MEAM 520
Lecture 8: Inverse Orientation Kinematics

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Logistics/Observations based on Pre-Lab/Lab 1

- Don't worry if you got points taken off because the robot's joint 5 was flipped – the TAs are fixing this

  Let us know only if you do not see corrections when Lab 1 grades come out

- Regrades: Post **privately** on Piazza and tag with 'regrade' and the lab number. Include a few sentences on your reasoning to direct the TA's where to look in your submission.

- Do NOT post comments on Canvas – we don't see these

- For pairs, both students will not receive credit UNLESS you use the 'Lab #-' groups already created in Canvas

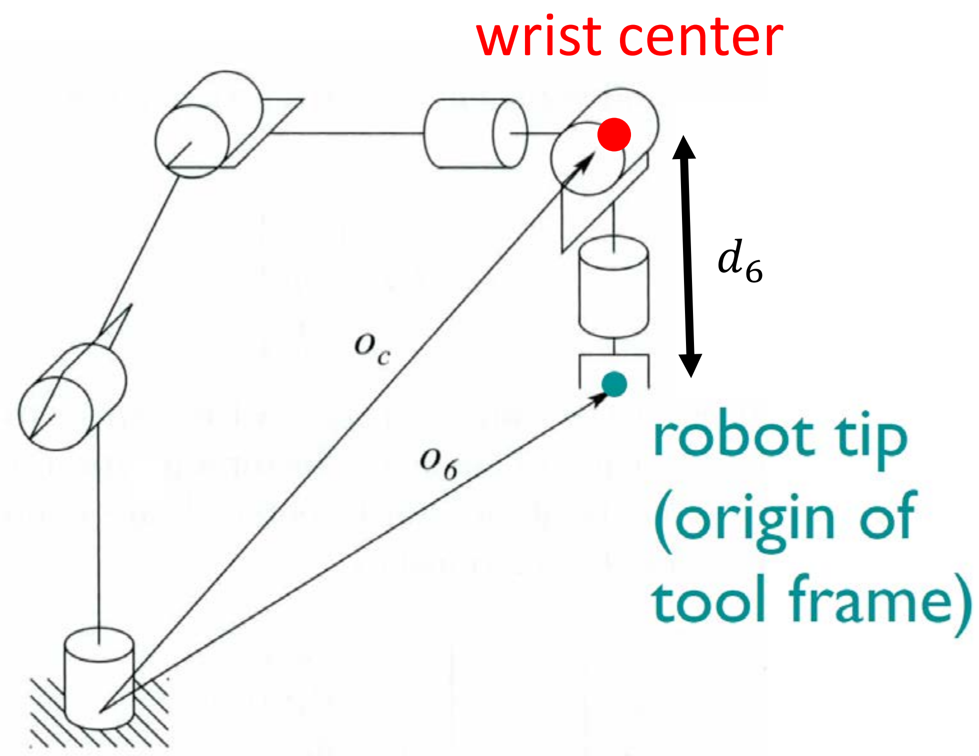
  TAs will correct these issues as they see them for Lab 1 – you must assign yourself correctly for future labs

- In general, follow the instructions. If you have questions about the instructions, ASK!

Given $\mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$ and a certain manipulator with *n* joints,
find $q_1,\ldots,q_n$ such that $\mathbf{T}_n^0(q_1,\ldots,q_n) = \mathbf{H}$

UGLY!

Last Time: Kinematic Decoupling



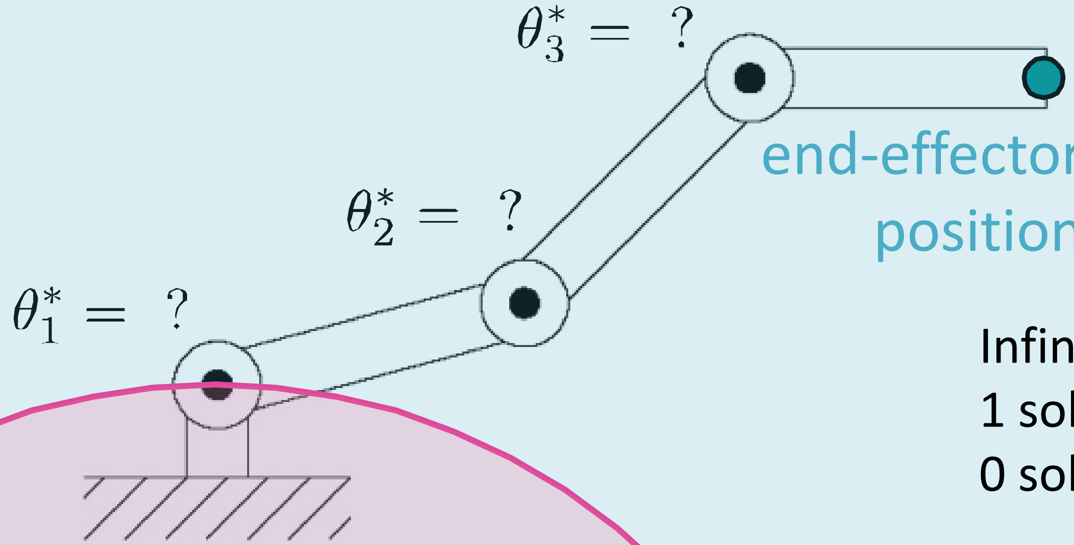
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1}\mathbf{R} = (\mathbf{R}_3^0)^{\mathrm{T}}\mathbf{R}$$

orientation

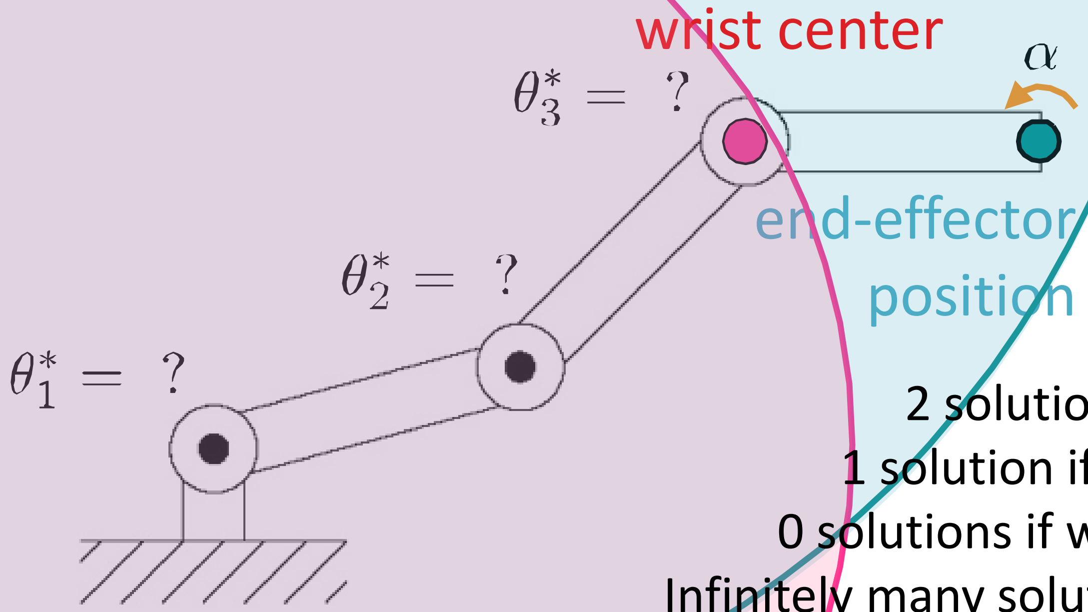
Multiple IK solutions



workspace

end-effector position $\begin{bmatrix} o_x \\ o_y \\ 0 \end{bmatrix}$

Infinitely many solutions within 3-link workspace
1 solution on the 3-link workspace boundary
0 solutions outside the 3-link workspace



