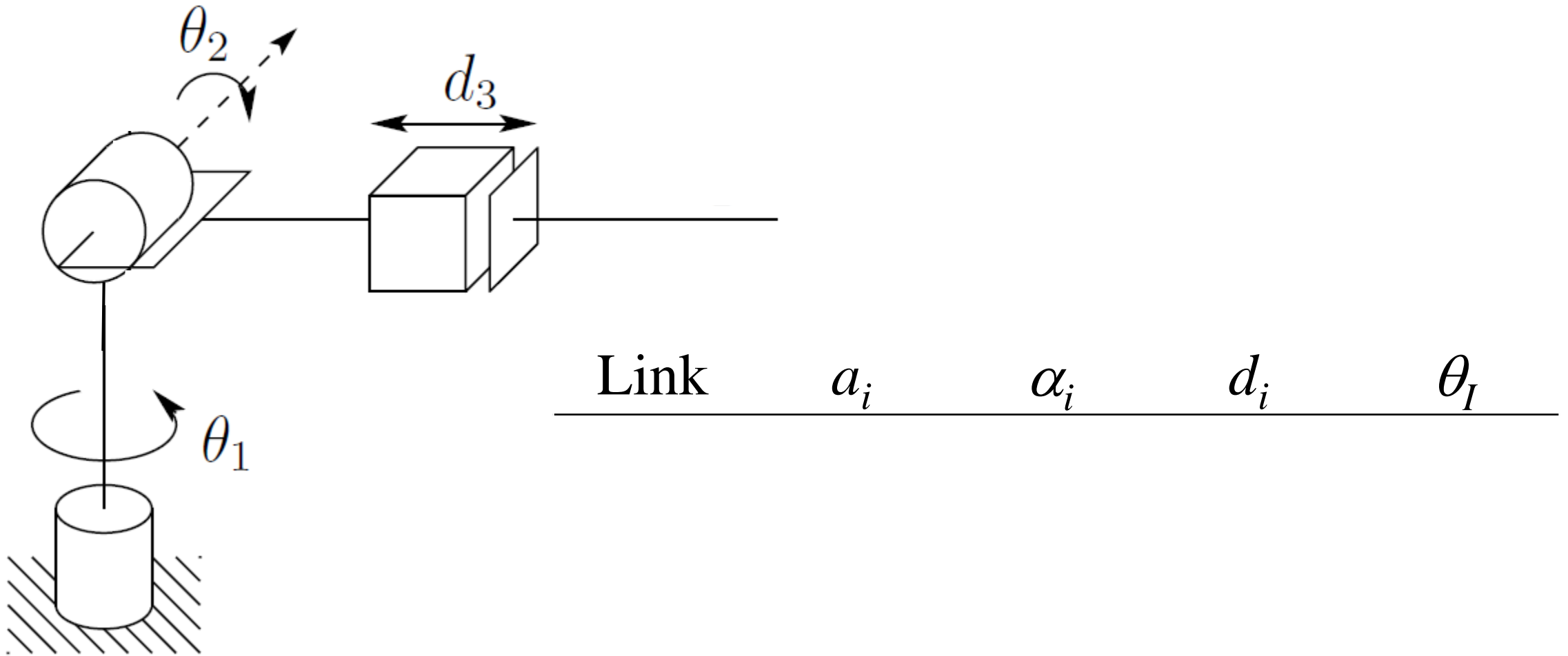


**Q: Are there other options?**



cylindrical

# Your coordinate frame may not lie on the robot?!?!



# You Practice

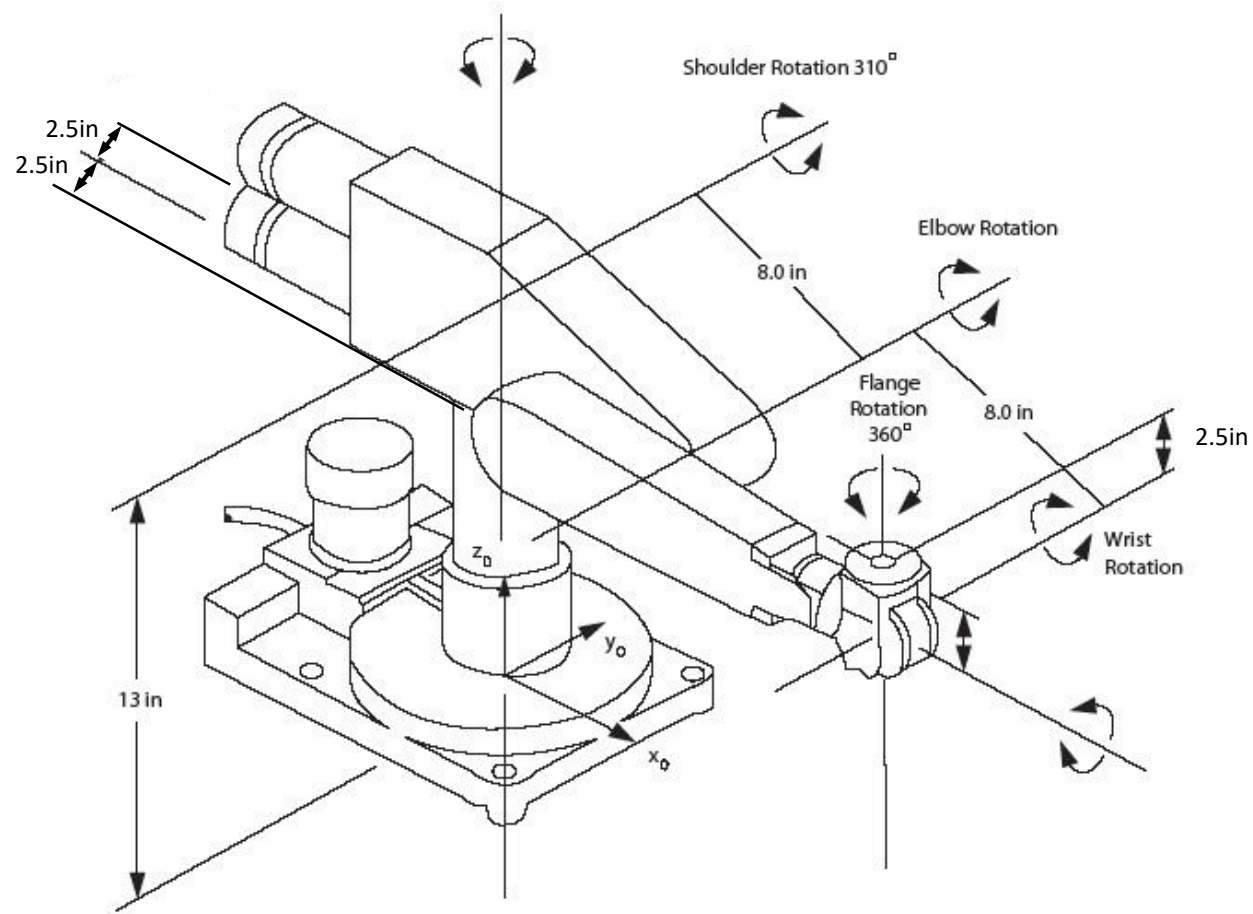
