MEAM 520
Lecture 21: Joint Space Dynamics

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Previously: Manipulator Jacobian

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

(3 x n) linear velocity Jacobian

(3 x n) angular velocity Jacobian

(6 x n) Jacobian
a.k.a. manipulator Jacobian
a.k.a. geometric Jacobian

$$J_v(\vec{q}) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

forward velocity kinematics

$$\xi = J(q)\dot{q}$$

(6 x 1) body velocity

(6 x n) Jacobian

(n x 1) joint velocities

$$J_\omega = [\rho_1 \hat{\mathbf{z}} \quad \rho_2 \mathbf{R}_1^0 \hat{\mathbf{z}} \quad \rho_3 \mathbf{R}_2^0 \hat{\mathbf{z}} \quad \cdots \quad \rho_n \mathbf{R}_{n-1}^0 \hat{\mathbf{z}}]$$

$\rho_i =$ 0 for prismatic
1 for revolute

inverse velocity kinematics

$$\dot{q} = J^{-1}\xi$$

Previously: Static Force/Torque Relationships

(n x 1) (n x 6) (6 x 1)

$$\vec{\tau} = J^T(\vec{q})\vec{F}$$

joint forces and torques

Jacobian matrix transpose

endpoint forces and torques

Simplest to think about for a 3-DOF robot with all revolute joints.
We want to output a force at the tip.

(3 x 1) (3 x 3) (3 x 1)

$$\vec{\tau} = J^T(\vec{q})\vec{F}$$

joint torques

Jacobian matrix transpose

endpoint forces

Derivation

$$\vec{\tau}^\top d\vec{q} = \vec{F}^\top d\vec{x}$$

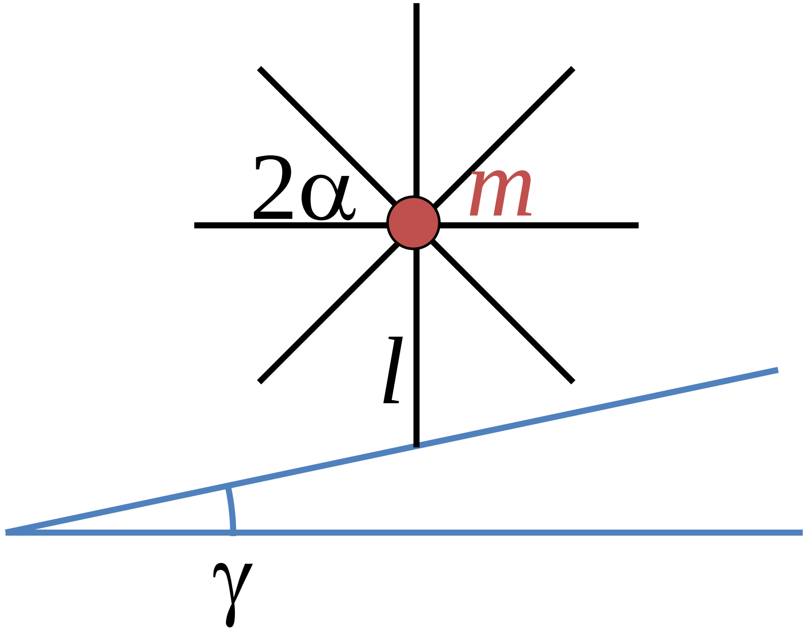
$$d\vec{x} = J_v d\vec{q}$$

$$\vec{\tau}^\top \cancel{d\vec{q}} = \vec{F}^\top J_v \cancel{d\vec{q}}$$

$$\vec{\tau}^\top = \vec{F}^\top J_v$$

$$\vec{\tau} = J_v^\top \vec{F}$$

Last Time: Walking is highly dynamic!



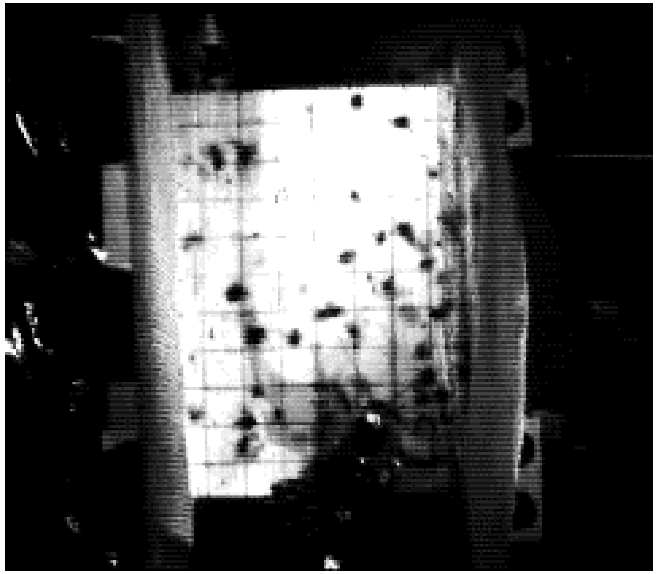
- Pin joint at foot
- Collision is inelastic and impulsive (no bouncing)

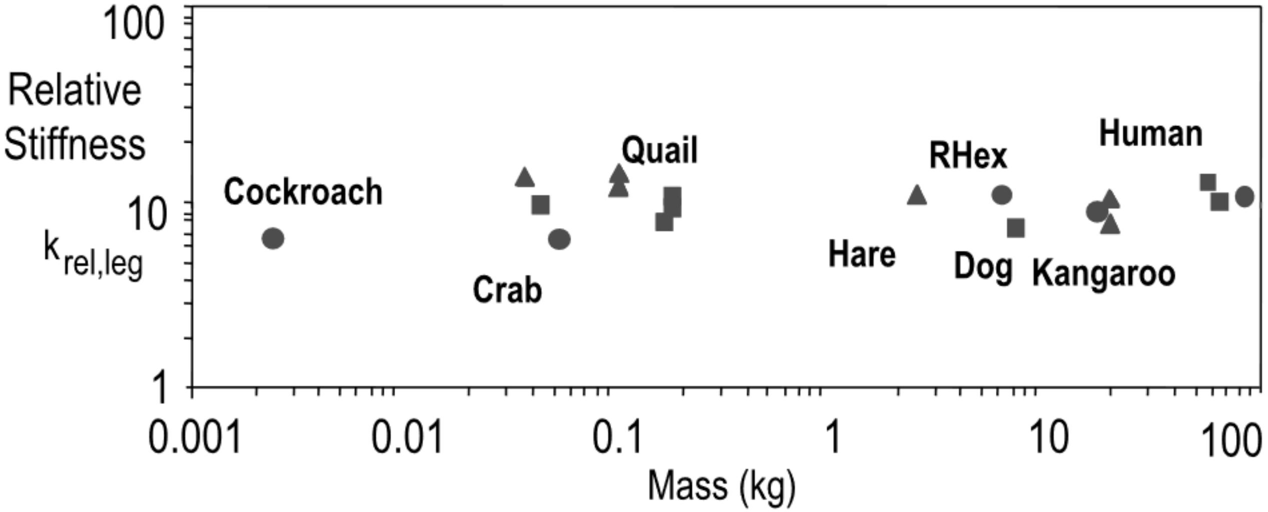
This is an inverted pendulum!