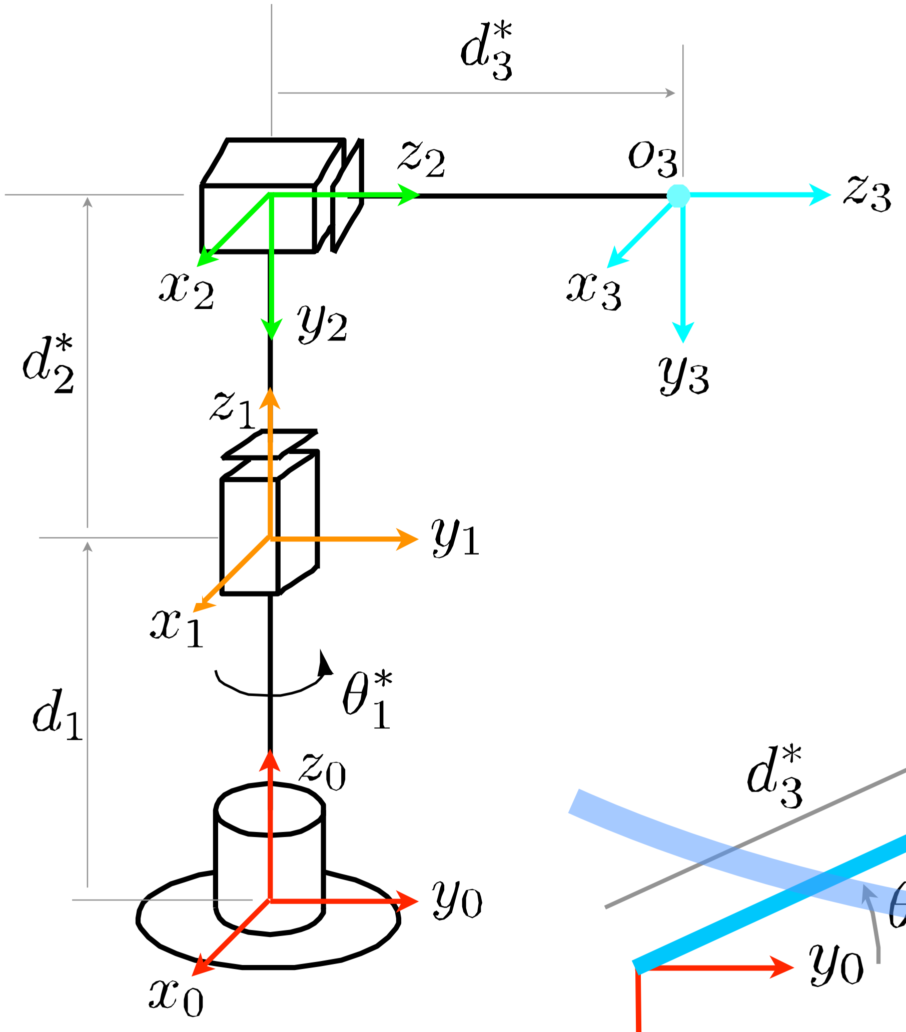
end-effector orientation

end-effector position $\begin{bmatrix} o_x \\ o_y \\ 0 \end{bmatrix}$

Kinematic Decoupling

2 solutions if wrist center is inside 2-link workspace
1 solution if wrist center is on 2-link workspace boundary
0 solutions if wrist center needs to be outside 2-link workspace
Infinitely many solutions if wrist center is origin and links are equal length

Last Time: Two Inverse Position Kinematics Approaches



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x = -d_3^* \sin(\theta_1^*)$$

$$y = d_3^* \cos(\theta_1^*)$$

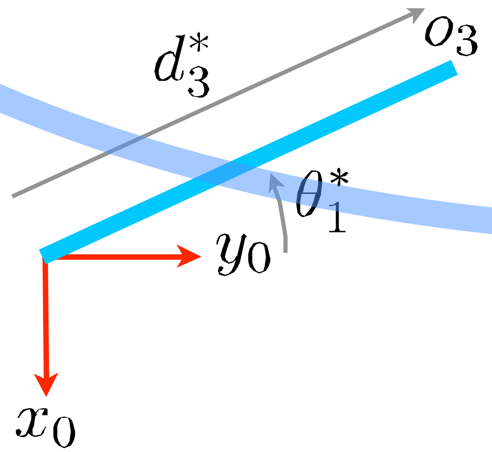
$$z = d_1 + d_2^*$$

$\theta_1^* = ?$

$d_2^* = ?$

$d_3^* = ?$

Algebra



Geometry

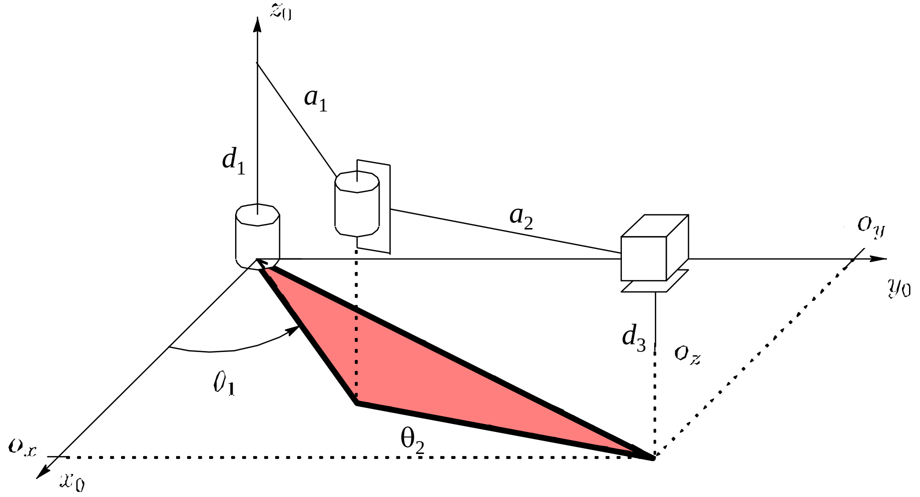
$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

$$\boxed{d_2^* = z - d_1}$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

Complete SCARA IK Example

$$o_4^0 = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \Rightarrow \begin{aligned} \theta_1 &= ? \\ \theta_2 &= ? \\ d_3 &= ? \end{aligned}$$



Start with Forward Position Kinematics

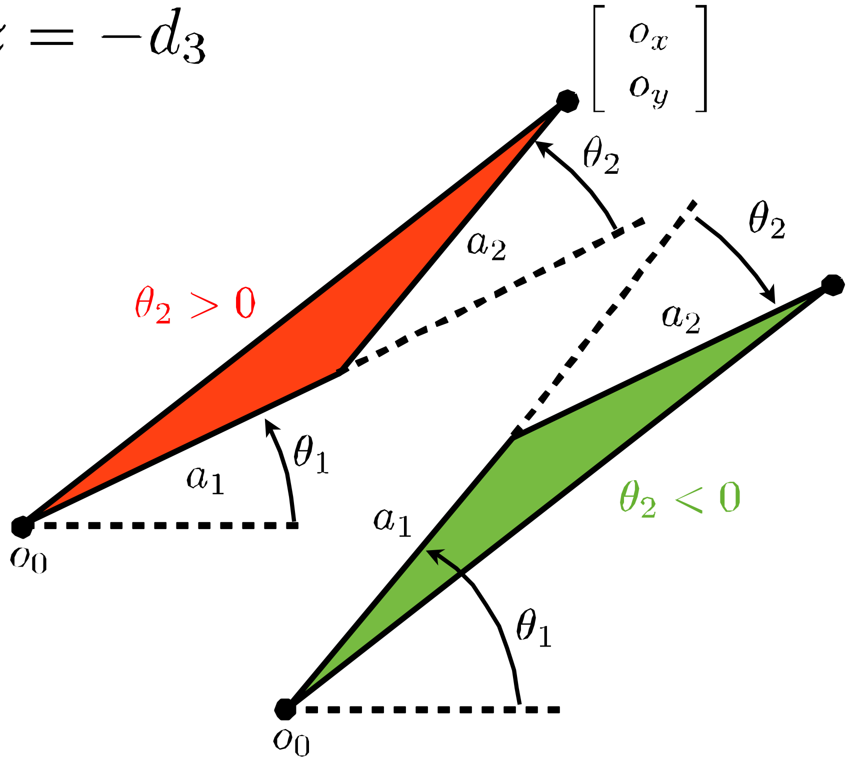
$$x = a_1 c_1 + a_2 c_{12}$$
$$y = a_1 s_1 + a_2 s_{12}$$
$$z = -d_3$$

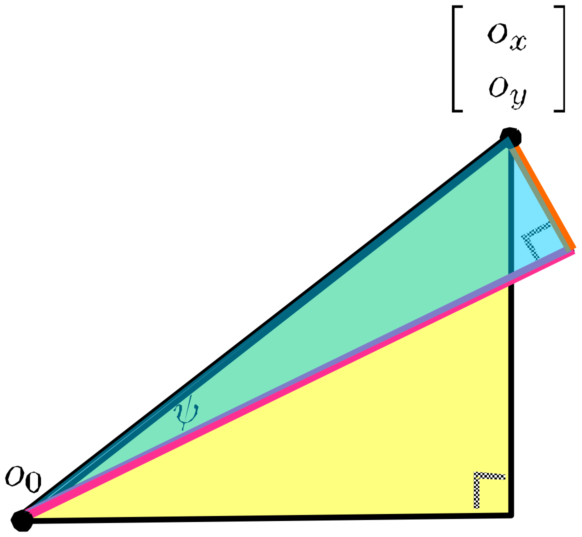
$$\boxed{d_3 = -o_z}$$

$$\boxed{\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}}$$

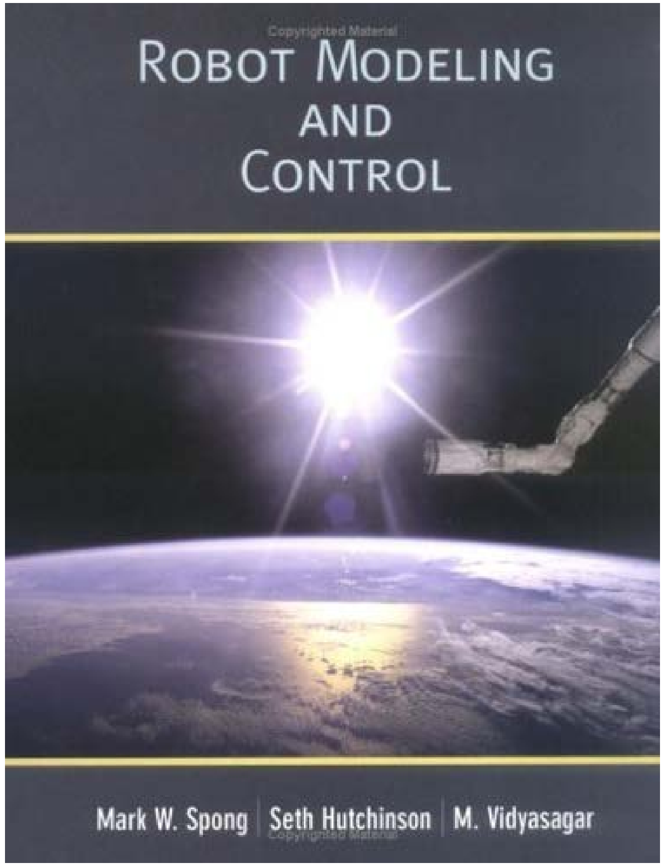
$$\boxed{\theta_2 = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)}$$

