Digital Signal Processing

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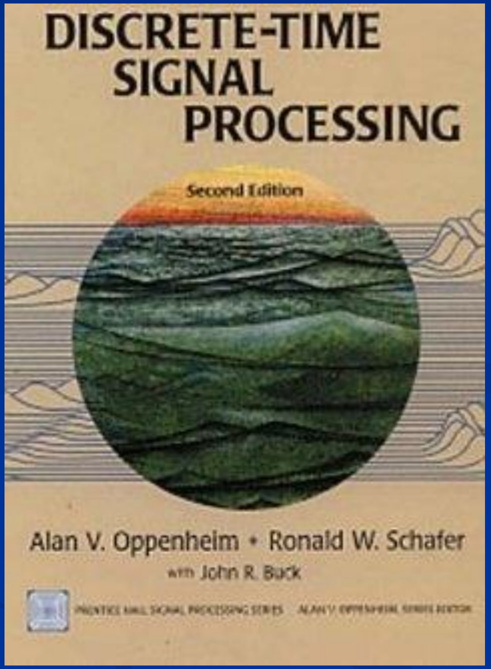
Bibliography

- **Textbook**
  - Alan. V. Oppenheim, Discrete Time Signal Processing, 2nd Ed., Prentice Hall, 1999
  - 刘树棠等译，《离散时间信号处理》，西安交通大学出版社
  - 胡广书编著，《数字信号处理一理论、算法与实现》，清华大学出版社
  - Vinay. K. Ingle， John.G Proakis， Digital Signal Processing Using MATLAB, PWS Publishing Company 1997



Examinations and Grading

Assignments

- Reference reading
- Method introduction
- Simulated and/or experimental applications
- Matlab, MS VB or VC program
- Original signals

Proposal & PPT

- Introduction
- Project objectives
- Background of research
- Proposed work
- Research plan
- Reference

Scientists discover the world that exists; Engineers create the world that never was.

– *Theodore Van Karman*

西奥多·冯·卡门（1881年~1963年），美籍匈牙利犹太人，20世纪最伟大的美国工程学家，开创了数学和基础科学在航空和航天和其他技术领域的应用，被誉为“航空航天时代的科学奇才”。他所在的加利福尼亚理工学院实验室后来成为美国国家航空和航天喷气实验室， 我国著名科学家钱伟长、钱学森、郭永怀都是他的亲传弟子。

Course Outlines

- Introduction
- Discrete-time signals and systems
- Sampling of continuous-time signals
- Filtering
- Fourier transform
- Signal processing using Matlab
- Joint time-frequency analysis
- Applications

Analog System and Digital System

Analog Signal
x(t)

Analog Signal
h(t)

Analog Signal
y(t)

Input
Terminal

Output
Terminal

A/D Convertor

D/A Convertor

Input
Terminal

Output
Terminal

Digital Signal
{x(m)}

Digital Signal
{h(m)}

Digital Signal
{y(m)}

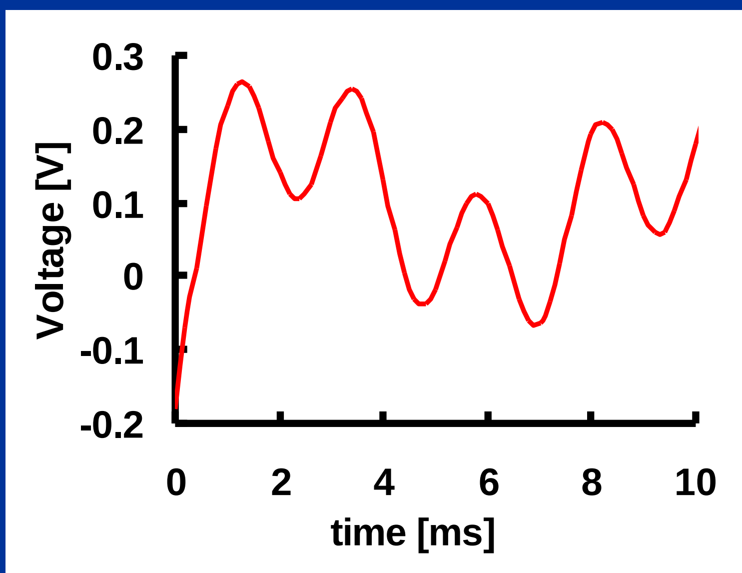
Analog & Digital Signals

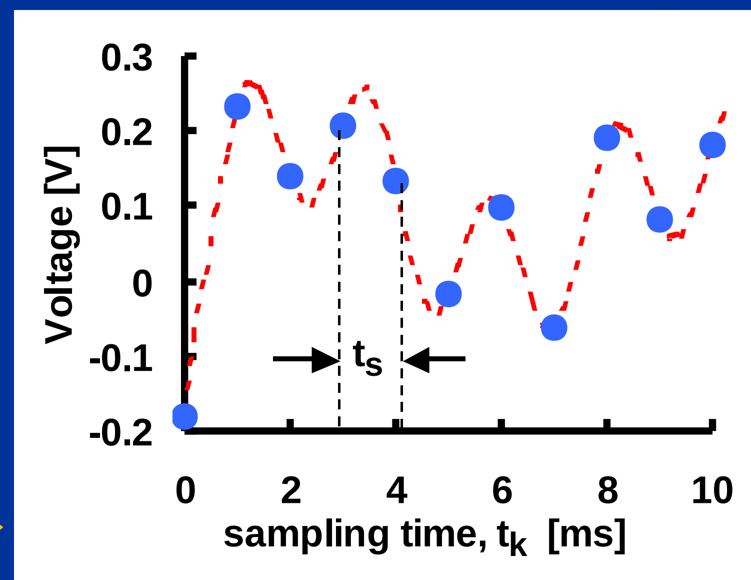
Analog

Continuous function V of *continuous* variable t (time, space etc) : V(t).

Digital

Discrete function V_k of *discrete* sampling variable t_k, with k = integer: $V_k = V(t_k)$.





Uniform (periodic) sampling.
Sampling frequency $f_S = 1/ t_S$

Digital System Example

General scheme !

Sometimes steps missing

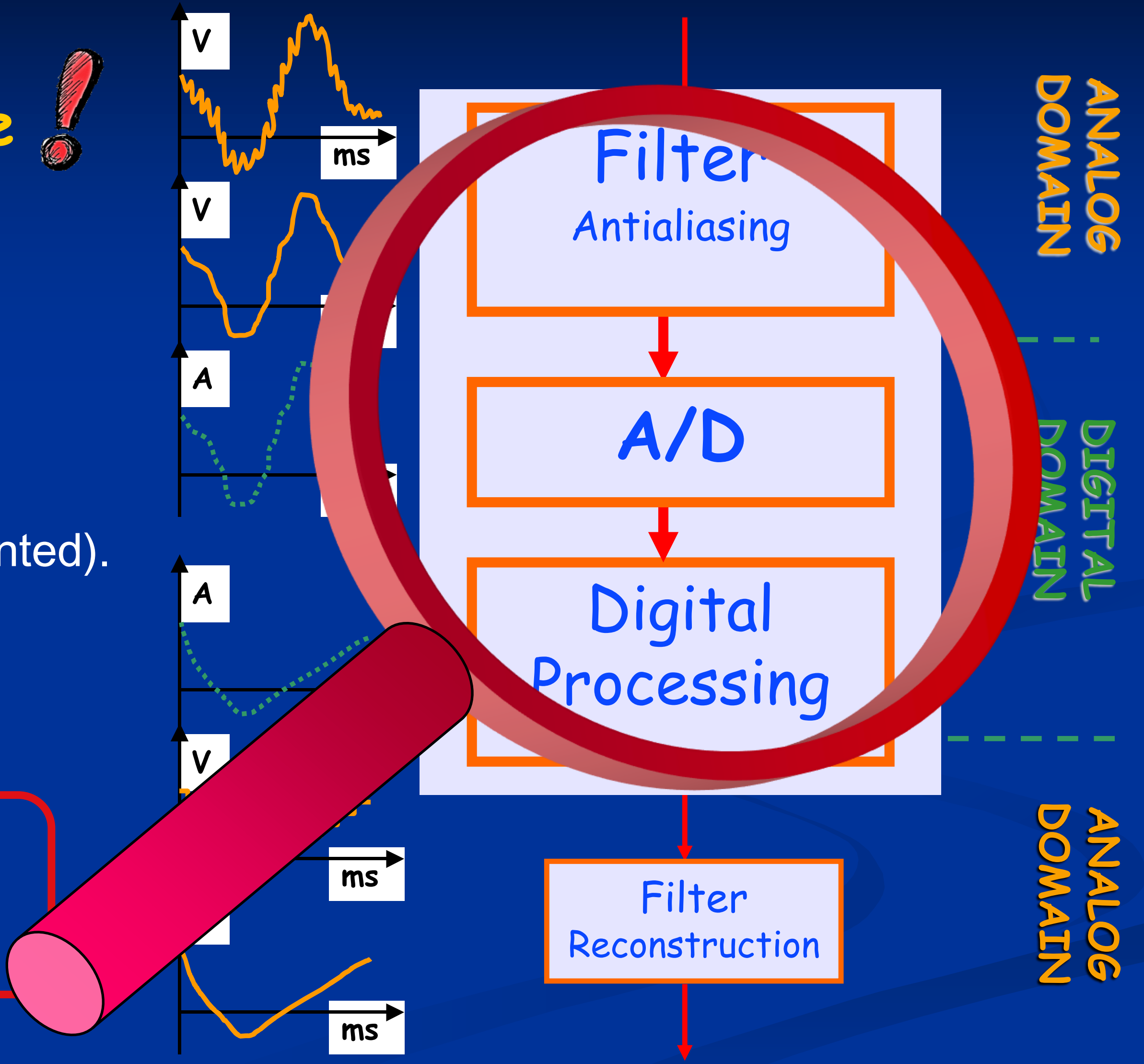
- Filter + A/D

  (ex: economics);

- D/A + filter

  (ex: digital output wanted).

Topics of this lecture.



Comparison of Digital and Analog Technique

- Digital techniques are less susceptible to noise and disturbance.
- The storage of digital signals is easier than that of analog signals.
- Digital techniques are more flexible and versatile than analog techniques.

The Advantage of Digital Signal Processing

- The performance of digital signal processing does not vary with environmental changes.
- The frequency response of a digital signal processor can be adjusted by using the programmable processor.
- Several input signals or channels can be processed by one digital processor.

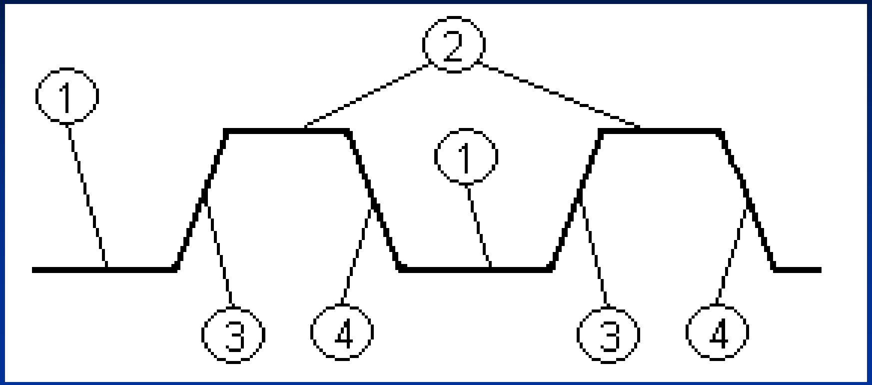
The Advantage of Digital Signal Processing

- Both processed, unprocessed and unfiltered data can be saved for further use.
- The digital processor can have small size, consume low power and keep the cost down.
- The precision achievable with analog signal processing is restricted, with digital processor the precision is limited only by the word length.

Disadvantages of DSP

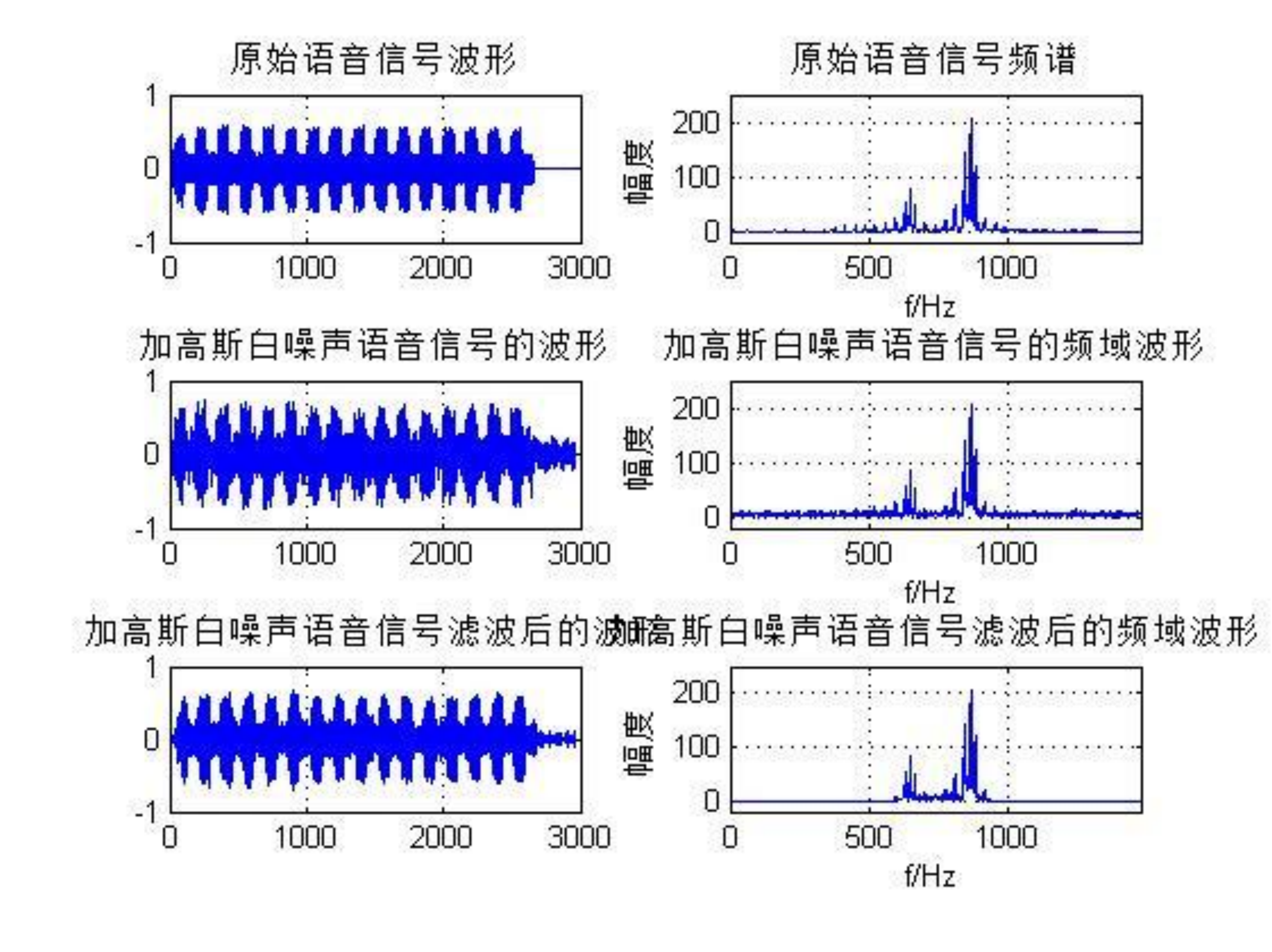
- Speed and Cost
  - DSP designed can be expensive especially when large bandwidth signal are involved.
- Design time
  - DSP design can be time consuming
- Finite word length problem

What is DSP?



