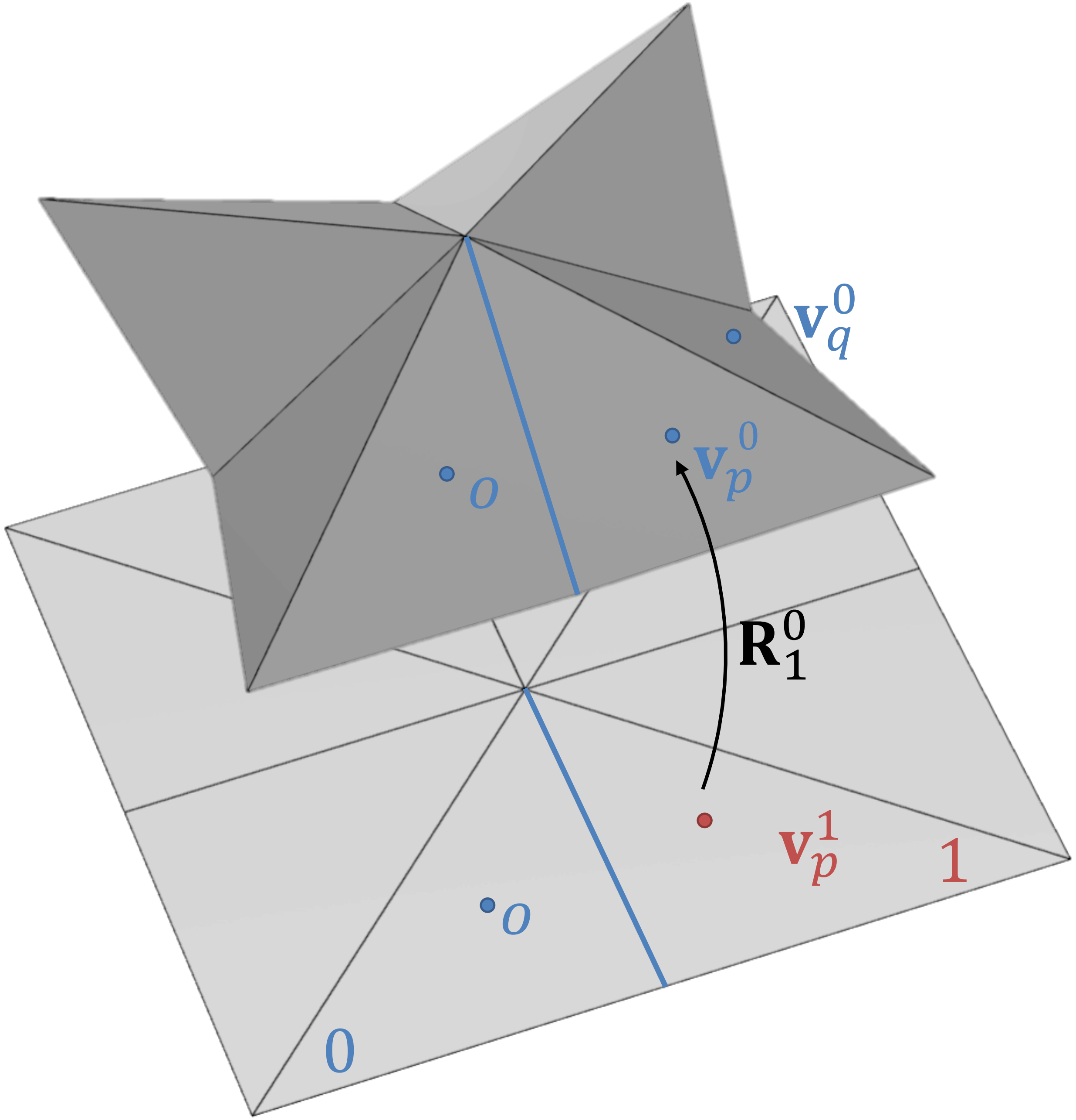
MEAM 520
Lecture 5: Forward Kinematics

Cynthia Sung, Ph.D.

Mechanical Engineering & Applied Mechanics

University of Pennsylvania



ORIGAMI SCIENCE MATHS EDUCATION
5th to 7th September 2018
OXFORD UK

$$\mathbf{v}_p^0 = \mathbf{R}_1^0 \mathbf{v}_p^1$$

$$\mathbf{R}_1^0 = [\hat{x}_1^0 \quad \hat{y}_1^0 \quad \hat{z}_1^0] = \begin{bmatrix} \hat{x}_0^1 \\ \hat{y}_0^1 \\ \hat{z}_0^1 \end{bmatrix}$$

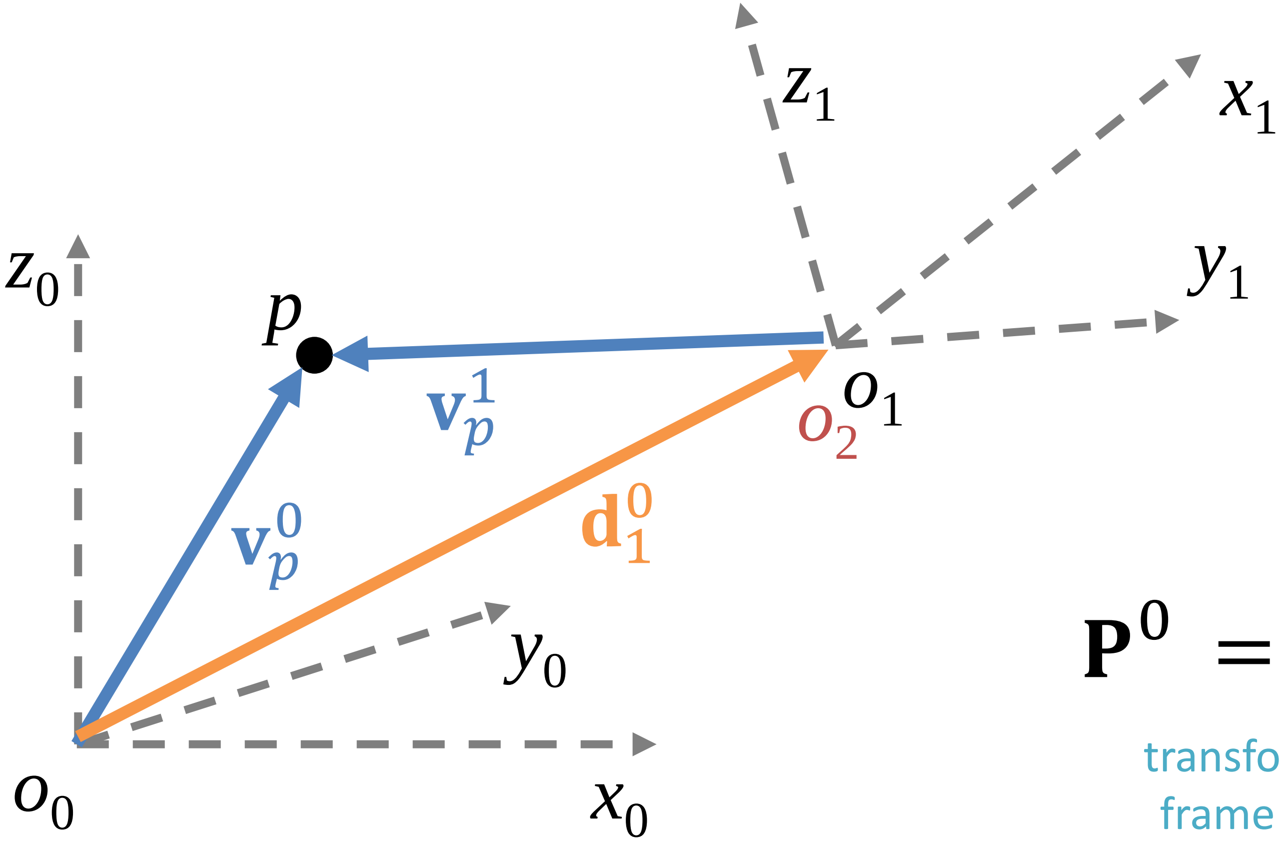
Representations:

- Euler Angles
- Yaw/Pitch/Roll
- Axis-Angle

Last Time: Rigid Motions

combine pure **rotation** and pure **translation**

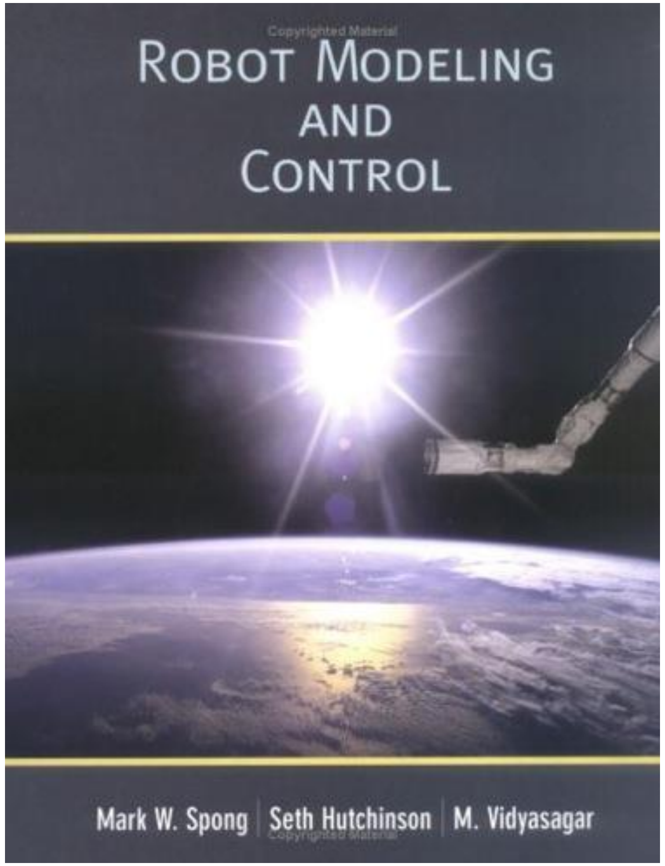
$$\mathbf{v}_p^0 = \mathbf{R}_1^0\mathbf{v}_p^1 + \mathbf{d}_1^0$$



$$\mathbf{P}^0 = \mathbf{H}_1^0\mathbf{P}^1 = \begin{array}{cc} \begin{array}{cccc} & \mathbf{R}_1^0 & & \mathbf{d}_1^0 \end{array} & \mathbf{P}^1 \\ \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \end{array}$$