$$\boxed{\theta_1 = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)}$$



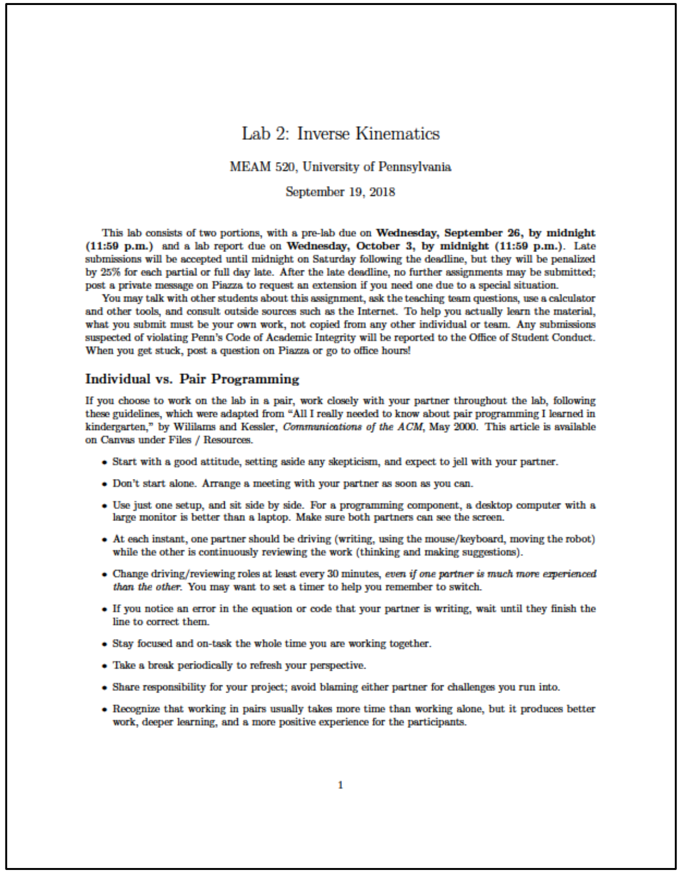


Today: Inverse Kinematics



Chapter 3: Forward and Inverse Kinematics

- Read Sec. 3.3 – 3.4



Lab 2: Inverse Kinematics

MEAM 520, University of Pennsylvania

September 19, 2018

This lab consists of two portions, with a pre-lab due on **Wednesday, September 26, by midnight (11:59 p.m.)** and a lab report due on **Wednesday, October 3, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

If you choose to work on the lab in a pair, work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Wililams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Resources.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Lab 2 posted yesterday

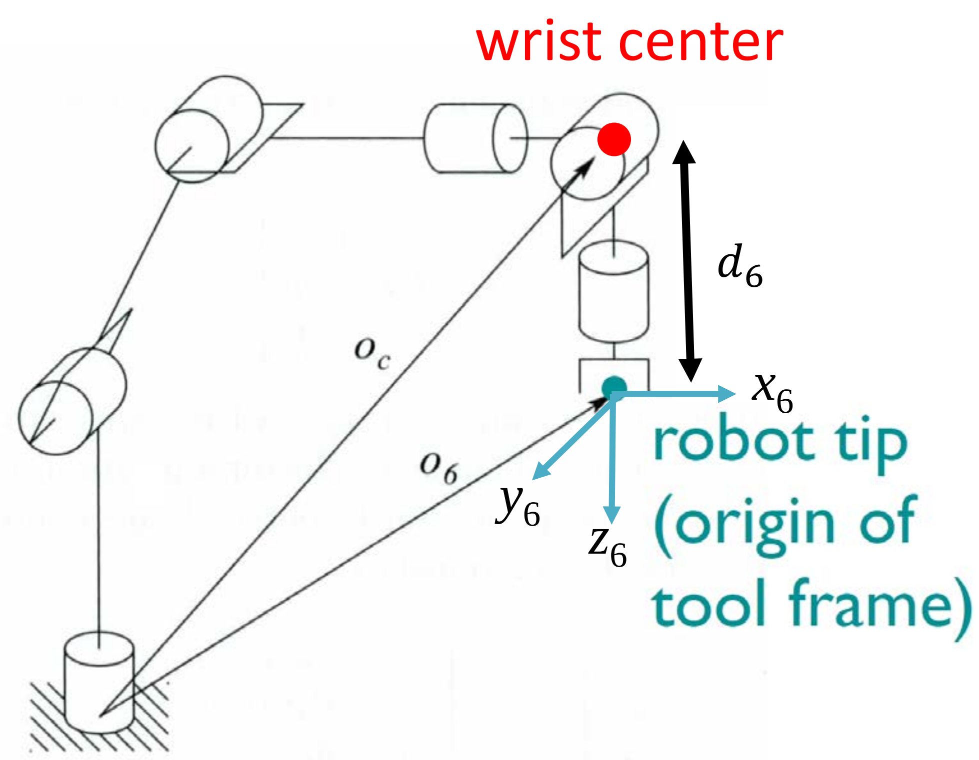
Prelab due 9/26, 11:59 p.m.

Show your work!

Given $\mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$ and a certain manipulator with n joints, find $q_1,\ldots,q_n$ such that $\mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$

UGLY!

Last Time: Kinematic Decoupling



Last Time

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

Today

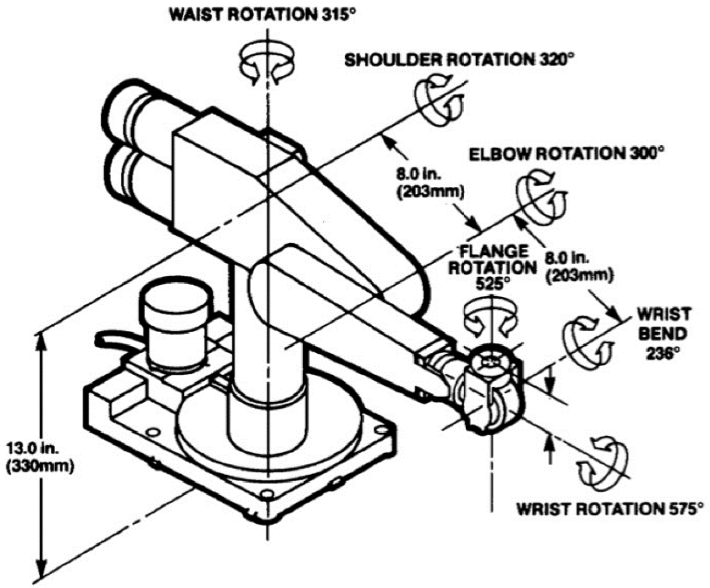
$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1}\mathbf{R} = (\mathbf{R}_3^0)^{\mathrm{T}}\mathbf{R}$$

orientation

Spherical Wrist

Intersecting RRR with middle joint perpendicular to the other two.

PUMA example



$a = 13.0$ in.

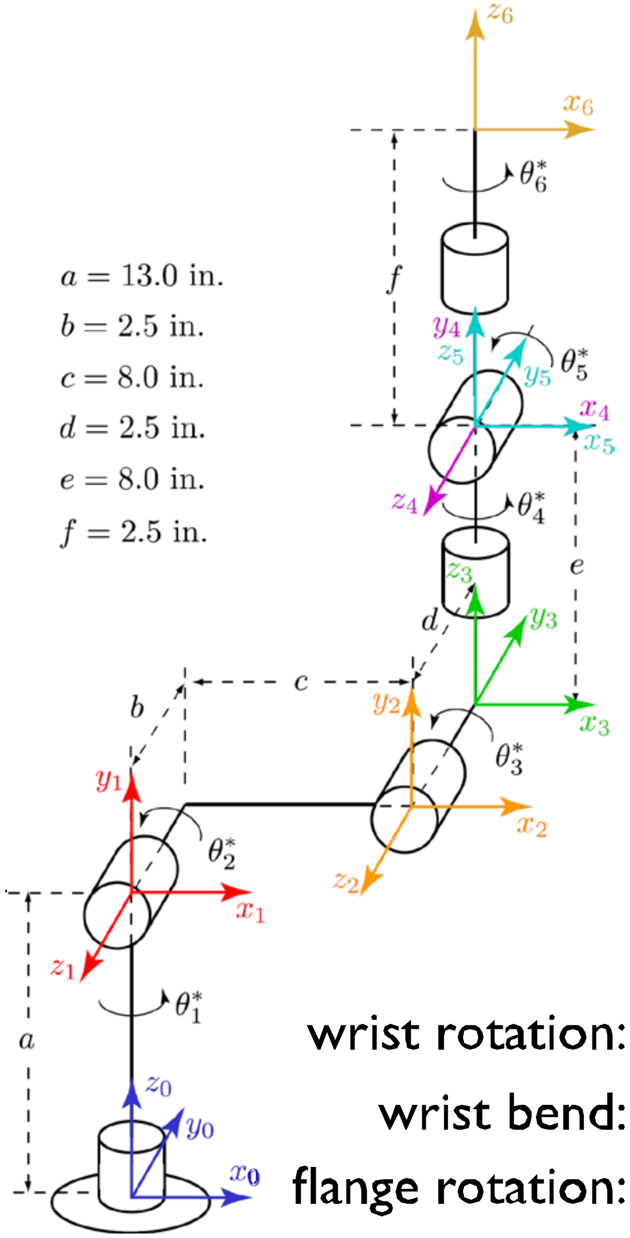
$b = 2.5$ in.

$c = 8.0$ in.

$d = 2.5$ in.

$e = 8.0$ in.

$f = 2.5$ in.