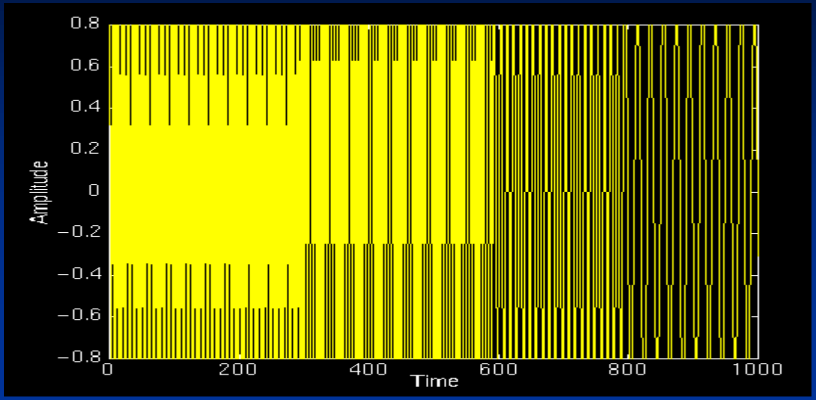
A digital signal waveform: (1) low level, (2) high level, (3) rising edge, and (4) falling edge.

What is DSP?

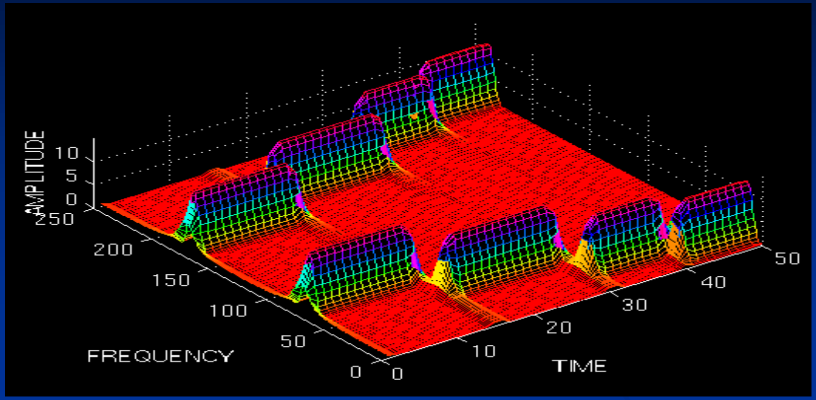


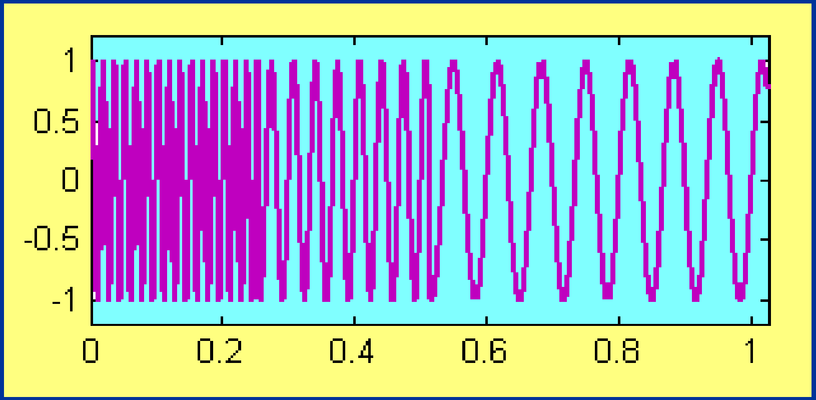
原始语音信号波形
原始语音信号频谱
加高斯白噪声语音信号的波形
加高斯白噪声语音信号的频域波形
加高斯白噪声语音信号滤波后的波
加高斯白噪声语音信号滤波后的频域波形
幅度
f/Hz
1
0
-1
0
1000
2000
3000
200
100
500

What is DSP?

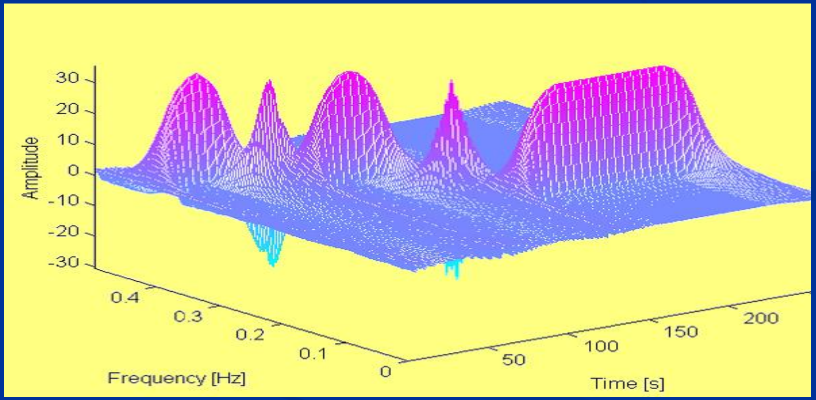


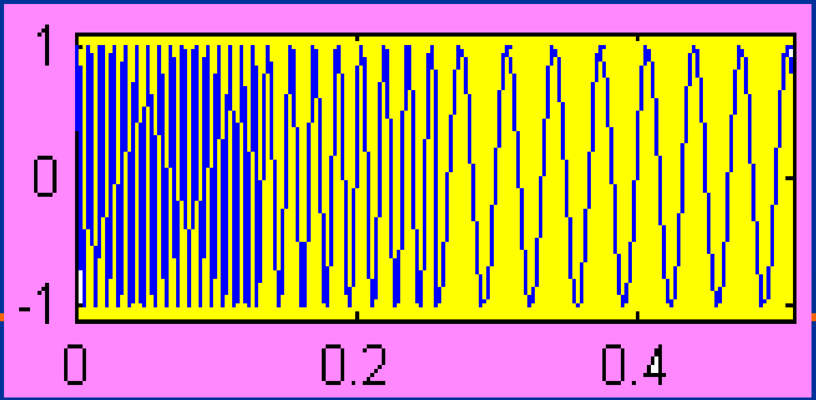
STFT



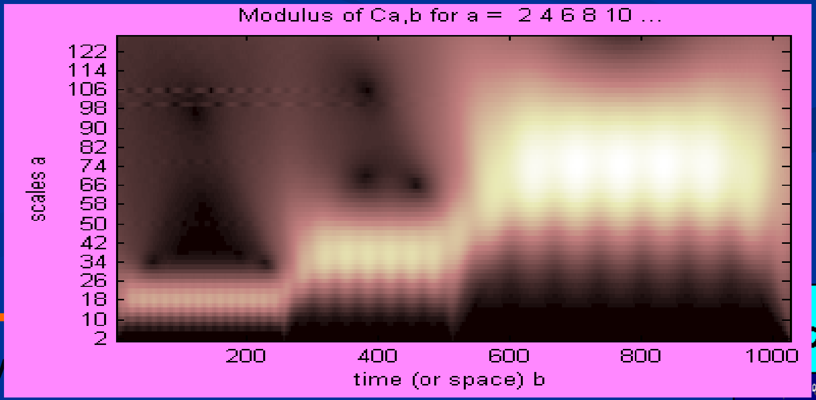


WVD

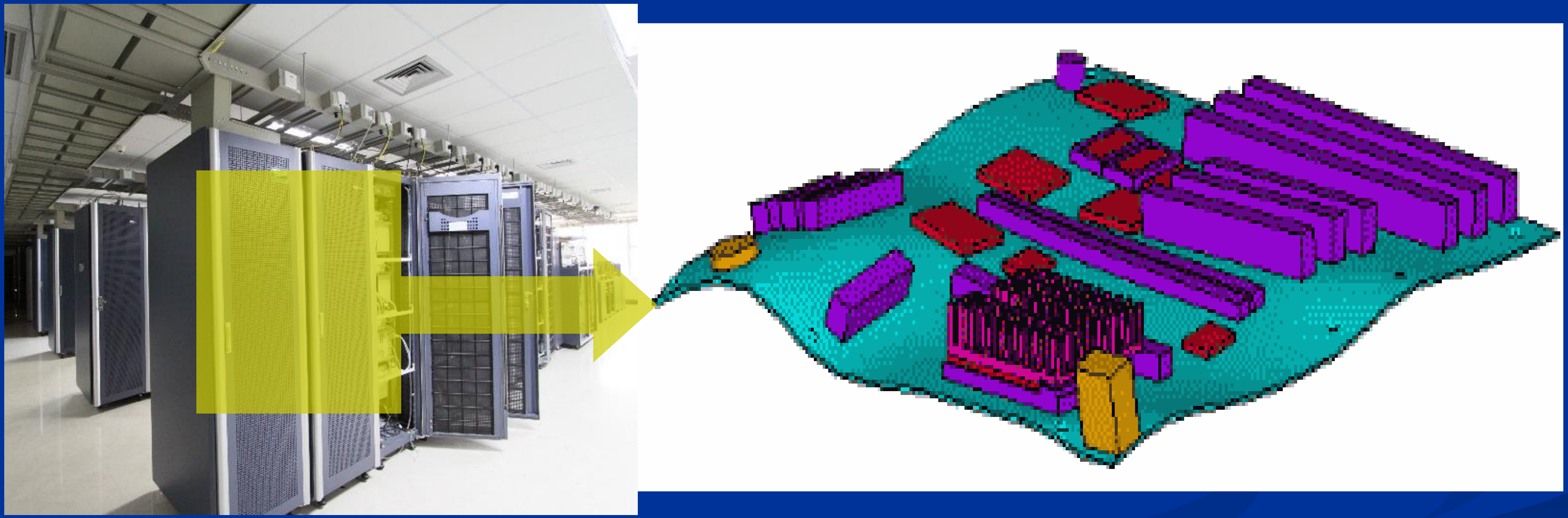




WT

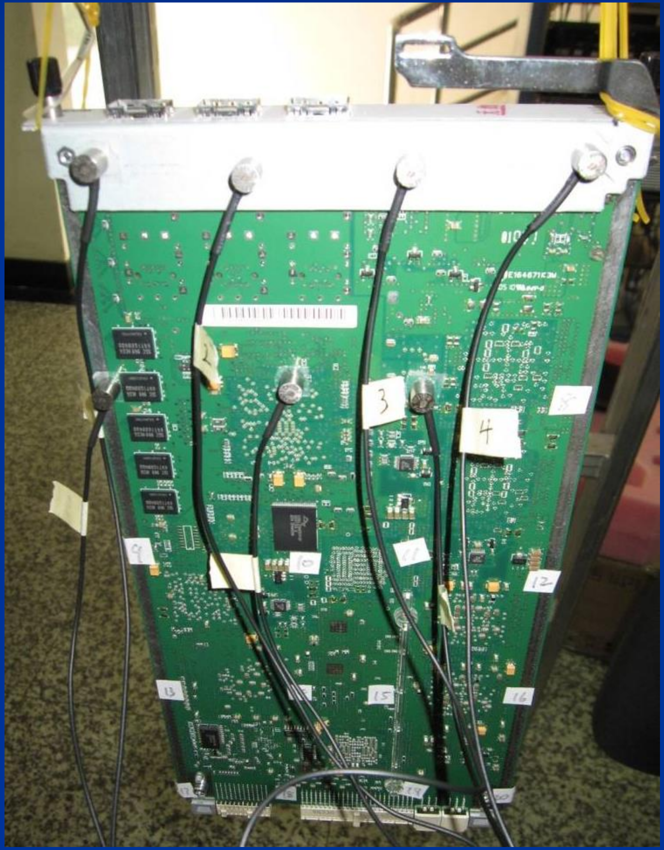


What is DSP?



What is DSP?

- Reliability analysis and design for printed circuit board (PCB)



Typical Applications of DSP Chips

(1) Voice/Speech

- Voice mail
- Speech vocoding
- Speech recognition
- Speaker verification
- Speech enhancement
- Speech synthesis
- Text to speech

(2) Graphics/Imaging

- 3-D rotation
- Robot vision
- Image transmission/compression
- Pattern recognition
- Image enhancement
- Homomorphic processing
- Animation/digital map

(3)Instrumentation

- Spectrum analysis
- Function generation
- Pattern matching
- Seismic processing
- Transient analysis
- Digital filtering
- Phase-locked loops

(4) Control

- Disk control
- Servo control
- Robot control
- Laser printer control
- Engine control
- Motor control

(5)Military

- Secure communications
- Radar processing
- Sonar processing
- Image processing
- Navigation
- Missile guidance
- Radio frequency modems

(6)Telecommunications

- Echo cancellation
- Channel multiplexing
- Adaptive equalizers
- Cellular telephone
- Digital speech
- Video conferencing
- Spread spectrum

(7)Automotive

- Engine control
- Vibration analysis
- Antiskid brakes
- Adaptive ride control
- Global positioning
- Navigation
- Voice commands
- Digital radio
- Cellular telephones

(8)Consumer

- Radar detectors
- Power tools
- Digital audio/TV
- Music synthesizer
- Educational toys

(9) Industrial

- Robotics
- Numeric control
- Security access
- Power line monitors

(10)Medical

- Hearing aids
- Patient monitoring
- Ultrasound equipment
- Diagnostic tools
- Prosthetics
- Fetal monitors

Graphics Imaging

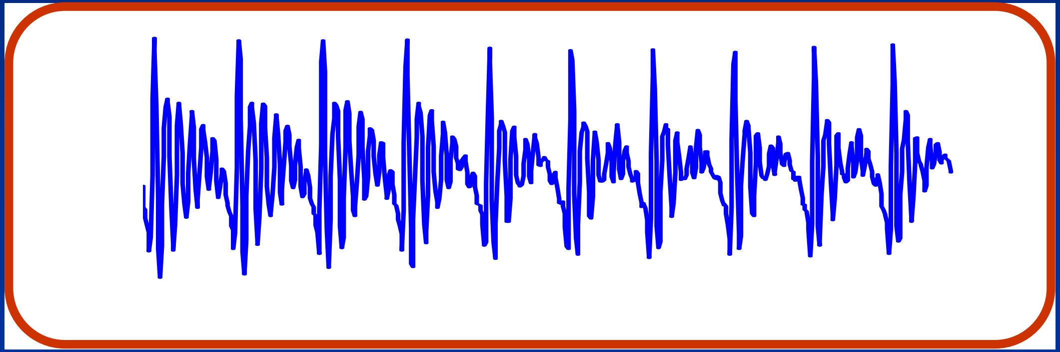
b. Pixel data

	Water			Hay		
Band 1	60	70	52	119	63	148
	46	45	30	101	123	156
	26	18				
Band 2	10	12	28	213	171	223
	31	36	50	204	201	218
	40	41				

c. Spectral signatures

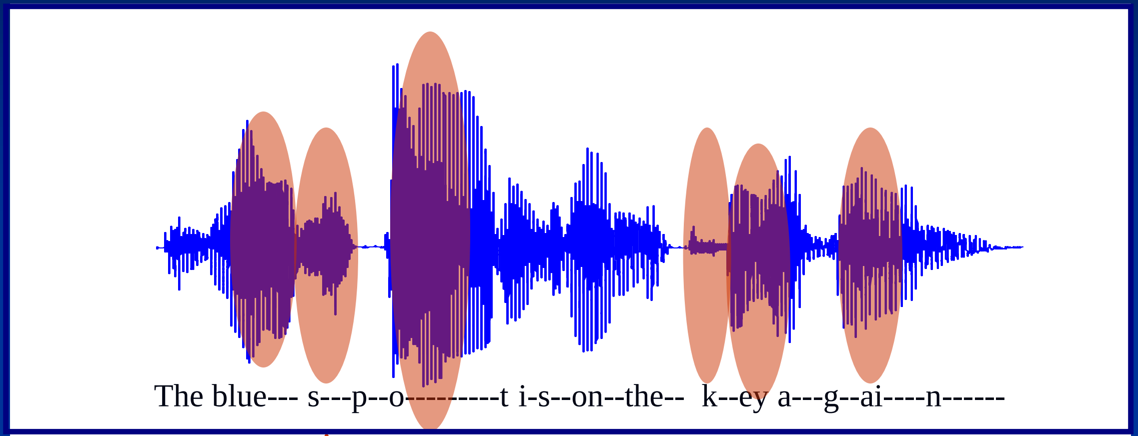
	Water				Hay			
	Min	Max	Mean	Std.dev.	Min	Max	Mean	Std.dev
Band 1	18	70	43.4	17.7	63	156	118.3	33.7
Band 2	10	50	31.0	14.0	171	223	205.0	18.6