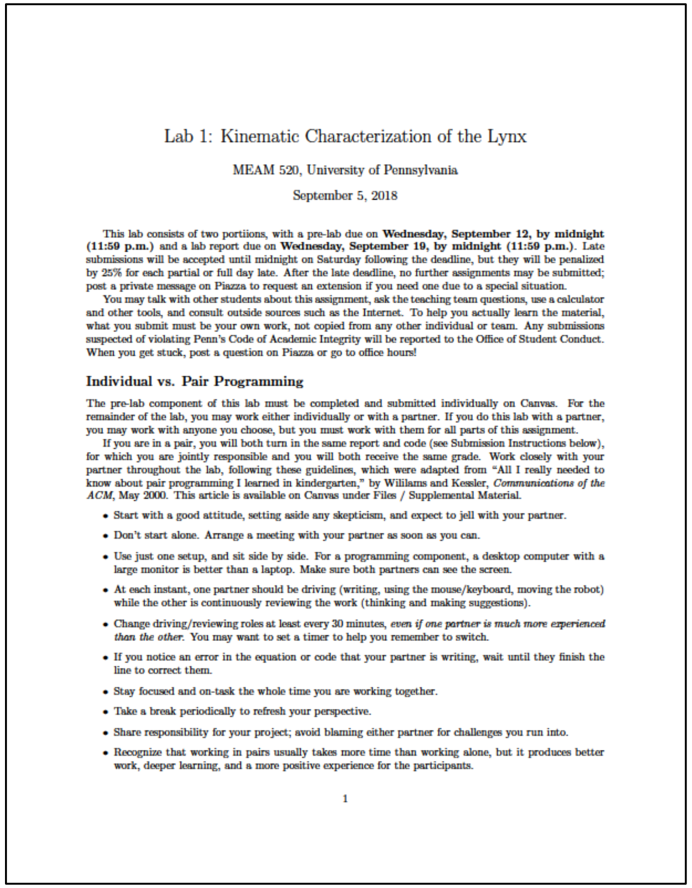
transformation from frame 1 to frame 0

Today: Forward Kinematics



Chapter 2: Rigid Motions

- Read Sec. 3.intro - 3.2



Lab 1: Kinematic Characterization of the Lynx

MEAM 520, University of Pennsylvania

September 5, 2018

This lab consists of two portiions, with a pre-lab due on **Wednesday, September 12, by midnight (11:59 p.m.)** and a lab report due on **Wednesday, September 19, by midnight (11:59 p.m.)**. Late submissions will be accepted until midnight on Saturday following the deadline, but they will be penalized by 25% for each partial or full day late. After the late deadline, no further assignments may be submitted; post a private message on Piazza to request an extension if you need one due to a special situation.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you submit must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. When you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

The pre-lab component of this lab must be completed and submitted individually on Canvas. For the remainder of the lab, you may work either individually or with a partner. If you do this lab with a partner, you may work with anyone you choose, but you must work with them for all parts of this assignment.

If you are in a pair, you will both turn in the same report and code (see Submission Instructions below), for which you are jointly responsible and you will both receive the same grade. Work closely with your partner throughout the lab, following these guidelines, which were adapted from "All I really needed to know about pair programming I learned in kindergarten," by Wililams and Kessler, *Communications of the ACM*, May 2000. This article is available on Canvas under Files / Supplemental Material.

- Start with a good attitude, setting aside any skepticism, and expect to jell with your partner.
- Don't start alone. Arrange a meeting with your partner as soon as you can.
- Use just one setup, and sit side by side. For a programming component, a desktop computer with a large monitor is better than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (writing, using the mouse/keyboard, moving the robot) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every 30 minutes, *even if one partner is much more experienced than the other*. You may want to set a timer to help you remember to switch.
- If you notice an error in the equation or code that your partner is writing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.
- Recognize that working in pairs usually takes more time than working alone, but it produces better work, deeper learning, and a more positive experience for the participants.

1

Pre-lab due tomorrow, 11:59 p.m.

Lab 1 due 9/19, 11:59 p.m.

Manipulator Terminology Review (SHV 1.1-1.3, 3.1)

Link : rigid body, 6 degrees of freedom

Joint : connection between two links,
allows 1 degree of freedom

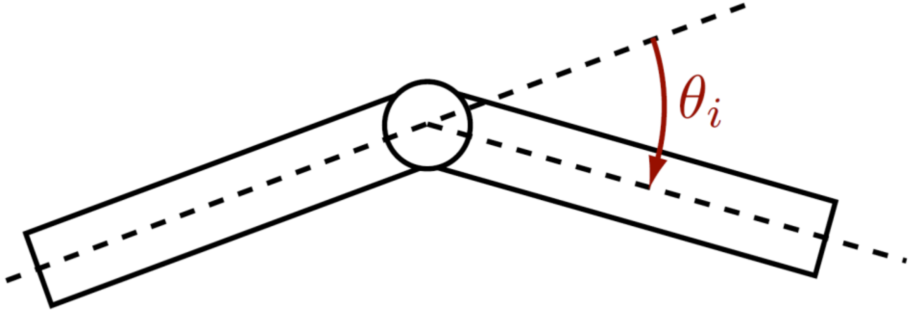
End-effector : interacts with the environment

Base : connected to ground

Joint Descriptions

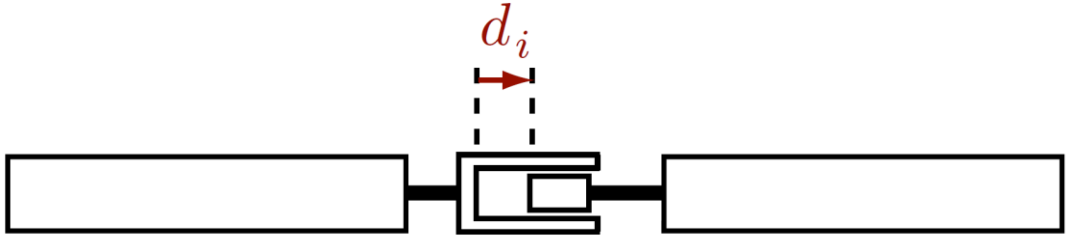
Revolute:

angular displacement between adjacent links



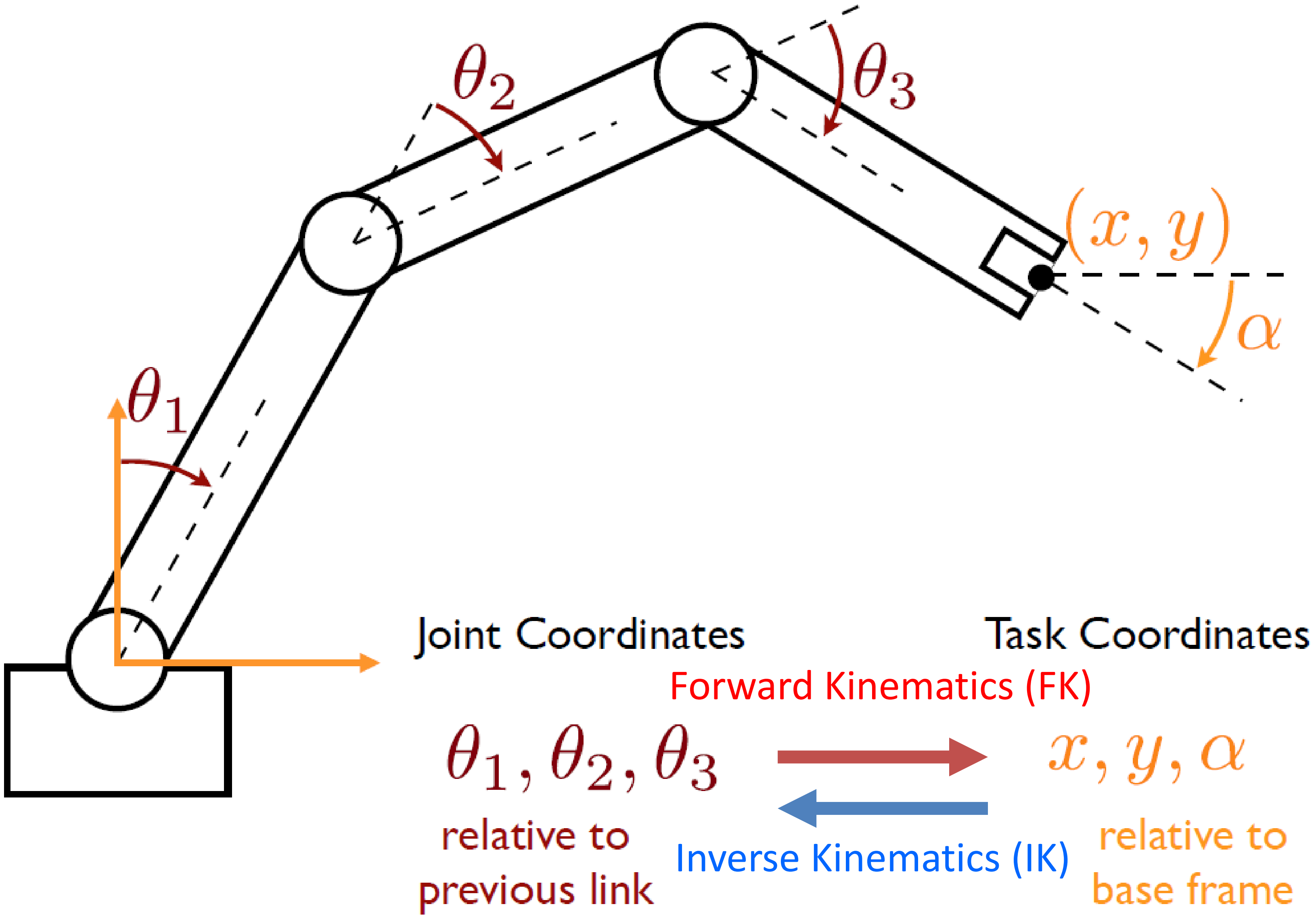
Prismatic:

linear displacement between adjacent links



We use z-axes to denote joint axes.

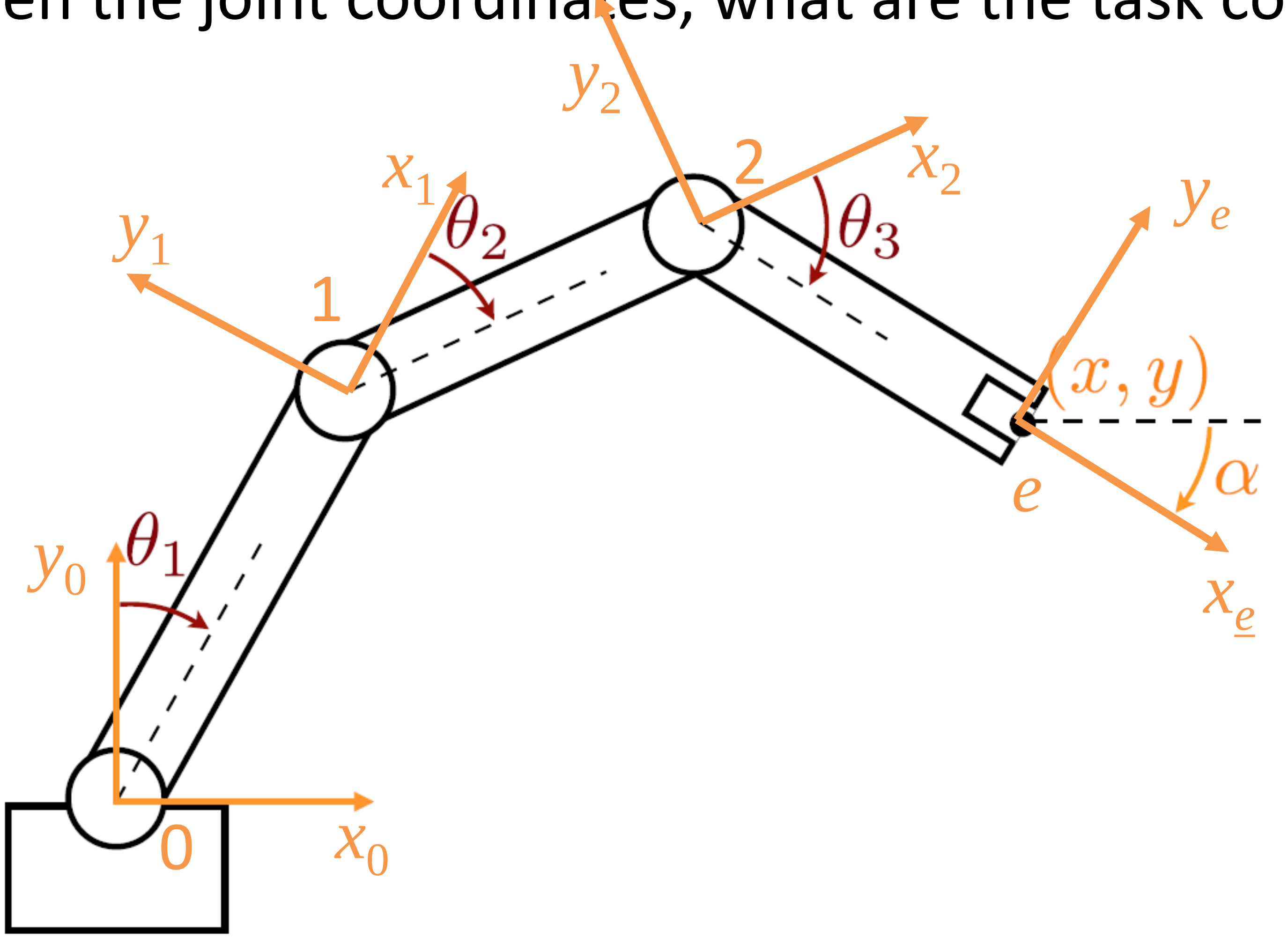
Kinematics



θ_2
θ_3
(x, y)
α
θ_1
Joint Coordinates
Task Coordinates
Forward Kinematics (FK)
$\theta_1, \theta_2, \theta_3$
x, y, α
relative to previous link
Inverse Kinematics (IK)
relative to base frame

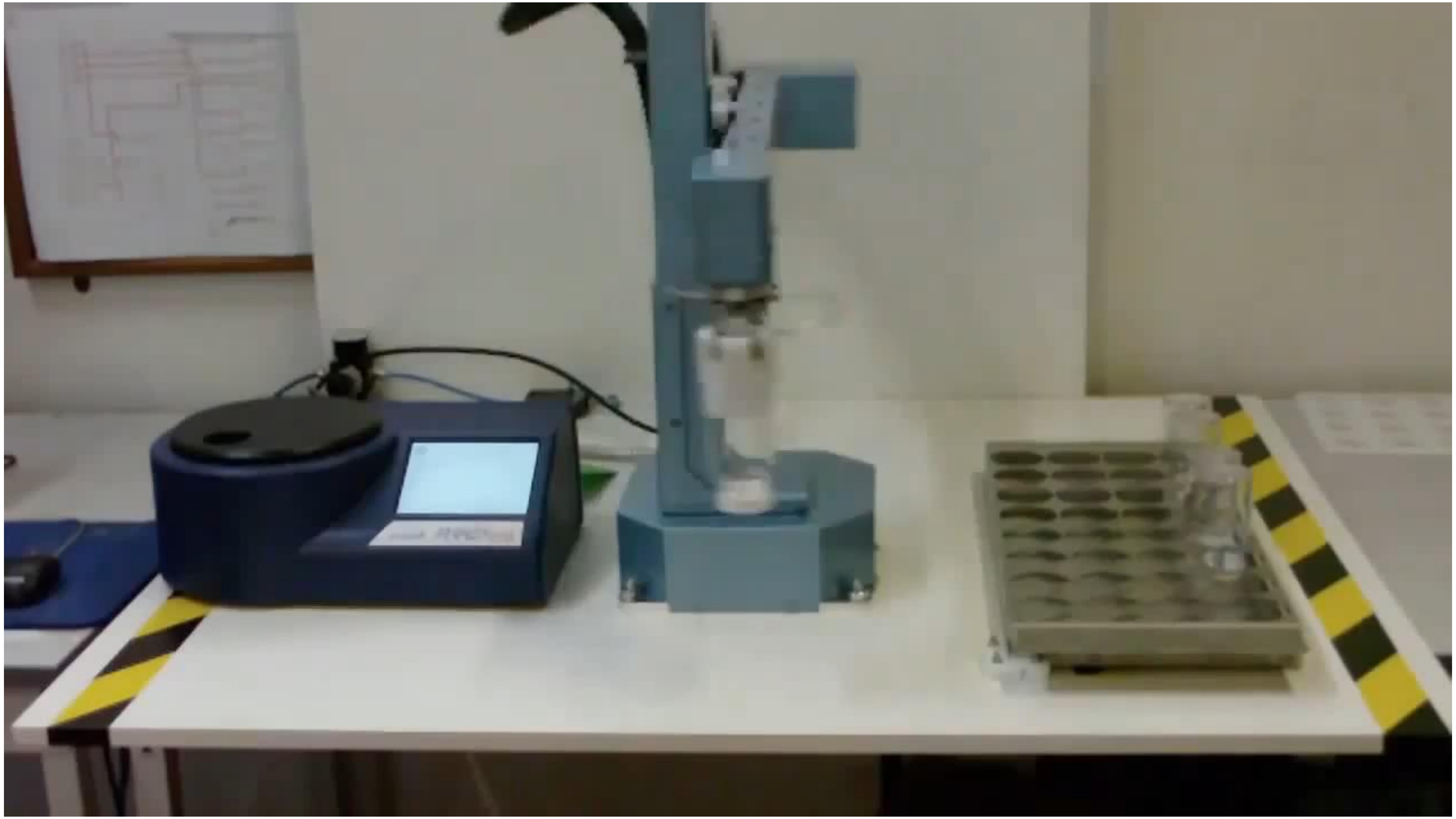
Forward Kinematics

Given the joint coordinates, what are the task coordinates?