$-180° < \theta_1^* < 110°$

$-75° < \theta_2^* < 240°$

$-235° < \theta_3^* < 60°$

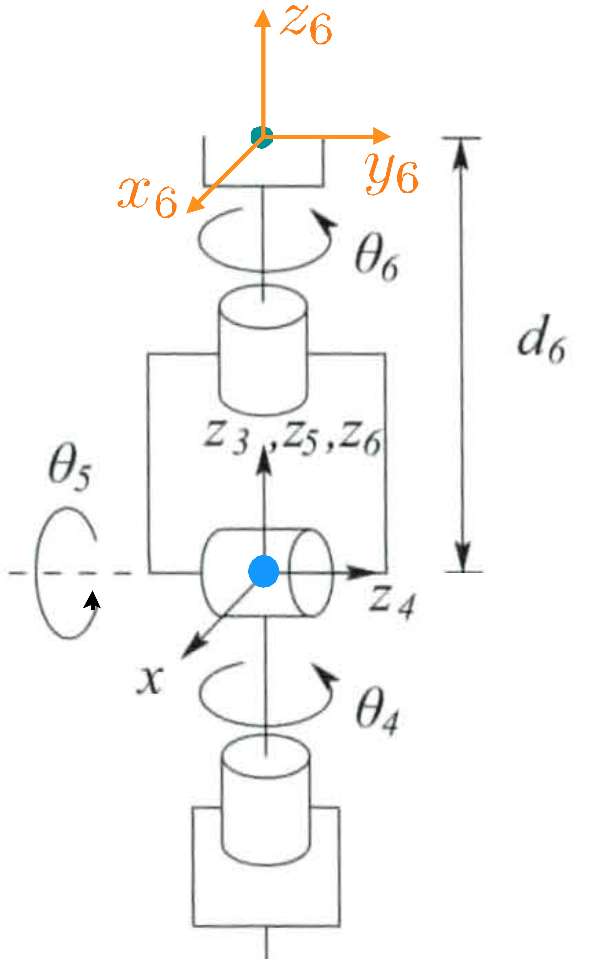
wrist rotation: $-580° < \theta_4^* < 40°$

wrist bend: $-120° < \theta_5^* < 110°$

flange rotation: $-215° < \theta_6^* < 295°$

Given a rotation matrix **R**, find a set of joint angles that puts the end-effector in the desired orientation.

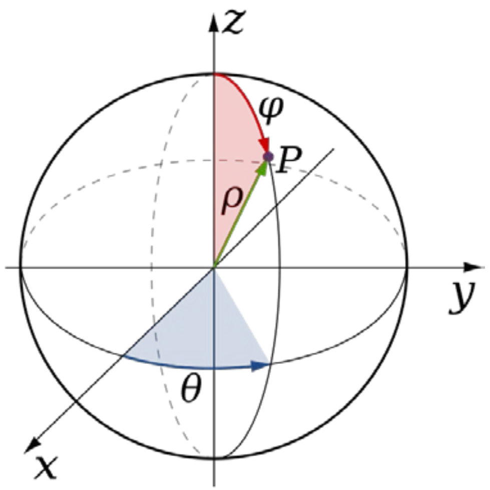
Spherical Wrist



Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

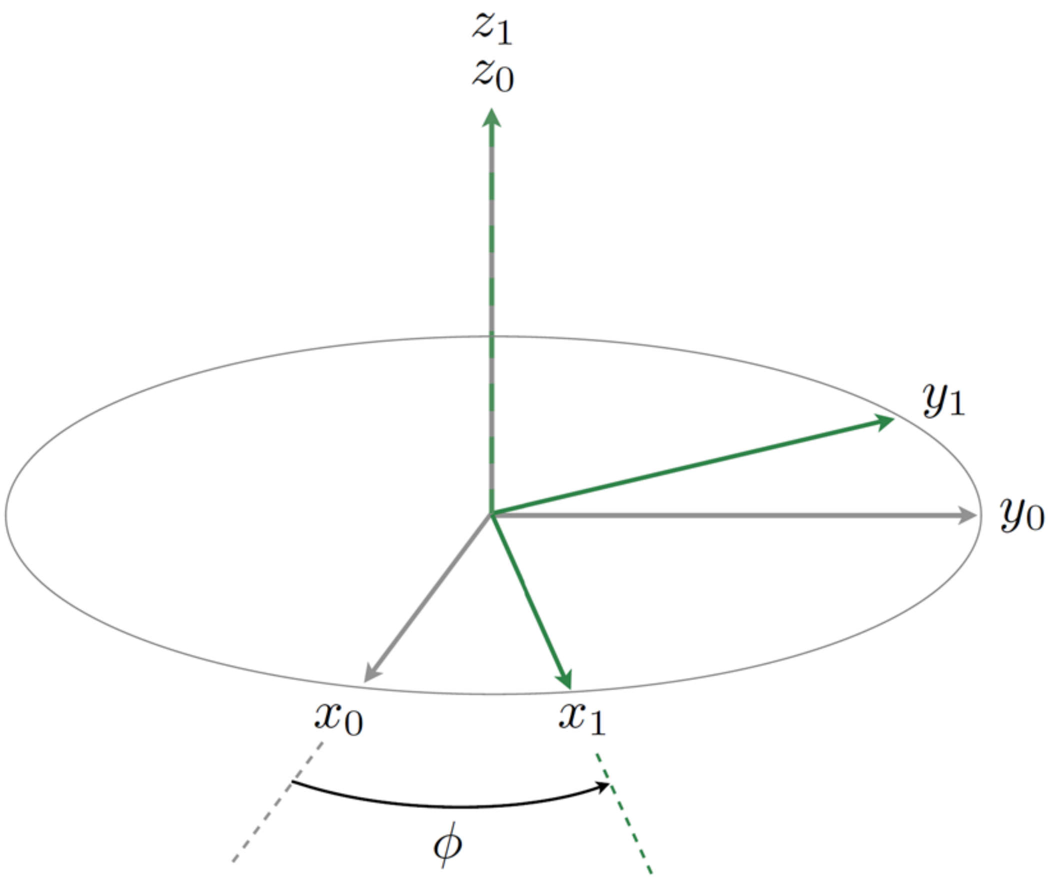


$$\begin{aligned} T_6^3 &= A_4 A_5 A_6 \\ &= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Our Focus Today

Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ, θ, ψ to go from one frame to another by rotating around the axes of the **current frame**.

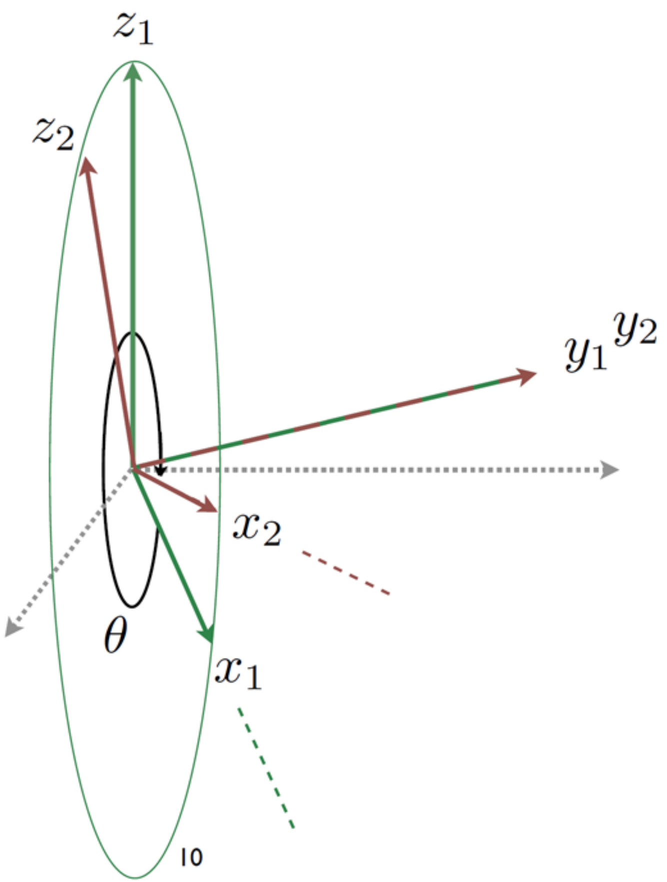


Using Z-Y-Z convention:

1. Rotate by ϕ about z_0

Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ, θ, ψ to go from 0 → 3
by rotating around the axes of the **current frame**.

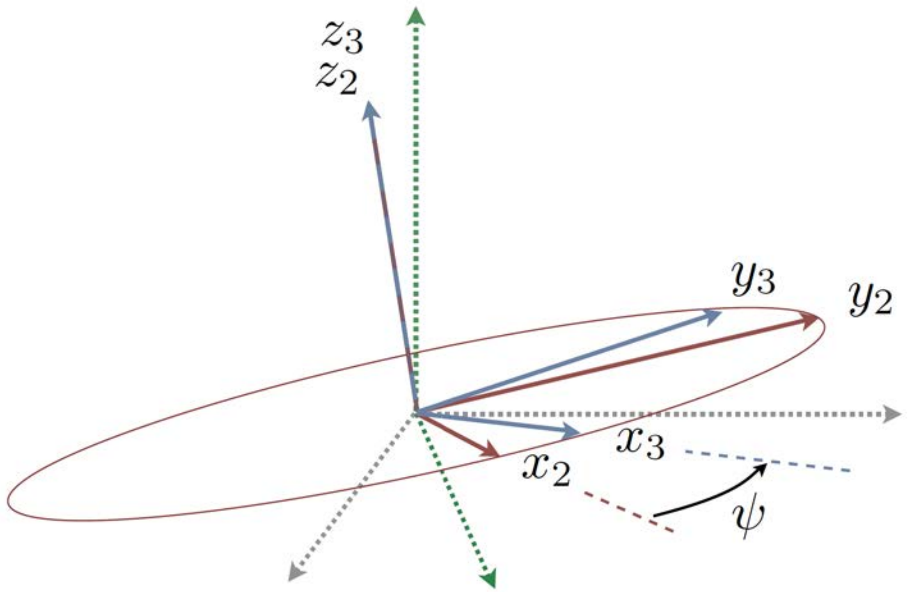


Using Z-Y-Z convention:

1. Rotate by ϕ about z_0
2. Rotate by θ about y_1

Euler Angles (from Lecture 3)

Define a set of 3 angles ϕ, θ, ψ to go from 0 → 3
by rotating around the axes of the **current frame**.