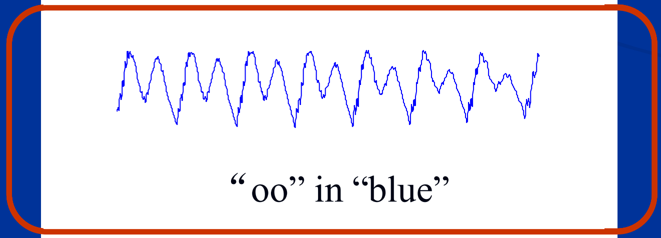
Speech Processing



- Speech coding/compression
- Speech synthesis
- Speech recognition

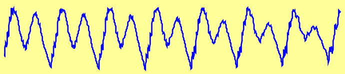
DSP Application in Speech



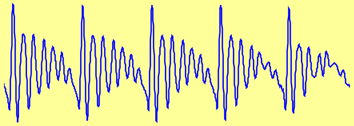


Speech Application (cont’d)

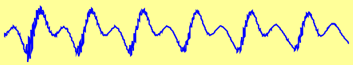
Vowels



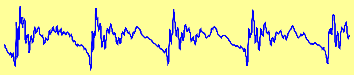
“ oo” in “blue”



“ o” in “spot”



“ ee” in “key”

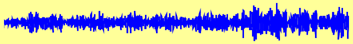


“ e” in “again”

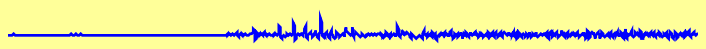
•Quasi-periodic

•Relatively high signal power

Consonants



“ s” in “spot”

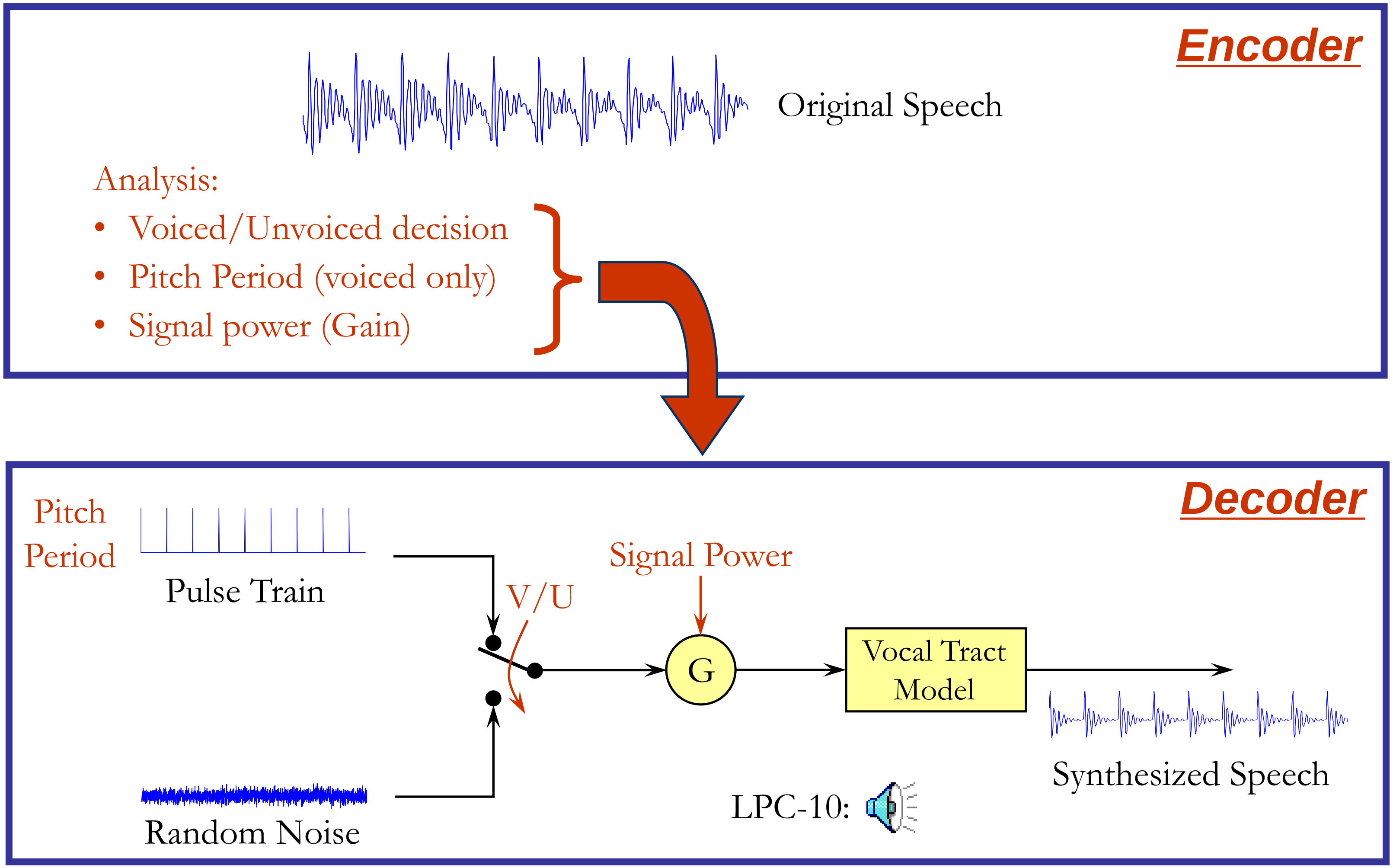


“ k” in “key”

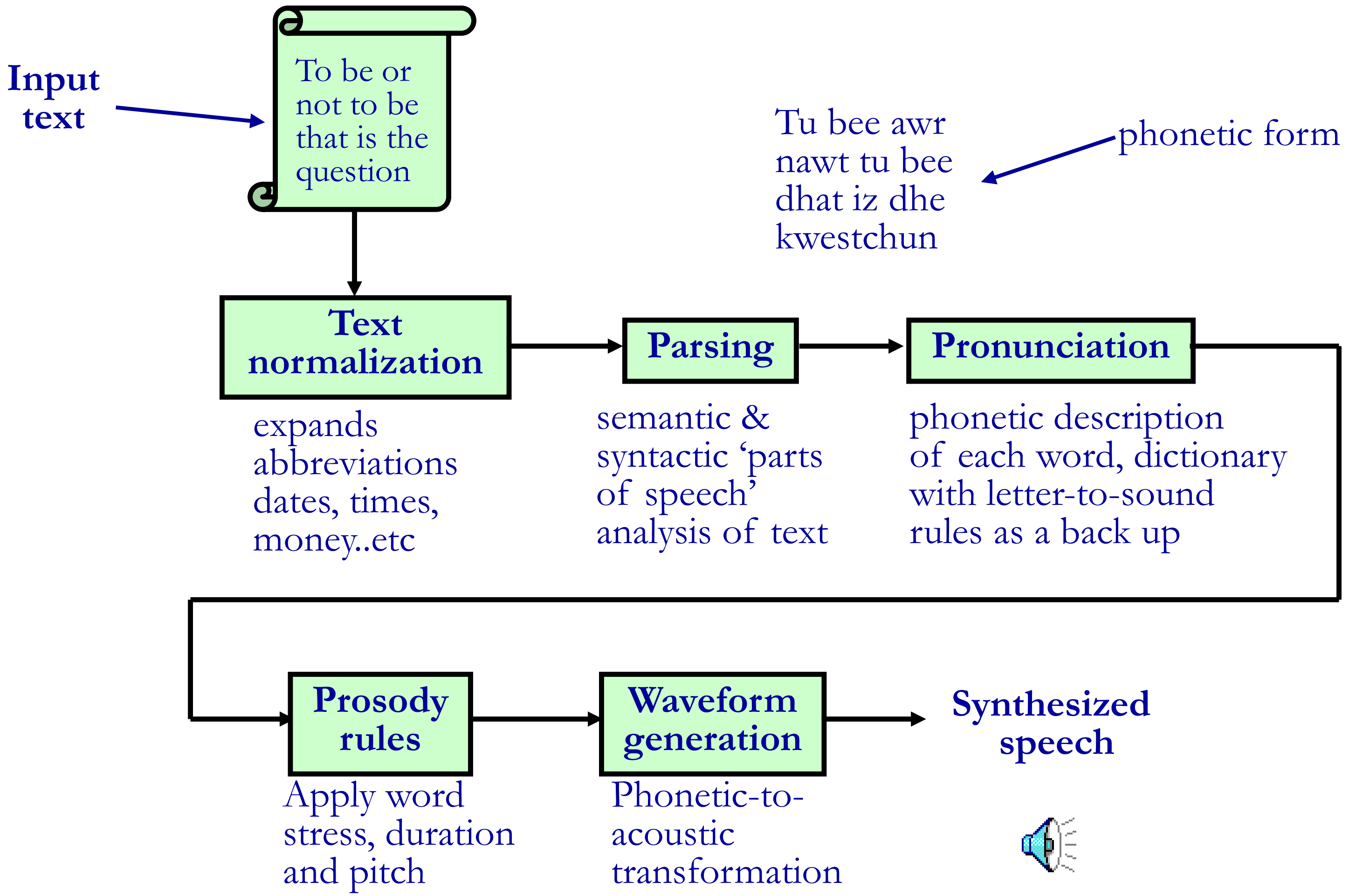
•Non-periodic (random)

•Relatively low signal power

Speech Coding – Vocoder



Text-to-Speech Synthesis



Text-to-speech synthesis sounds very natural these days.

Speech Synthesis Applications

- Speaking clocks
- Spoken (variable) announcements
- Talking emails + talking heads for mobile
- Synthesis of location-based information (e.g. traffic information)
- Interactive systems (e.g. catalogue ordering, Yellow Pages, ...)

Speech/Speaker Recognition

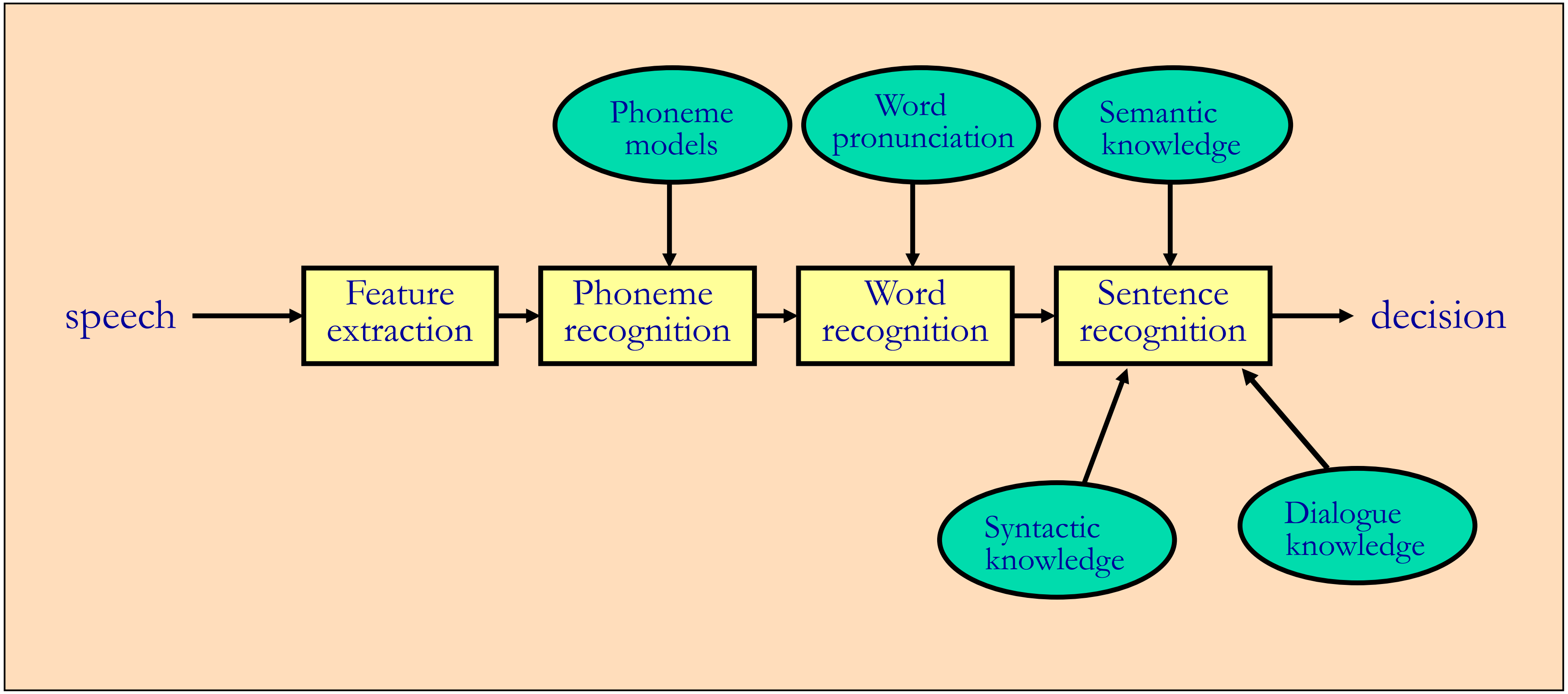
- Speech Recognition – What has been spoken?
  - Speaker dependent – Recognition system trained for a particular person's voice.
  - Speaker independent – Recognition system expected to deal with a wide variety of speakers.
- Speaker Recognition – Who has spoken?
- Not easy…

Sometimestherearenogapsbetweenwords.

Sometim esthereareg aps inthe mid dleofwords.

Acc*ents,* dialects and Stress.

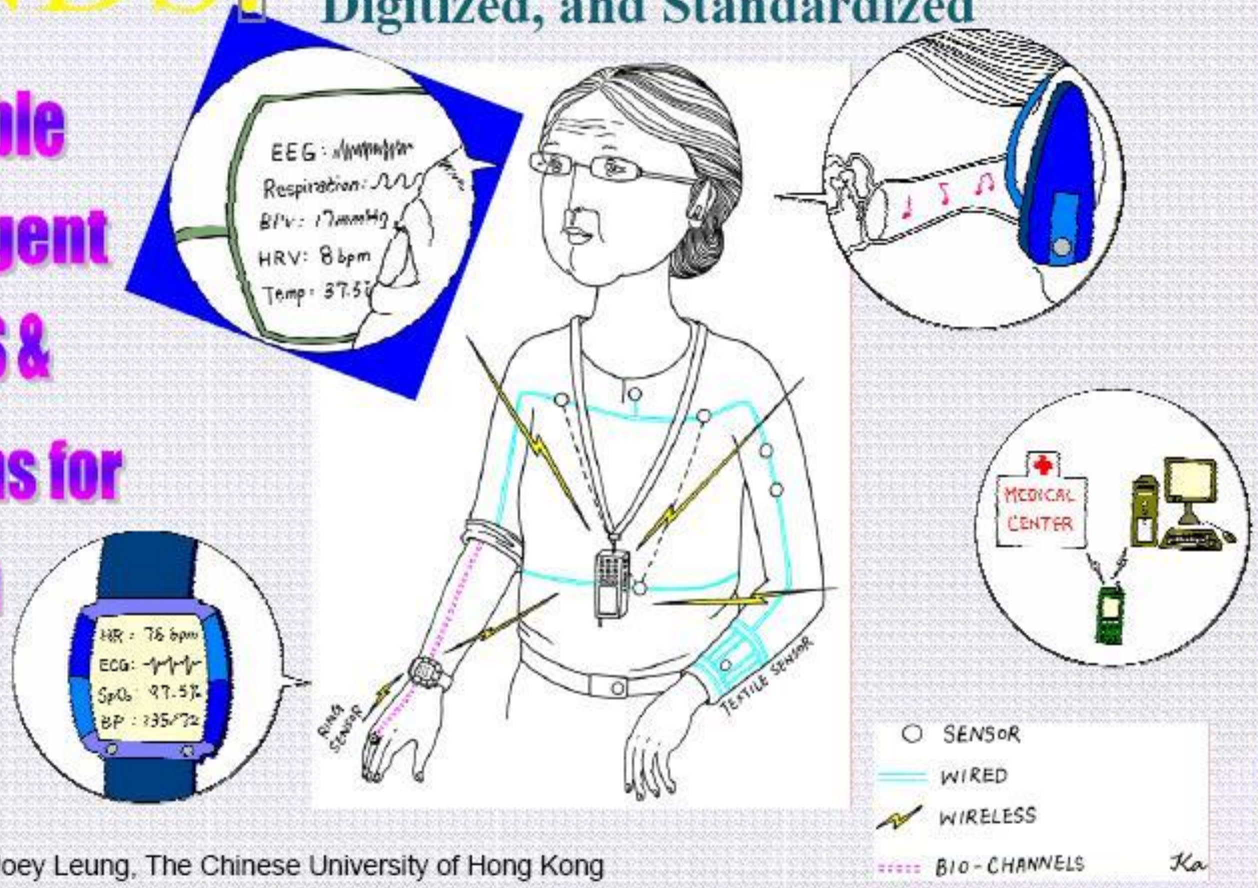
Speech Recognition System



Biomedical Application – Patient Monitoring

MINDS:
- Miniaturized, Integrated, Networked,
- Digitized, and Standardized

Wearable
Intelligent
Sensors &
Systems for
e-**H**ealth



Artwork: Courtesy of Joey Leung, The Chinese University of Hong Kong

Ref: Y.T. Zhang, C.C.Y. Poon, C.H. Chan, *et al.* "A health-shirt using e-textile materials for the continuous and cuffless monitoring of arterial blood pressure," in *Proc. 3rd IEEE-EMBS ISSS-MDBS*, Boston, 2006.

