MEAM 520 Lecture 23: Actuation and Control

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Previously: Manipulator Equation

We can write this as a matrix equation

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

SHV uses a bit of strange notation. Most people call this matrix *H* or *M*.

where

D(q) is the nxn mass matrix (inertia terms)

 $C(q,\dot{q})$ is the nxn matrix of centrifugal (square of joint velocities) and Coriolis (product of two different joint velocities) terms

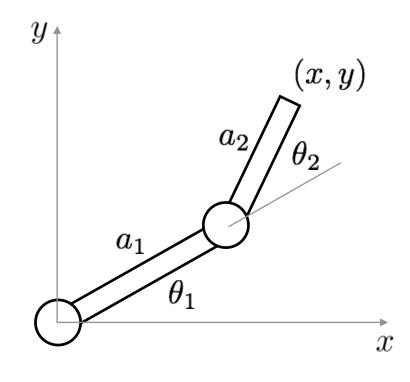
g(q) is a nx1 vector of gravitational terms

Previously: Euler-Lagrange Method

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

$$D = \sum_{i=1}^{N} \left(m_i J_{vci}^{\mathsf{T}} J_{vci} + J_{\omega i}^{\mathsf{T}} R_i I_i R_i^{\mathsf{T}} J_{\omega i} \right)$$

$$g = \frac{\partial}{\partial q} \sum_{i=1}^{N} m_i \vec{g} \cdot \vec{r}_i$$



$$(C\dot{q})_{k} = \sum_{i,j} \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_{i}} + \frac{\partial d_{ki}}{\partial q_{j}} - \frac{\partial d_{ij}}{\partial q_{k}} \right) \dot{q}_{i} \dot{q}_{j}$$
 or
$$c_{kj} = \sum_{i} \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_{i}} + \frac{\partial d_{ki}}{\partial q_{j}} - \frac{\partial d_{ij}}{\partial q_{k}} \right) \dot{q}_{i}$$

Previously: Newton-Euler (for revolute joints)

Start with
$$\omega_0 = 0$$
, $\alpha_0 = 0$, $a_{c,0} = 0$, $a_{e,0} = 0$

Solve kinematic constraints for *i* from 1 to *n*

Start with
$$f_{n+1} = 0$$
, $\tau_{n+1} = 0$

Solve force/moments for i from n to 1

Kinematic constraints

No forces/moments!
$$\begin{cases} a_i = R^i_{i-1} \alpha_{i-1} + z^i_{i-1} \dot{q}_i \\ \alpha_i = R^i_{i-1} \alpha_{i-1} + z^i_{i-1} \ddot{q}_i + \omega_i \times z^i_{i-1} \dot{q}_i \\ a_{e,i} = R^i_{i-1} a_{e,i-1} + \dot{\omega}_i \times r_{i,i+1} + \omega_i \times \left(\omega_i \times r_{i,i+1}\right) \\ a_{c,i} = R^i_{i-1} a_{e,i-1} + \dot{\omega}_i \times r_{i,c_i} + \omega_i \times \left(\omega_i \times r_{i,c_i}\right) \end{cases}$$

Forces/Moments

$$f_{i} - R_{i+1}^{i} f_{i+1} + m_{i} g_{i} = m_{i} a_{c,i}$$

$$\tau_{i} - R_{i+1}^{i} \tau_{i+1} + f_{i} \times r_{i,c_{i}} - (R_{i+1}^{i} f_{i+1}) \times r_{i+1,c_{i}} = I_{i} \dot{\omega}_{i} + \omega_{i} \times (I_{i} \omega_{i})$$

i-1 terms on the right

Previously: Method Comparisons

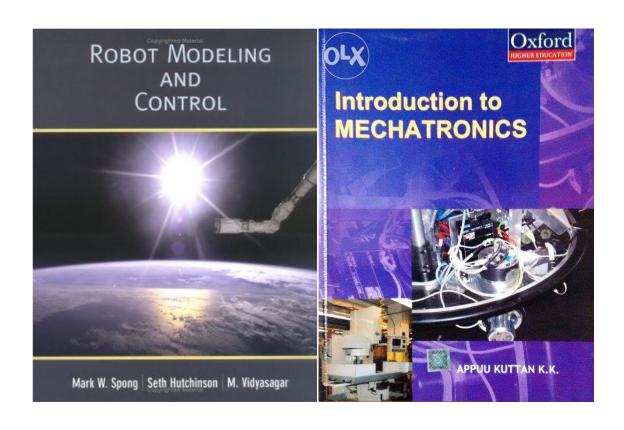
Newton-Euler

- Complete solution for all forces and kinematic variables
- Inefficient when only a few of the system's forces need to be solved for

Euler-Lagrange

- Disregard all interactive and constraint forces that do not perform work
- Need to differentiate scalar energy functions
- Inefficient for large multibody systems

How do we make a robot move to a particular location?



Chapter 6: Independent Joint Control

Read 6.intro-6.4

AKKK: 7.2 - 7.3

More details in
ESE 500 Linear Systems
MEAM 513 Control Systems

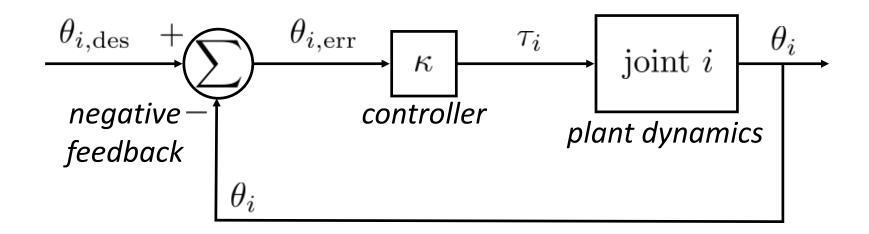
Desired Joint Angles

Actual Joint Angles

$$\theta_{1,\text{des}}, \theta_{2,\text{des}}, \theta_{3,\text{des}} \dots$$

$$\theta_1, \theta_2, \theta_3 \dots$$

Proportional Feedback Controller



Desired Joint Angles

Actual Joint Angles

$$\theta_{1,\text{des}}, \theta_{2,\text{des}}, \theta_{3,\text{des}} \dots$$

$$\theta_1, \theta_2, \theta_3 \dots$$

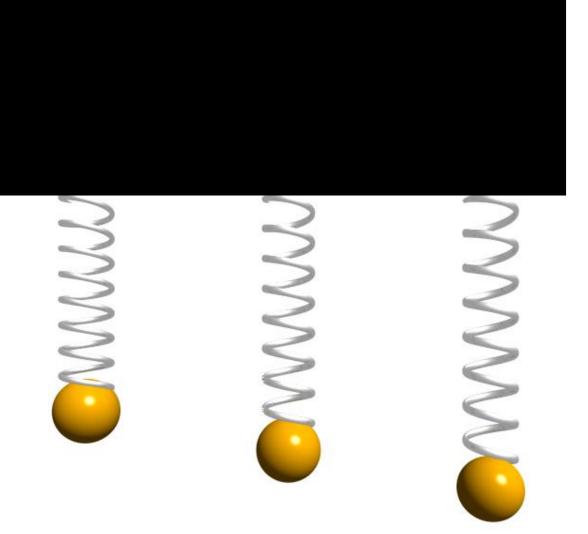
Proportional Feedback Controller

$$\begin{array}{c} \tau_1 \\ \text{joint} \\ \tau_2 \\ \text{torques} \end{array} = \begin{array}{c} \kappa(\theta_{1,\mathrm{des}} - \theta_1) \\ \kappa(\theta_{2,\mathrm{des}} - \theta_2) \\ = \kappa(\theta_{3,\mathrm{des}} - \theta_3) \end{array} \text{joint angle}$$

proportional gain in Nm / rad



Proportional feedback acts like a torsional spring with linear stiffness, pulling each joint to the desired angle.

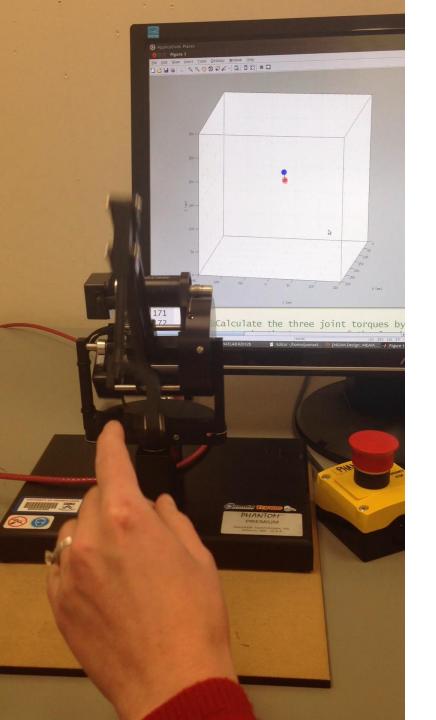


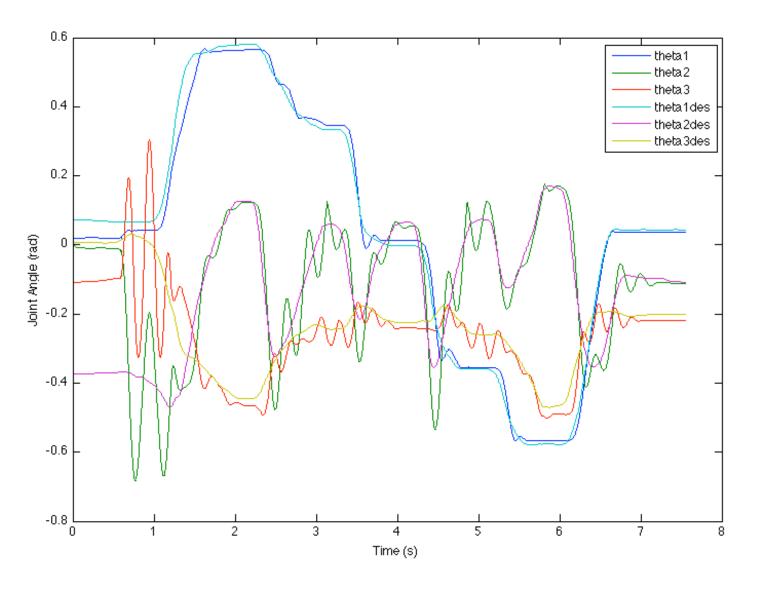
Mass on a spring: simple harmonic oscillator