Ruina, Cornell, https://www.youtube.com/watch?v=durPmC42MHM

Honda P2 (1997)

https://www.youtube.com/watch?v=d2BUO4HEhvM https://www.youtube.com/watch?v=x-a8cCXtpbM

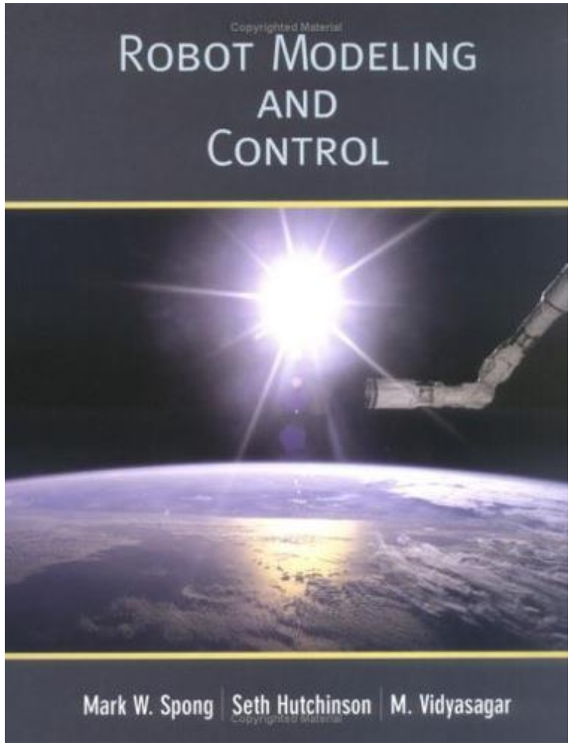




Koditschek &al, Arthropod Structure and Development, 2004

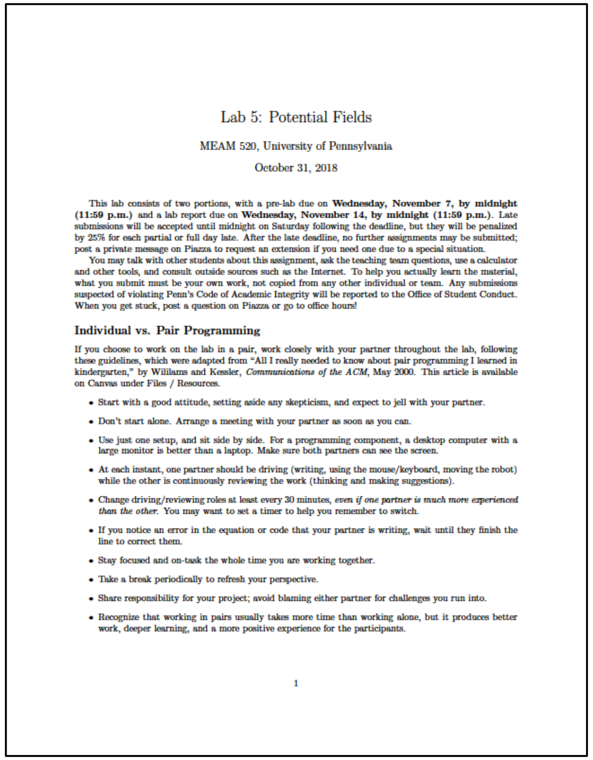
https://www.youtube.com/watch?v=cNZPRsrwumQ

Today: Dynamics



Chapter 7: Dynamics

- Read 7.1-7.3



Lab 5: Potential Fields

MEAM 520, University of Pennsylvania

October 31, 2018

This lab consists of two portions, with a pre-lab due on **Wednesday, November 7, by midnight (11:59 p.m.)** and a lab report due on **Wednesday, November 14, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

If you choose to work on the lab in a pair, work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Wililams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

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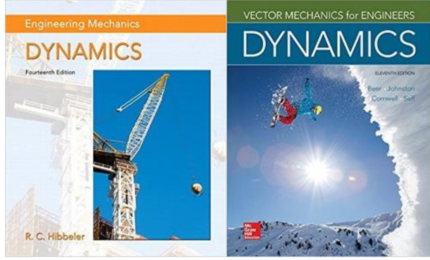
Lab 5 (last lab!) due 11/14

Dynamics

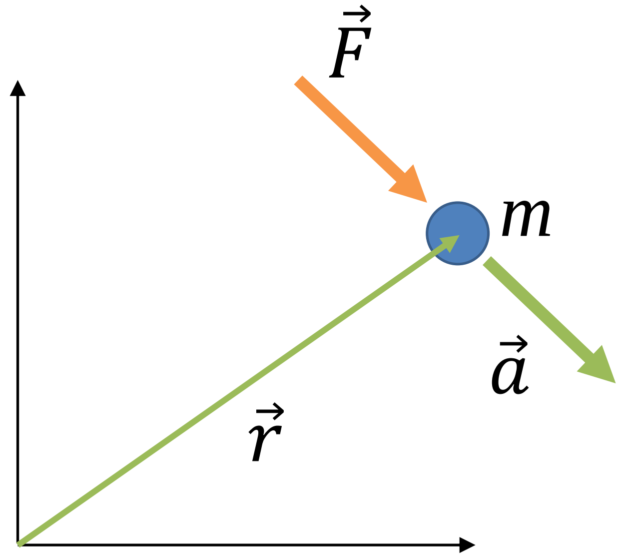
Kinematics: motion of the robot without consideration of the forces/torques producing motion

Dynamics: Relationship between forces and motion

Particle Dynamics



Hibbeler Ch. 13.1-13.2
Beer Ch. 12.1



Position: $\vec{r}(t)$

Velocity: $\vec{v}(t) = \dfrac{d\vec{r}}{dt}$

Acceleration: $\vec{a}(t) = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

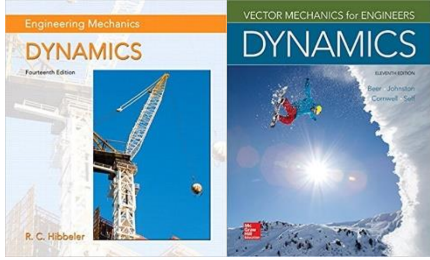
Newton's Second Law: $\vec{F}(t) = m\vec{a}(t)$

$$\vec{F}(t) = m\frac{d\vec{v}}{dt}$$

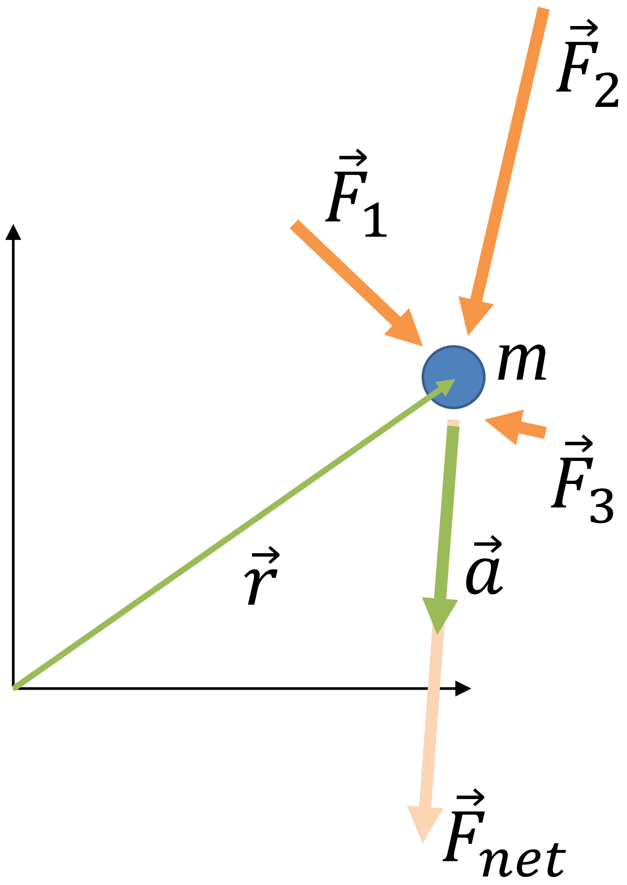
$$\vec{F}(t) = \frac{d\vec{p}}{dt}$$

linear momentum

Particle Dynamics



Hibbeler Ch. 14
Beer Ch. 13.1-13.2



$$\sum_i \vec{F}_i(t) = m\vec{a}(t)$$

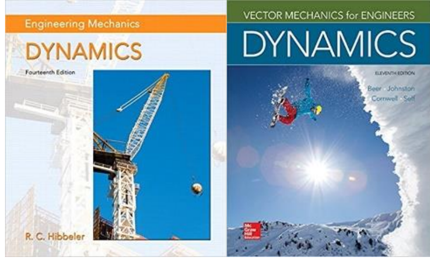
Kinetic Energy: $K = \frac{1}{2}m\vec{v}^{\top}\vec{v}$