Health Monitoring for Welders

Biomedical Application – Patient Monitoring

- Herbalist doctor

The Basic DSP Operations

- Sampling
- Convolution
- Transformations
- Filtering
- Windowing
- Correlation
- Modulation
- Multiplexing
- Coding

Signal Processing

- Fundamental of Signal Processing
  - Systems
  - Basic signals/functions
  - Basic algorithm: convolution sum
- Engineering Signal Processing
  - Systems
  - Signal processing, feature extraction and explanation based on basic signals/functions

Chapter 2. Discrete-Time Signals and Systems

- **Signal** – Something that conveys information

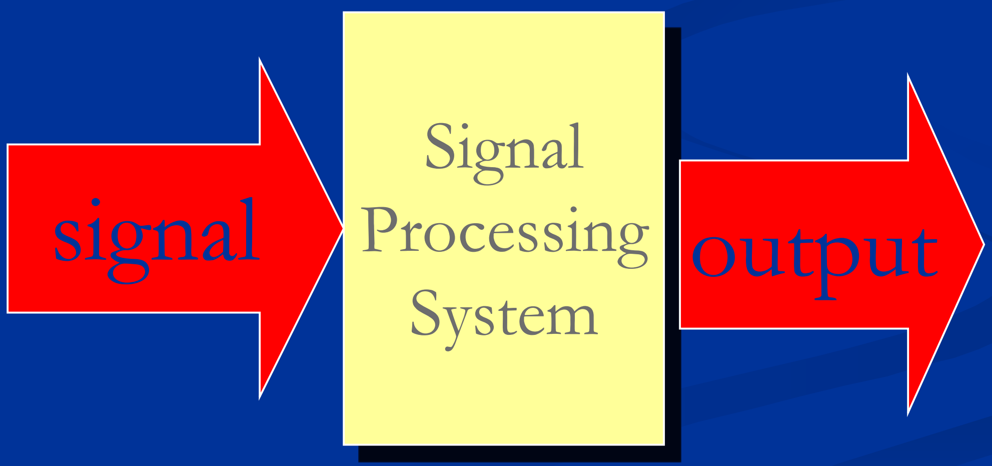
		Amplitude	
		Continuous	Discrete
Time	Continuous	analog signals	continuous-time signals
	Discrete	discrete-time signals	digital signals

- Signals: functions of one or more independent variables
  - A speech signal is a function of time
  - A photographic image is a brightness function of two spatial variables
- Common convention: the independent variable of the mathematical representation of a signal is ***time***

Signal Process Systems

Facilitate the extraction of desired information e.g.,

- Filters
- Parameter estimation



Signal Process Systems

continuous-time signal

signal → analog system → output

continuous-time signal

discrete-time signal

signal → discrete-time system → output

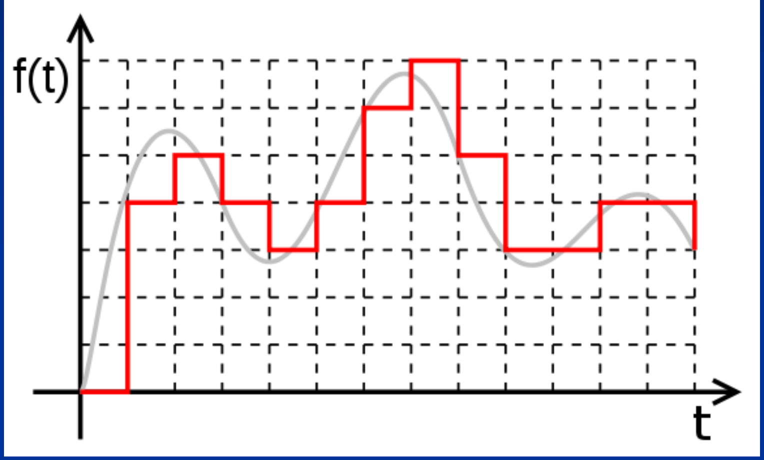
discrete-time signal

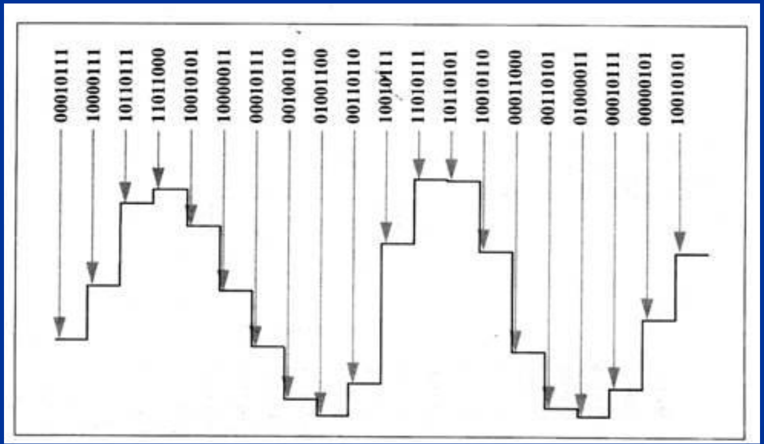
digital signal

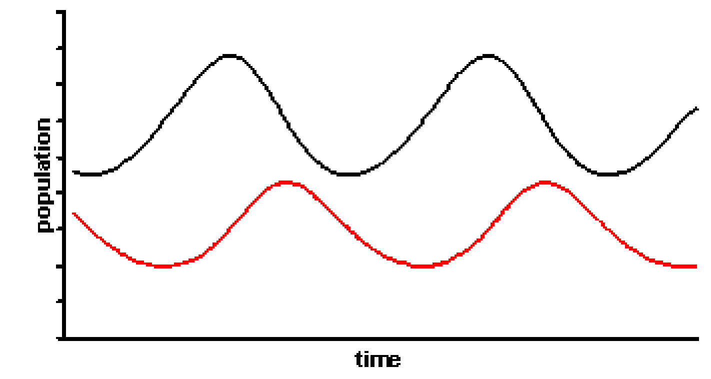
signal → digital system → output

digital signal

- Continuous, discrete and digital signals

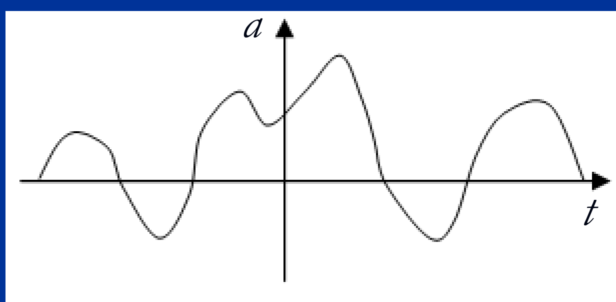




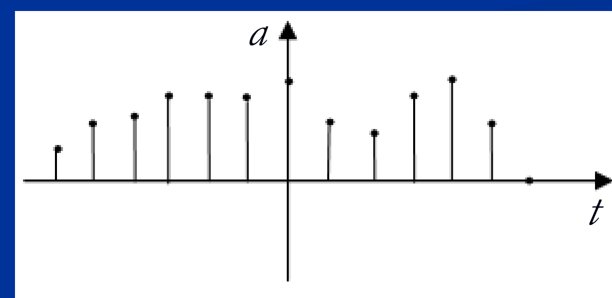


y is the number of wolves
x is the number of sheep

- **Independent variable** in the mathematical representation of a signal may be:
  - Continuous, termed *continuous-time signal* (CTS)
  - Discrete, termed *discrete-time signal* (DTS)

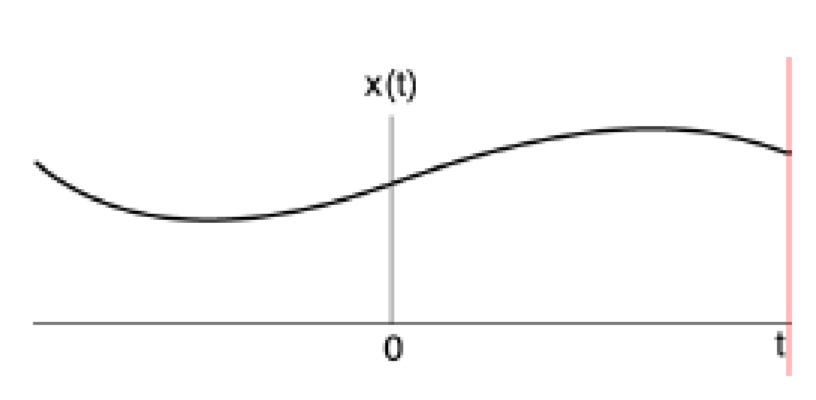


Continuous-time signals: defined along a continuum of times, often referred to as analog signals



Discrete-time signal

Continuous-Time Signals



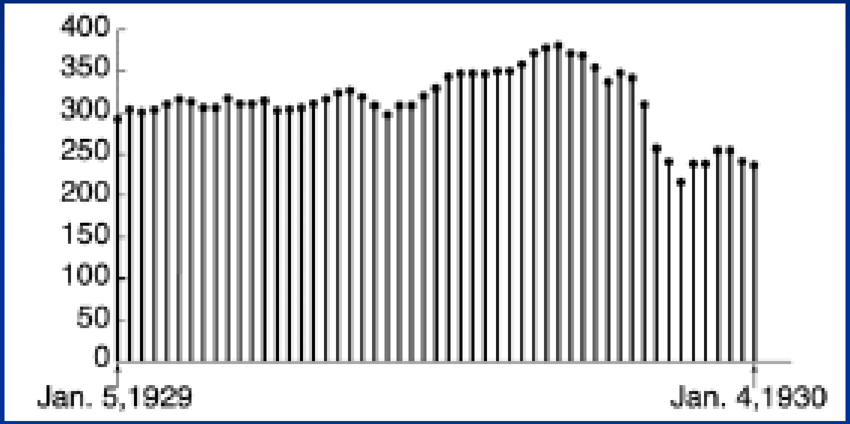
- Most of the signals in the physical world are CT signals—E.g. voltage & current, pressure, temperature, velocity, etc.
- Continuous in time and amplitude

Discrete-Time Signal

- Discrete-time signals can arise in several ways.
  - <u>Periodically sampling a continuous time signal</u>.
  - Some signals are inherently discrete-time.

Many human-made DT Signals

Ex.#1 Weekly Dow-Jones industrial average

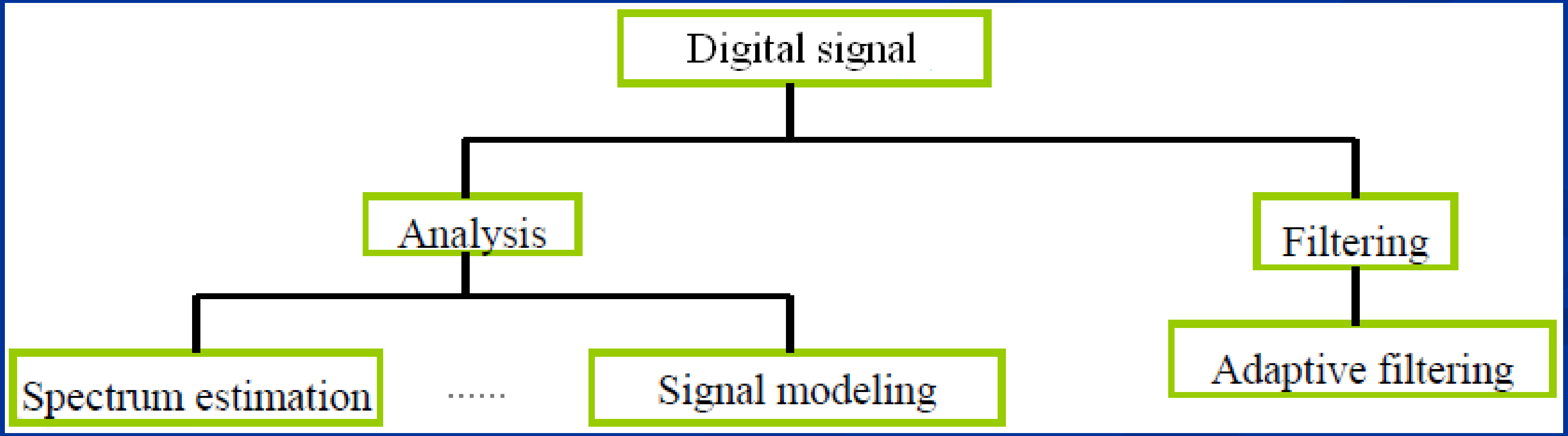


Ex.#2 digital image

Why DT? — Can be processed by modern <u>digital</u> computers and <u>digital</u> signal processors (DSPs).

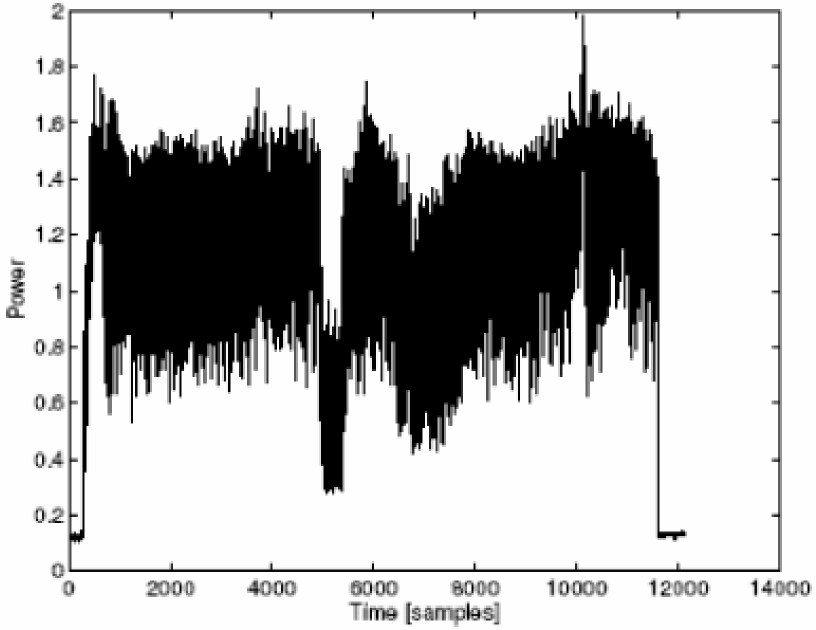
Overview

- Signals are carriers of information, both ***useful*** and ***unwanted***.
- Signal processing techniques can be classified as:
  - **Signal analysis**: Extracting useful information.
  - **Signal filtering**: Enhance the quality of a signal.