$$\tau_i = \kappa (\theta_{i,des} - \theta_i)$$

$$f_1(q)\ddot{q} + f_2(q,\dot{q}) = \kappa(\theta_{i,des} - \theta_i)$$

$$f_1(q)\ddot{q} = \left[\kappa(\theta_{i,des} - \theta_i) - f_2(q, \dot{q})\right]$$





It's pretty oscillatory.

How can we improve the controller's tracking?

Add derivative feedback – virtual damping.

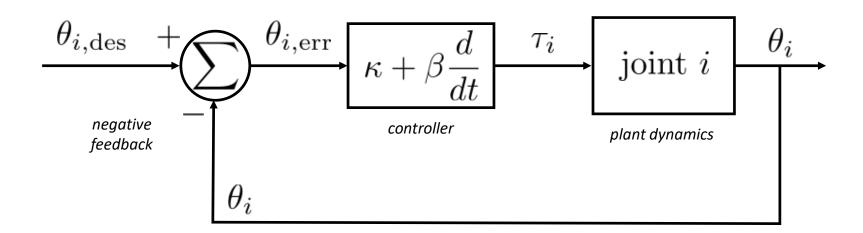
Desired Joint Angles

Actual Joint Angles

$$\theta_{1,\text{des}}, \theta_{2,\text{des}}, \theta_{3,\text{des}} \dots$$

$$\theta_1, \theta_2, \theta_3 \dots$$

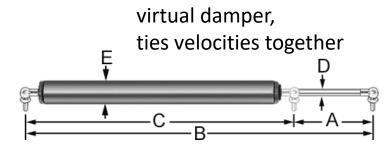
Proportional Derivative Feedback Controller



Add a derivative term to our position feedback controller, making it a Proportional Derivative (PD) controller.

virtual spring, ties positions together





$$\tau_1 = \kappa(\theta_{1,\text{des}} - \theta_1) + \beta(\omega_{1,\text{des}} - \omega_1)$$

$$\tau_2 = \kappa (\theta_{2,\text{des}} - \theta_2) + \beta (\omega_{2,\text{des}} - \omega_2)$$

The gains are typically tuned separately for each joint.

$$\tau_3 = \kappa(\theta_{3,\text{des}} - \theta_3) + \beta(\omega_{3,\text{des}} - \omega_3)$$

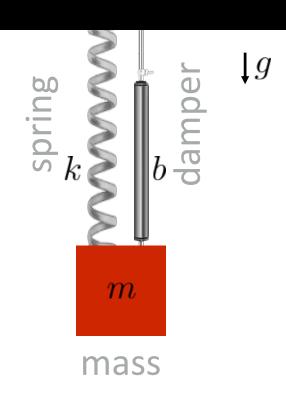
proportional gain in Nm / rad

derivative gain in Nm / (rad/s)

$$\theta_{i,\text{err}} = \theta_{i,\text{des}} - \theta_i$$

$$\dot{\theta}_{i,\mathrm{err}} = \omega_{i,\mathrm{des}} - \omega_i$$

$$\tau_i = \kappa \, \theta_{i, \text{err}} + \beta \, \dot{\theta}_{i, \text{err}}$$



$$\Sigma F_y = m\ddot{y}$$

$$-mg - ky - b\dot{y} = m\ddot{y}$$

$$-mg = m\ddot{y} + b\dot{y} + ky$$

$$-g = \ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y$$
Second-order system $\frac{k}{m} = \omega_n^2$

$$-g = \ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2y \qquad natural$$

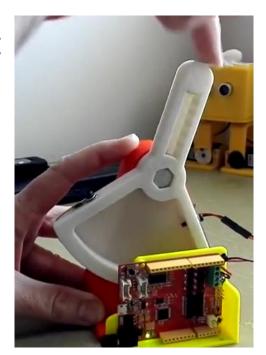
$$frequency$$

$$\frac{b}{m} = 2\zeta\omega_n \qquad k_{\rm controller} = m\omega_{n,{\rm desired}}^2$$

 $b_{
m controller}=2m\zeta_{
m desired}\omega_n-b_{
m robot}$ usually, $\zeta_{
m desired}=1$ damping ratio

What are the effects of the gains?

- The system goes unstable if either k_p or k_d are negative
- The system is critically damped if $\zeta = \frac{b}{2m\omega_n} = 1$
- For a fast response, k_p should be as high as possible, subject to saturation, chattering, etc.
- With a constant disturbance D (e.g., gravity), the constant offset with PD control is $-\frac{D}{k_p}$



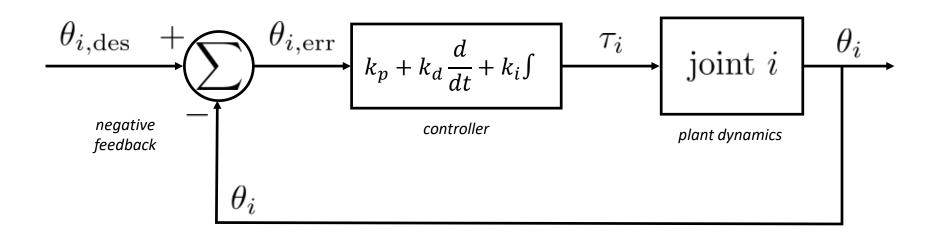
Desired Joint Angles

Actual Joint Angles

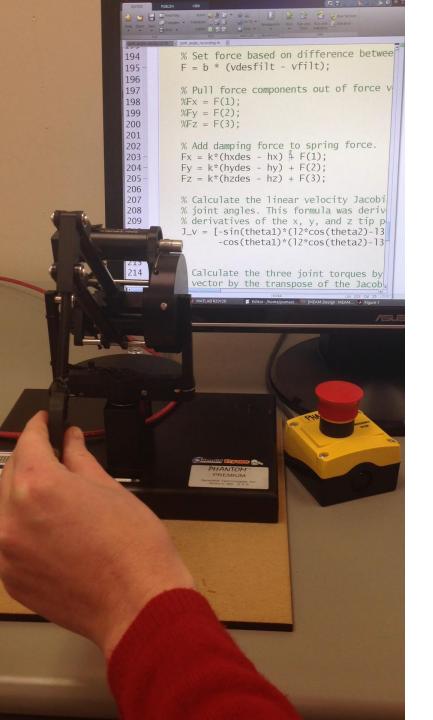
$$\theta_{1,\text{des}}, \theta_{2,\text{des}}, \theta_{3,\text{des}} \dots$$

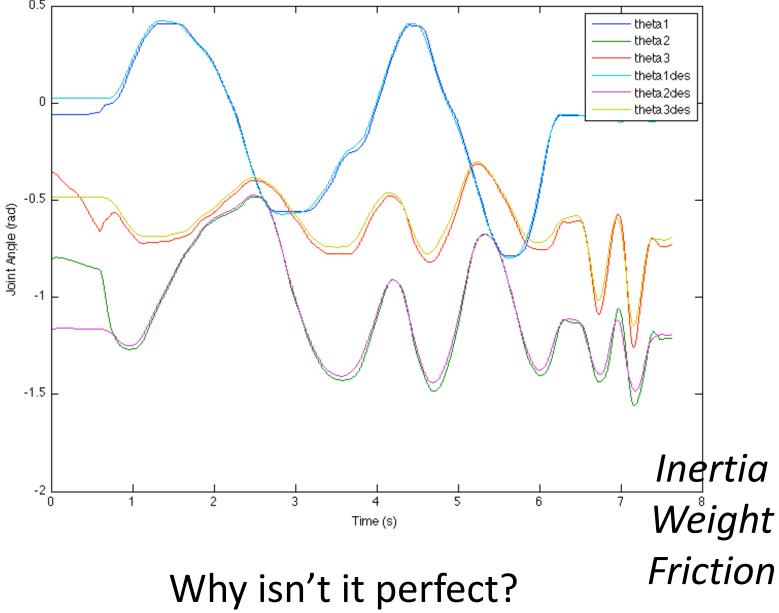
$$\theta_1, \theta_2, \theta_3 \dots$$

Proportional Integral Derivative Feedback Controller



$$\tau_i = k_p (\theta_{i,des} - \theta_i) + k_d (\omega_{i,des} - \omega_i) + k_i \int \theta_{i,des} - \theta_i$$



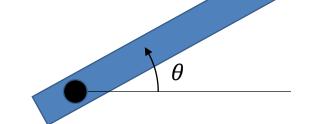


Why isn't it perfect?