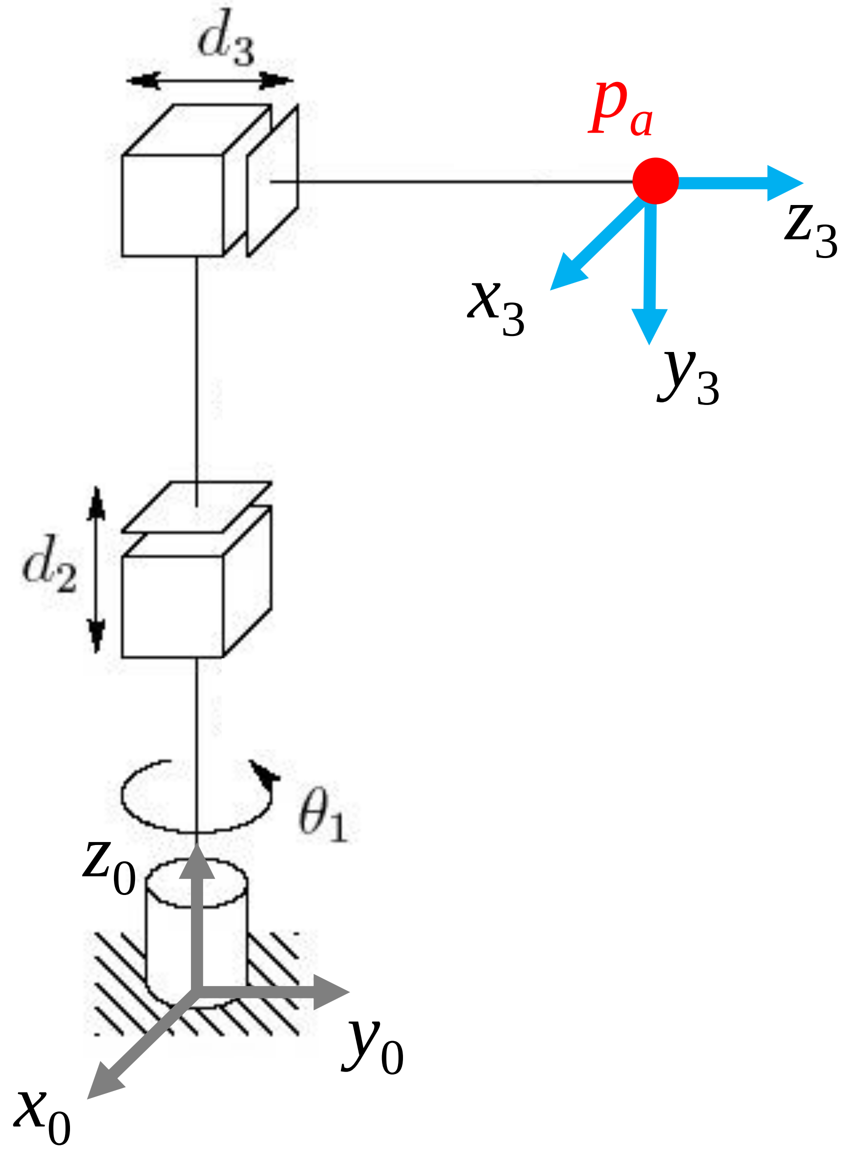
Strategy: Break up the robot into its links

Forward Kinematics of the RPP Cylindrical Robot



https://www.youtube.com/watch?v=Hj7PxjeH5y0

Forward Kinematics of the RPP Cylindrical Robot



3 links plus ground

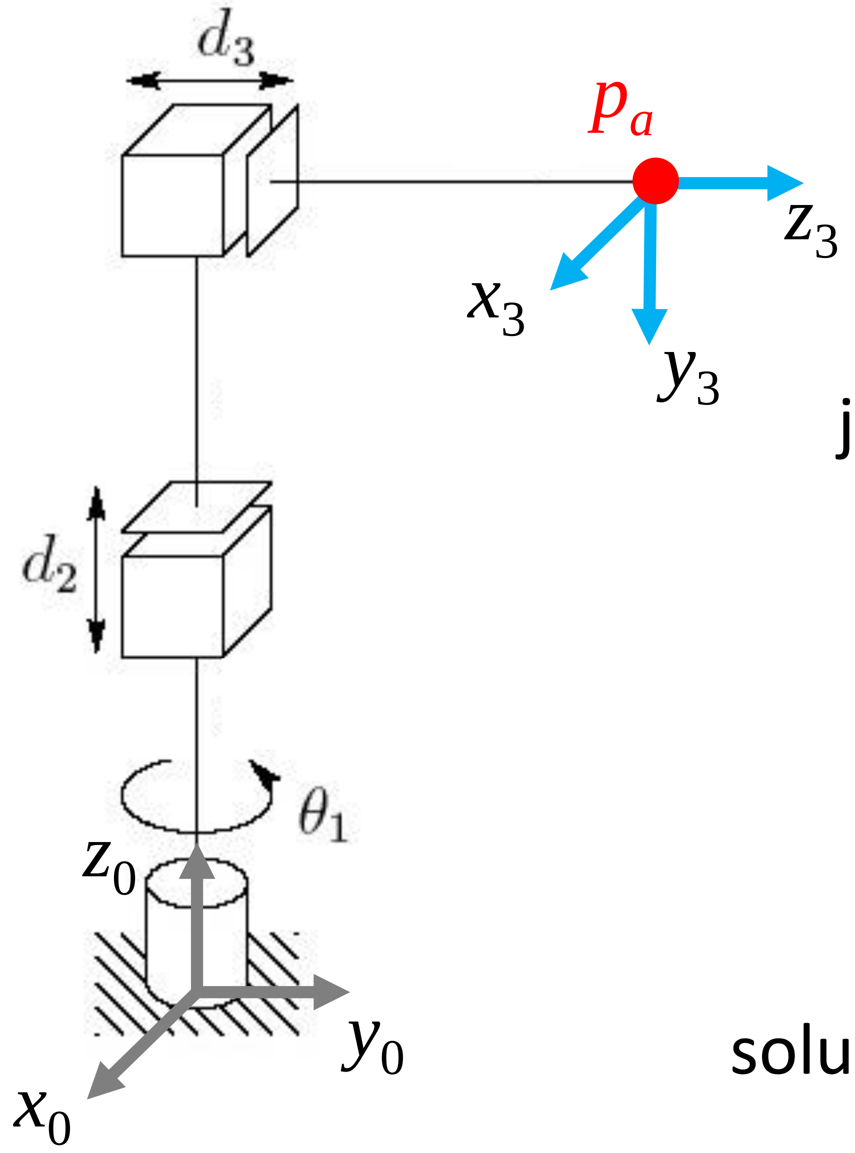
3 joints

3 joint variables (q_1, q_2, q_3)

shown with $\theta_1 = 0$, $d_2 > 0$, $d_3 > 0$

Q: Given (q_1, q_2, q_3), where is the tip of the robot?

Forward Kinematics of the RPP Cylindrical Robot

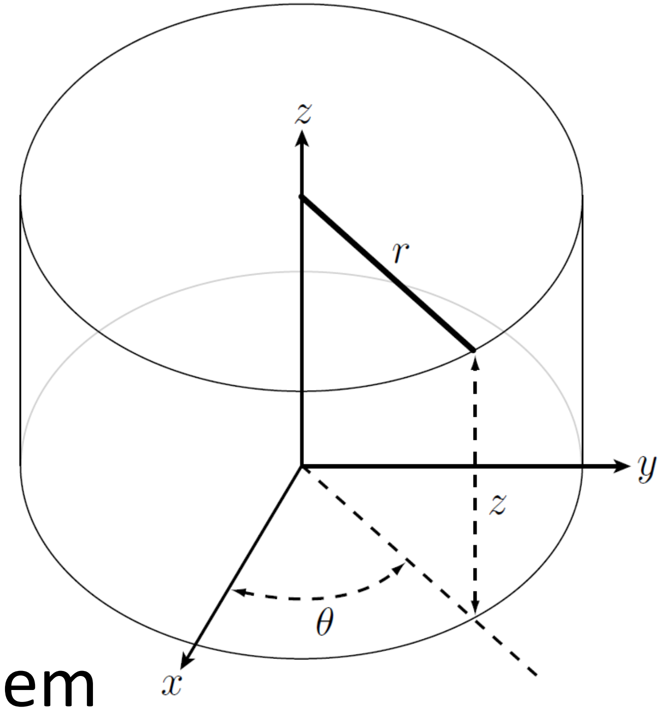


Q: Given (q_1, q_2, q_3), where is the tip of the robot?