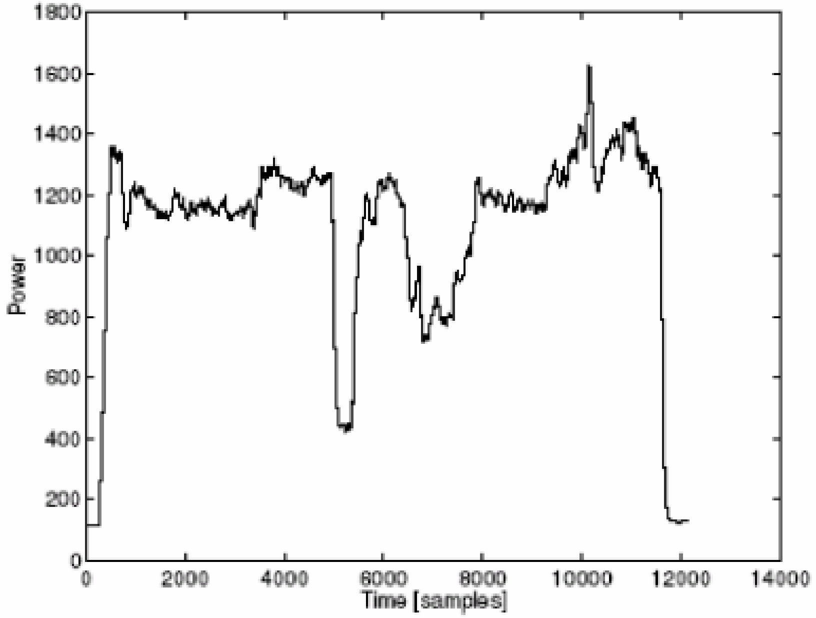
Application - 1

- Adaptive filtering for noise attenuation
  - The noise is considerably attenuated (to be useful) by an adaptive filter.



A raw engine power signal in kW

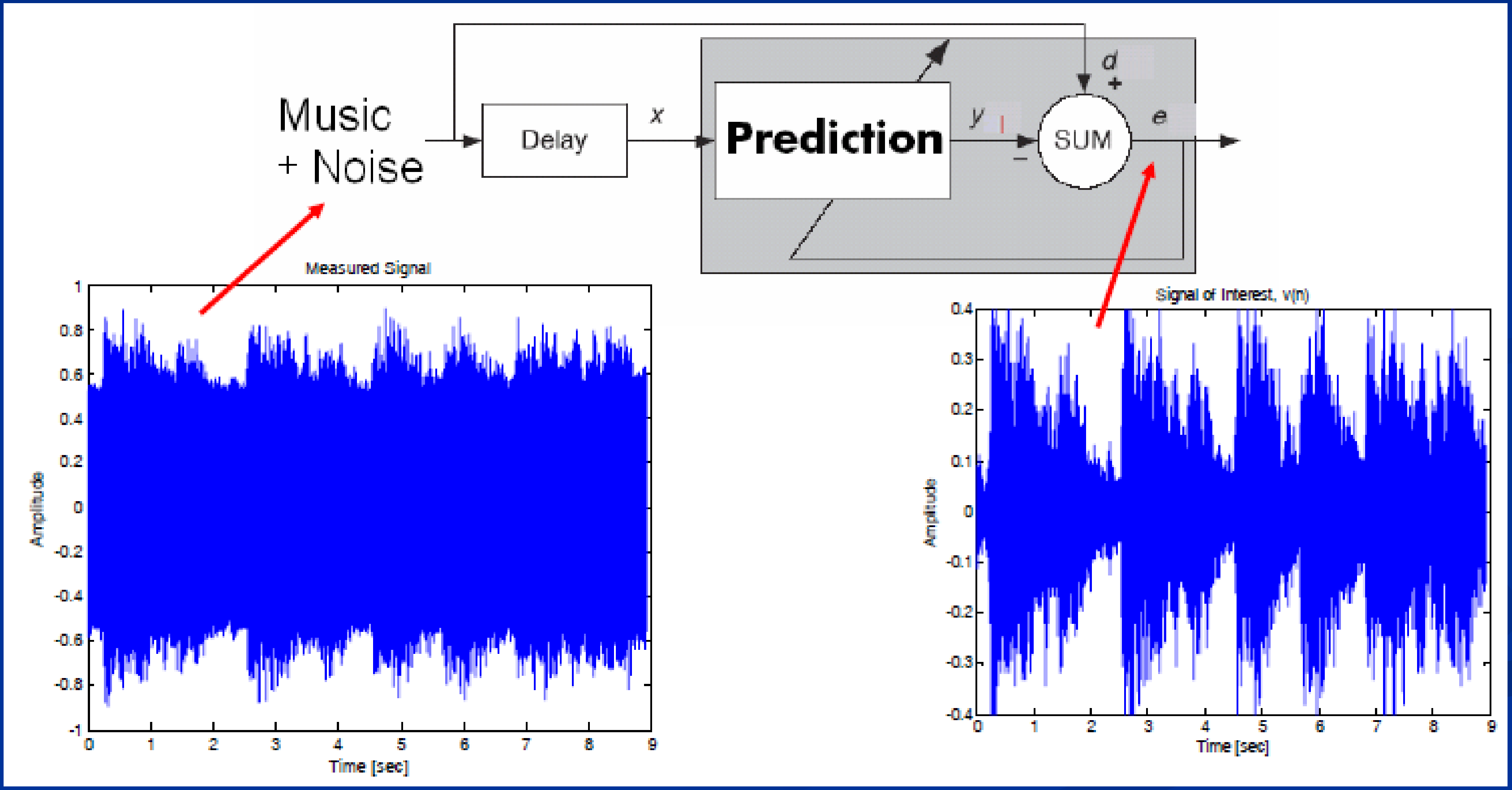
(from a paper refinery at Sund's Defibrator AB, Sundsvall, Sweden)



Output of an adaptive filter

Application - 2

- Adaptive filtering for noise suppression in music
  - (See the details at www.mathworks.com/products/filterdesign/)

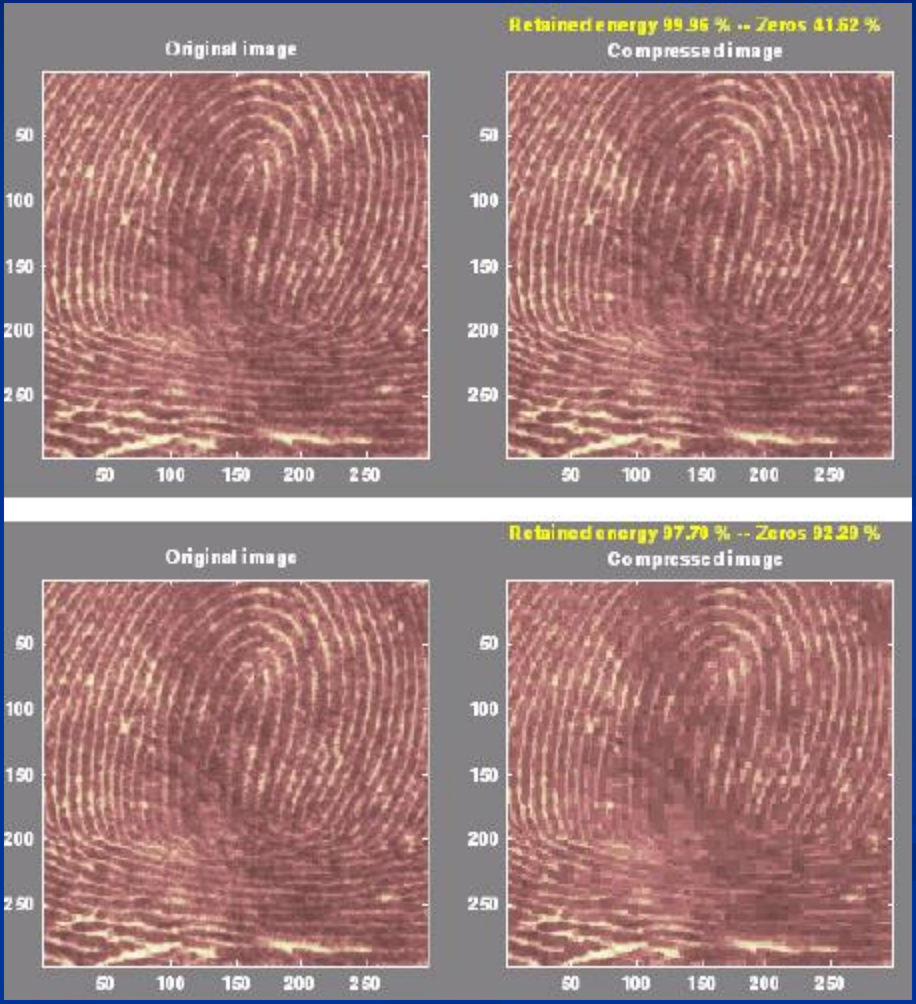


Application - 3

- **Compressing images of fingerprints for FBI using the wavelet transform**

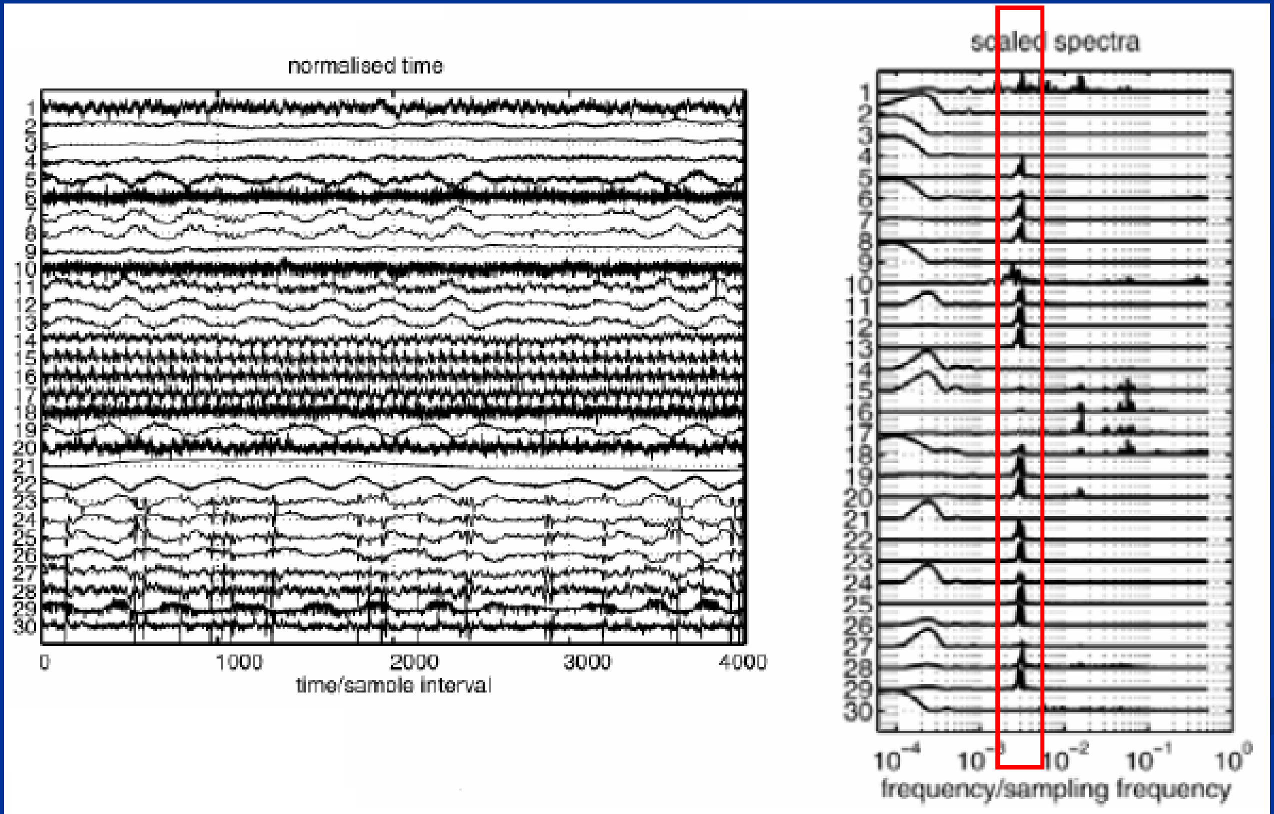
42% zeros
99.96% retained energy

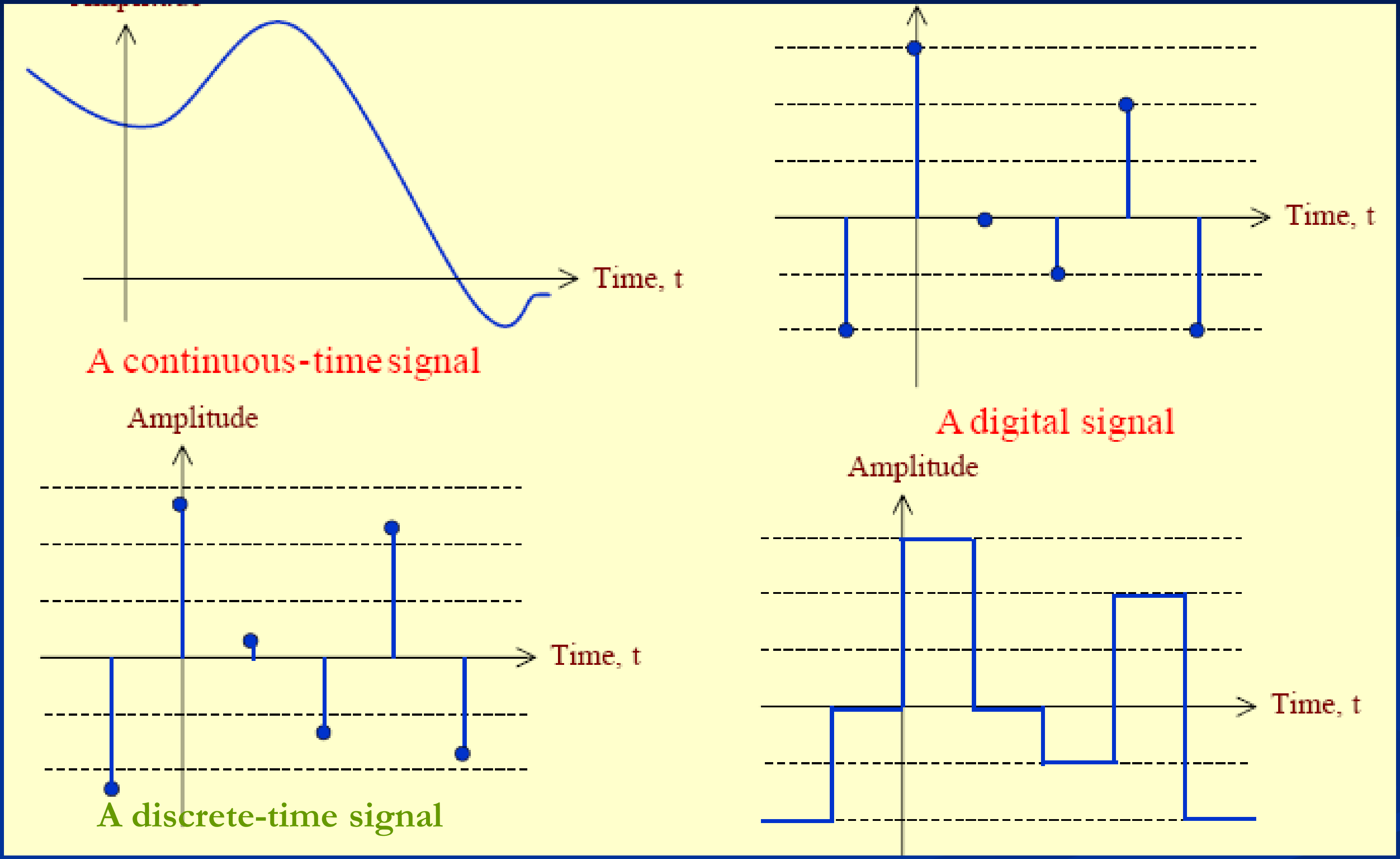
92% zeros
97.7% retained energy



Application - 4

- Spectrum estimation for detection of plant-wide oscillation
  - One oscillation can propagate to cause oscillation throughout a plant.
  - It is difficult and time-consuming to detect the oscillation from the time trends, but relatively easy and quick to do so from the spectrum peaks.