The robot's dynamics interfere with tracking.

Example: Single Link Manipulator (i.e., Pendulum)

EOM:
$$m\ddot{\theta} + b_m\dot{\theta} = \tau + mgl\cos\theta$$



Controller:
$$\tau = k_p \theta_e + k_i \int \theta_e dt + k_d \dot{\theta}_e$$

System dynamics:

$$m\ddot{\theta}_e + (b_m + k_d)\dot{\theta}_e + k_p\theta_e + k_i\int\theta_e dt = mgl\cos\theta$$

The inertia, weight, and friction of the robot all interfere with tracking.

Robot designers generally try to minimize the inertia (mass & mass distribution) of the robot so it can accelerate more quickly and be less affected by gravity.

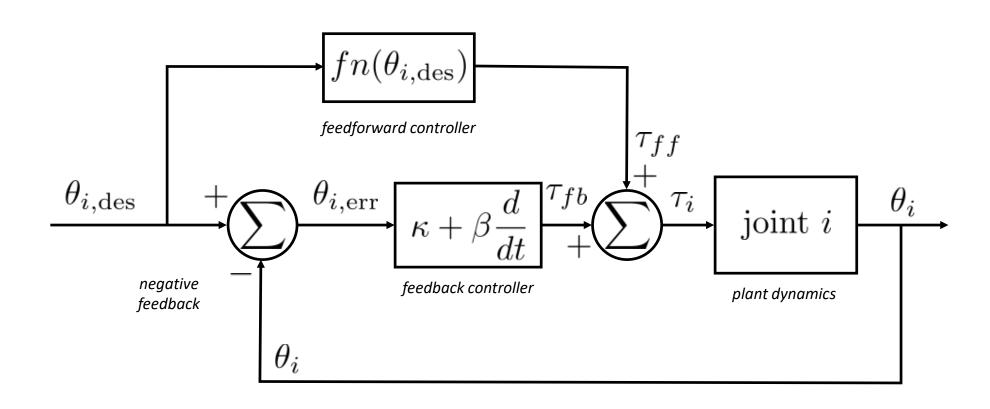
Similarly, robot designers try to minimize the **friction** of the robot so that the start of motion is **smooth** and sustained motion doesn't require much torque.

How can we improve the controller's tracking?

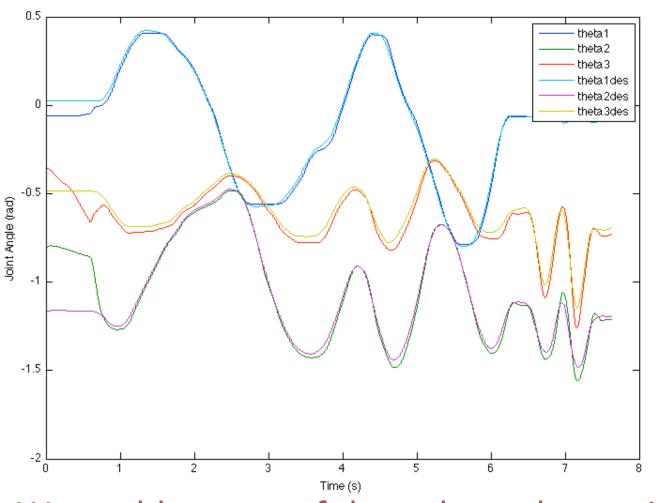
Try to compensate for the robot's dynamics in advance, instead of just reacting to errors when they occur.

This approach is called **feedforward control**, and it's very powerful for tracking time-varying trajectories.

Adding a Feedforward Term to the PD Controller



Which aspect of the robot dynamics can we feedforward?



We could try any of them, but robot weight is generally the easiest and most useful.

Inertia
Weight
Friction

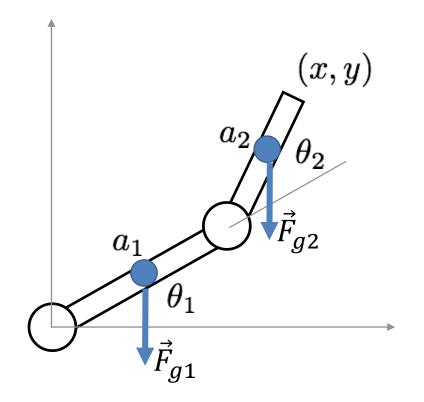
Previously: Gravitational Force/Torque

$$\vec{\tau}^{\mathsf{T}} d\vec{q} = \vec{F}^{\mathsf{T}} d\vec{x}$$

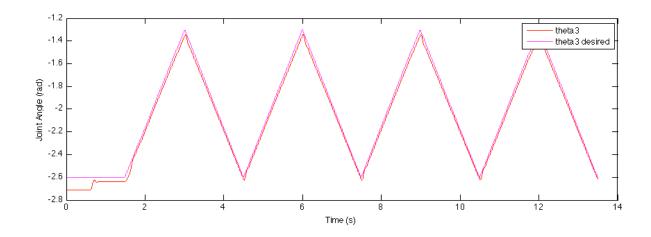
$$\vec{\tau}^{\mathsf{T}} d\vec{q} = \sum_{i=1}^{n} \vec{F}_{gi}^{\mathsf{T}} d\vec{x}_{i}$$

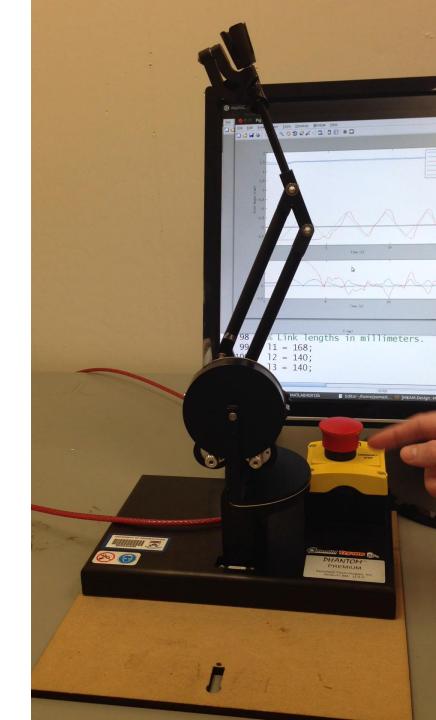
$$\vec{\tau}^{\mathsf{T}} d\vec{q} = \sum_{i=1}^{n} \vec{F}_{gi}^{\mathsf{T}} J_{i} d\vec{q}$$

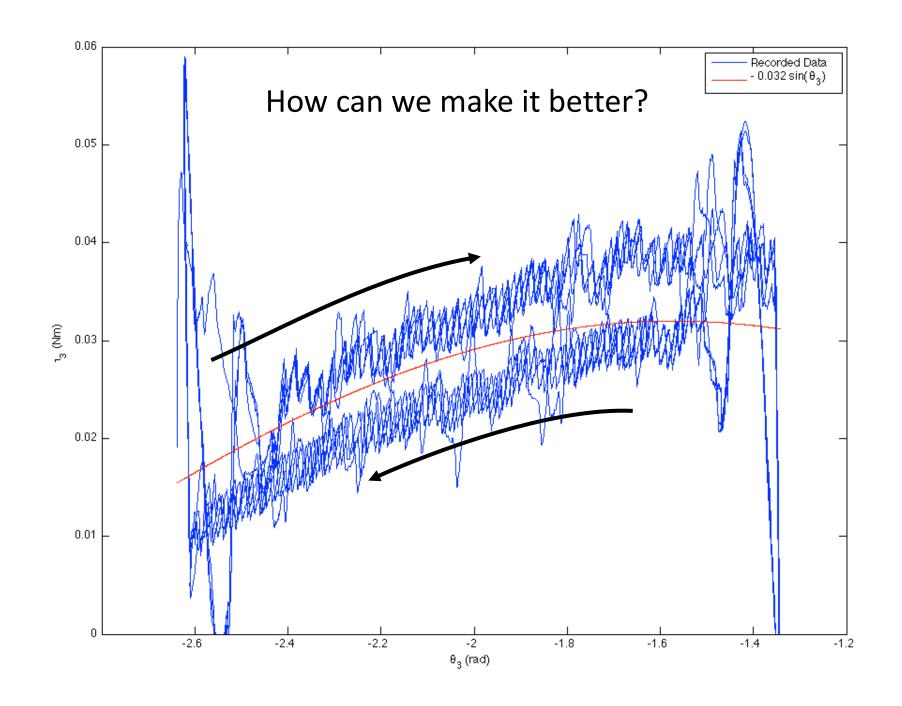
$$\vec{\tau} = \sum_{i=1}^{n} J_{i}^{\mathsf{T}} \vec{F}_{gi}$$



Another Option: Record torque on robot through a trajectory



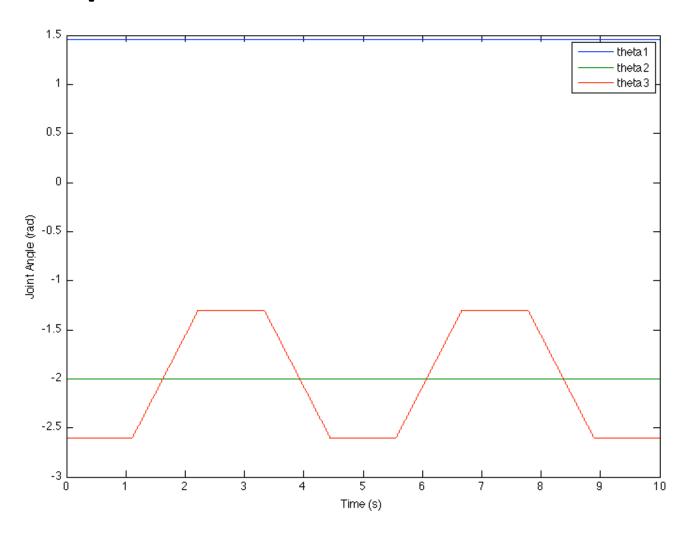




Inertia Weight Friction

How does each of these dynamic properties affect this test?