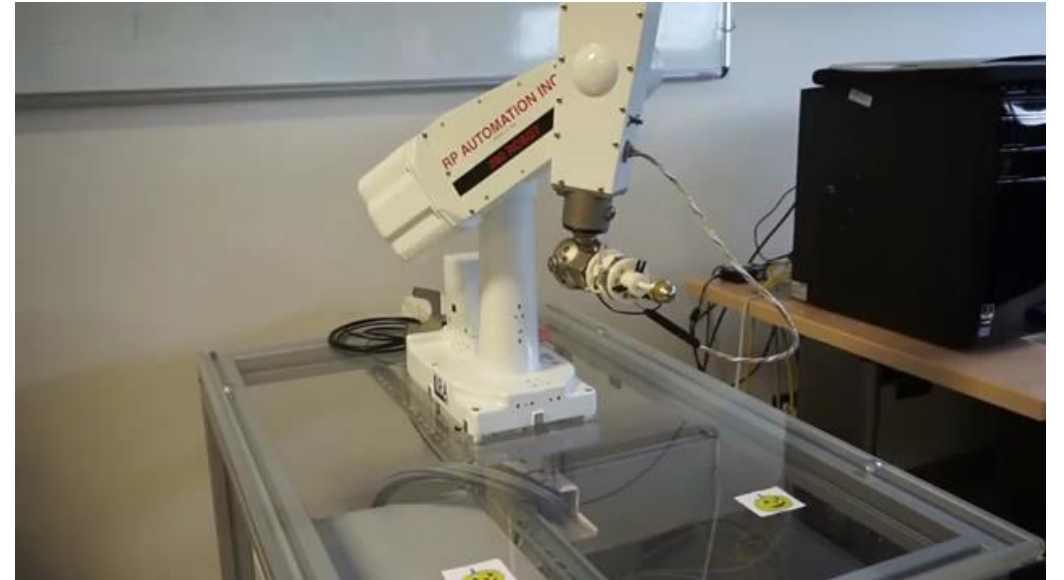


Figure 3.31: PUMA 260 manipulator.



<https://www.youtube.com/watch?v=3zhTopWv2oI>



# Next time: Inverse Position Kinematics

## Chapter 3: Forward and Inverse Kinematics

- Read 3.3 – 3.4