Work: $W = \int \vec{F}_{net} \cdot d\vec{r}$

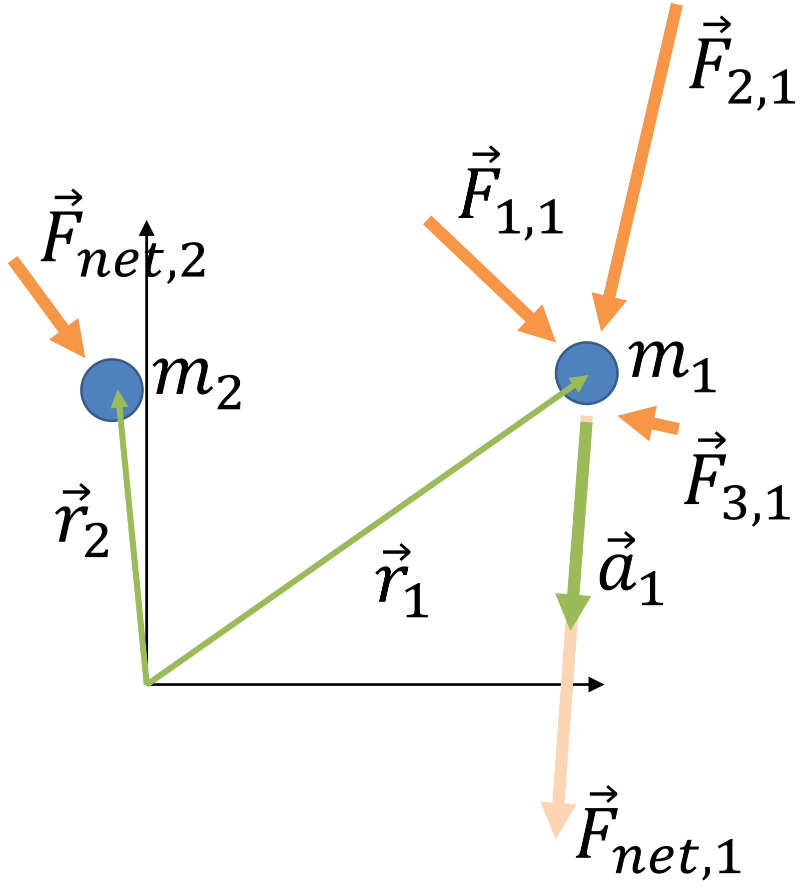
$$W = \int \vec{F}_C \cdot d\vec{r} + \int \vec{F}_{NC} \cdot d\vec{r}$$

Potential Energy: $P = -\int \vec{F}_C \cdot d\vec{r}$

Multiple Particles



Hibbeler Ch. 13.3, 14.3
Beer Ch. 14

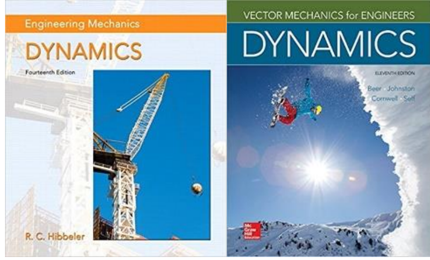


Particle j: $\vec{F}_{net,j}(t) = m_j \vec{a}_j(t)$

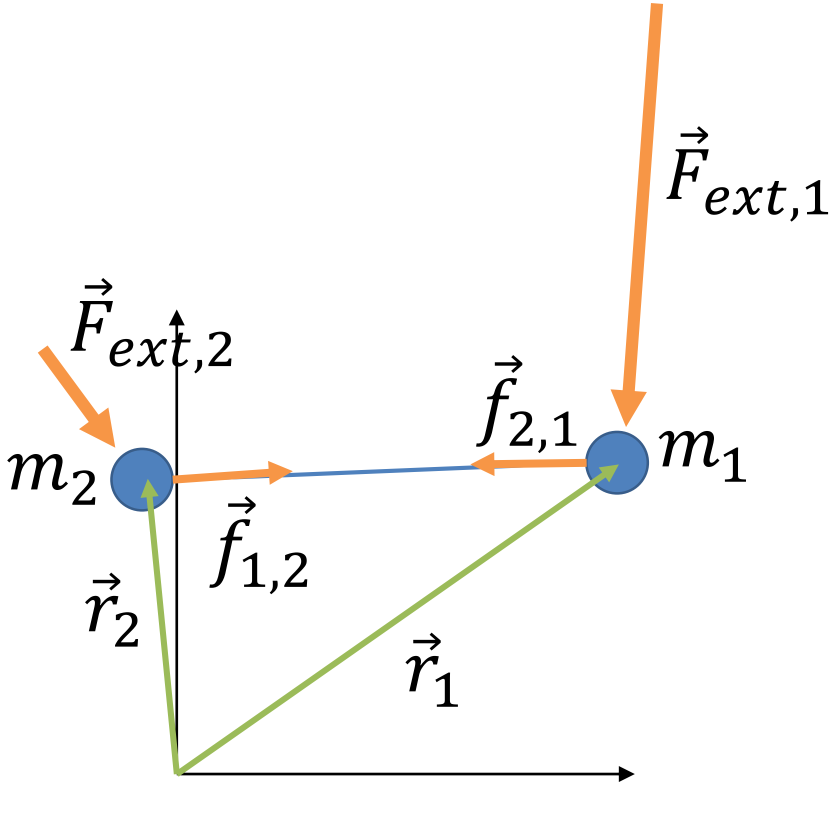
Kinetic Energy: $K = \sum_j \frac{1}{2} m_j \vec{v}_j^\top \vec{v}_j$

Potential Energy: $P = -\sum_j \int \vec{F}_{C,j} \cdot d\vec{r}_j$

Particles with Constraints



Hibbeler Ch. 13.3
Beer Ch. 14



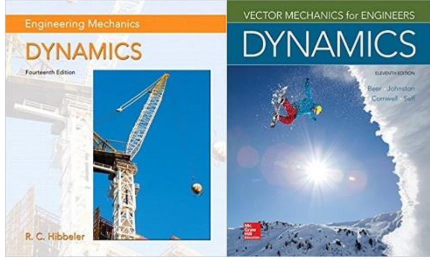
Internal forces: $\vec{f}_{i,j}(t) = -\vec{f}_{j,i}(t)$