Flatten the sharp corners

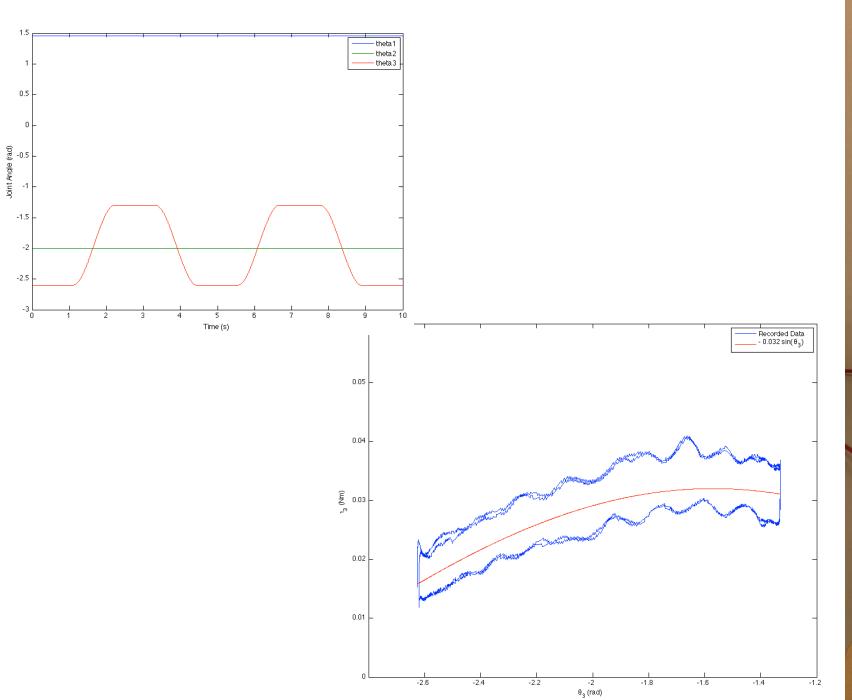


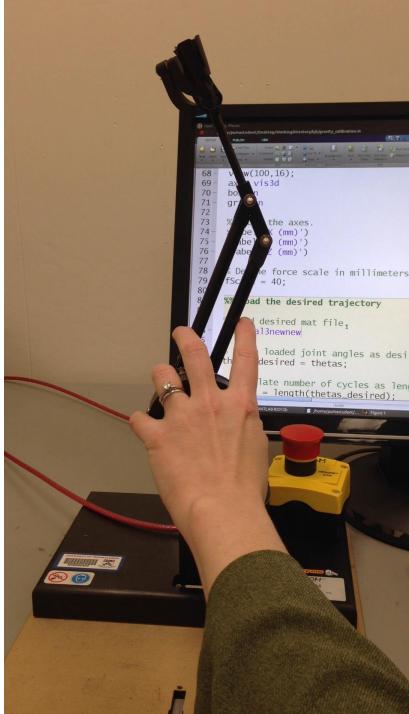
Trajectory Generation

start end
$$q(t_0)=q_0 \longrightarrow q(t_f)=q_f$$
 $\dot{q}(t_0)=v_0 \longrightarrow \dot{q}(t_f)=v_f$

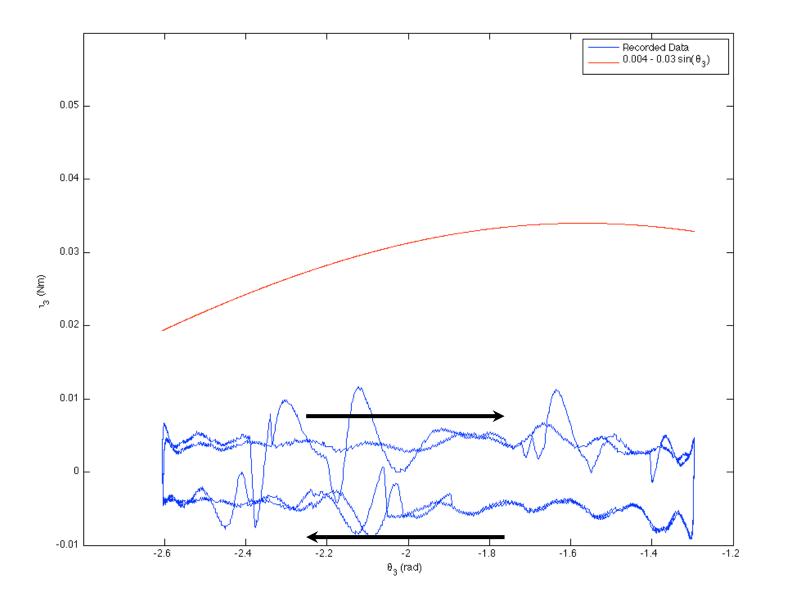
$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

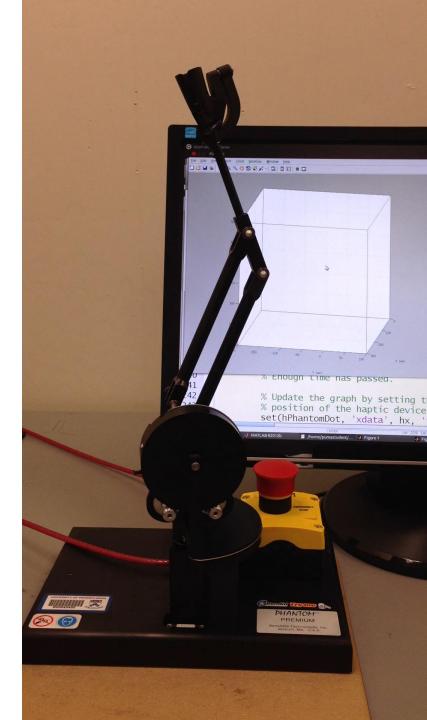
$$\left[egin{array}{c} q_0 \ v_0 \ q_f \ v_f \end{array}
ight] = \left[egin{array}{cccc} 1 & t_0 & t_0{}^2 & t_0{}^3 \ 0 & 1 & 2t_0 & 3{t_0}^2 \ 1 & t_f & t_f{}^2 & t_f{}^3 \ 0 & 1 & 2t_f & 3{t_f}^2 \end{array}
ight] \left[egin{array}{c} a_0 \ a_1 \ a_2 \ a_3 \end{array}
ight]$$



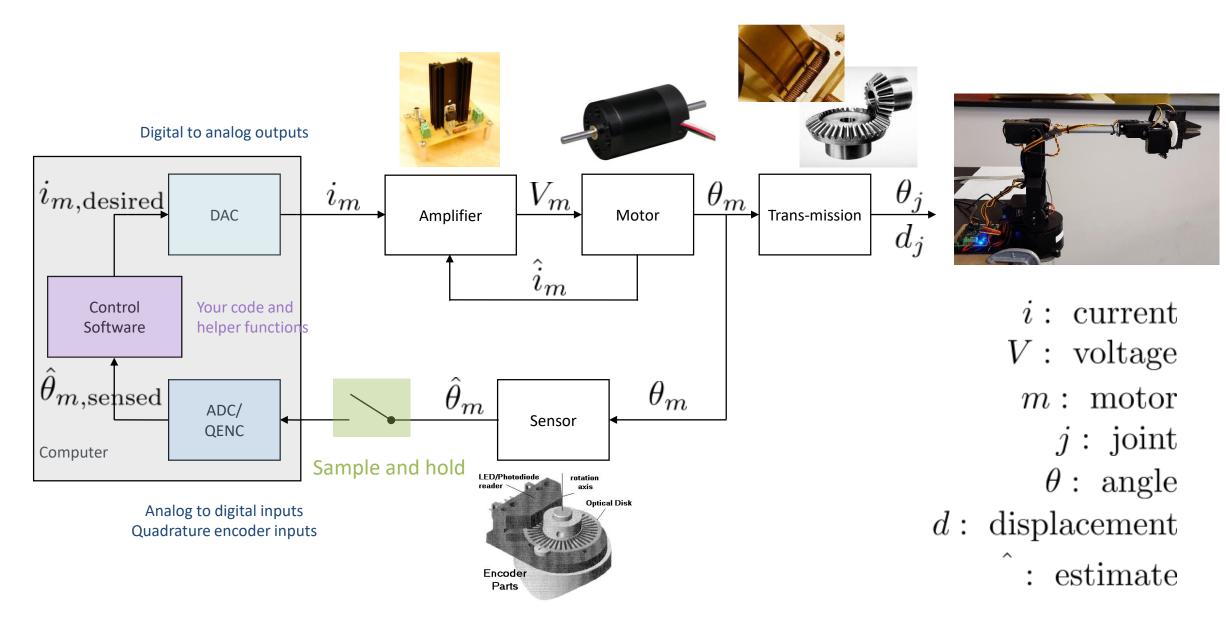


You can also use this strategy to compensate for friction.