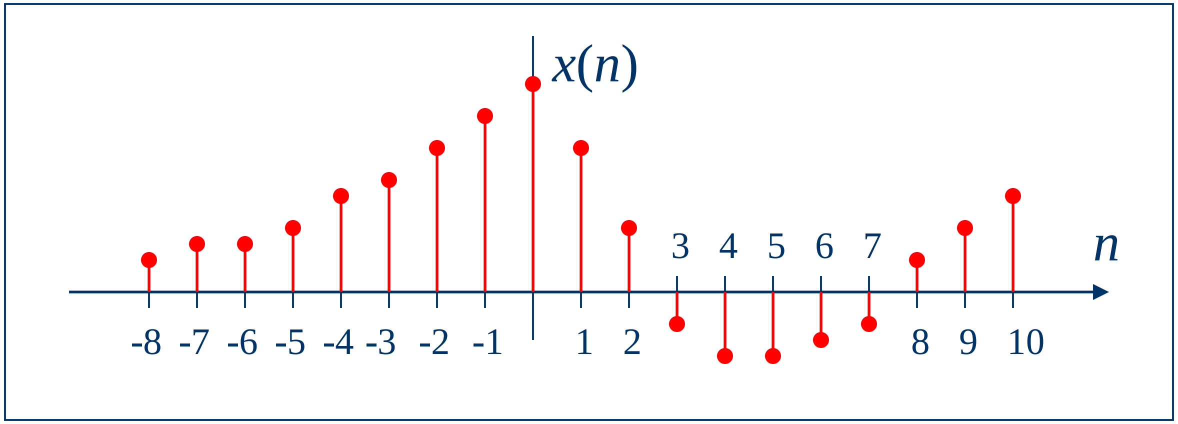
The Concepts of discrete signal and digital signal are considered the sense in many references

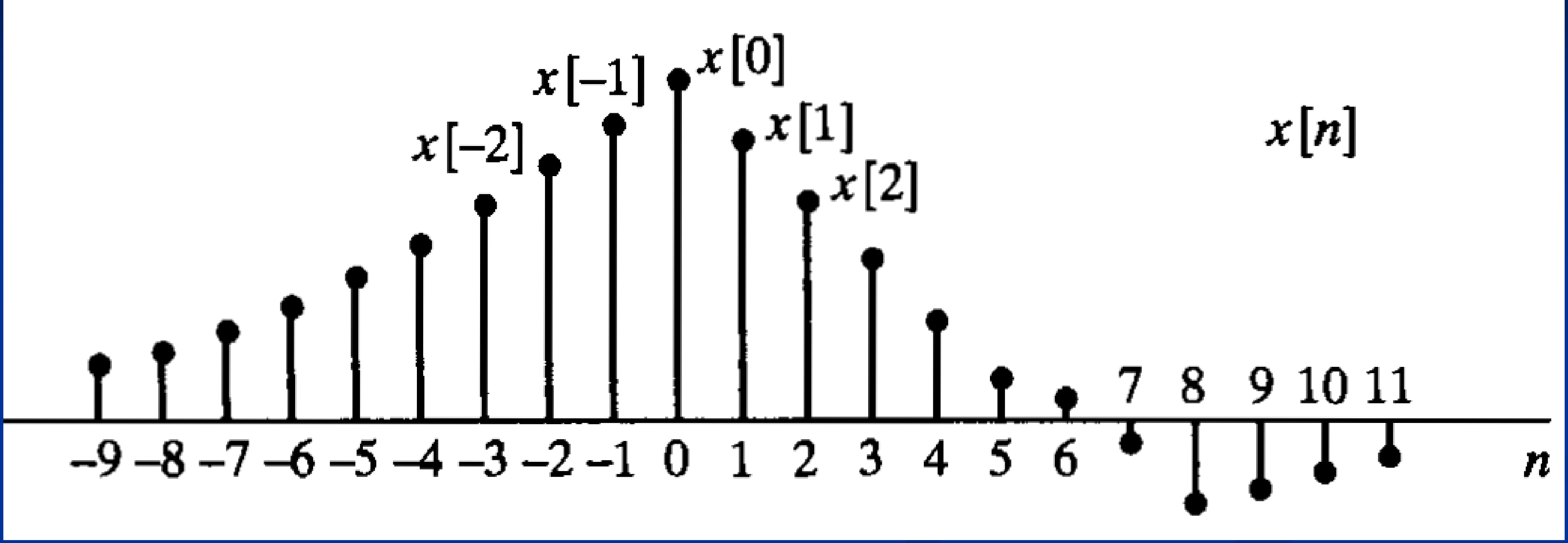
- **Digital signals** are represented mathematically as sequence of numbers, $x=\{x[n]\}$
- Often arise from periodic sampling of an analog signal, $x[n]=x_a(nT)$.
- The quantity T is termed the *sampling period*, and its reciprocal is the *sampling frequency*.

Representation by a Sequence

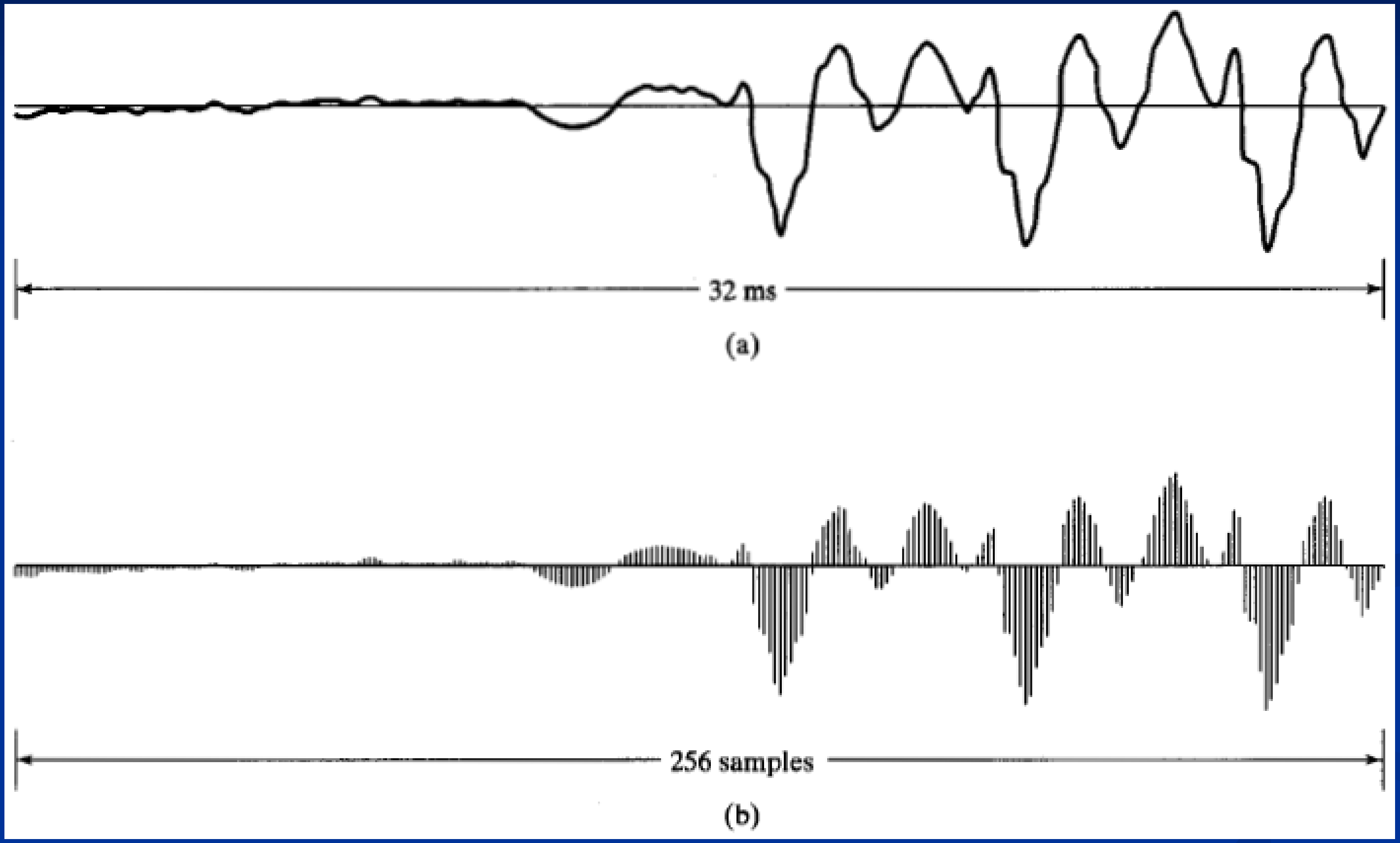
- Discrete-time system theory
  - Concerned with processing signals that are represented by *sequences*.

$$x = \{x(n)\}, \qquad -\infty < n < \infty$$





- Discrete time signal: only when *n* is integer



Segment of a continuous time speech signal

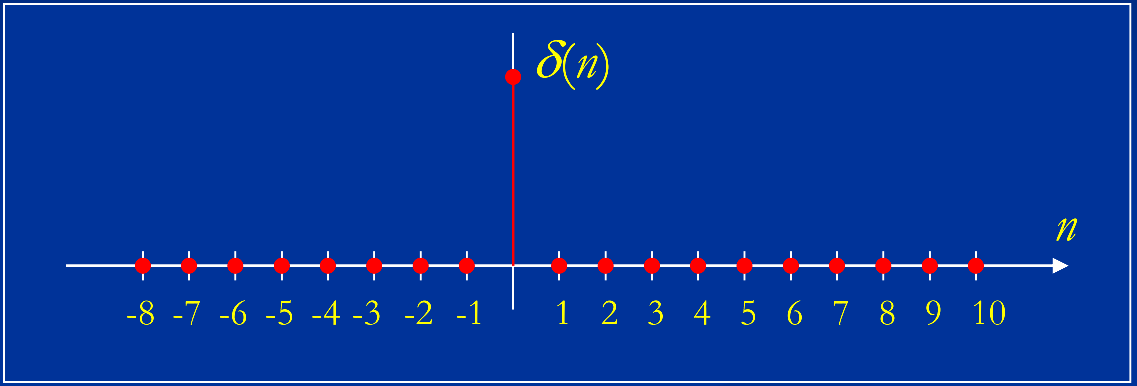
Sequence of samples with T=125μs, fs=8000 Hz

Operations on Sequences

- Sum $$x + y = \{x(n) + y(n)\}$$
- Product $$x \cdot y = \{x(n)y(n)\}$$
- Multiplication $$\alpha x = \{\alpha x(n)\}$$
- Shift $$y(n) = x(n - n_0)$$

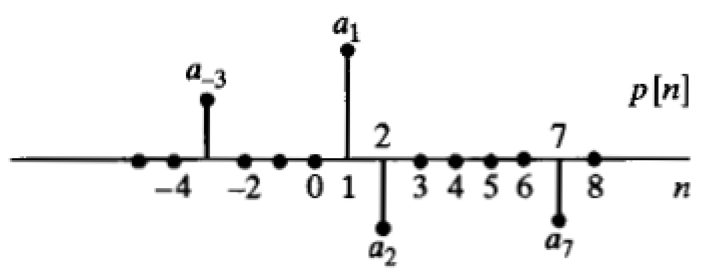
The unit sample sequence/an impulse

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



- **An arbitrary sequence can be represented as a sum of scaled, delayed impulse.**

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$



$$p[n] = a_{-3}\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-2] + a_7\delta[n-7]$$

The Unit Step Sequence

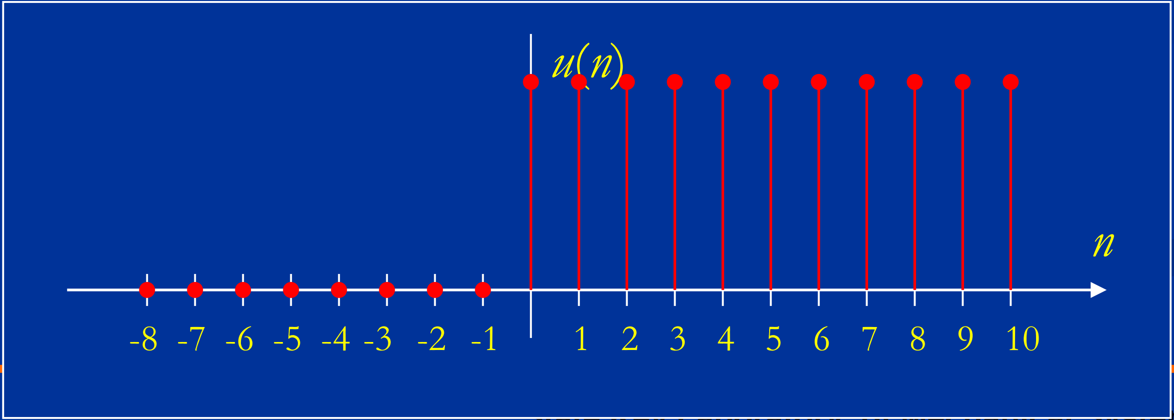
$$u(n)=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$$

- Fact:

$$\delta(n)=u(n)-u(n-1)$$

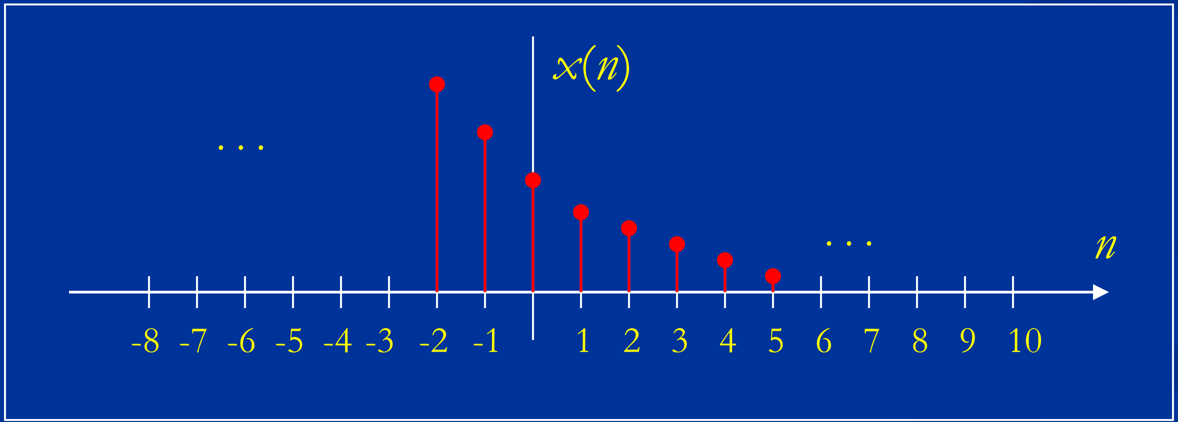
- An alternative representation of the unit: ***the sum delayed impulse***