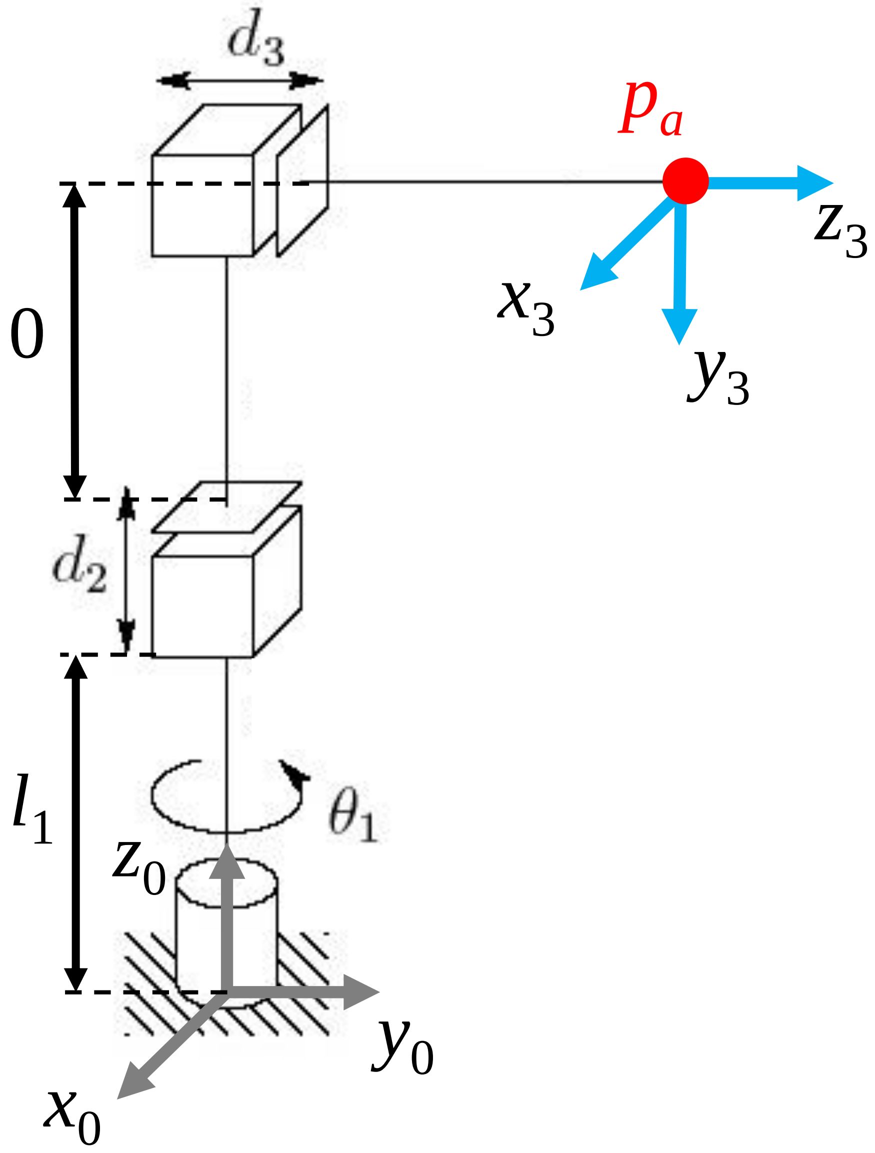
joint coordinates map to cylindrical coordinates

$$\theta_1 \longleftrightarrow \theta$$
$$d_2 \longleftrightarrow z$$
$$d_3 \longleftrightarrow r$$

but this is not a general solution to this type of problem



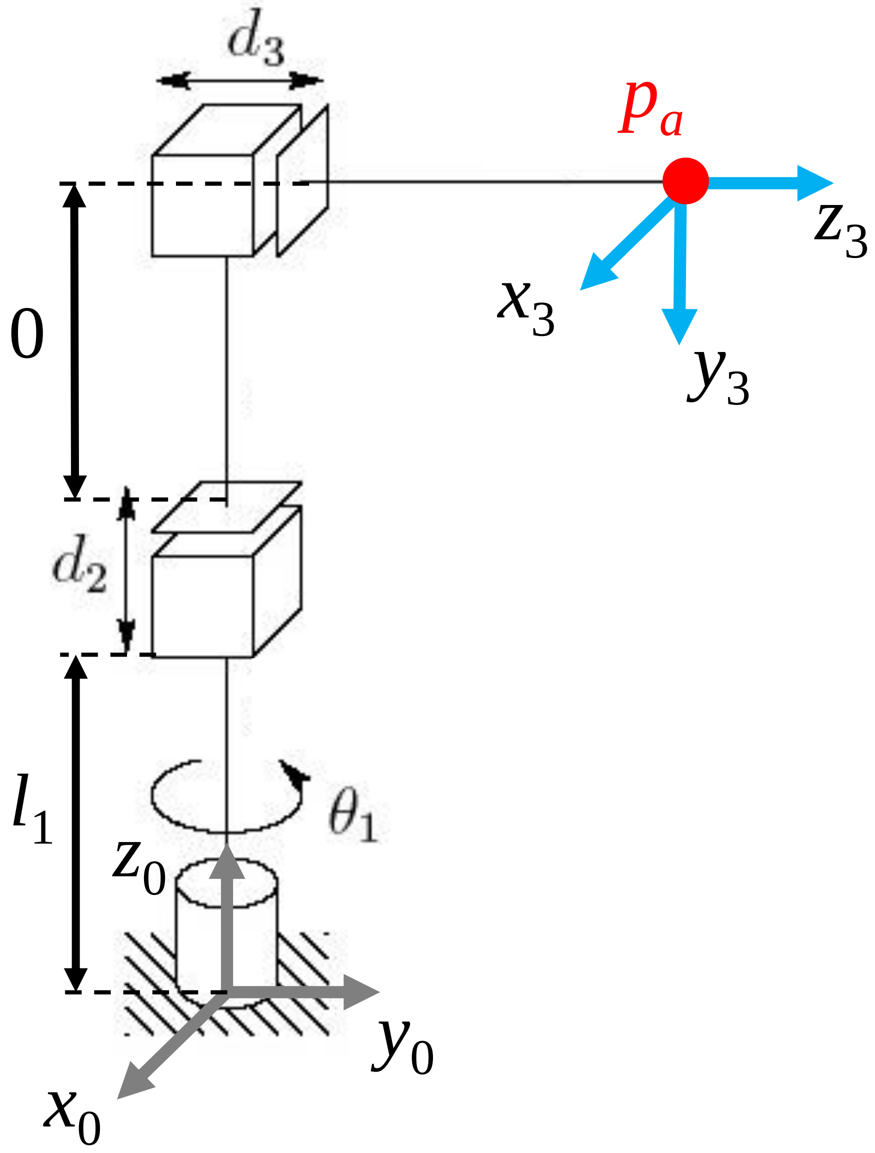
Forward Kinematics of the RPP Cylindrical Robot



Q: Given (q_1, q_2, q_3), where is the tip of the robot?

$$\mathbf{T}_3^0 = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

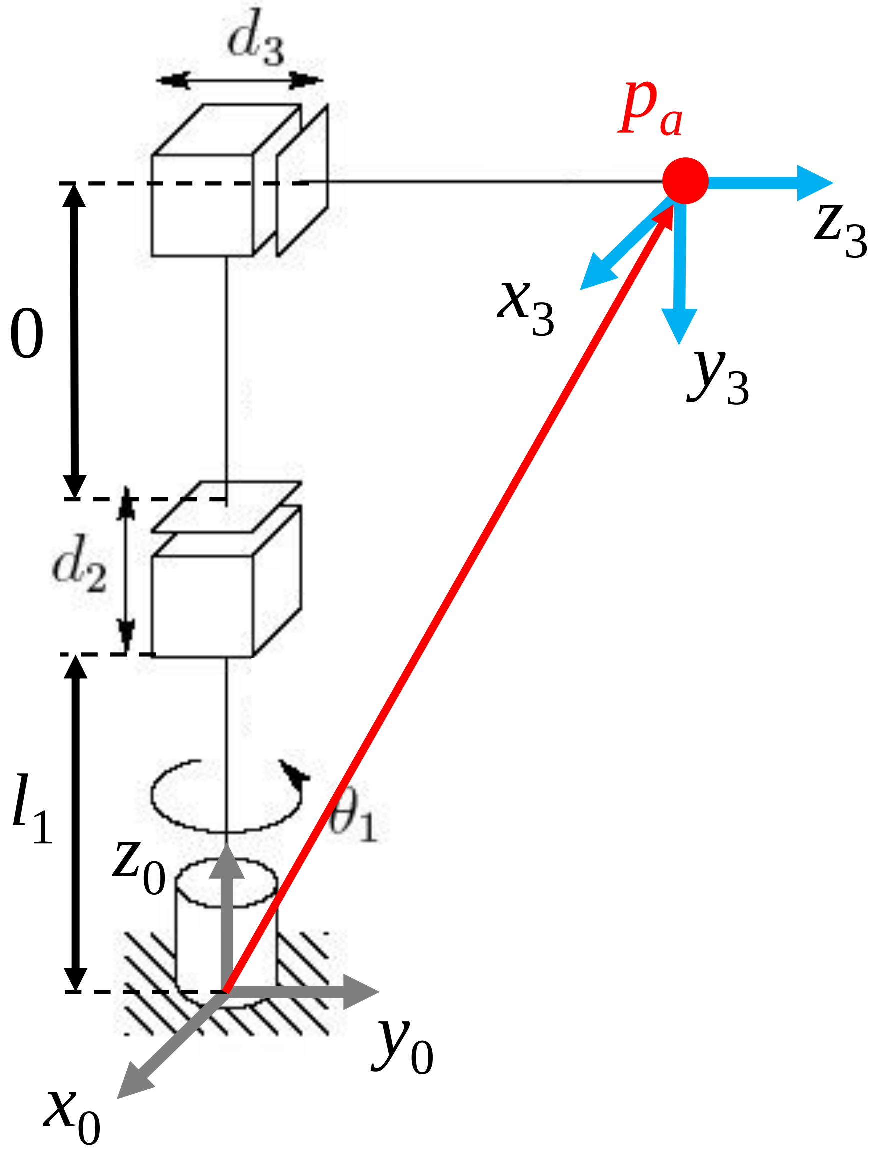
Forward Kinematics of the RPP Cylindrical Robot