$$\sum_j \sum_i \vec{f}_{i,j} = 0$$

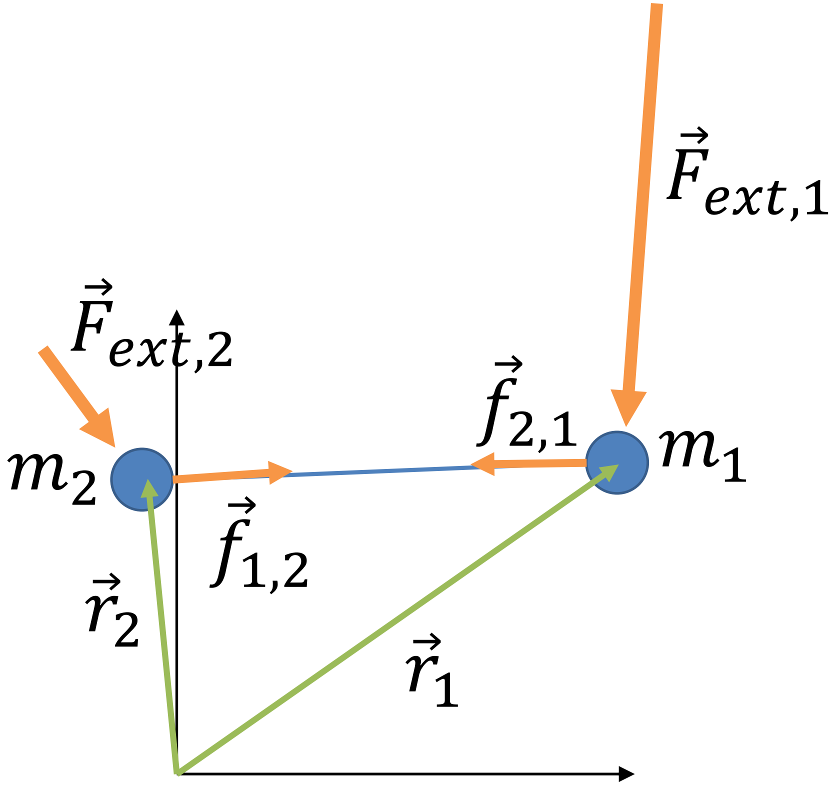
$$\vec{F}_{net,sys} = \sum_j \left(\vec{F}_{ext,j} + \cancel{\sum_i \vec{f}_{i,j}} \right)$$

$$\vec{F}_{net,sys} = \sum_j \vec{F}_{ext,j}$$

Particles with Constraints



Hibbeler Ch. 13.3
Beer Ch. 14



$$\vec{F}_{net,sys} = \sum_j \vec{F}_{ext,j} = \underbrace{\sum_j m_j \vec{a}_j}_{\text{Newton's 2nd Law}}$$

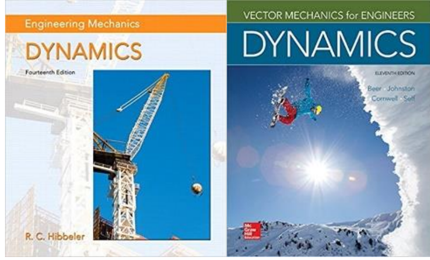
$$\vec{F}_{net,sys} = m_{tot} \sum_j \frac{m_j \vec{a}_j}{m_{tot}}$$

$$= m_{tot} \sum_j \frac{m_j \left(\frac{d^2 \vec{r}_j}{dt^2}\right)}{m_{tot}}$$

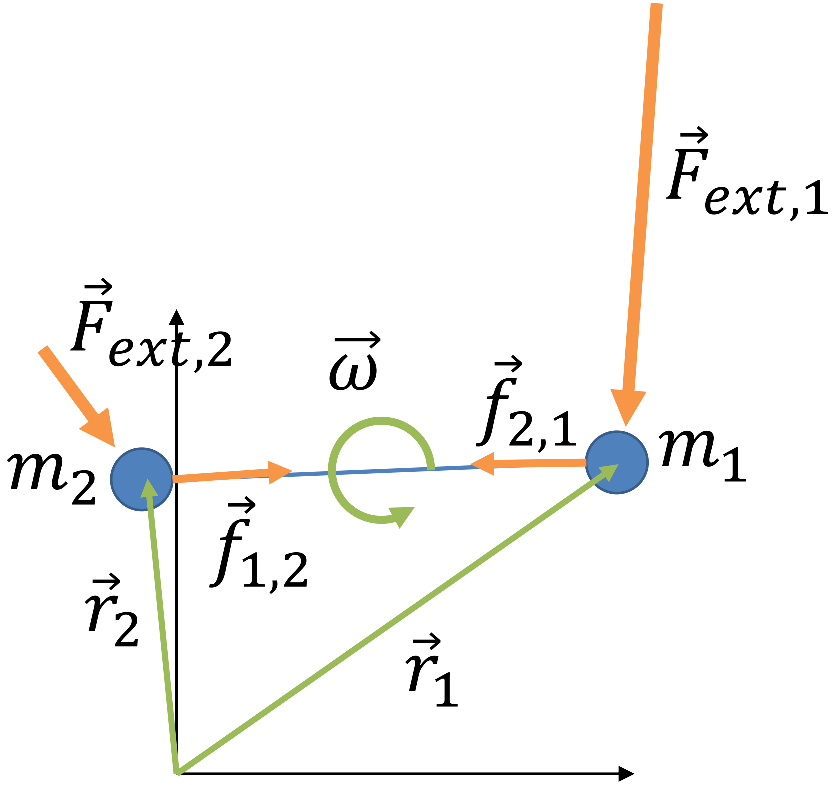
$$= m_{tot} \frac{d^2}{dt^2} \boxed{\frac{\sum_j m_j \vec{r}_j}{m_{tot}}}$$

Center of Mass

Particles with Constraints



Hibbeler Ch. 17
Beer Ch. 15

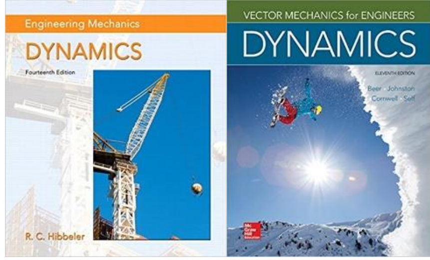


Inertia tensor: $[I]_{3\times3}$

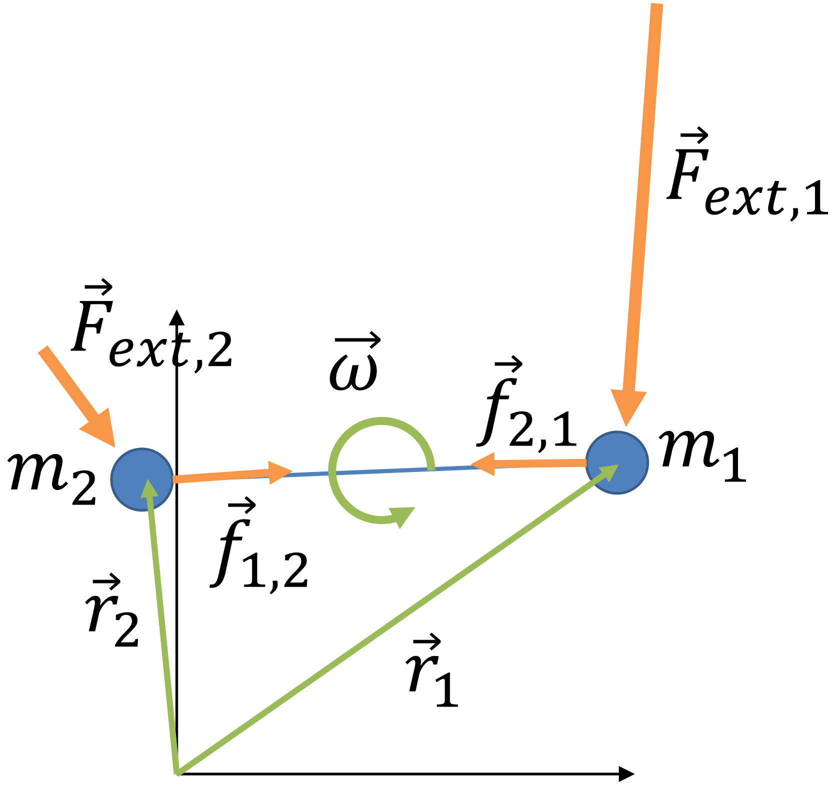
Euler Equation: $\sum \vec{\tau}_{COM} = [I]_{COM}\vec{\alpha}$

Euler Equation: $\sum \vec{\tau}_p = [I]_p\vec{\alpha} + \vec{r}_{p/COM} \times m_{tot}\vec{a}_p$