$$u[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\cdots$$



- ***Exponential Sequences***

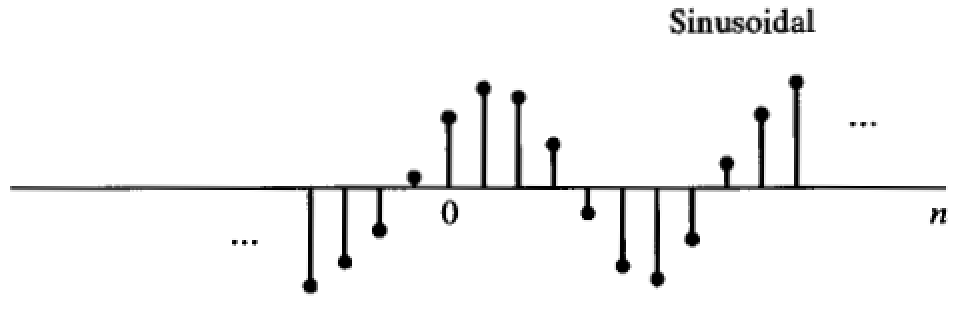
$$x(n) = a^n$$



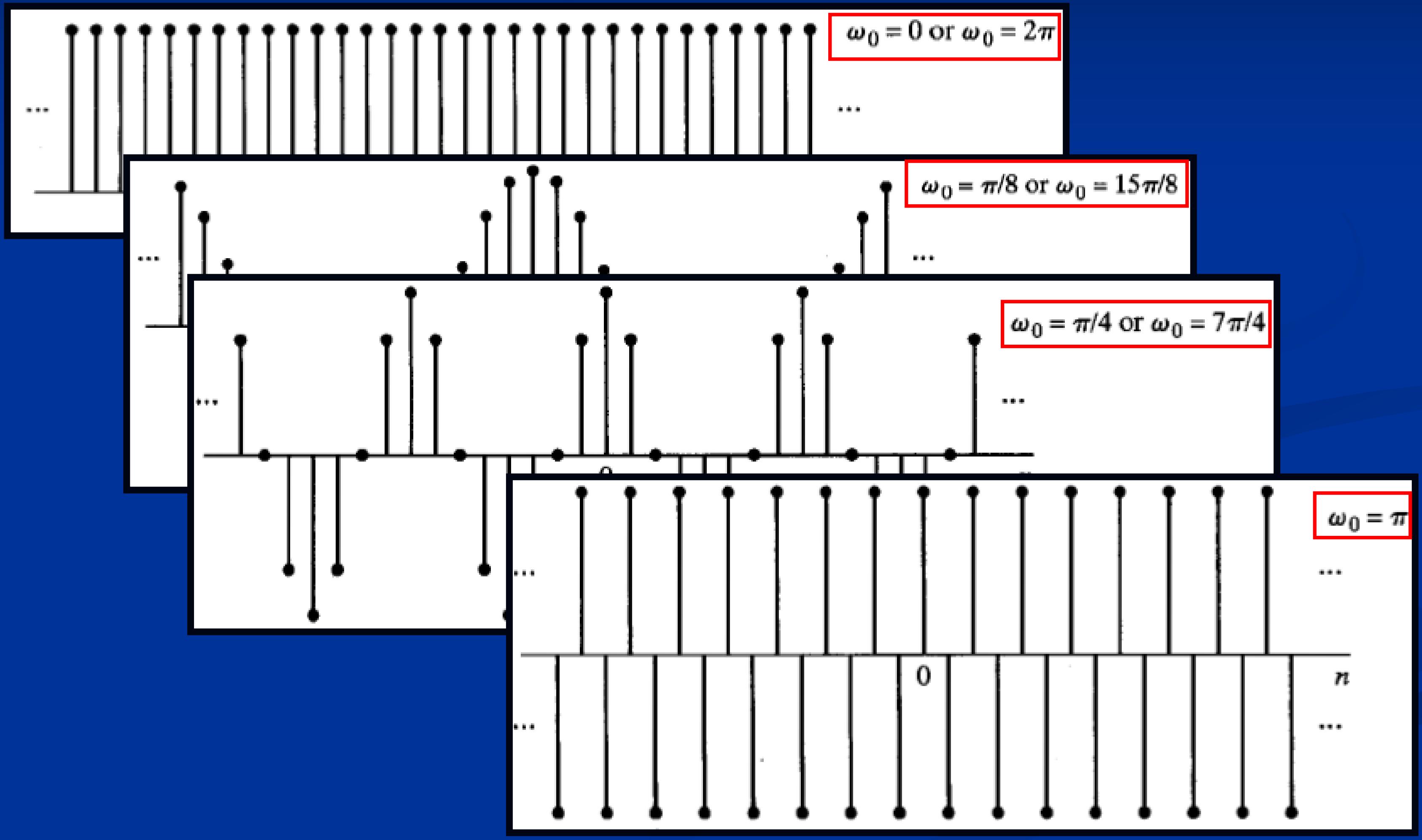
- ***Sinusoidal Sequences***
  - Sinusoidal sequences are also very important. General form of sinusoidal sequence is

$$x[n] = A\cos(\omega_0 n + \phi), \qquad \text{for all } n,$$

  - When A and φ are real constants:

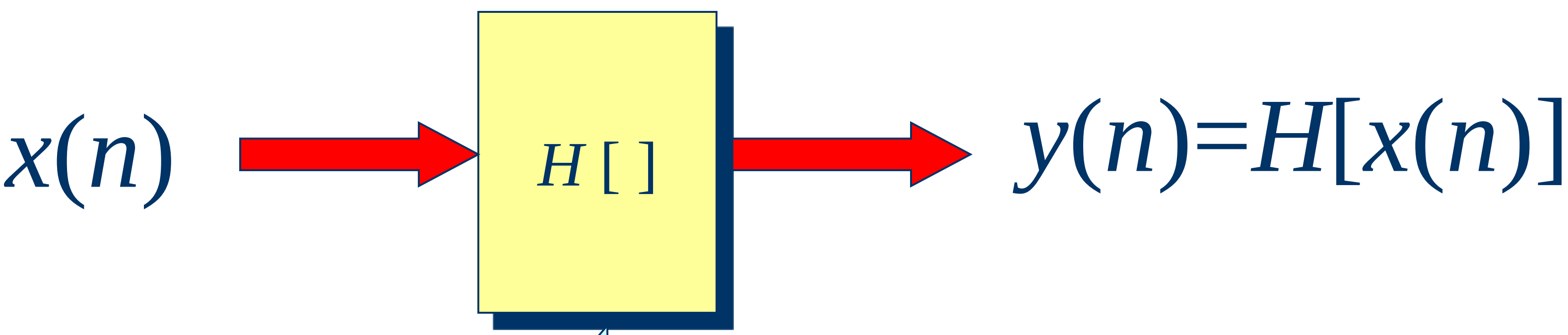


- For the case of $\varphi=0$, $x[n]=A\cos(\omega_0 n)$



- A **discrete-time system** can be denoted as $y[n] = H\{x[n]\}$ and pictorially indicated as

Systems



Mathematically modeled as a unique *transformation* or *operator*.

Example #1 Discrete-time systems

- *The Ideal Delay System*

$$y[n] = x[n - n_d]$$

n_d is a fixed positive integer called the delay of the system

The ideal delay system simple *shifts* the input sequence to the right by n_d samples to form the output

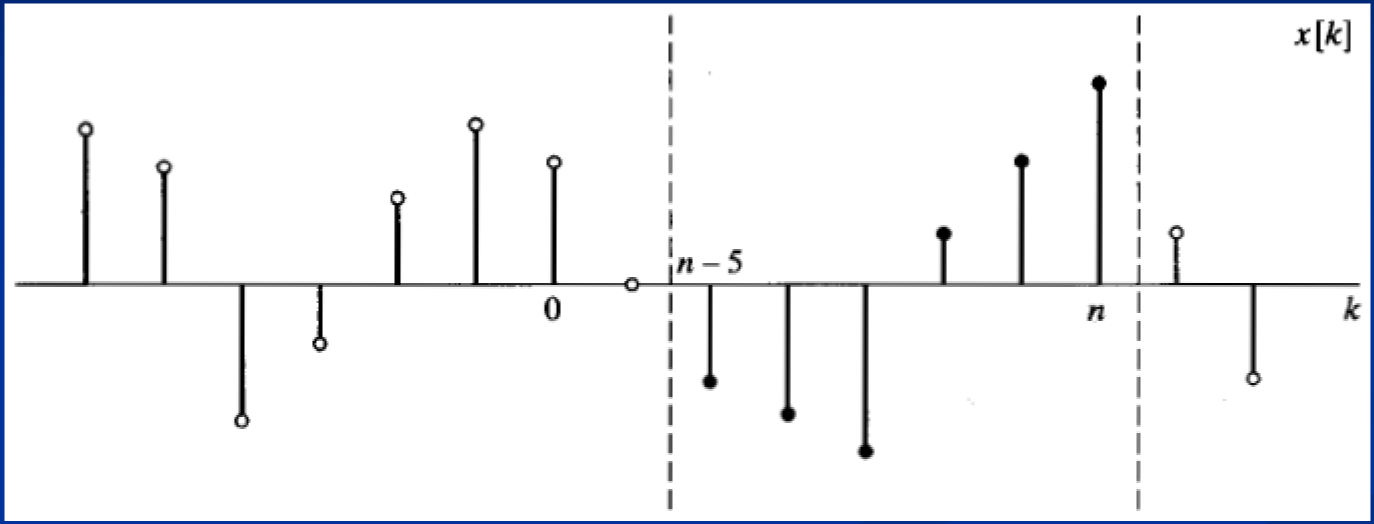
Example #2 Discrete-time systems

- ***Moving Average***

$$
\begin{aligned}
y[n] &= \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k] \\
&= \frac{1}{M_1 + M_2 + 1} \{x[n+M_1] + x[n+M_1-1] + \cdots + x[n] \\
&\quad + x[n-1] + \cdots + x[n-M_2]\}.
\end{aligned}
$$

The system computers the n^{th} sample of the output sequence as the average of $(M_1 + M_2 + 1)$ samples of the input sequence around the n^{th} sample.

Example #2 (continued)



For $n = 7$, $M_1 = 0$, and $M_2 = 5$, the output sample $y[7]$ is equal to one-sixth of the sum of all the samples between the vertical dotted lines.

- ***Memoryless Systems***
  - Output $y[n]$ at every value of n depends only on the input $x[n]$ at the same value of n.

$$y[n] = (x[n])^2, \qquad \text{for each value of } n.$$

Review: The Course

General scheme

Sometimes steps missing

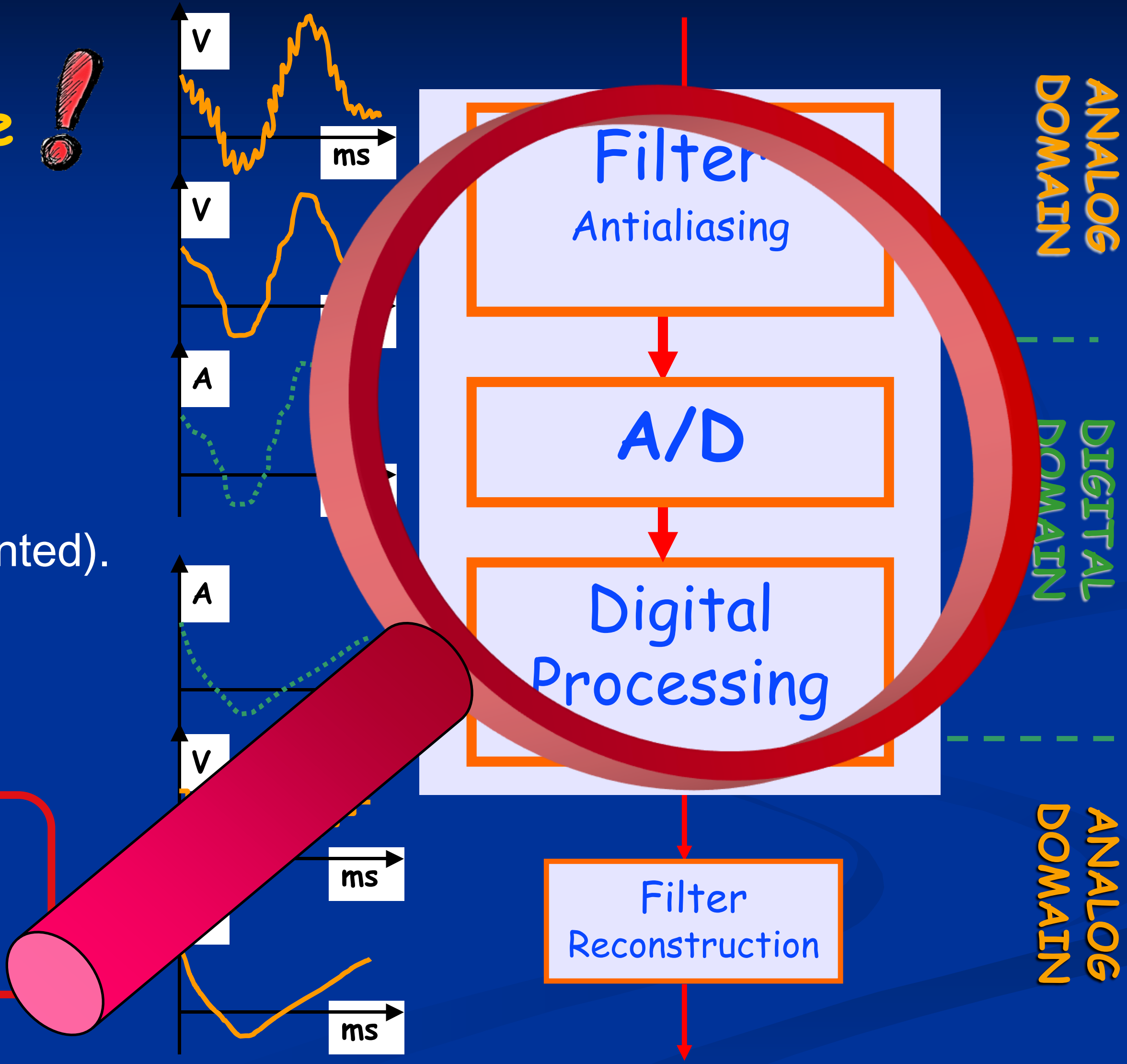
- Filter + A/D

  (ex: economics);

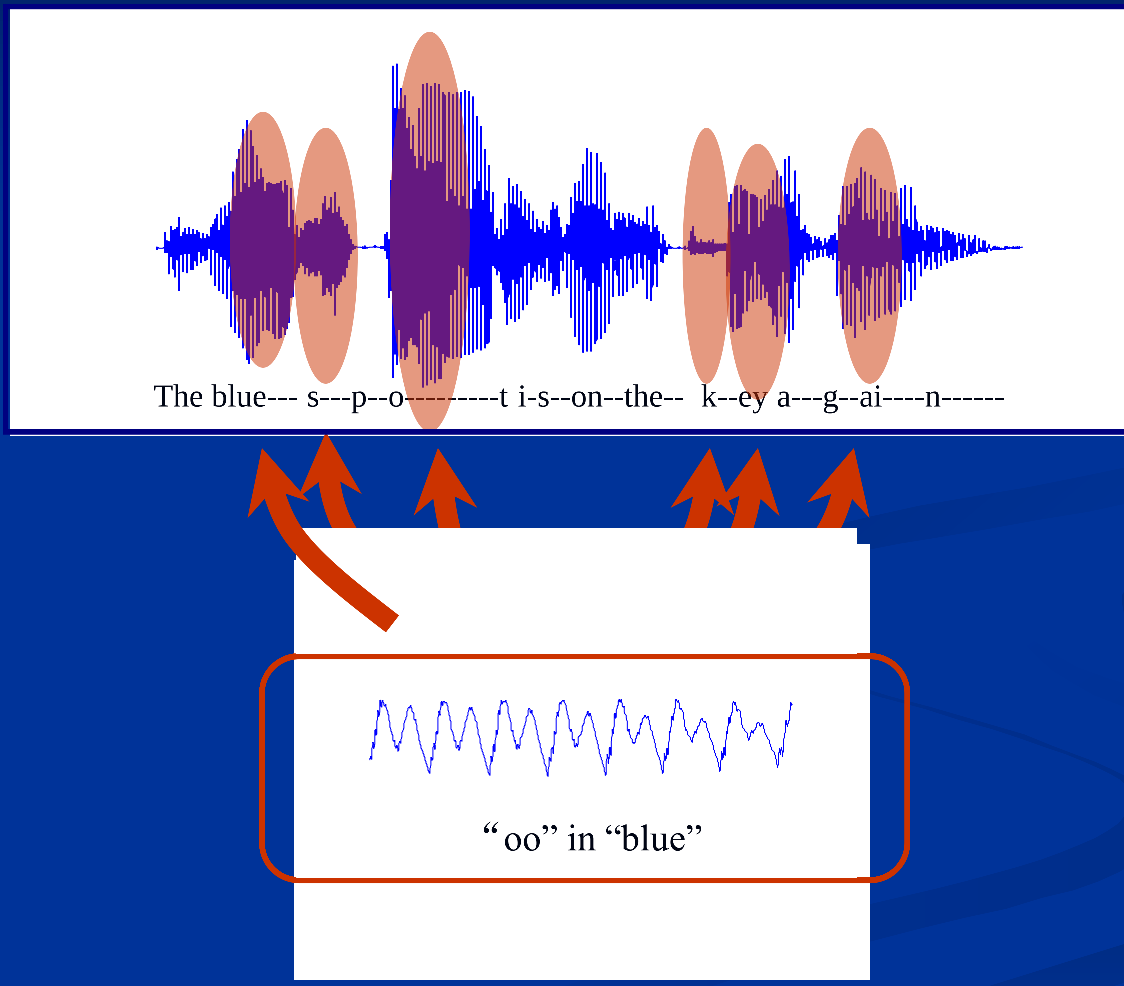
- D/A + filter

  (ex: digital output wanted).