Q: Given (q_1, q_2, q_3), where is the tip of the robot?

$$\mathbf{T}_3^0 = \begin{bmatrix} \hat{x}_3^0 & \hat{y}_3^0 & \hat{z}_3^0 & o_3^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics of the RPP Cylindrical Robot

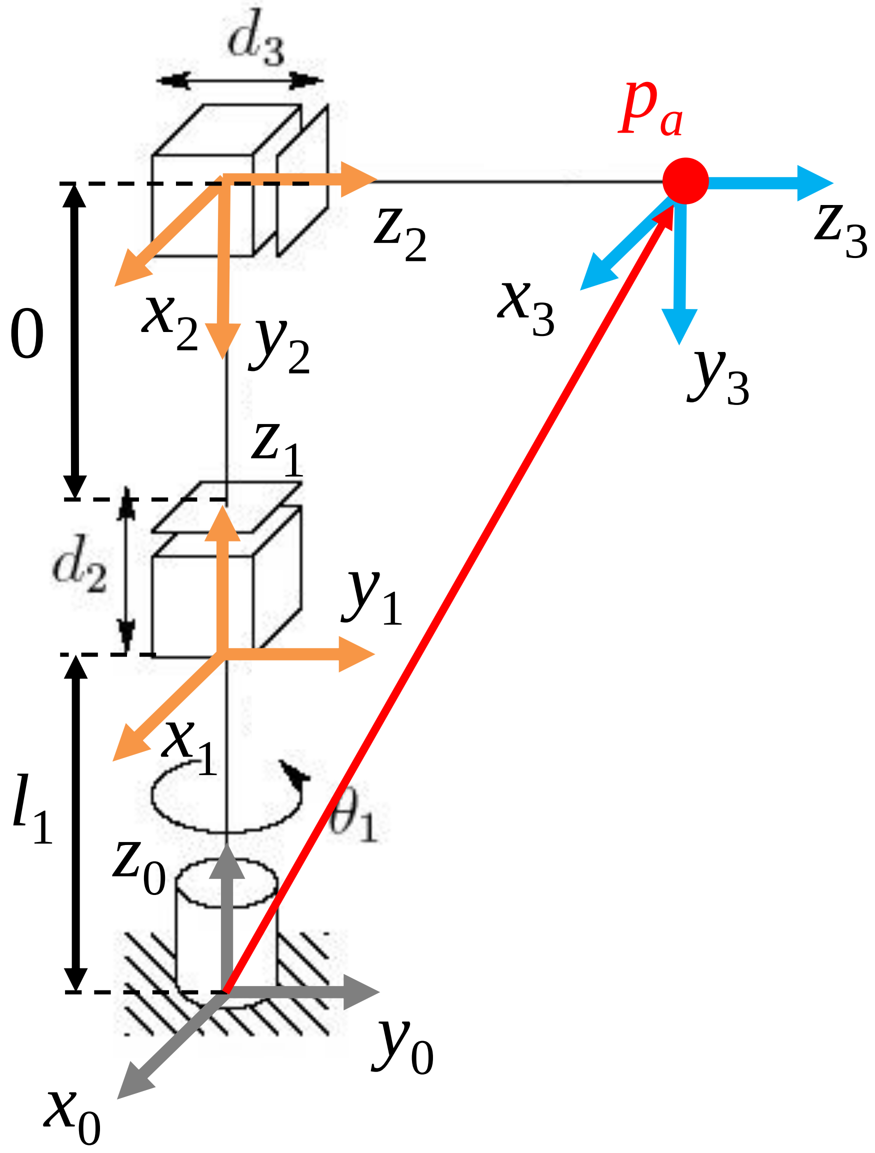


Q: Given (q_1, q_2, q_3), where is the tip of the robot?

$$\mathbf{T}_3^0 = \begin{array}{cc} \color{teal}{\mathbf{R}_3^0} & \color{orange}{\mathbf{d}_3^0} \\ \left[\begin{array}{ccc} c_1^* & 0 & -s_1^* \\ s_1^* & 0 & c_1^* \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right. & \left.\begin{array}{c} -d_3^* s_1^* \\ d_3^* c_1^* \\ d_2^* + l_1 \\ 1 \end{array}\right] \end{array}$$

Superscript * marks joint variables, which vary over time

Forward Kinematics of the RPP Cylindrical Robot

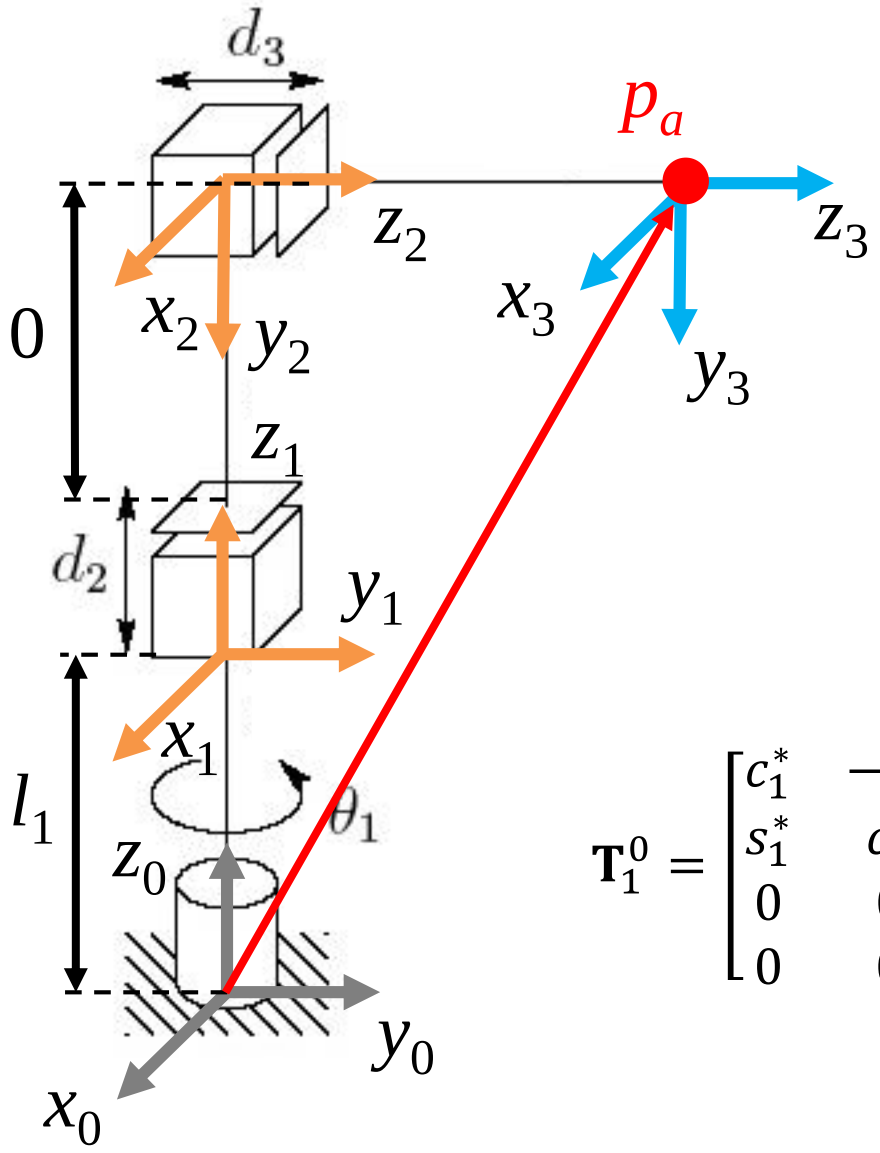


Q: Given (q_1, q_2, q_3), where is the tip of the robot?

It's hard to write transformation matrices by inspection when robots are more complicated.

$$\mathbf{T}_3^0 = \mathbf{T}_1^0(q_1)\mathbf{T}_2^1(q_2)\mathbf{T}_3^2(q_3)$$

Forward Kinematics of the RPP Cylindrical Robot



Q: Given (q_1, q_2, q_3), where is the tip of the robot?

It's hard to write transformation matrices by inspection when robots are more complicated.

$$\mathbf{T}_3^0 = \mathbf{T}_1^0(q_1)\mathbf{T}_2^1(q_2)\mathbf{T}_3^2(q_3)$$

$$\mathbf{T}_1^0 = \begin{bmatrix} c_1^* & -s_1^* & 0 & 0 \\ s_1^* & c_1^* & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is the general idea of **forward kinematics** for manipulators.

There are many **choices** one must make regarding placement of intermediate frames and definition of joint variables, which means there are many equally good ways to reach the same final solution.