Particles with Constraints



Hibbeler Ch. 20
Beer Ch. 18.2



Position: $\vec{r}(t)$

Velocity: $\vec{v}(t) = \frac{d\vec{r}}{dt}$

Acceleration: $\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

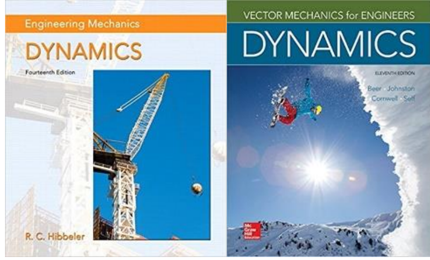
Kinematic Constraints

$$\vec{r}_2 = \vec{r}_1 + \vec{r}_{2/1}$$

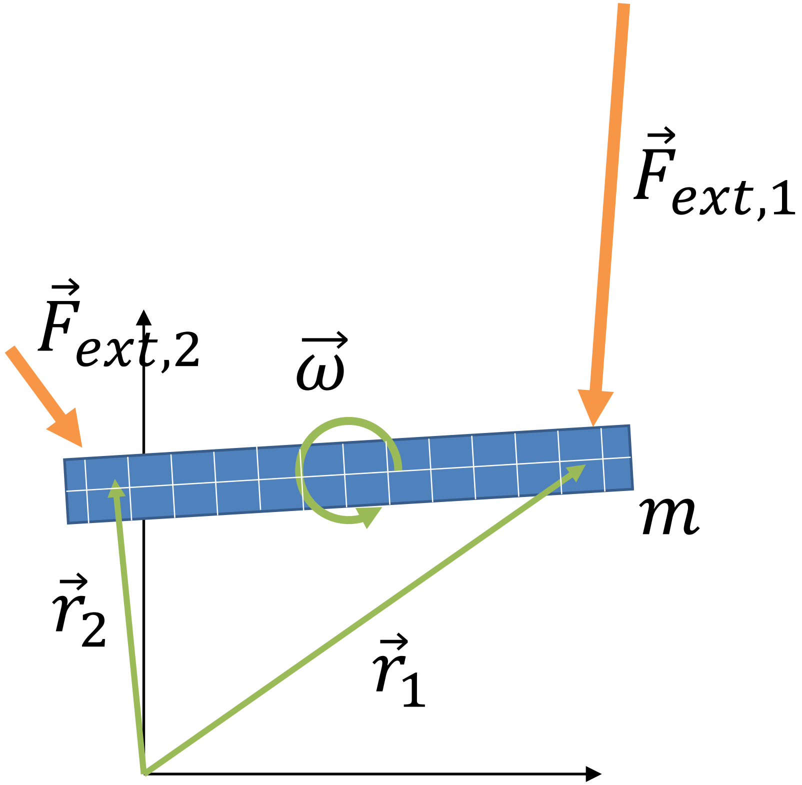
$$\vec{v}_2 = \frac{d\vec{r}_2}{dt} = \vec{v}_1 + \vec{\omega} \times \vec{r}_{2/1}$$

$$\vec{a}_2 = \frac{d\vec{v}_2}{dt} = \vec{a}_1 + \vec{\alpha} \times \vec{r}_{2/1} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{2/1}\right)$$

Rigid Bodies



Hibbeler Ch. 21.1
Beer Ch. 18.2



$$\vec{r}_{COM} = \frac{1}{m}\int \vec{r}dm$$

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

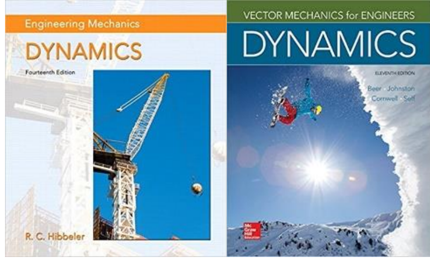
I depends on your frame!

$$I_{xx} = \iiint (y^2 + z^2)dm \qquad I_{xy} = I_{yx} = -\iiint xydm$$

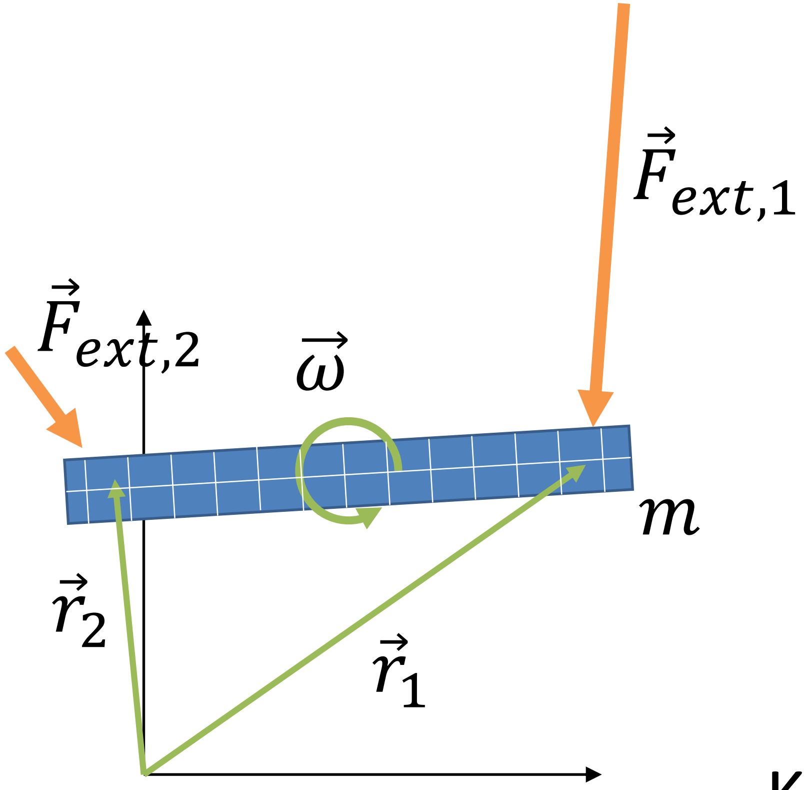
$$I_{yy} = \iiint (x^2 + z^2)dm \qquad I_{xz} = I_{zx} = -\iiint xzdm$$

$$I_{zz} = \iiint (x^2 + y^2)dm \qquad I_{yz} = I_{zy} = -\iiint yzdm$$

Rigid Bodies



Hibbeler Ch. 21.3
Beer Ch. 18.1

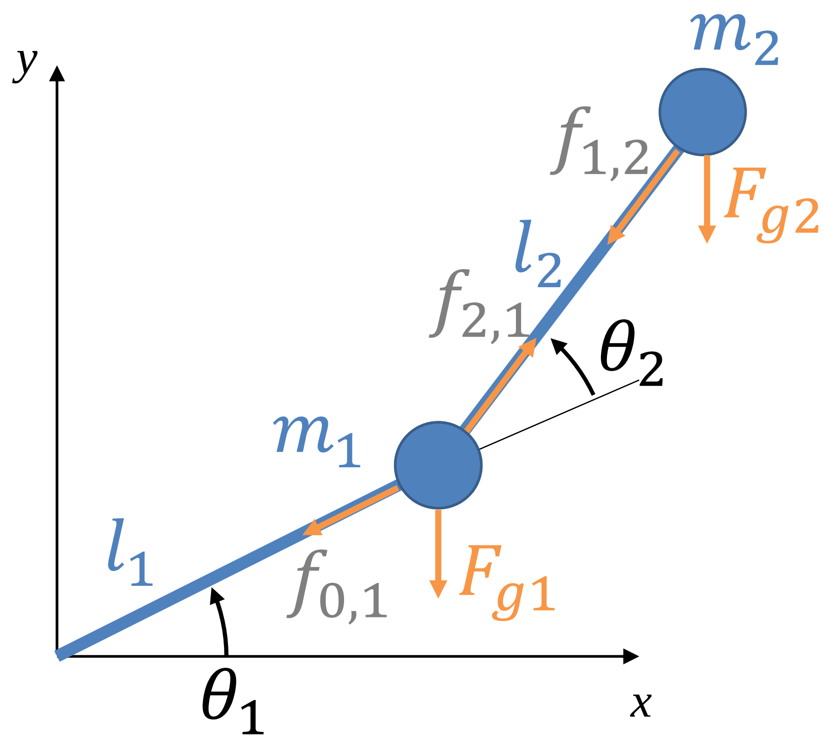


Kinetic Energy: $K = \frac{1}{2} m \vec{v}_{COM}^{\top} \vec{v}_{COM} + \frac{1}{2} I_{COM} \vec{\omega}^{\top} \vec{\omega}$