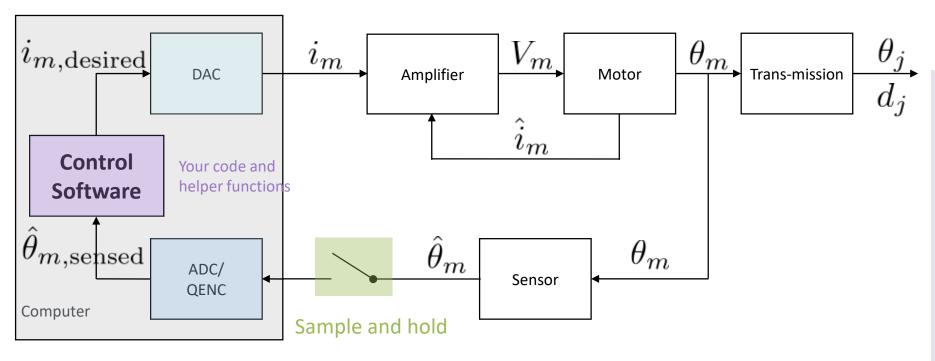


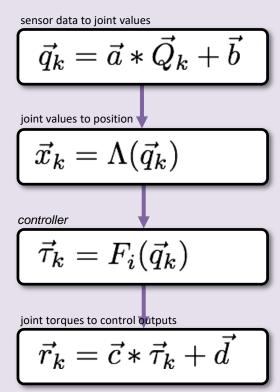
How most real robots work



Control Software

Digital to analog outputs

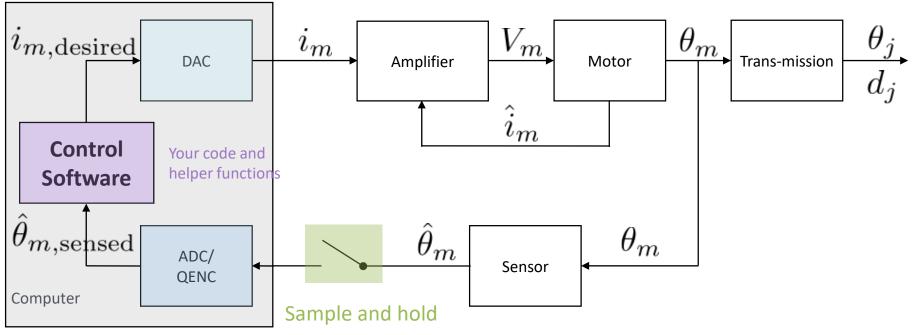




Joint Angles to Robot Position

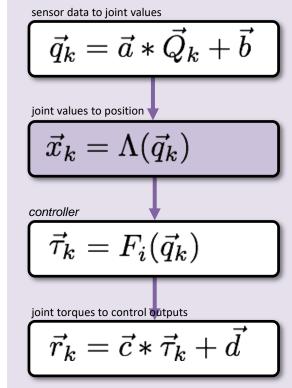
How can we estimate robot pose from $\hat{\theta}_j$? Forward kinematics!





Derive using DH or geometry.

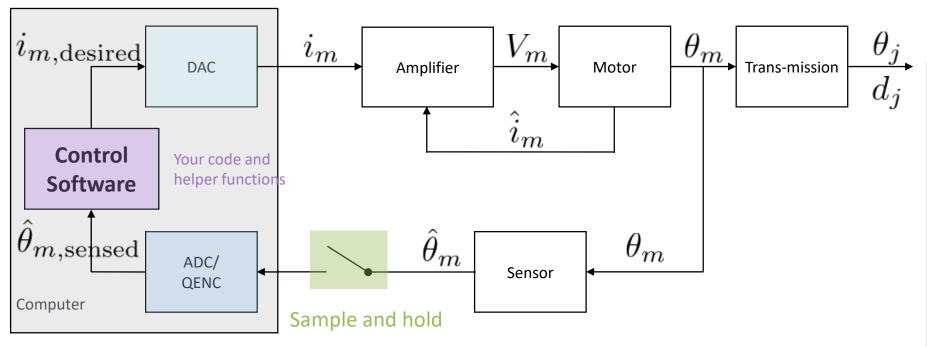
zero pose, link lengths, and link offsets matter!



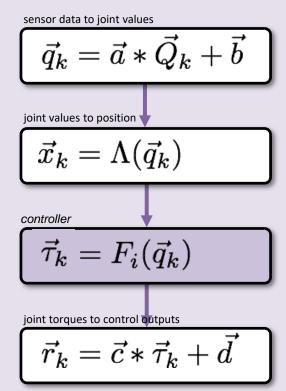
Robot Positions to Torques

How can we calculate joint torques from positions? PID control!





$$\tau_{i} = k_{p}(\theta_{i,des} - \theta_{i}) + k_{d}(\omega_{i,des} - \omega_{i}) + k_{i} \int \theta_{i,des} - \theta_{i}$$

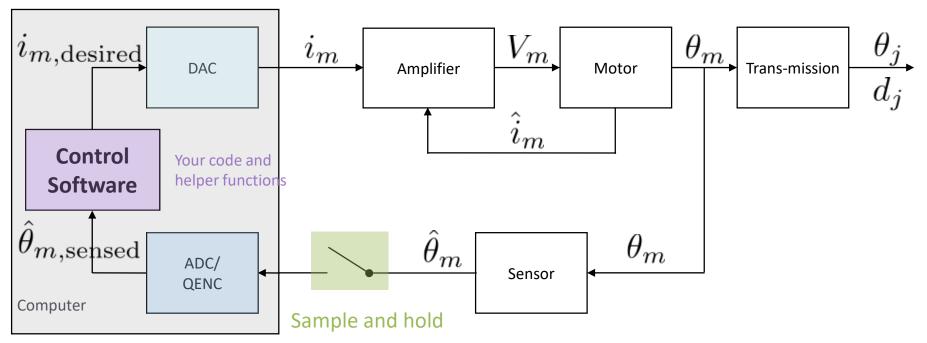


Alternatively, Robot Positions to Forces to Torques

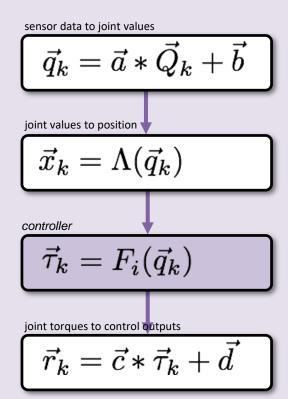
How can we calculate joint torques from desired forces?

Manipulator equation!



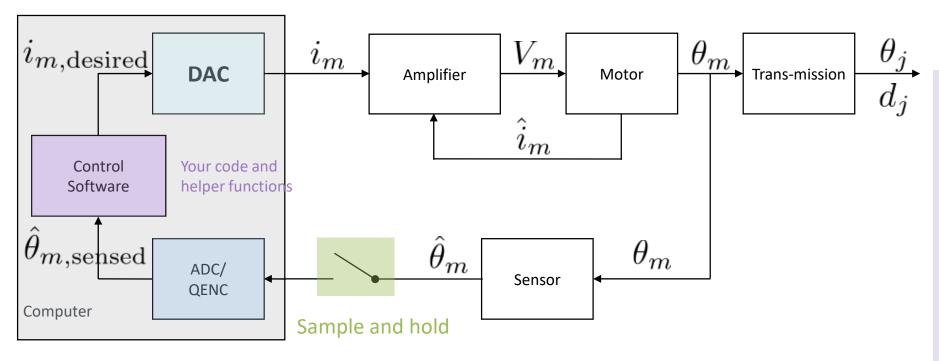


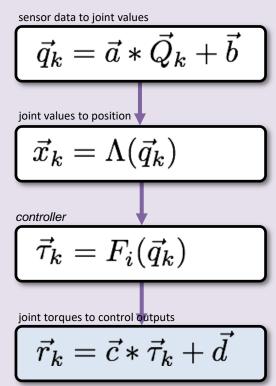
$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$