The robotics community has agreed on a set of conventions to ensure uniformity:

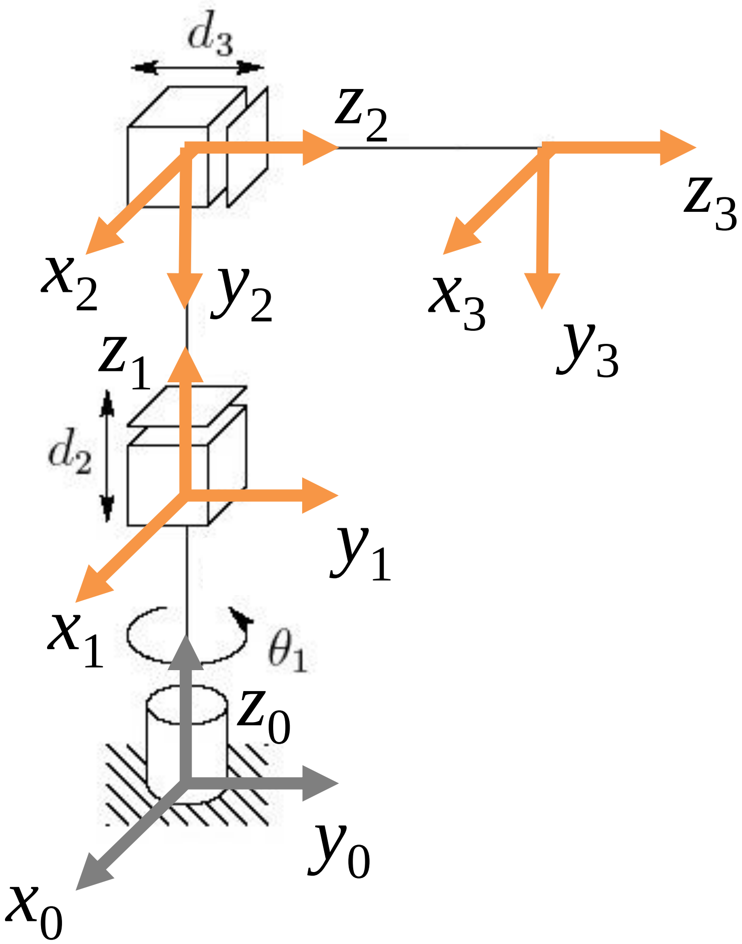
The **Denavit-Hartenberg (DH)** Convention

Denavit-Hartenberg (DH) Parameters

J. Denavit and R. S. Hartenberg, “A kinematic notation for lower pair mechanisms based on matrices,” *ASME Journal of Applied Mechanics*, 22 (1955): 215–221.

Denavit-Hartenberg Convention

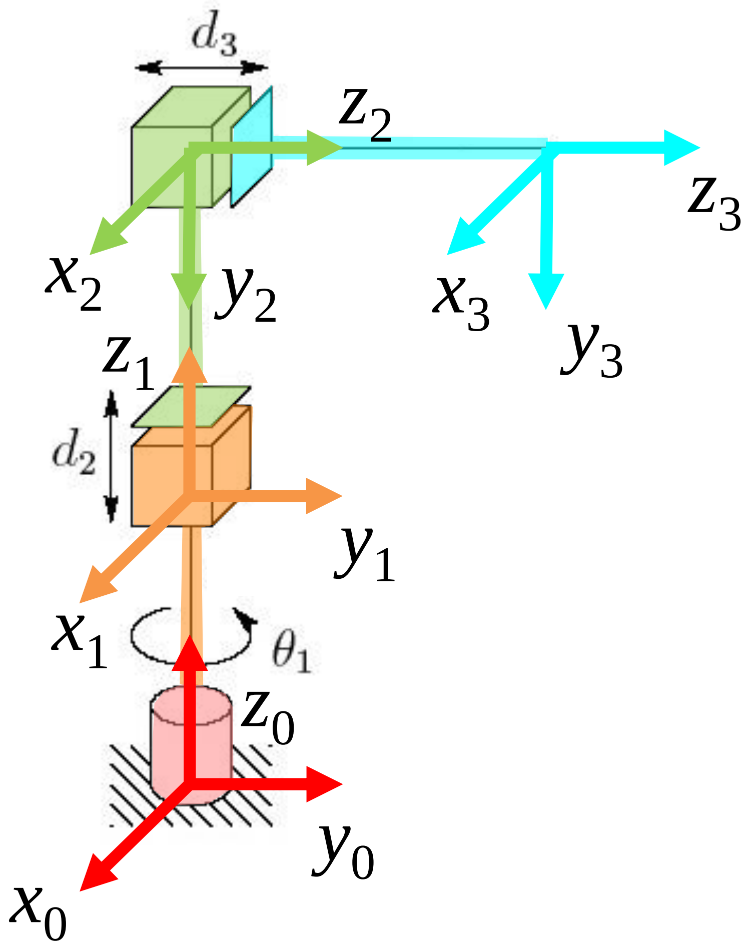
Defines **four** parameters and some rules to help characterize arbitrary kinematic chains



1. Start with a schematic of the robot in its zero pose
2. Attach one frame to each link:
   a) Joint variable for joint i+1 acts along z_i
   b) Orientation of z_i defines positive direction
   c) Axis x_i is perpendicular to and intersects $z_{i\text{-}1}$
3. Also choose a location for base (0) frame:
   a) Origin on z_0. x_0, y_0 chosen for convenience

Denavit-Hartenberg Convention

Defines **four** parameters and some rules to help characterize arbitrary kinematic chains



1. Start with a schematic of the robot in its zero pose
2. Attach one frame to each link: Frame i-1 is at the start of link i and frame i is at the end
   a) Joint variable for joint i+1 acts along z_i
   b) Orientation of z_i defines positive direction
   c) Axis x_i is perpendicular to and intersects z_{i-1}
3. Also choose a location for base (0) frame:
   a) Origin on z_0. x_0, y_0 chosen for convenience

Denavit-Hartenberg Convention

Defines **four** parameters and some rules to help characterize arbitrary kinematic chains

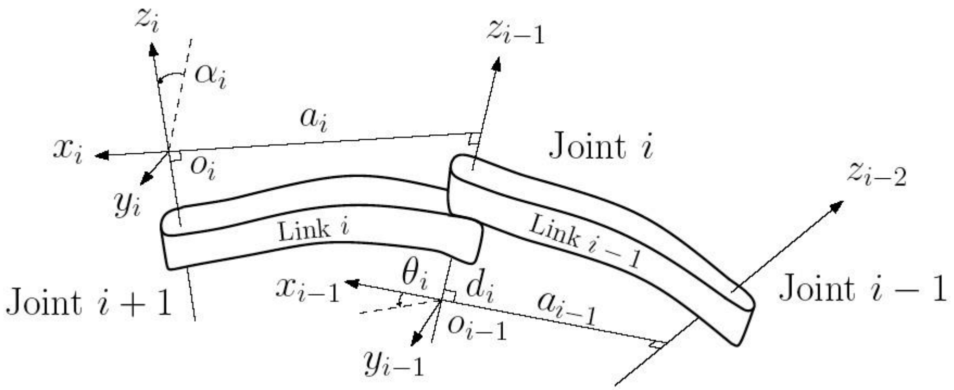


Figure 3.4: Denavit-Hartenberg frame assignment.

Coordinate frames do not need to be located at the actual joints of the robot; the z-axis defines the joint's effective action.

x step	a_i Link Length	distance between z_{i-1} and z_i, measured along x_i
	α_i Link Twist	angle between z_{i-1} and z_i, measured in the plane normal to x_i (right hand rule)
z step	d_i Link Offset	distance between x_{i-1} and x_i, measured along z_{i-1}
	θ_i Joint Angle	angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1} (right hand rule)

a_i Link Length	distance between z_{i-1} and z_i, measured along x_i
α_i Link Twist	angle between z_{i-1} and z_i, measured in the plane normal to x_i (RHR)
d_i Link Offset	distance between x_{i-1} and x_i, measured along z_{i-1}
θ_i Joint Angle	angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1} (RHR)

In what order are the transformations applied?