Gravitational Potential Energy: $P_g = m z_{COM}$

Example: RR Manipulator w/ mass concentrated at ends of links

AKA the double pendulum



Newton:

$$m_1\vec{a}_1 = \vec{F}_{g1} + \vec{f}_{0,1} + \vec{f}_{2,1}$$

$$m_2\vec{a}_2 = \vec{F}_{g2} + \vec{f}_{1,2}$$

9 unknowns
9 equations

Constraints:

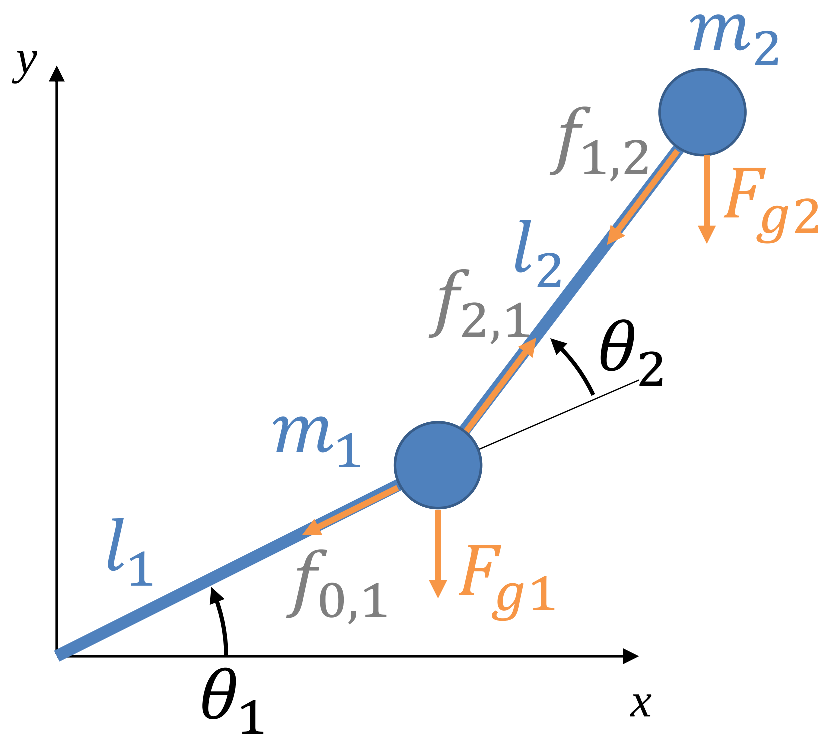
$$\vec{a}_1 = \vec{\alpha}_1 \times \vec{r}_1 - \omega_1^2\vec{r}_1$$

$$\vec{a}_2 = \vec{a}_1 + \vec{\alpha}_2 \times \vec{r}_{2/1} - \omega_2^2\vec{r}_{2/1}$$

$$f_{1,2} = f_{2,1}$$

Example: RR Manipulator w/ mass concentrated at ends of links

AKA the double pendulum



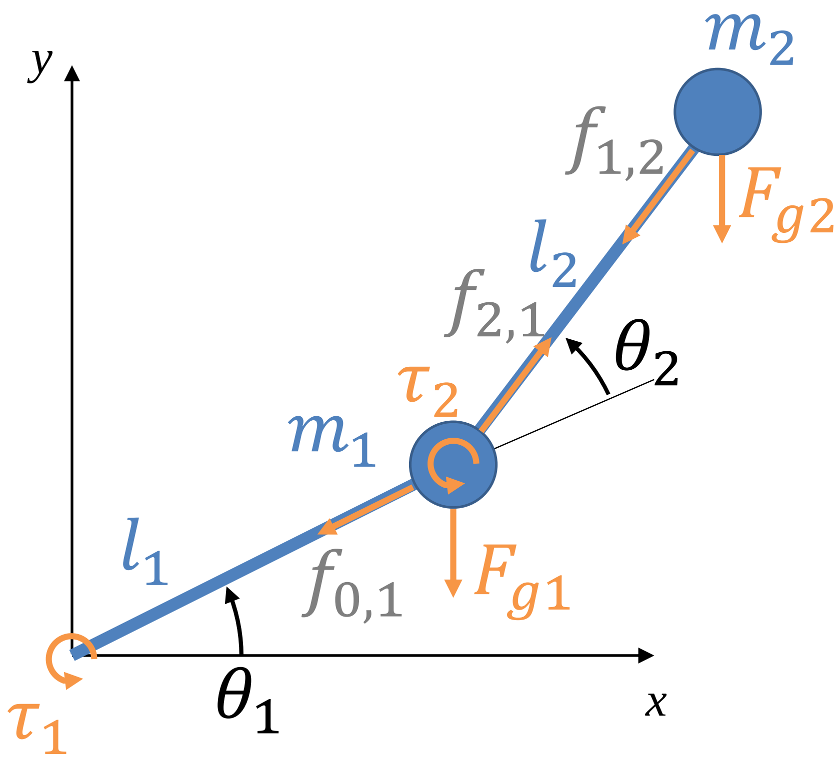
EOM (sub $\alpha = \ddot{\theta}$, $\omega = \dot{\theta}$):

$$
\begin{aligned}
&[m_1 l_1^2 + m_2(l_1^2 + l_2^2 + 2l_1 l_2 c_2)]\ddot{\theta}_1 \\
&\quad + [m_2(l_2^2 + l_1 l_2 c_2)]\ddot{\theta}_2 - m_2 l_1 l_2 s_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) \\
&\quad + m_1 g l_1 c_1 + m_2 g(l_1 c_1 + l_2 c_2) = 0
\end{aligned}
$$

$$
\begin{aligned}
&[m_2(l_2^2 + l_1 l_2 c_2)]\ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \\
&\quad + m_2 g l_2 c_{12} = 0
\end{aligned}
$$

Example: RR Manipulator w/ mass concentrated at ends of links

AKA the double pendulum



EOM (sub $\alpha = \ddot{\theta}$, $\omega = \dot{\theta}$):

coefficients of $\ddot{q}_i$ depend only on q

$$\boxed{[m_1 l_1^2 + m_2(l_1^2 + l_2^2 + 2l_1 l_2 c_2)]}\ddot{\theta}_1 + \boxed{[m_2(l_2^2 + l_1 l_2 c_2)]}\ddot{\theta}_2 \boxed{- m_2 l_1 l_2 s_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right)} + \boxed{m_1 g l_1 c_1 + m_2 g(l_1 c_1 + l_2 c_2)} = \tau_1$$

$$\boxed{[m_2(l_2^2 + l_1 l_2 c_2)]}\ddot{\theta}_1 + \boxed{m_2 l_2}\ddot{\theta}_2 + \boxed{m_2 l_1 l_2 s_2 \dot{\theta}_1^2} + \boxed{m_2 g l_2 c_{12}} = \tau_2$$

centrifugal and Coriolis terms depend on q and $\dot{q}$

gravitational terms depend only on q

The Manipulator Equation

We can write this as a matrix equation

$$\tau = \underbrace{D(q)}\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

SHV uses a bit of strange notation.
Most people call this matrix *H* or *M*.

where

$D(q)$ is the *nxn* mass matrix (inertia terms)

$C(q,\dot{q})$ is the *nxn* matrix of centrifugal (square of joint velocities) and Coriolis (product of two different joint velocities) terms