Using Z-Y-Z convention:

1. Rotate by ϕ about z_0
2. Rotate by θ about y_1
3. Rotate by ψ about z_2

Euler Angles (from Lecture 3)

Post-multiply using the **basic rotation matrices**

$$\mathbf{R} = \mathbf{R}_{z,\phi}\mathbf{R}_{y,\theta}\,\mathbf{R}_{z,\psi}$$

$$s_\theta = sin\,\theta\,, c_\theta = \cos\theta$$

$$\mathbf{R} = \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}\begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Euler Angles vs Spherical Wrists

$$
\begin{aligned}
T_6^3 &= A_4 A_5 A_6 \\
&= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$\theta_4 = \phi \qquad \theta_5 = \theta \qquad \theta_6 = \psi$$

$$
\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}
$$

How to calculate the three angles given **R**? See Lecture 3 and SHV pages 55-56.

Rotation Matrices to Euler Angles

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Plug in to solve for ϕ

Plug in to solve for ψ

Solve for θ

Two solutions for θ because sign of s_θ is not known.

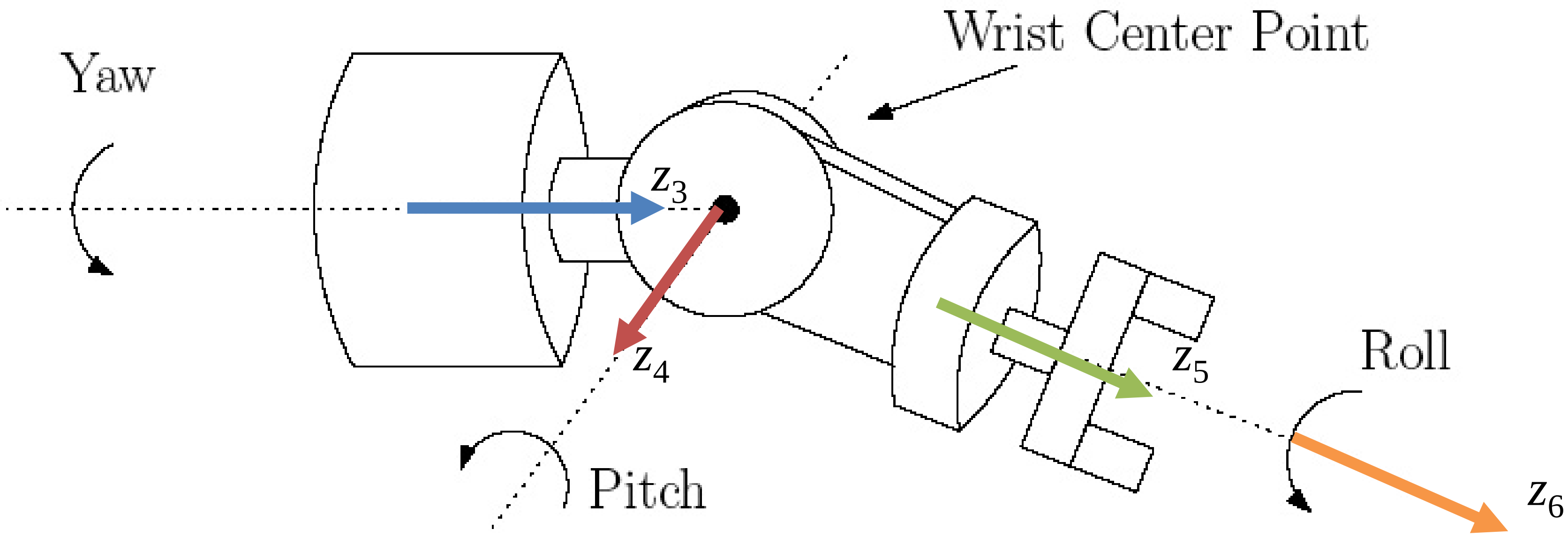
Geometric Interpretation of Solution Method

$$\begin{array}{cccc} & \hat{x}_6^3 & \hat{y}_6^3 & \hat{z}_6^3 \\ \mathbf{R}_6^3 = & \left[\begin{array}{c} c_\phi c_\theta c_\psi - s_\phi s_\psi \\ s_\phi c_\theta c_\psi + c_\phi s_\psi \\ -s_\theta c_\psi \end{array}\right. & \begin{array}{c} -c_\phi c_\theta s_\psi - s_\phi c_\psi \\ -s_\phi c_\theta s_\psi + c_\phi c_\psi \\ s_\theta s_\psi \end{array} & \left.\begin{array}{c} c_\phi s_\theta \\ s_\phi s_\theta \\ c_\theta \end{array}\right] \end{array}$$

Plug in to solve for ϕ

Plug in to solve for ψ

Solve for θ



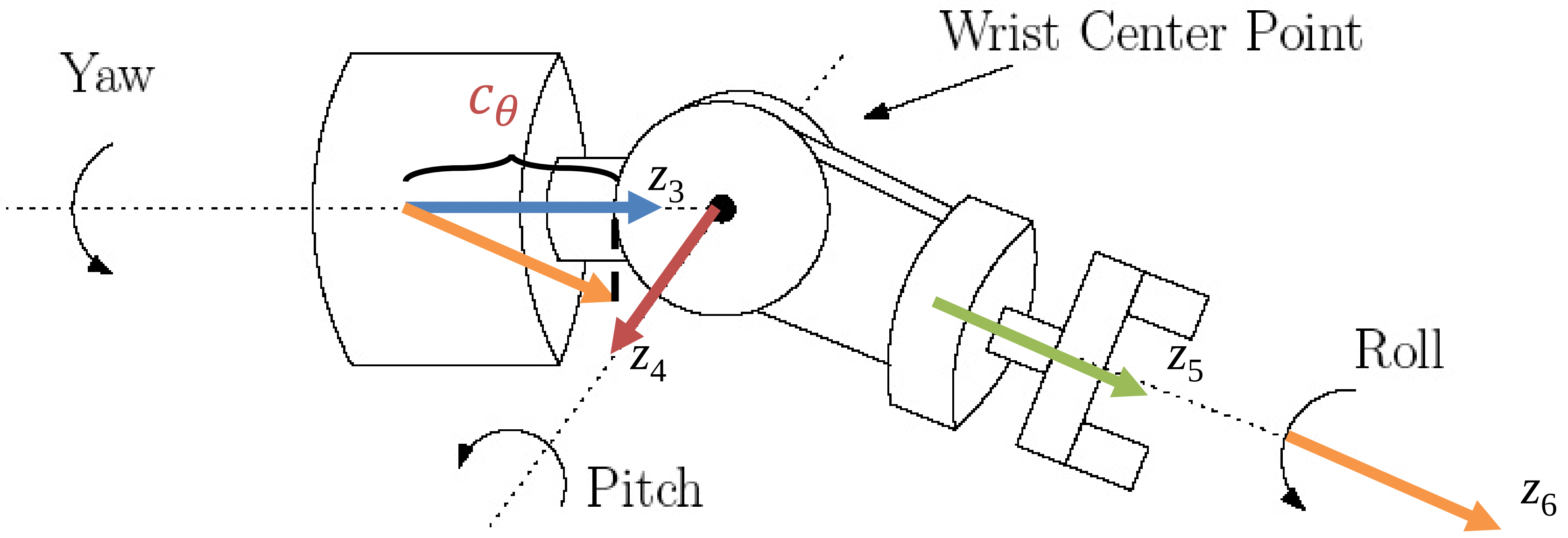
Geometric Interpretation of Solution Method

$$\begin{array}{cccc} & \hat{x}_6^3 & \hat{y}_6^3 & \hat{z}_6^3 \\ \mathbf{R}_6^3 = & \left[\begin{matrix} c_\phi c_\theta c_\psi - s_\phi s_\psi \\ s_\phi c_\theta c_\psi + c_\phi s_\psi \\ -s_\theta c_\psi \end{matrix}\right. & \begin{matrix} -c_\phi c_\theta s_\psi - s_\phi c_\psi \\ -s_\phi c_\theta s_\psi + c_\phi c_\psi \\ s_\theta s_\psi \end{matrix} & \left.\begin{matrix} c_\phi s_\theta \\ s_\phi s_\theta \\ c_\theta \end{matrix}\right] \end{array}$$