Intermediate frames

Post-multiply

$$A_i =$$

$$A_i =$$

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

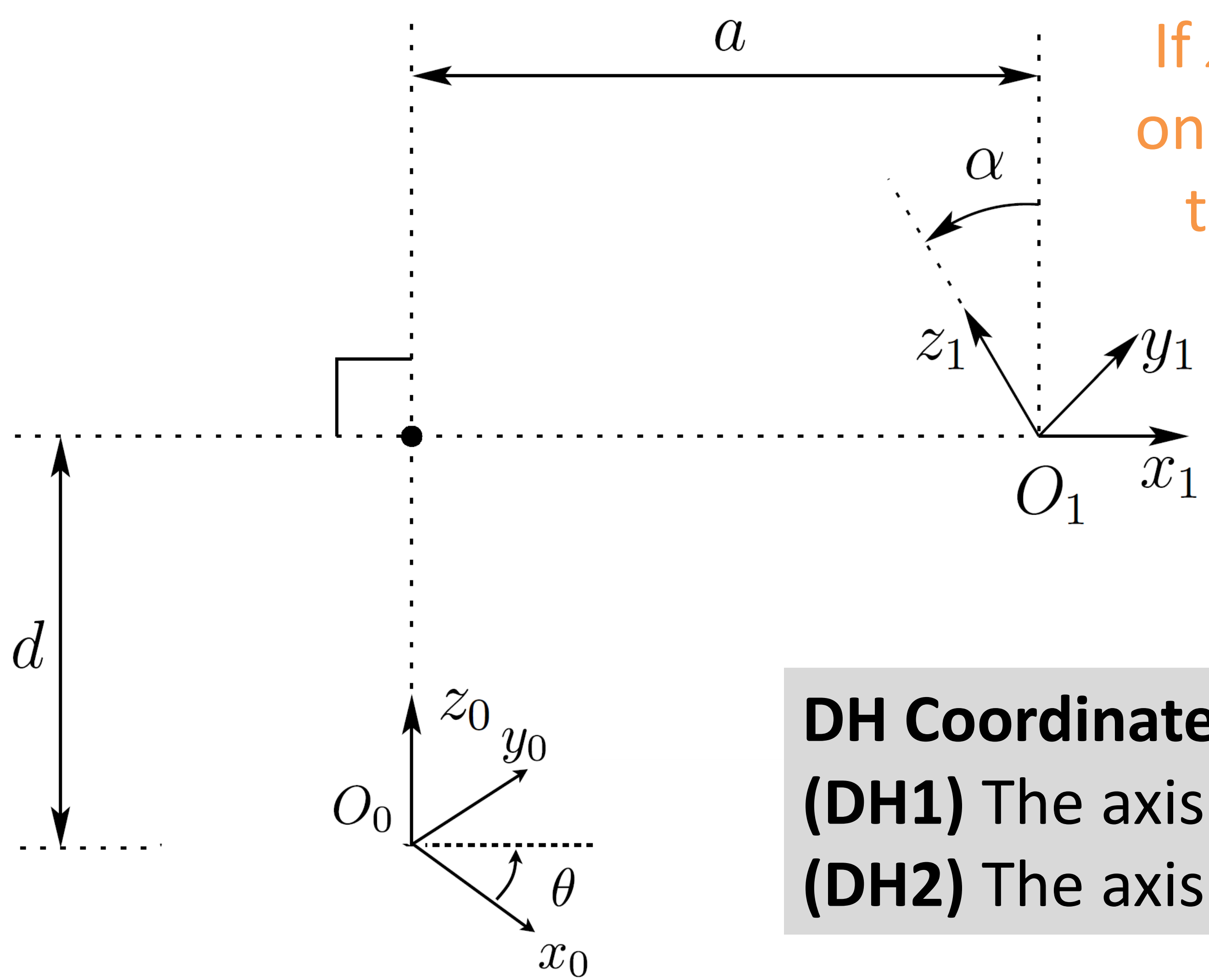
Rotate about z by theta

Translate along z by d

Translate along new x by a

Rotate around new x by alpha

DH parameters can be used to represent transformations between any two rigid bodies, provided the frames are chosen following the convention.



If x_1 satisfies these requirements for one robot configuration, will it satisfy them for all robot configurations?

DH Coordinate Frame Assumptions

(DH1) The axis x_1 is perpendicular to the axis z_0

(DH2) The axis x_1 intersects the axis z_0

Great List of DH Steps on Pages 110-111 in SHV

3.4 CHAPTER SUMMARY

In this chapter we studied the relationships between joint variables, q_i and the position and orientation of the end effector. We begin by introducing the Denavit-Hartenberg convention for assigning coordinate frames to the links of a serial manipulator. We may summarize the procedure based on the DH convention in the following algorithm for deriving the forward kinematics for any manipulator.

Step l: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.

For $i = 1, \ldots, n-1$, perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n-th joint is revolute, set $z_n = a$ along the direction z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.

Step 7: Create a table of link parameters a_i, d_i, α_i, θ_i.

$a_i =$ distance along x_i from o_i to the intersection of the x_i and z_{i-1} axes.

$d_i =$ distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

$\alpha_i =$ the angle between z_{i-1} and z_i measured about x_i.

$\theta_i =$ the angle between x_{i-1} and x_i measured about z_{i-1}. θ_i is variable if joint i is revolute.

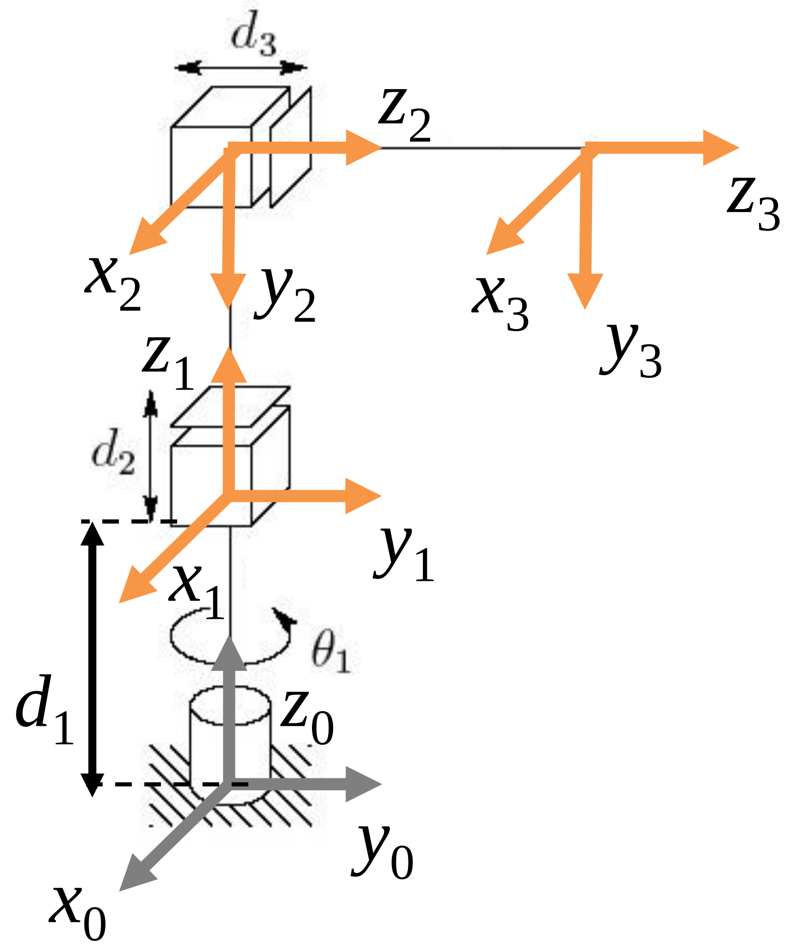
Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into (3.10).

Step 9: Form $T_n^0 = A_1 \cdots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

Denavit-Hartenberg Convention

a_i Link Length	distance between z_{i-1} and z_i, measured along x_i
α_i Link Twist	angle between z_{i-1} and z_i, measured in the plane normal to x_i (RHR)
d_i Link Offset	distance between x_{i-1} and x_i, measured along z_{i-1}
θ_i Joint Angle	angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1} (RHR)

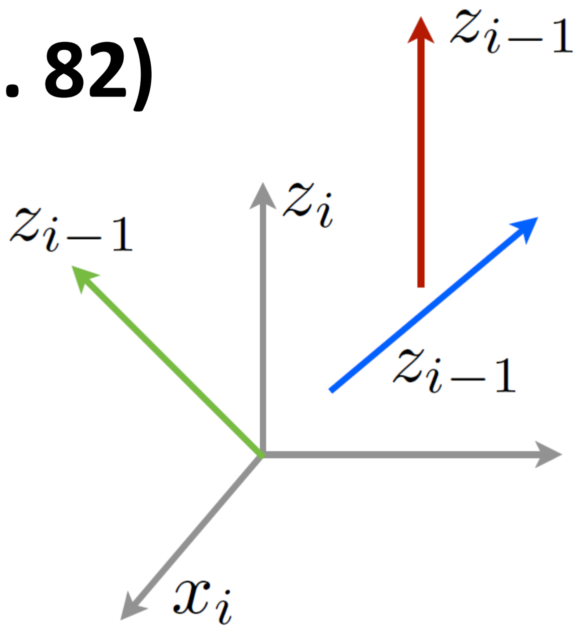
Defines **four** parameters and some rules to help characterize arbitrary kinematic chains



Link	x step		z step	
	a_i	α_i	d_i	θ_i

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Conventions that make x placement easier (SHV p. 82)



If z_{i-1} is parallel to z_i	orient x_i toward z_i
if z_{i-1} intersects z_i	orient x_i normal to the plane formed by z_{i-1} and z_i
if z_{i-1} is not coplanar with z_i	orient x_i along normal with z_{i-1}, toward z_i

Conventions that make x placement easier (SHV p. 82)

Satisfied by Construction:

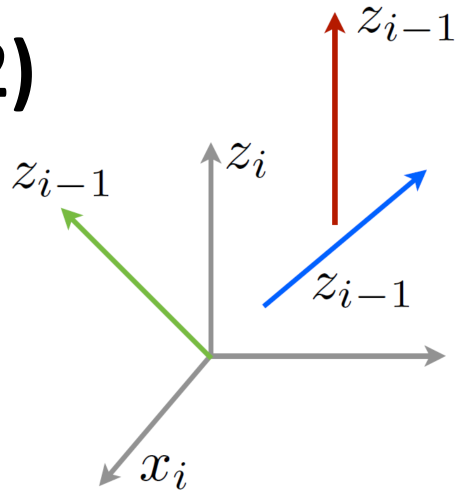
(DH0) The axis x_i is perpendicular to the axis z_i

DH Coordinate Frame Assumptions

(DH1) The axis x_i is perpendicular to the axis $z_{i\text{-}1}$

(DH2) The axis x_i intersects the axis $z_{i\text{-}1}$

Your coordinate frames may not be on your robot!



if $z_{i\text{-}1}$ is not coplanar with $z_{i,}$ orient x_i along normal with $z_{i\text{-}1}$, toward z_i

If $z_{i\text{-}1}$ is parallel to $z_{i,}$ orient x_i toward z_i

if $z_{i\text{-}1}$ intersects $z_{i,}$ orient x_i normal to the plane formed by $z_{i\text{-}1}$ and z_i

The **Denavit-Hartenberg transform** results from successive rotations and translations via the four DH parameters

a parameterization for homogeneous transformations

The transform from i to i-1 is

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Three DH parameters will be **constant** for each joint's transformation, and one will **vary**.

Plug DH parameters into the above formula to find each joint's transformation matrix.

The final transformation matrix from tip to base is

$$\mathbf{T}_n^0 = A_1(q_1) \cdots A_n(q_n)$$

Next time: More DH Parameters

Chapter 3: Forward and Inverse Kinematics

- Read 3.2