$g(q)$ is a *nx1* vector of gravitational terms

Another Method: Euler-Lagrange Equation

Derivation SHV 7.1.3

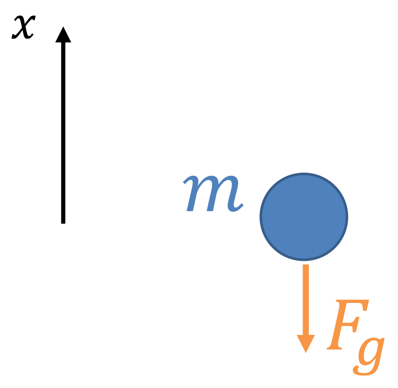
Lagrangian: $L = K - P$

EOM: $$\frac{d}{dt}\frac{\partial}{\partial \dot{q}}L - \frac{\partial}{\partial q}L = \tau$$

generalized coordinates

generalized force

Example: Particle under Gravity



Kinetic energy: $K = \frac{1}{2} m \dot{x}^2$

Potential energy: $P = mgx$

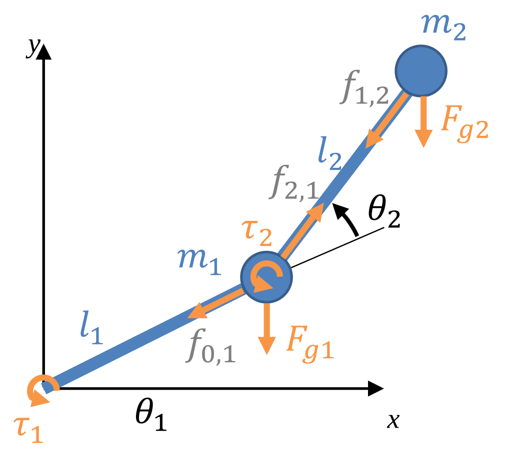
Lagrangian: $L = K - P = \frac{1}{2} m \dot{x}^2 - mgx$

$$\frac{\partial}{\partial x} L = -mg \qquad \frac{\partial}{\partial \dot{x}} L = m\dot{x}$$

EOM: $\frac{d}{dt} \frac{\partial}{\partial \dot{q}} L - \frac{\partial}{\partial q} L = \tau$

$$m\ddot{x} + mg = \tau$$

Euler-Lagrange Equation



Kinetic Energy K

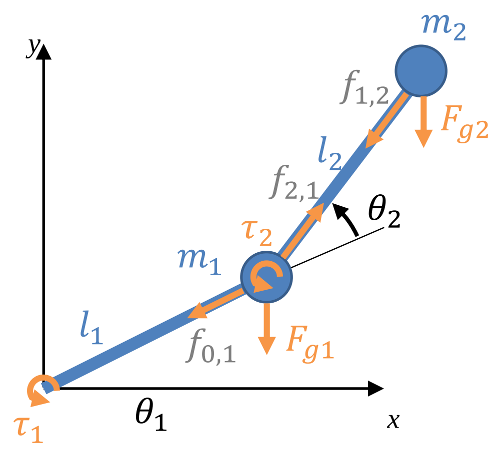
$$\text{Link 1: } \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \end{bmatrix} \Longrightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 \\ l_1 c_1 \dot{\theta}_1 \end{bmatrix}$$

$$\text{Link 2: } \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix} \Longrightarrow \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} (-l_1 s_1 - l_2 s_{12})\dot{\theta}_1 - l_2 s_{12} \dot{\theta}_2 \\ (l_1 c_1 + l_2 c_{12})\dot{\theta}_1 + l_2 c_{12} \dot{\theta}_2 \end{bmatrix}$$

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left[(l_1^2 + 2 l_1 l_2 c_2 + l_2^2)\dot{\theta}_1^2 + 2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_1 \dot{\theta}_2 + l_2^2 \dot{\theta}_2 \right]$$

Example: RR manipulator



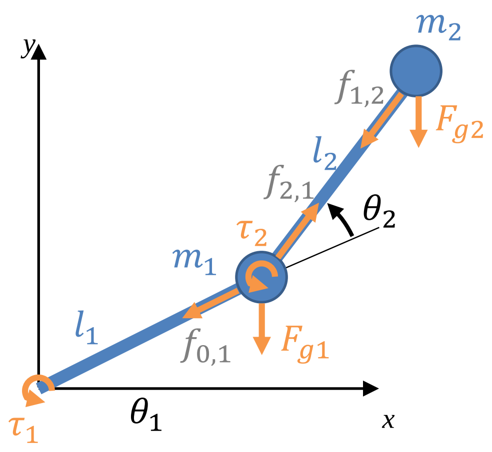
Potential Energy P

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix}$$

$$P = m_1 g y_1 + m_2 g y_2$$

$$= m_1 g l_1 s_1 + m_2 g (l_1 s_1 + l_2 s_{12})$$

Example: RR manipulator



Equation of Motion

$$L = K - P \qquad \tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$$

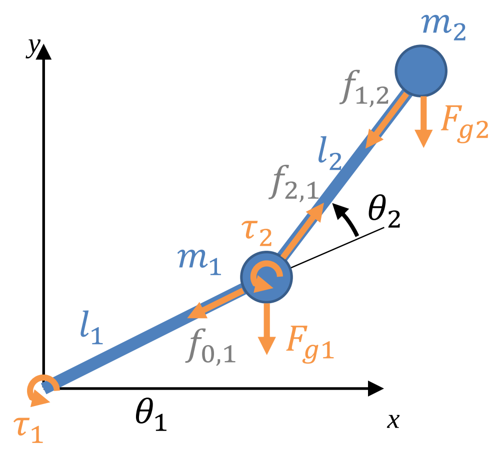
$$K = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + + \frac{1}{2}m_2\left[(l_1^2 + 2l_1 l_2 c_2 + l_2^2)\dot{\theta}_1^2 + 2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_1\dot{\theta}_2 + l_2^2\dot{\theta}_2\right]$$

$$P = m_1 g l_1 s_1 + m_2 g(l_1 s_1 + l_2 s_{12})$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial K}{\partial \dot{q}} = \begin{bmatrix} m_1 l_1^2\dot{\theta}_1 + m_2(l_1^2 + 2l_1 l_2 c_2 + l_2^2)\dot{\theta}_1 + m_2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_2 \\ m_2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_1 + m_2 l_2^2\dot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial L}{\partial q} = \begin{bmatrix} -m_1 g l_1 c_1 - m_2 g l_1 c_1 - m_2 g l_2 c_{12} \\ -m_2 l_1 l_2 s_2\dot{\theta}_1^2 - m_2 l_1 l_2 s_2\dot{\theta}_1\dot{\theta}_2 - m_2 g l_2 c_{12} \end{bmatrix}$$

Example: RR manipulator



Equation of Motion

$$L = K - P \qquad \tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$$

$$\frac{\partial L}{\partial \dot{q}} = \begin{bmatrix} m_1 l_1^2 \dot{\theta}_1 + m_2(l_1^2 + 2l_1 l_2 c_2 + l_2^2)\dot{\theta}_1 + m_2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_2 \\ m_2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_1 + m_2 l_2^2 \dot{\theta}_2 \end{bmatrix}$$

$$\frac{\partial L}{\partial q} = \begin{bmatrix} -m_1 g l_1 c_1 - m_2 g l_1 c_1 - m_2 g l_2 c_{12} \\ -m_2 l_1 l_2 s_2 \dot{\theta}_1^2 - m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 g l_2 c_{12} \end{bmatrix}$$

$$\tau = \begin{bmatrix} m_1 l_1^2 \ddot{\theta}_1 + m_2(l_1^2 + 2l_1 l_2 c_2 + l_2^2)\ddot{\theta}_1 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2(l_2^2 + l_1 l_2 c_2)\dot{\theta}_2 \\ -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 + m_1 g l_1 c_1 + m_2 g l_2 c_{12} \\ \\ m_2(l_2^2 + l_1 l_2 c_2)\ddot{\theta}_2 + m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 g l_2 c_{12} \end{bmatrix}$$

$$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$$c

Observations

Kinetic Energy

$$K = \frac{1}{2}m_1\vec{v}_1^\top\vec{v}_1 + \frac{1}{2}m_2\vec{v}_2^\top\vec{v}_2$$

$$K = \frac{1}{2}m_1(J_{v1}\dot{q})^\top(J_{v1}\dot{q}) + \frac{1}{2}m_2(J_{v2}\dot{q})^\top(J_{v2}\dot{q})$$