Plug in to solve forϕ

Plug in to solve for ψ

Solve for θ



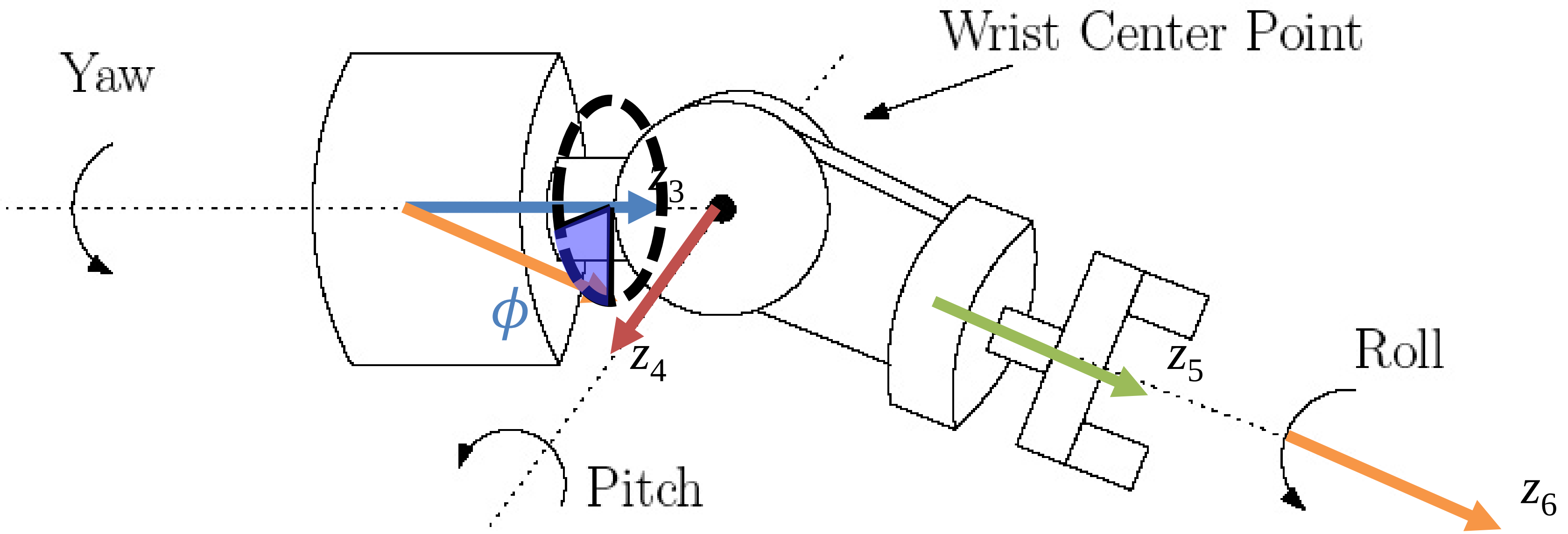
Geometric Interpretation of Solution Method

$$\mathbf{R}_6^3 = \begin{array}{c} \begin{array}{ccc} \hat{x}_6^3 & \hat{y}_6^3 & \hat{z}_6^3 \end{array} \\ \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix} \end{array}$$

Plug in to solve forϕ

Plug in to solve for ψ

Solve for θ



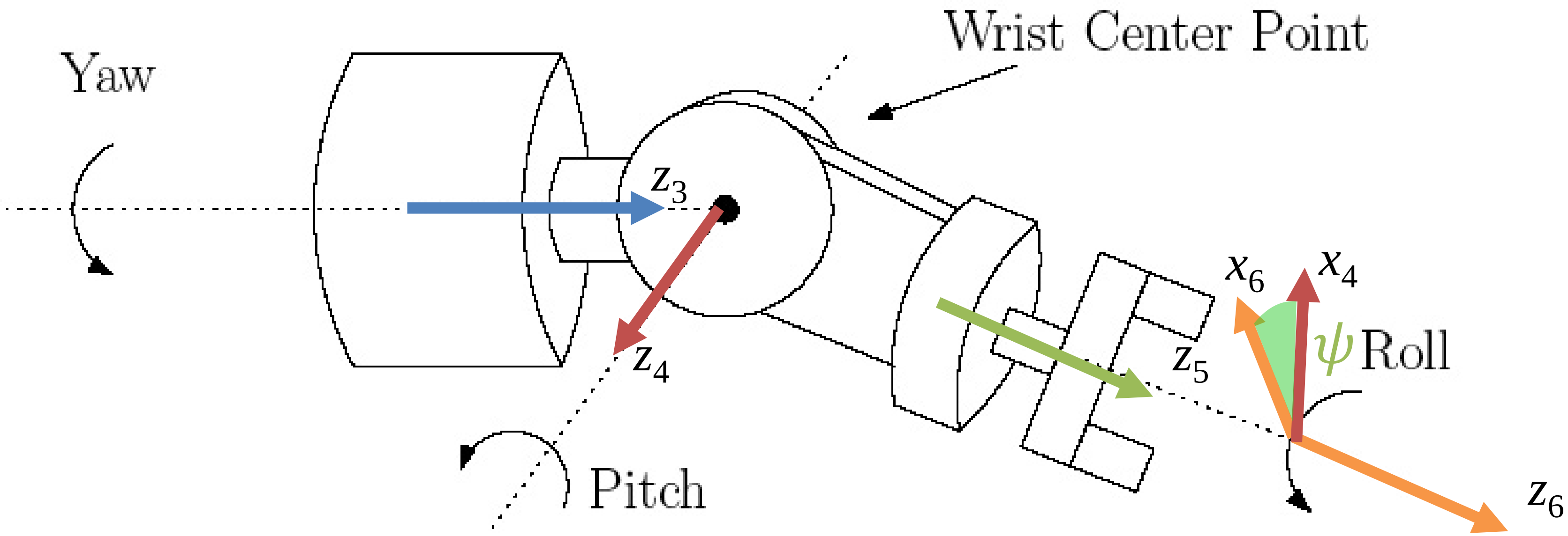
Geometric Interpretation of Solution Method

$$\begin{array}{cccc} & \hat{x}_6^3 & \hat{y}_6^3 & \hat{z}_6^3 \\ \mathbf{R}_6^3 = & \multicolumn{3}{c}{\begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}} \end{array}$$

Plug in to solve for ϕ

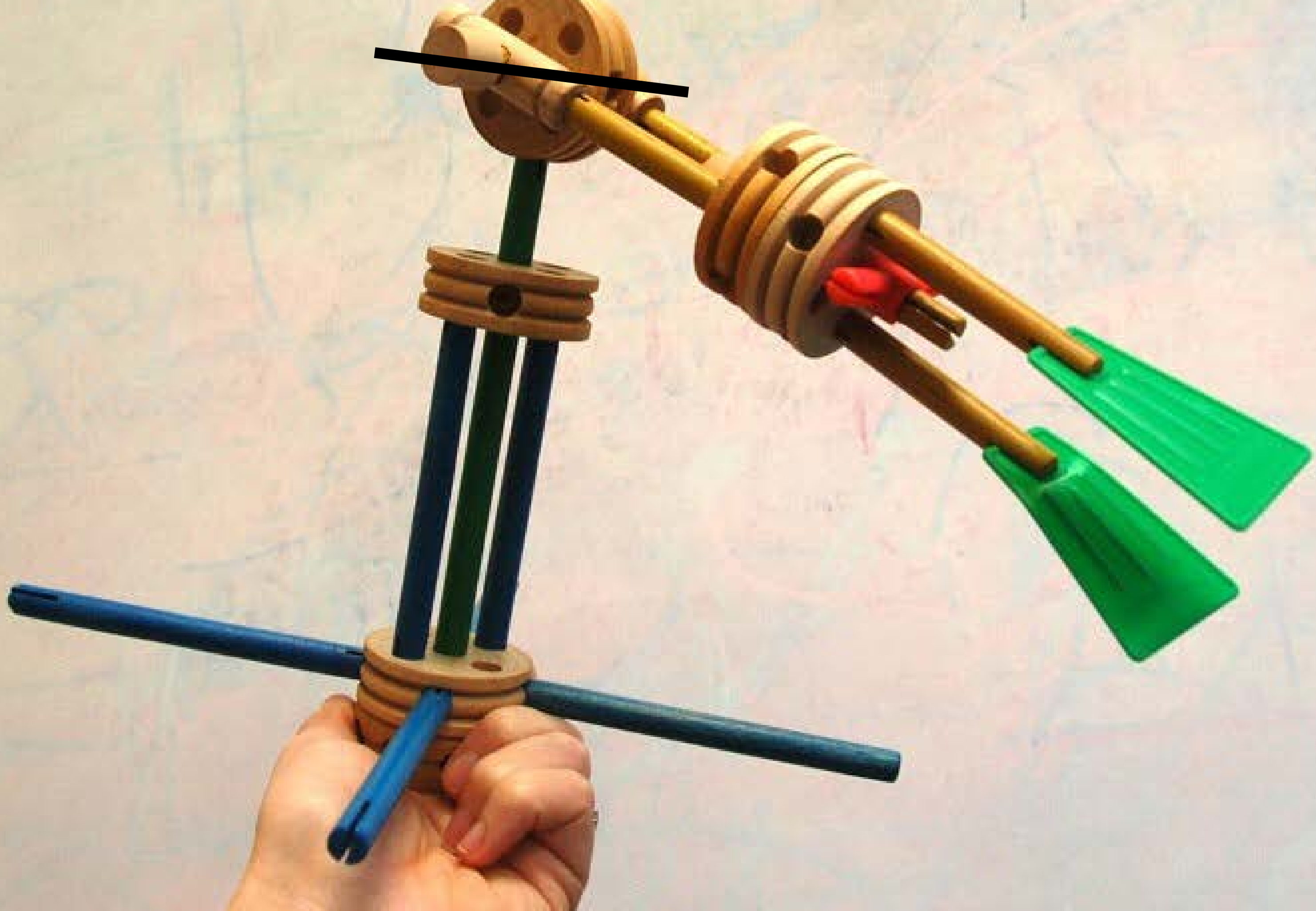
Plug in to solve for ψ

Solve for θ



What do the two solution sets mean geometrically?