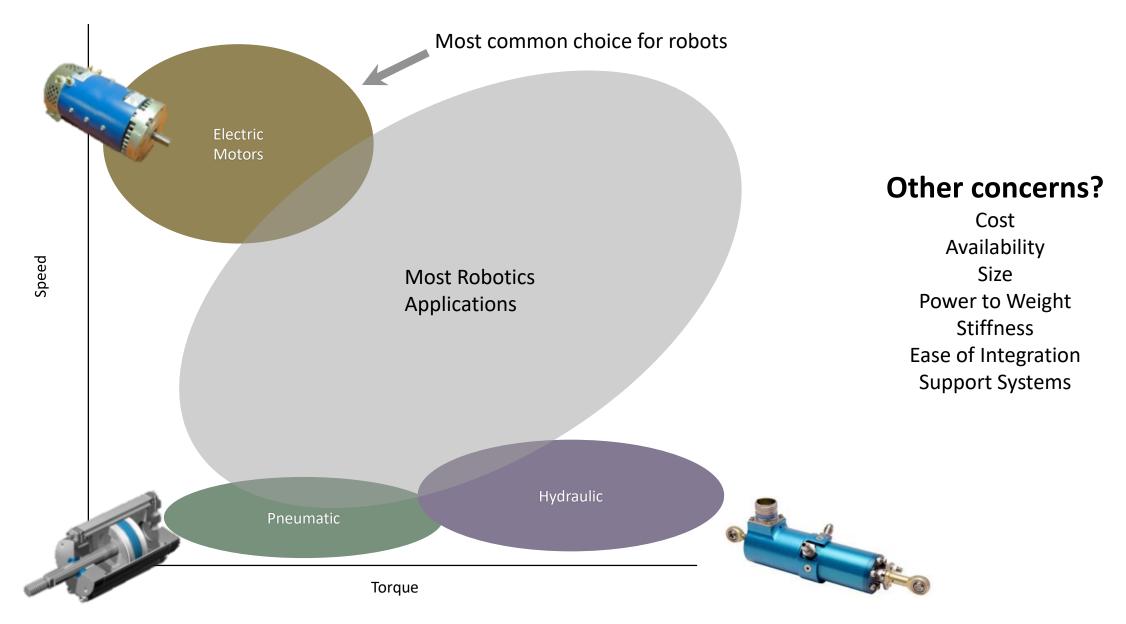
What goes in here?

Digital to analog outputs

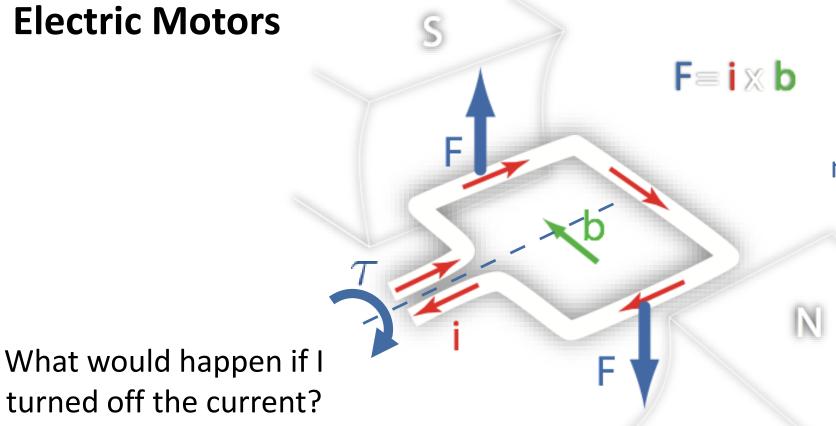




Types of Actuators



Electric Motors



Pair of offset equalmagnitude forces causes a torque (a.k.a. couple, moment) around the axis of rotation

What would happen if I flipped the sign of the current?

What if I kept the current but rotated coil to another position?



AC

Magnetic Rotor
Coil Stator
Output speed is a sub-multiple of voltage supply frequency



DC Brushed

Most common!

Coil Rotor
Magnetic Stator
Brushes carry current to the rotor



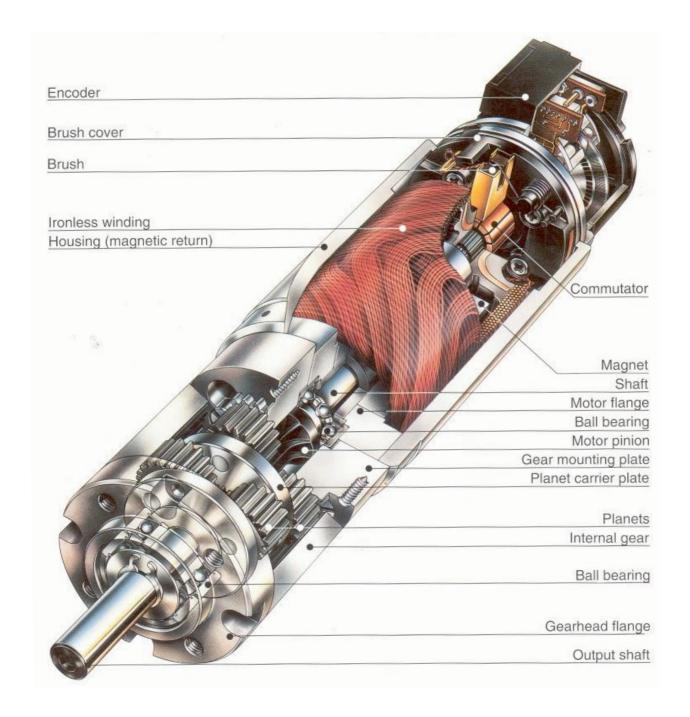
DC Brushless

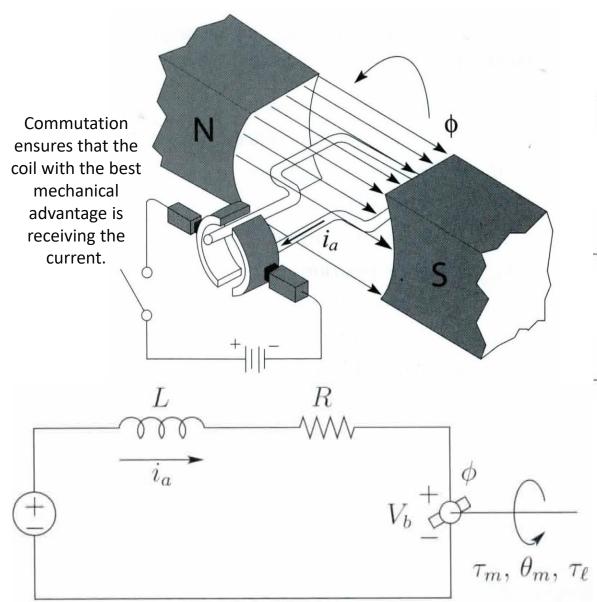
Magnetic Rotor
Coil Stator
Similar in construction to AC, but electrically commutated
Requires a position sensor (commonly built in)

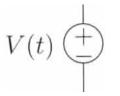


Stepper

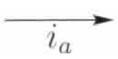
Toothed Magnetic Rotor Multi-Coil Stator Capable of open-loop positioning Requires a controller







Time-varying **voltage** supply.
Compare **voltage** to water pressure.



Current through the motor armature. Compare **current** to water flow.



Electrical resistance of the armature.

Compare **resistance** to water flow resistance in pipes (small diameter) Follows Ohm's Law.

Nilows Onm s Law. V=iR

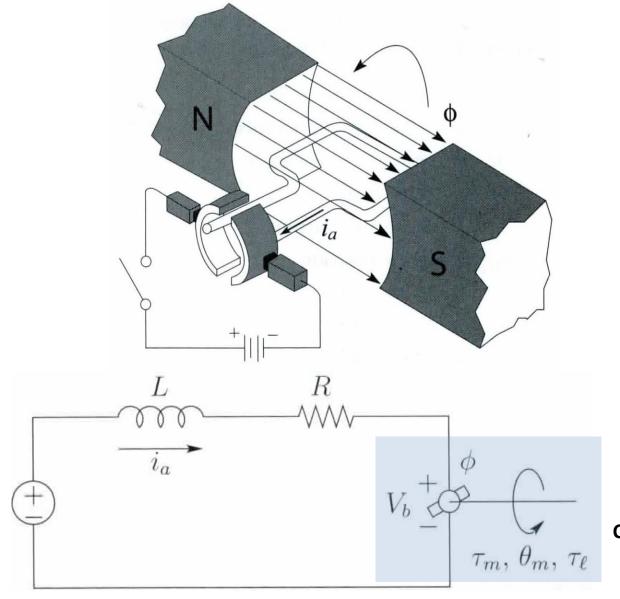


Electrical inductance of armature.

Compare **inductance** to the momentum of the water flowing in a pipe. Follows constitutive equation.

$$V = L \frac{di}{dt}$$

Circuit representation for a DC motor driven by a time-varying voltage.



Time-varying voltage caused by the back electro-motive force (back-emf).

Back-emf voltage is proportional to the rotational speed of the motor and opposes the voltage that would drive the motor in the direction in which it is rotating.

$$V_b = k_v \, \omega_m$$

Circuit representation for a DC motor driven by a time-varying voltage.

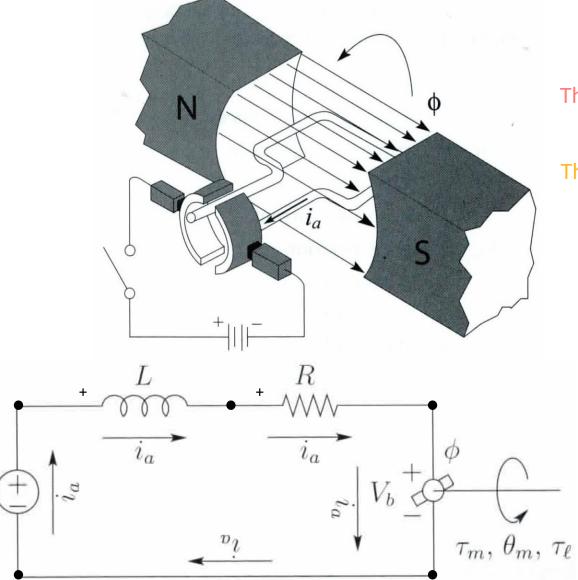






Turning one motor creates back-emf (a voltage across that motor's terminals).