Linear velocity Jacobian: $v_i = J_{vi}\dot{q}$

$$K = \frac{1}{2}m_1\dot{q}^\top J_{v1}^\top J_{v1}\dot{q} + \frac{1}{2}m_2\dot{q}^\top J_{v2}^\top J_{v2}\dot{q}$$

$(AB)^\top = B^\top A^\top$

$$K = \frac{1}{2}\dot{q}^\top\underbrace{(m_1 J_{v1}^\top J_{v1} + m_2 J_{v2}^\top J_{v2})}_{\text{Function of } q}\dot{q}$$

Function of q $\quad \Rightarrow \frac{\partial}{\partial \dot{q}}(\quad) = 0$

$$\frac{\partial}{\partial \dot{q}}K = \frac{1}{2}[(m_1 J_{v1}^\top J_{v1} + m_2 J_{v2}^\top J_{v2})\dot{q}]^\top + \frac{1}{2}\dot{q}^\top(m_1 J_{v1}^\top J_{v1} + m_2 J_{v2}^\top J_{v2})$$

$$= \dot{q}^\top\boxed{(m_1 J_{v1}^\top J_{v1} + m_2 J_{v2}^\top J_{v2})}$$

Inertia Matrix D $\left\{ \begin{array}{l} \text{symmetric} \\ \text{positive definite} \end{array} \right.$

$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$c

Observations

Lagrangian: $L = K - P = \frac{1}{2}\boxed{\dot{q}^\top D \dot{q}} - \boxed{P}$

all terms contain $\dot{q}$ depends only on q

Manipulator equation: $\tau = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q)$

computed using D only

$\frac{\partial}{\partial q} P$

$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$c

N-link manipulator w/ mass concentrated at ends of links

Inertia:

$$N = 2: \; D = m_1 J_{v1}^{\top} J_{v1} + m_2 J_{v2}^{\top} J_{v2}$$

general case: $D = \sum_{i=1}^{N} m_i J_{vi}^{\top} J_{vi}$

Gravity:

$$N = 2: P = m_1 g l_1 s_1 + m_2 g (l_1 s_1 + l_2 s_{12})$$

general case: $P = \sum_{i=1}^{N} m_i \vec{g} \cdot \vec{r}_i$

$$g(q) = \frac{\partial}{\partial q} P$$

$$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}\mathrm{c}$$

What about C?

$$L = \frac{1}{2}\dot{q}^\top D\dot{q} - P = \frac{1}{2}\sum_{i,j} d_{ij}\dot{q}_i\dot{q}_j - P$$

$$\frac{\partial}{\partial q_k}L = \frac{1}{2}\sum_{i,j}\frac{\partial}{\partial q_k}d_{ij}\dot{q}_i\dot{q}_j \boxed{-\frac{\partial}{\partial q_k}P}$$

gravitational terms – ignore from here on

$$\frac{\partial}{\partial \dot{q}_k}L = \sum_j d_{kj}\dot{q}_j$$

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_k}L = \sum_j d_{kj}\ddot{q}_j + \sum_j \boxed{\frac{d}{dt}d_{kj}}\dot{q}_j$$

$$= \boxed{\sum_j d_{kj}\ddot{q}_j} + \sum_{i,j}\frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

inertia terms – ignore from here on

$$(C\dot{q})_k = \sum_{i,j}\left(\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i\dot{q}_j$$

$$(C\dot{q})_k = \sum_{i,j}\frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i\dot{q}_j$$

Christoffel symbols

Manipulator Equation

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

$$D = \sum_{i=1}^{N} m_i J_{vi}^{\top} J_{vi}$$

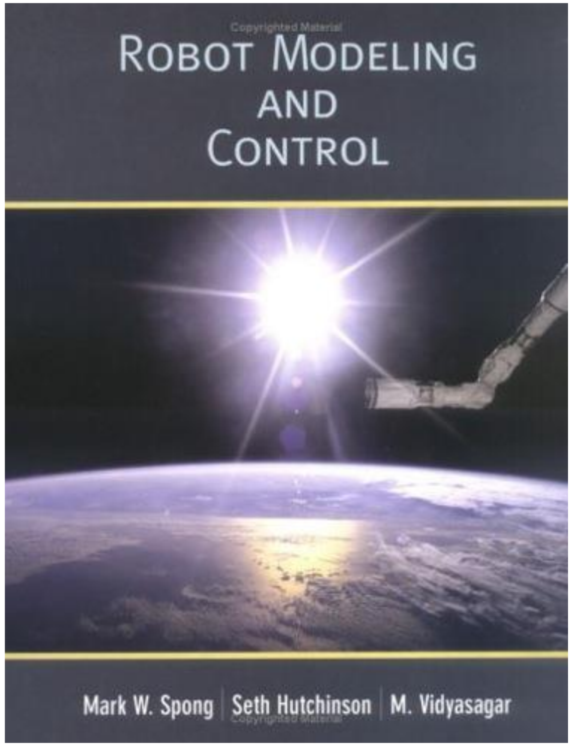
$$g = \frac{\partial}{\partial q} \sum_{i=1}^{N} m_i \vec{g} \cdot \vec{r}_i$$

$$(C\dot{q})_k = \sum_{i,j} \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i \dot{q}_j$$

or

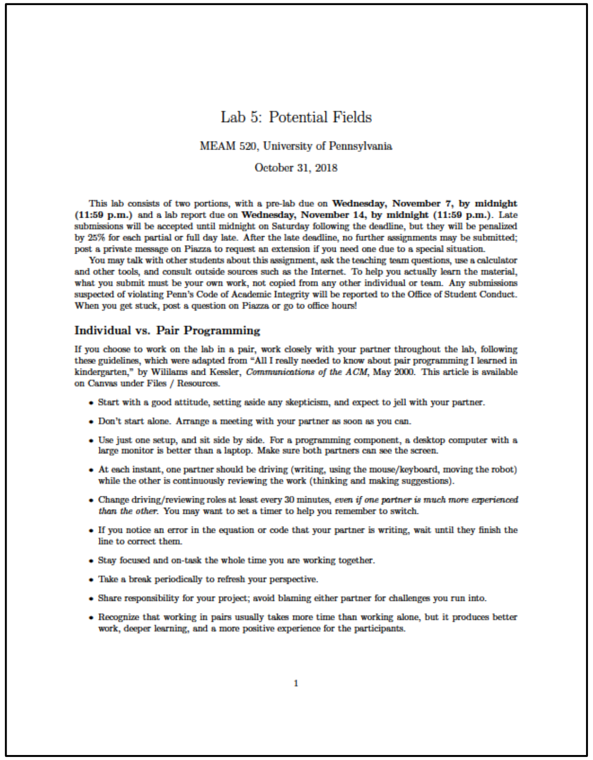
$$c_{kj} = \sum_{i} \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i$$

Next time: More Joint Space Dynamics



Chapter 7: Dynamics

- Read 7.4-7.7



Lab 5: Potential Fields

MEAM 520, University of Pennsylvania

October 31, 2018

This lab consists of two portions, with a pre-lab due on **Wednesday, November 7, by midnight (11:59 p.m.)** and a lab report due on **Wednesday, November 14, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

If you choose to work on the lab in a pair, work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Wililams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Lab 5: Potential Fields due 11/14