Talk to a partner



Special case for $\cos\boldsymbol{\theta} = \pm\mathbf{1}$

To find a solution for this problem we break it down into two cases. First, suppose that not both of r_{13}, r_{23} are zero. Then from Equation (2.26) we deduce that $s_\theta \neq 0$, and hence that not both of r_{31}, r_{32} are zero. If not both r_{13} and r_{23} are zero, then $r_{33} \neq \pm 1$, and we have $c_\theta = r_{33}$, $s_\theta = \pm\sqrt{1 - r_{33}^2}$ so

$$\theta = \mathrm{Atan2}\left(r_{33}, \sqrt{1 - r_{33}^2}\right) \tag{2.28}$$

or

$$\theta = \mathrm{Atan2}\left(r_{33}, -\sqrt{1 - r_{33}^2}\right) \tag{2.29}$$

where the function Atan2 is the **two-argument arctangent function** defined in Appendix A.

If we choose the value for θ given by Equation (2.28), then $s_\theta > 0$, and

$$\phi = \mathrm{Atan2}(r_{13}, r_{23}) \tag{2.30}$$
$$\psi = \mathrm{Atan2}(-r_{31}, r_{32}) \tag{2.31}$$

If we choose the value for θ given by Equation (2.29), then $s_\theta < 0$, and

$$\phi = \mathrm{Atan2}(-r_{13}, -r_{23}) \tag{2.32}$$
$$\psi = \mathrm{Atan2}(r_{31}, -r_{32}) \tag{2.33}$$

Thus, there are two solutions depending on the sign chosen for θ.

If $r_{13} = r_{23} = 0$, then the fact that R is orthogonal implies that $r_{33} = \pm 1$, and that $r_{31} = r_{32} = 0$. Thus, R has the form

$$R = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix} \tag{2.34}$$

If $r_{33} = 1$, then $c_\theta = 1$ and $s_\theta = 0$, so that $\theta = 0$. In this case, Equation (2.26) becomes

$$\begin{bmatrix} c_\phi c_\psi - s_\phi s_\psi & -c_\phi s_\psi - s_\phi c_\psi & 0 \\ s_\phi c_\psi + c_\phi s_\psi & -s_\phi s_\psi + c_\phi c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\phi+\psi} & -s_{\phi+\psi} & 0 \\ s_{\phi+\psi} & c_{\phi+\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, the sum $\phi + \psi$ can be determined as

$$\phi + \psi = \mathrm{Atan2}(r_{11}, r_{21}) = \mathrm{Atan2}(r_{11}, -r_{12}) \tag{2.35}$$

Since only the sum $\phi + \psi$ can be determined in this case, there are infinitely many solutions. In this case, we may take $\phi = 0$ by convention. If $r_{33} = -1$, then $c_\theta = -1$ and $s_\theta = 0$, so that $\theta = \pi$. In this case Equation (2.26) becomes

$$\begin{bmatrix} -c_{\phi-\psi} & -s_{\phi-\psi} & 0 \\ s_{\phi-\psi} & c_{\phi-\psi} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{2.36}$$

The solution is thus

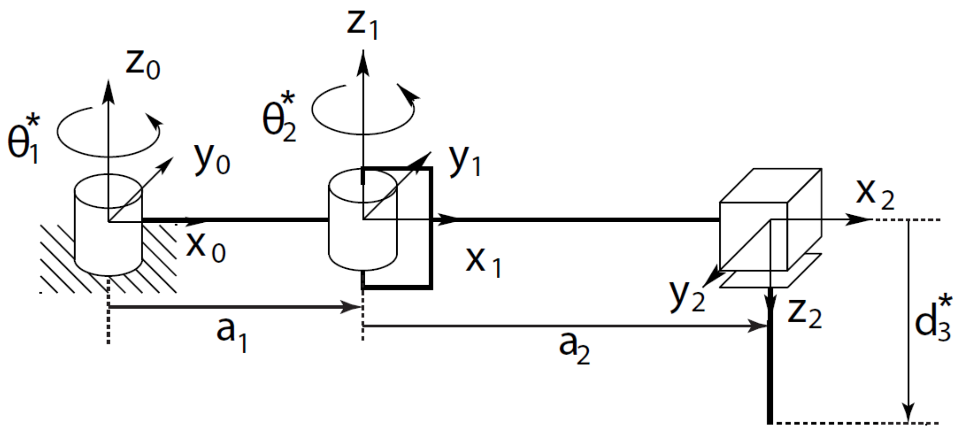
$$\phi - \psi = \mathrm{Atan2}(-r_{11}, -r_{12}) \tag{2.37}$$

As before there are infinitely many solutions.

What does it mean when $\cos\boldsymbol{\theta} = +\mathbf{1}$?

What does it mean when $\cos\boldsymbol{\theta} = -\mathbf{1}$?

SCARA + spherical wrist tracing out a circle



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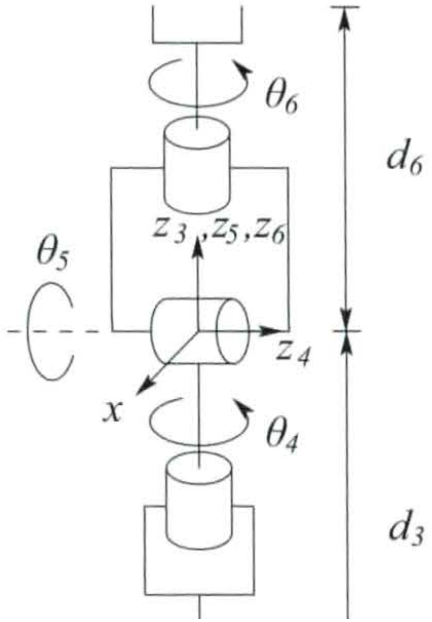


SCARA + spherical wrist tracing out a circle

```
%% scara_circle_starter.m
%
% This Matlab script provides solves the SCARA robot's inverse kinematics.
% It was modified from the original starter code provided by
% Professor Katherine J. Kuchenbecker (kuchenbe@seas.upenn.edu)
% for Homework 6 in MEAM 520.
%


%% SETUP

% Define our time vector.
tStart = 0;    % The time at which the simulation starts, in seconds.
tStep = 0.01; % The simulation's time step, in seconds.
tEnd = 2*pi;     % The time at which the simulation ends, in seconds.
t = (tStart:tStep:tEnd)';  % The time vector (a column vector).

% Set whether to animate the robot's movement and how much to slow it down.
pause on;  % Set this to off if you don't want to watch the animation.
GraphingTimeDelay = 0.002; % How long Matlab should pause between positions when graphing, if


%% ROBOT PARAMETERS

% This problem is about the first three joints (RRP) of a SCARA
% manipulator.  This robot's forward kinematics are worked out on
% pages 91 to 93 of the SHV textbook, although we are replacing the
% fourth joint with a full spherical wrist.

% Define robot link lengths.
a1 = 1.0; % Distance between joints 1 and 2, in meters.
a2 = 0.7 * a1; % Distance between joints 2 and 3, in meters.
d6 = 0.25 * a1; % Offset from wrist center to end-effector, in meters.

```

SCARA + spherical wrist tracing out a circle

```
%% DEFINE CIRCULAR MOTION

% We want the SCARA to draw a vertical circle parallel to the x-z
% plane.

% Define the radius of the circle.
radius = .5; % meters

% Define the y-value for the plane that contains the circle.
y_offset = -1; % meters

% Define the x and z coordinates for the center of the circle.
x_center = -1; % meters
z_center = -1; % meters

% Set the desired x, y, and z positions over time given the circle
% parameters.
ox_history = x_center + radius * sin(t);
oy_history = y_offset * ones(size(t));
oz_history = z_center + radius * cos(t);
```

SCARA + spherical wrist tracing out a circle

```
% Step through the time vector to animate the robot.
for i = 1:length(t)

    % Pull the current values of ox, oy, and oz from their histories.
    ox = ox_history(i);
    oy = oy_history(i);
    oz = oz_history(i);

    % Calculate where the wrist center position needs to be.
    oc = [ox oy oz]' - d6*R(:,3);

    % Pull out the components of wrist center and store in ox, oy, oz.
    ox = oc(1);
    oy = oc(2);
    oz = oc(3);

    % Calculate theta1, theta2, and d3 given the robot's parameters
    % (a1 and a2) and the current desired position for its tip ([ox oy
    % oz]').

    % Calculate the cosine of theta2 using law of cosines.
    c2 = (ox.^2 + oy.^2 - a1^2 - a2^2)/(2*a1*a2);

    % Calculate the positive and negative solutions for theta2.
    theta2_pos = atan2(sqrt(1-c2^2),c2); % Corrected solution from our derivation.
    theta2_neg = atan2(-sqrt(1-c2^2),c2); % Corrected solution from our derivation.

    % Choose the positive solution.
    theta2 = theta2_pos;