If two motors are connected terminal to terminal, turning the first motor makes the other motor turn.



How do we analyze this circuit?

Kirchoff's Current Law (KCL)

The sum of the currents flowing into (positive) and out of (negative) a node in the circuit is zero.

The current flowing through all elements of our circuit is the same.

Kirchoff's Voltage Law (KVL)

The sum of the voltage drops around any loop in the circuit is zero.

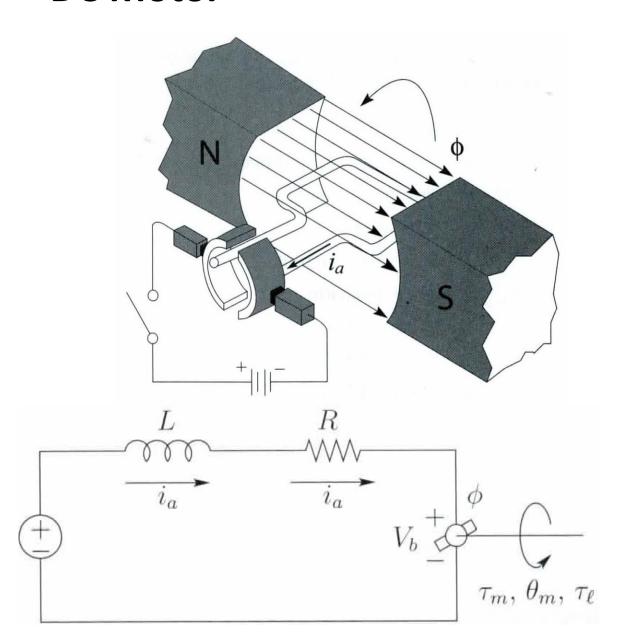
Draw a + sign where current enters each element other than voltage sources.

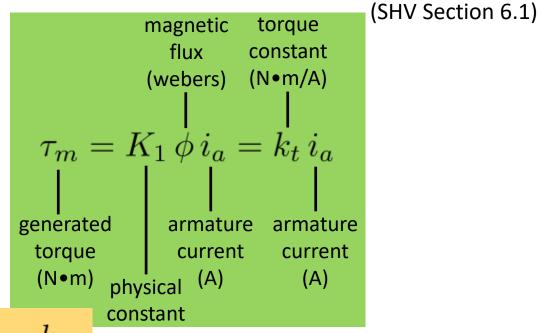
$$V(t) - V_L - V_R - V_b = 0$$

$$V = L \frac{di}{dt} \qquad V_b = k_v \omega_m$$

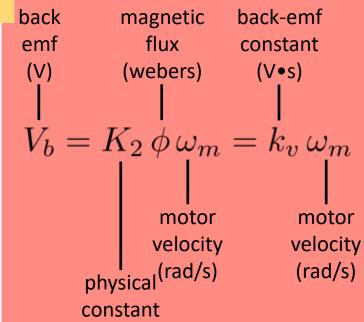
$$V = iR$$

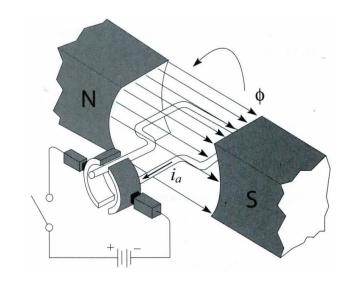
$$V(t) = L\frac{di_a}{d_t} + R i_a + k_v \,\omega_m$$





 $k_t = k_v$ f using meters, kilograms and seconds





Electrical Dynamics

$$V(t) = L\frac{di_a}{dt} + Ri_a + k_v \frac{d\theta_m}{dt}$$

Physical Dynamics

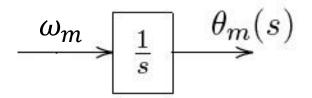
SHV shows the load torque in the wrong direction and confusingly calls gear ratio "r"

$$J_m \frac{d^2 \theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m + \tau_{ext} = \frac{k_t i_a}{k_t i_a} + \tau_{ext}$$

external disturbances from connections

input

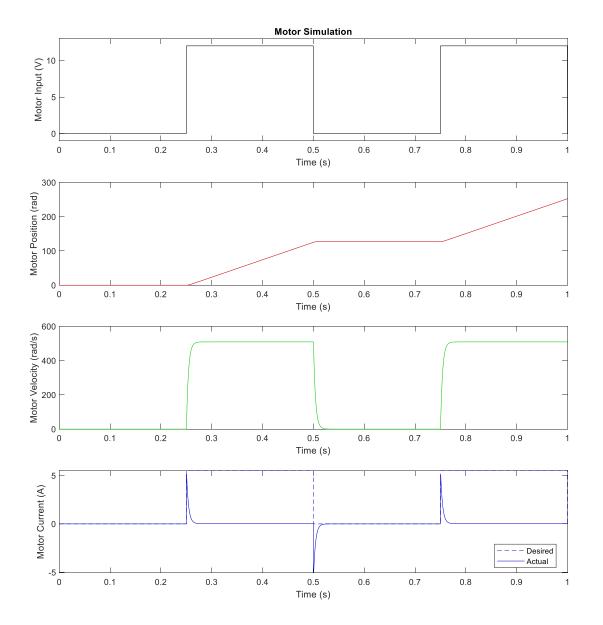
torque constant



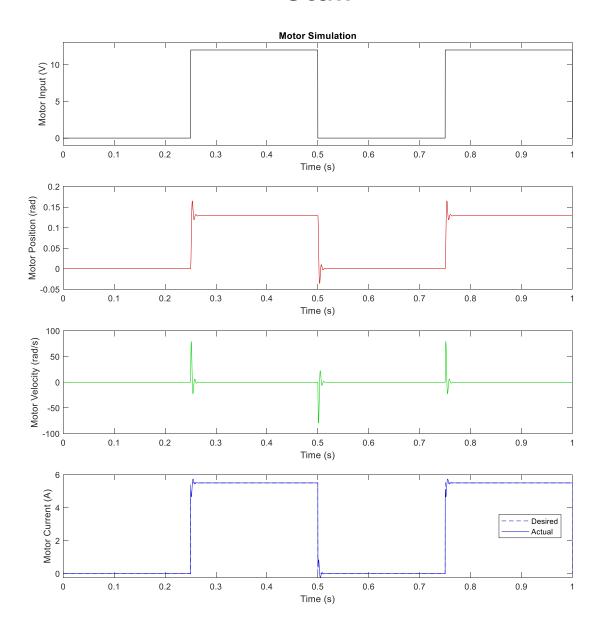
electrical dynamics

motor torque motor physics

No Load



Stall

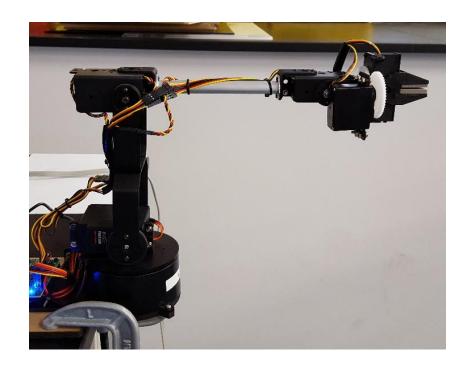


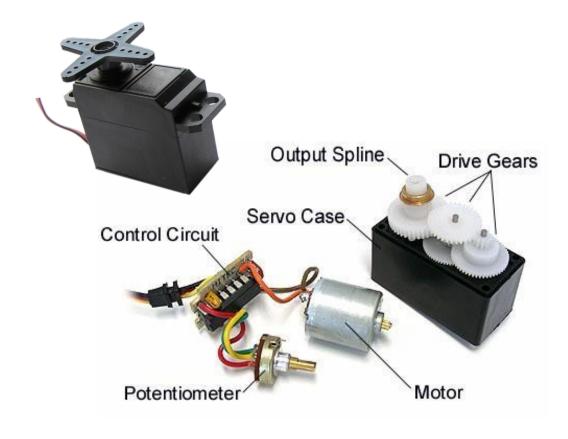


The best brushed DC motors are made by Maxon. They are rather expensive, but they work quite well.

- Smooth torque output, independent of motor angle. In other words, very low cogging and torque ripple.
- Low friction, both at low and high speeds, due to high quality bearings and low eddy currents.
- Relatively high stall torque, which is the torque the motor can deliver when it is not rotating.
- Larger motors create higher torques, but they also have higher inertia, higher friction, and higher cost.

Position Control: Servomotors





Typically, roboticists treat each joint independently, as a single-input/single-output (SISO) model.

This is adequate for applications that don't involve very fast motions, which helps decouple the links from one another.

DC brushed motor with integrated sensor and controller

Transmission

Most DC motors are designed for high-speed low-torque output.

In order to create robots that can bear load, we need a transmission system.

Common Transmissions

Direct Drive: simplest implementation

Band/Belt Drive:

- move actuator mass away from joint
- smoothest drive but unstable when belts are long

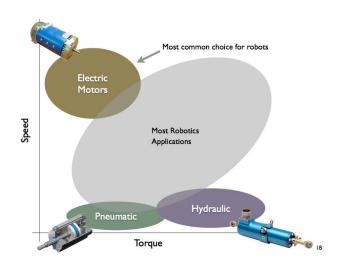
Gear Drive:

- high torque low speed
- backlash

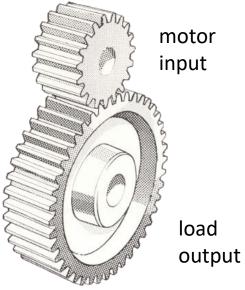












motor input

gear ratio

$$N = rac{n_{
m out}}{n_{
m in}} = rac{r_{
m out}}{r_{
m in}} = rac{\omega_{
m in}}{\omega_{
m out}} = rac{ au_{
m out}}{ au_{
m in}}$$

Teeth

Radius

Speed

Torque

Same equations apply for belts, pulleys, and friction drive

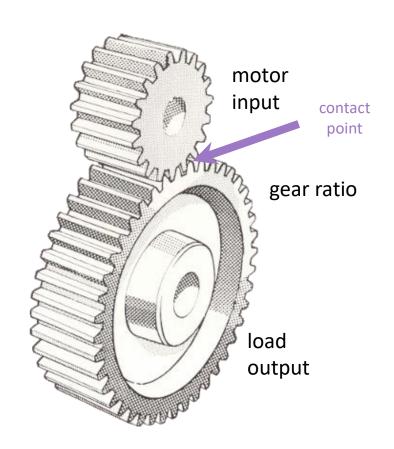
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m in}} = rac{\omega_{
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m in}}$$

Teeth

Radius

Speed

Torque



Imagine the small gear rotates an angle equivalent to 5 gear teeth.

How far does the large gear rotate?

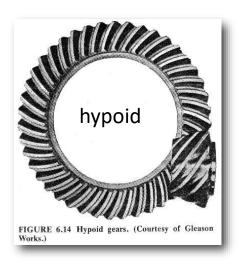
Imagine the you apply a torque of 1 Nm to the small gear while holding the large gear still.

What torque do you feel on the other gear?

Types of Gears





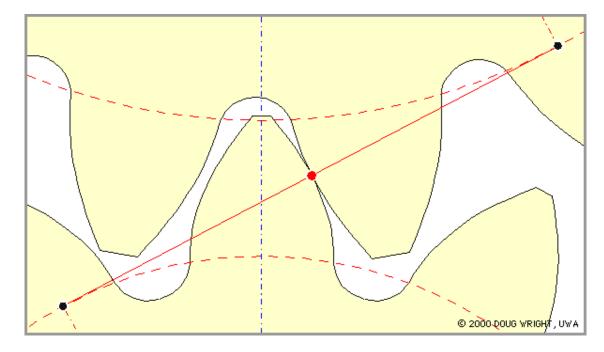


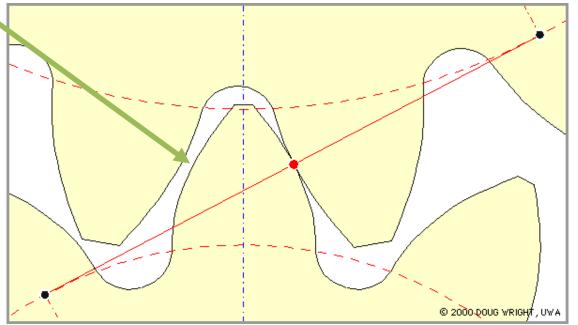




Backlash

Gap between teeth means that if the rotation changes direction, one gear can move a small amount without making the other move.





Capstan Drives

Many high-performance robots use a capstan drive with thin stranded cables.

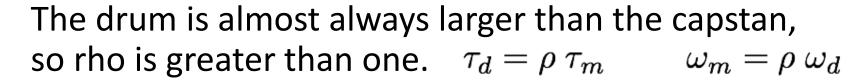
- The rotation of the motor shaft is coupled to the rotation of a larger drum or the motion of a linear stage by wrapping cables around a capstan.
- When pre-tensioned, cables provide a very stiff connection with zero backlash.
- The motion is smooth and efficient, without vibrations.





Capstan Drives

The gear ratio is the ratio of the diameters $ho = rac{d_d}{d_d}$ (or equivalently the ratio of the radii).



The drum torque is greater than the motor torque.

The motor speed is greater than the drum speed.

A drawback is that the output feels amplified versions of the motor's inertia and friction. This amplification goes with the gear ratio squared.



Next time:

2017 IEEE International Conference on Robotics and Automation (ICRA) Singapore, May 29 - June 3, 2017

Autonomous Robotic Stone Stacking with Online next Best Object Target Pose Planning

Fadri Furrer*,1, Martin Wermelinger*,2, Hironori Yoshida* Fabio Gramazio3, Matthias Kohler3, Roland Siegwart1, Marco Hutter2

prelabrication in structured indoor environments with standard building materials. Our work, on the other hand, focuses on utilities; pregular materials found on-site, such as rubble and rocks, for autonomous construction. We present a pipeline that detects randomly placed objects in a scene that are used by our next best stacking pone searching method employing gradient descent with a random initial orientation, a pipeling a philo-decent with a random initial orientation, as pipeling a philo-siste, a probler painting and the properties of the properties of some a probler painting to the constructive balancies settical using a robotic manipulator by constructing balancing vertical stacks without mortars and adhesives. We show the results usine four arbitrarily in front of the robot placed rocks.

I. INTRODUCTION

Over the last decade, robotics has been introduced to architectural construction not only for safer and more efficient construction, but also for exploring diverse forms [1]. However, there are still intensive manual labor works involved for on-site assembly of these components [2].

Digital fabrication has explored applications of autonomous robots in on-site operation scenarios [3], but are restricted to build with regular materials. Building structures with irregular shaped objects was presented in [4], however they apply glue to increase stability. To reduce the environmental impact we aim to use such material without additional adhesives, to build dry-stack compositions. Therefore, our work focuses on developing an automated fabrication process the trajectories to replicate the proposed stack with the using irregular objects, which are not processed but found

As a case study, discrete rigid elements, such as stones or concrete rubble, are targeted as a building material. Our goal is to construct a balancing vertical tower with found objects. while maintaining the structure in static equilibrium using a errors between a desired target pose and an actual stacked robotic manipulator. To achieve this, we developed a holistic work-flow including precise object detection, motion control, and planning the next target pose (see Figure 1). As part of this work-flow, we describe an algorithm suggesting stable poses for stacking, validated by an implementation of this autonomous stacking work-flow in a real-world experiment. Due to the instability of vertical tower, it is natural to observe

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Fig. 1: In an offline step we scan a set of objects (top). These objects, or a subset of it, can be distributed arbitrarily on the work-space and get detected by our object detection pipeline (middle-left). From the detected objects the presented pose searching algorithm proposes the next stable stack (middleright). A motion planner (bottom-right) is used to generate robot arm (bottom-left). After placing the object, its pose is measured and used as base for the subsequent pose searching

pose. Thus, the work-flow puts emphasis on the resultant pose evaluation and a dynamic re-planning of the target pose. This paper makes the following contributions regarding

- a pose searching algorithm considering structural stability using a physics engine

handling irregularly shaped objects:

 an object detection pineline - an autonomous system for constructing balancing vertical towers using a manipulator

Research in architecture and digital fabrication investigates novel production techniques in which material behavior is linked to fabrication and assembly tasks [5]. Recently, it has

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Final Project

MEAM 520, University of Pennsylvania

November 4, 2018

Project Guidelines

The final project will be an open-ended project. Intro to Robotics is a fast-paced overview of many different topics in robotics. The purpose of the final project is for you to explore one of these topics in more depth.

What to do: The final project can take many forms depending on your interest. Possible final projects

- · Implementation and evaluation of a method or procedure we learned in class but did not use in lab. This option should have a substantial evaluation and analysis leading to conclusions of when the method works or does not work
- · Literature review on recent results in an area discussed in class. I would expect you to read and
- . Research project on a open-ended robotics problem. This is ambitious. I suggest you couple this with a small lit review so you have something to report

Topics: Possible topics include any topics on the course schedule. Sample topics are listed at the end of

Scope: The project should be equivalent in effort to 1/2 lab per person.

Partners: The project can be done in groups of up to four people. Although not required, I encourage you to do the project in a group so that you can get outside feedback.

Overlap with projects outside of class: This project does NOT have to be an isolated effort. You are strongly encouraged to do a project that you are actually interested in. I support doing a project related to your work in another course (such as MEAM 510 or senior design) or in a research group.

Project Proposal (due 11/16)

Your project proposal should have the following information:

- · Goal: Summarize what you want to accomplish
- · Approach: Describe what techniques you plan to use
- · Relation to the course: Explain the relevance to the course material

The project proposal is a completion grade worth 5 pts on Canvas. Make sure to submit the proposal as a group, following the Submission Instructions below. After the due date, I will provide you with feedback

"Autonomous Robot Stone Stacking..." (ICRA 2017)

Final project proposal due 11/16