```

$$d_3 = -o_z$$

$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\theta_2 = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)$$

$$\theta_1 = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

SCARA + spherical wrist tracing out a circle

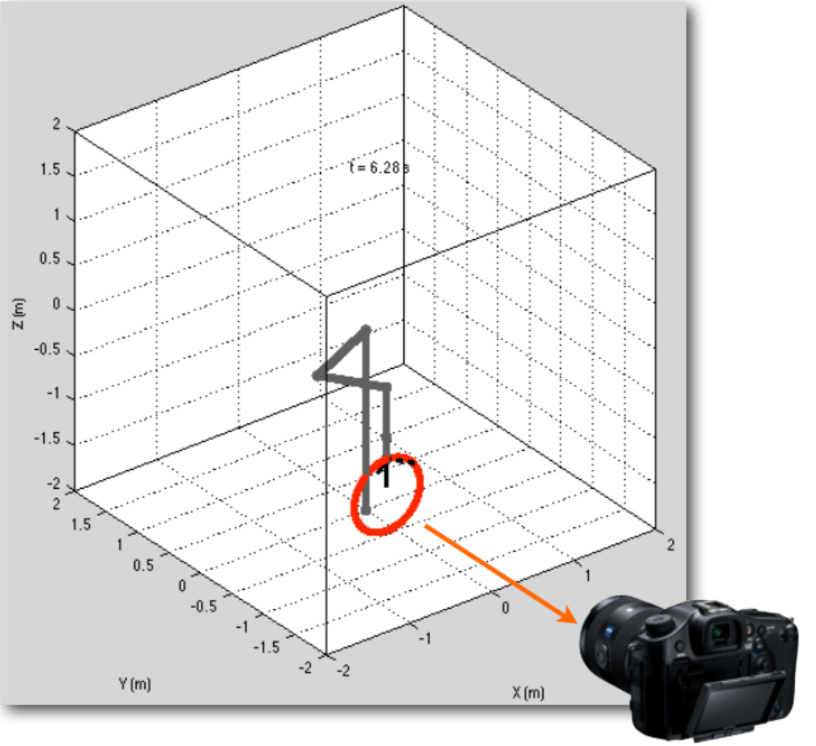
```
    % Choose the positive solution.
    theta2 = theta2_pos;

    % Uncomment this line to choose the negative solution instead.
    %theta2 = theta2_neg;

    % Calculate theta1 using a pair of inverse tangents.  Note that
    % MATLAB atan2 takes the numerator and then the denominator.
    theta1 = atan2(oy,ox) - atan2(a2*sin(theta2),a1+a2*cos(theta2));

    % Calculate d3.
    d3 = -oz; % Corrected solution from our derivation.

    % ************************************************************
    % All of your code should go between the lines of asterisks.

    % Solve for the values of theta4, theta5, and theta6 that will put
    % the end-effector frame at the desired orientation R relative to
    % the base frame, given the present values of theta1, theta2, and
    % d3.

    % Set the values of theta4, theta5, and theta6 arbitrarily to
    % zero. You will need to change this!
    theta4 = 0;
    theta5 = 0;
    theta6 = 0;
```

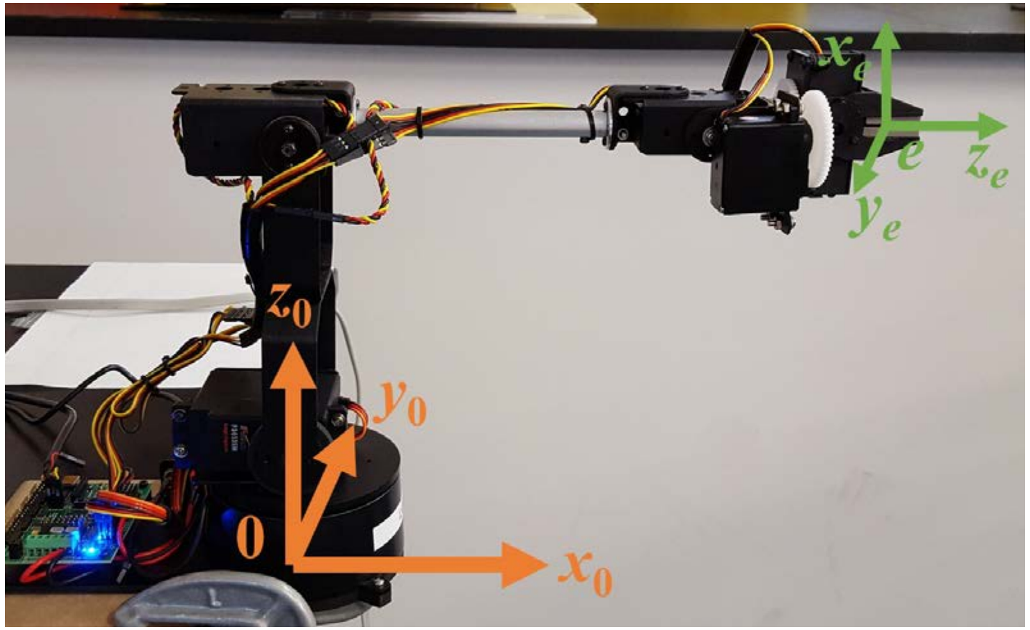
Questions for you

- The SCARA has been programmed to place its wrist center at the location needed to draw a flat circle. When instead it keeps all three of its wrist joints at zero, how does the orientation of the end-effector frame change over time? Sketch / explain.

- What shape will the tip of the SCARA draw if we set the 5th joint at 90 deg. And keep joints 4 and 6 at 0 deg.?

- We want the end-effector's z axis to point out toward the camera with its y-axis straight up (vertical). What rotation matrix **R** should we give to the inverse orientation kinematics?



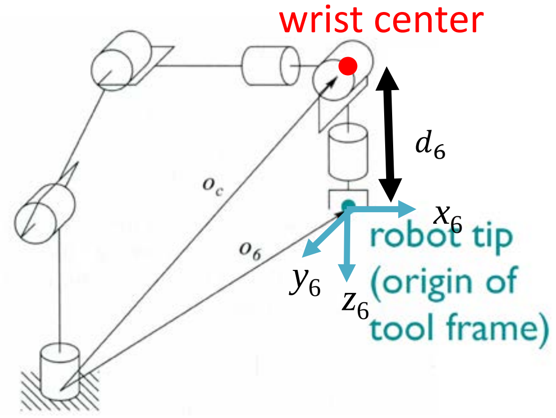


What if the wrist is not spherical?

Can we use Kinematic Decoupling?

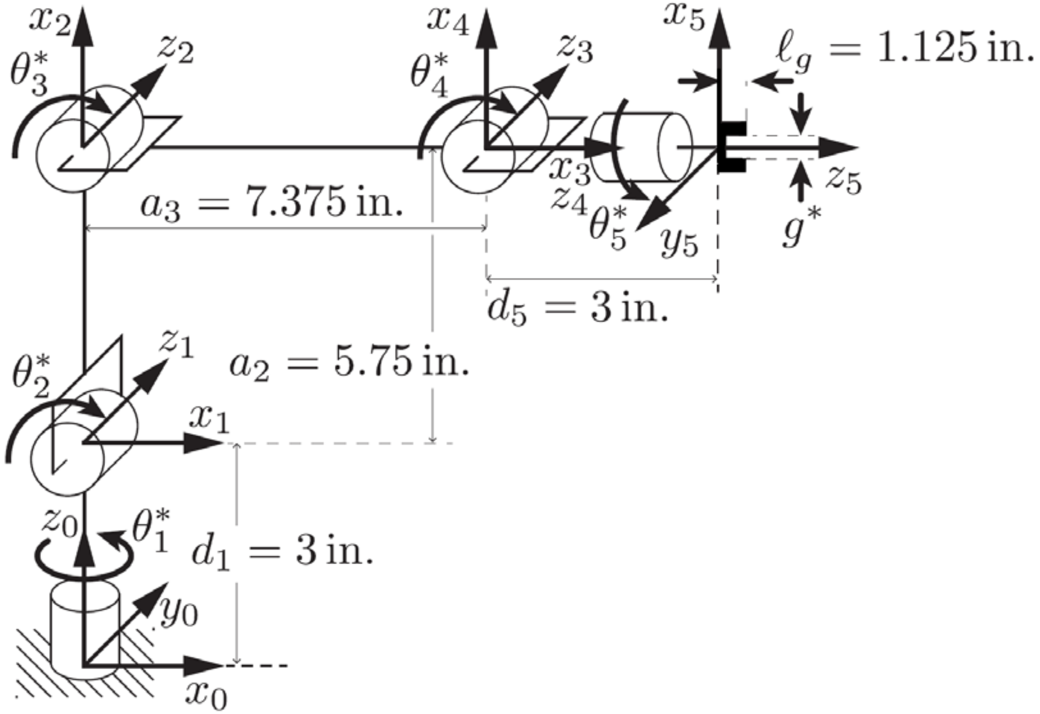
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position



$$\mathbf{R}_6^3 = (\mathbf{R}_3^0)^{-1}\mathbf{R} = (\mathbf{R}_3^0)^{\mathrm{T}}\mathbf{R}$$

orientation



Next time: Quaternions!

Funda, J. and Paul, R.P. "A comparison of transforms and quaternions in robotics." Proceedings of the IEEE Conference on Robotics and Automation (ICRA). 1998. pp 886-891. doi: 10.1109/ROBOT.1988.12172

A Comparison of Transforms and Quaternions in Robotics

James Funda   Richard P. Paul

University of Pennsylvania, Philadelphia

"A comparison of transforms and quaternions in robotics" (ICRA 1998)

Lab 2: Inverse Kinematics

MEAM 520, University of Pennsylvania

September 18, 2017

This exercise is due on **Wednesday, October 4, by midnight (11:59 p.m.)** Late submissions will be accepted until midnight on Friday, October 6, but they will be penalized by 10% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation such as illness. This assignment is worth 25 points.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

You may do this assignment either individually or with a partner. If you do this lab with a partner, you may work with anyone you choose, but you must work with them for all parts of this assignment. Looking for a partner? Try the 'Search for Teammates!' tool on Piazza.

If you are in a pair, you will both turn in the same report and code (see Submission Instructions below), for which you are jointly responsible and you will both receive the same grade. Work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Wililams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Supplemental Material.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other.* You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Lab 2: Inverse Kinematics due 10/3

- You can now do all the tasks