Topics of this lecture.

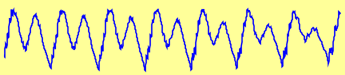


Review: DSP Application in Speech

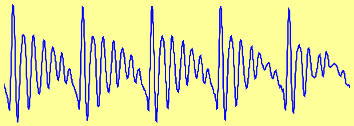


Review: Speech Application (cont’d)

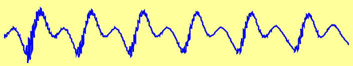
Vowels



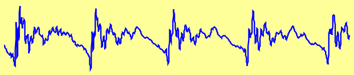
“oo” in “blue”



“o” in “spot”



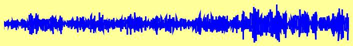
“ee” in “key”



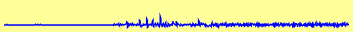
“e” in “again”

- Quasi-periodic
- Relatively high signal power

Consonants



“s” in “spot”



“k” in “key”

- Non-periodic (random)
- Relatively low signal power

Review: Signal Processing

- Fundamental of Signal Processing
  - Systems
  - Basic signals/functions
  - Basic algorithm: convolution sum
- Engineering Signal Processing
  - Systems
  - Signal processing, feature extraction and explanation based on basic signals/functions

Review: Discrete-Time Signal

- Discrete-time signals can arise in several ways.
  - <u>Periodically sampling a continuous time signal</u>.
  - Some signals are inherently discrete-time.

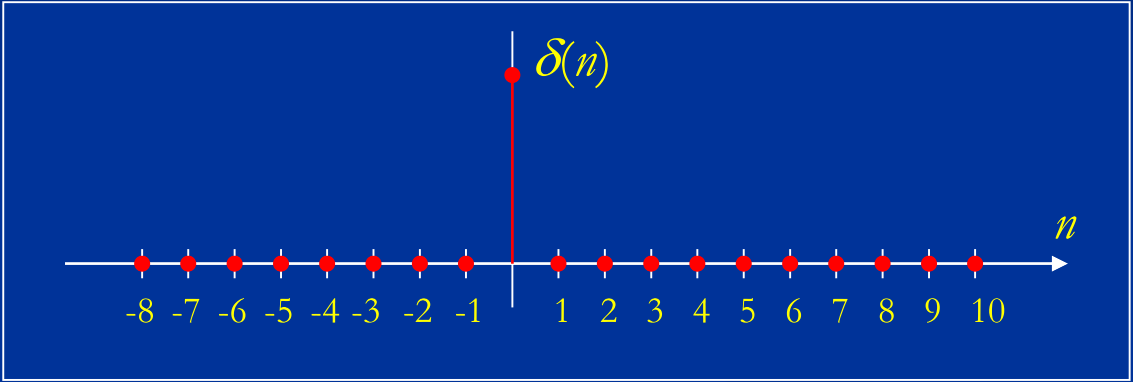
Review: Operations on Sequences

- Sum $$x + y = \{x(n) + y(n)\}$$
- Product $$x \cdot y = \{x(n)y(n)\}$$
- Multiplication $$\alpha x = \{\alpha x(n)\}$$
- Shift $$y(n) = x(n - n_0)$$

Review: Basic Sequences

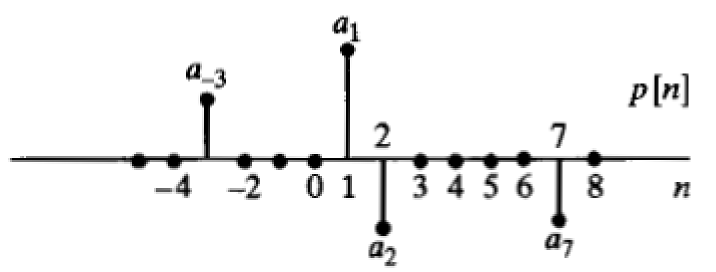
- *The unit sample sequence/an impulse*

$$\delta(n)=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$$



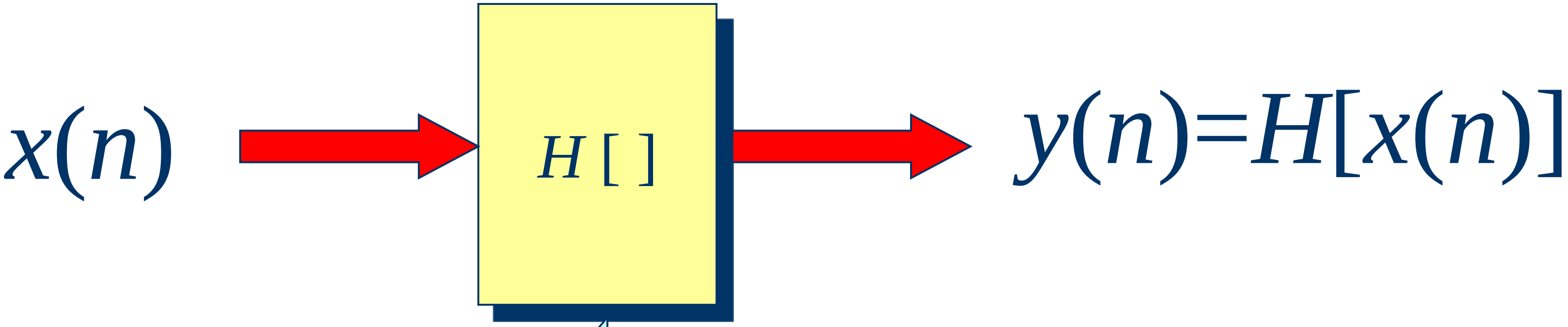
  - **An arbitrary sequence can be represented as a sum of scaled, delayed impulse.**

$$x(n)=\sum_{k=-\infty}^{\infty}x(k)\delta(n-k)$$



$$p[n]=a_{-3}\delta[n+3]+a_1\delta[n-1]+a_2\delta[n-2]+a_7\delta[n-7]$$

Review: Systems



Mathematically modeled as a unique *transformation* or *operator*.

System Properties
(Causality, Linearity, Time-invariance, etc.)

WHY?

- Important practical/physical implications
- Offer insight and structure to analyze and understand systems more deeply.

Causality

- Causal systems --- output for $y(n_0)$ depends only on $x(n)$ with $n \leq n_0$.
- A causal system whose impulse response $h(n)$ satisfies

$$\boxed{h(n) = 0 \quad \text{for } n < 0}$$

Causal or Noncausal

$$y(t) = x^2(t-1)$$

E.g. $y(5)$ depends on $x(4)$ ... causal

$$y(t) = x(t+1)$$

E.g. $y(5) = x(6), y$ depends on future $\Rightarrow$ noncausal

$$y[n] = x[-n]$$

E.g. $y[5] = x[-5]$ ok, but

$y[-5] = x[5], y$ depends on future $\Rightarrow$ noncausal

$$y[n] = \left(\tfrac{1}{2}\right)^{n+1} x^3[n-1]$$

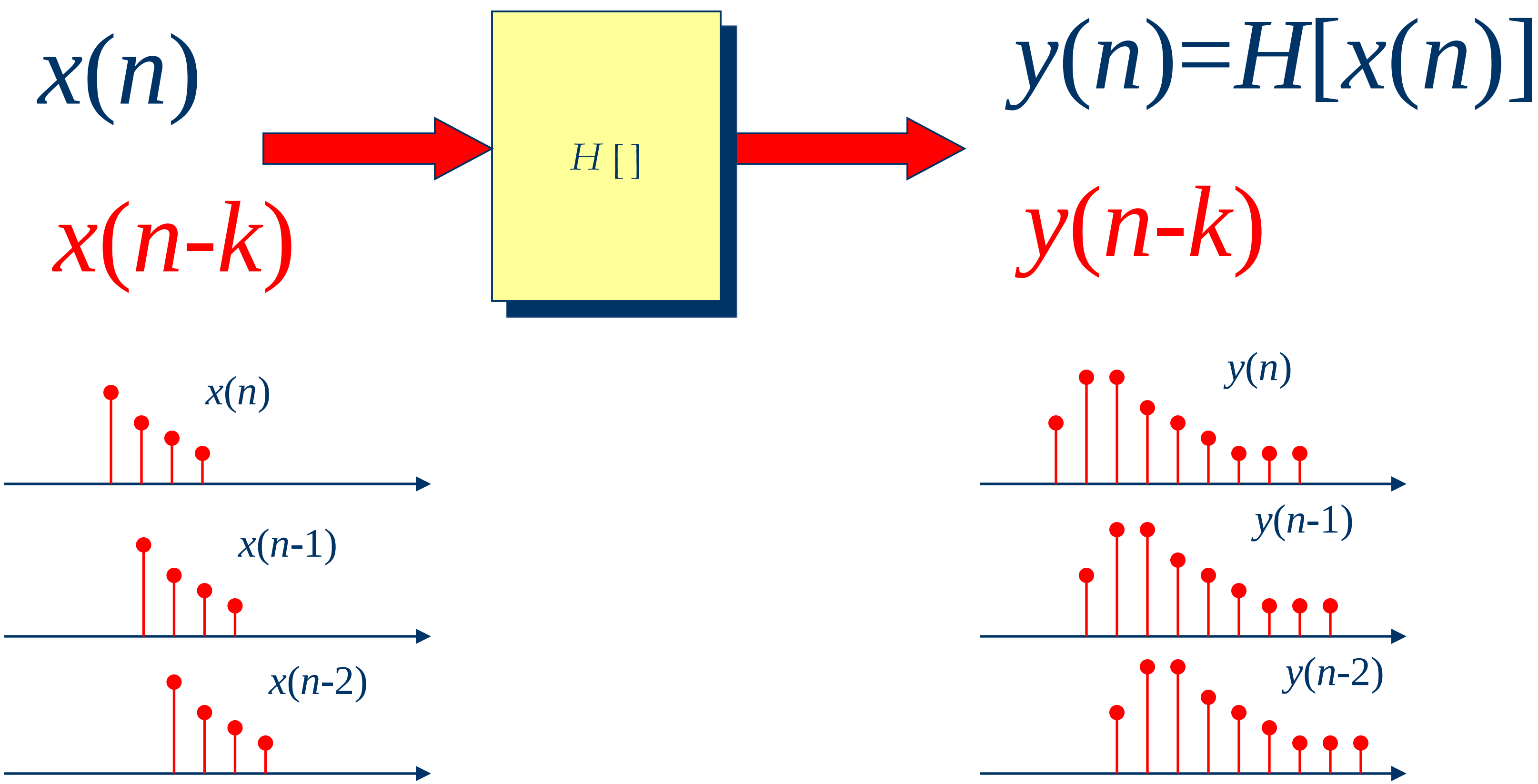
E.g. $y[5]$ depends on $x[4]$ ... causal

Stability

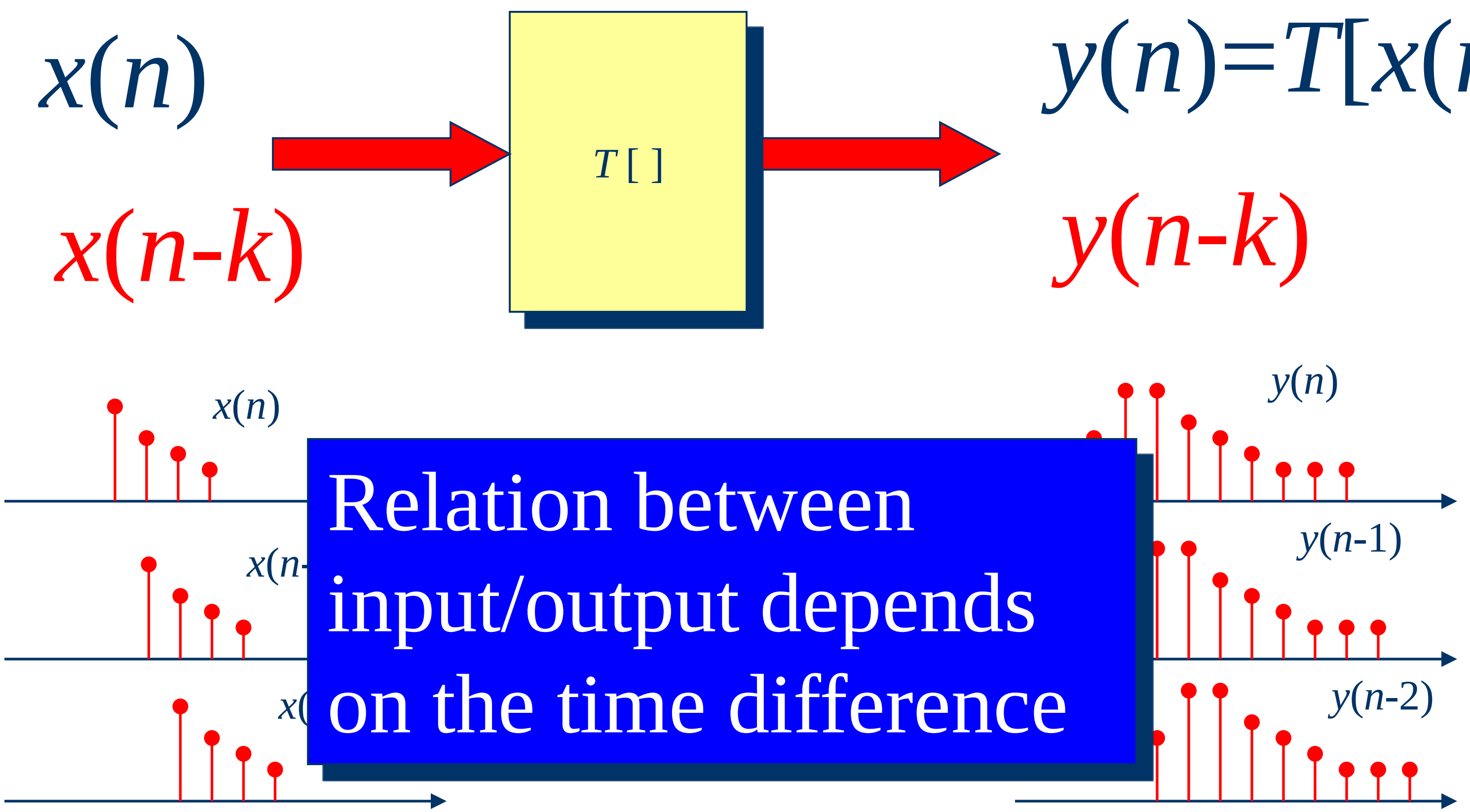
- Stable systems --- every *bounded input* produce a *bounded output* (BIBO)
- Necessary and sufficient condition for a BIBO

$$S \stackrel{\Delta}{=} \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Shift/time-Invariant Systems



Shift/time-Invariant Systems



Relation between input/output depends on the time difference

Now We Can Deduce Somthing!

Fact: If the input to a TI System is periodic, then the output is periodic with the same period.

"Proof": Suppose $x(t+T) = x(t)$

and $x(t) \rightarrow y(t)$

Then by TI

$$x(t+T) \rightarrow y(t+T).$$

These are the same input!

So these must be the same output, i.e., $y(t) = y(t+T)$.

Linear and Nonlinear System

- Many systems are nonlinear. E.g. many circuit elements (e.g., diodes, 二极管), dynamics of aircraft …

- We focus exclusively on *linear* systems.

  - Linear models represent accurate representations of behavior of many systems

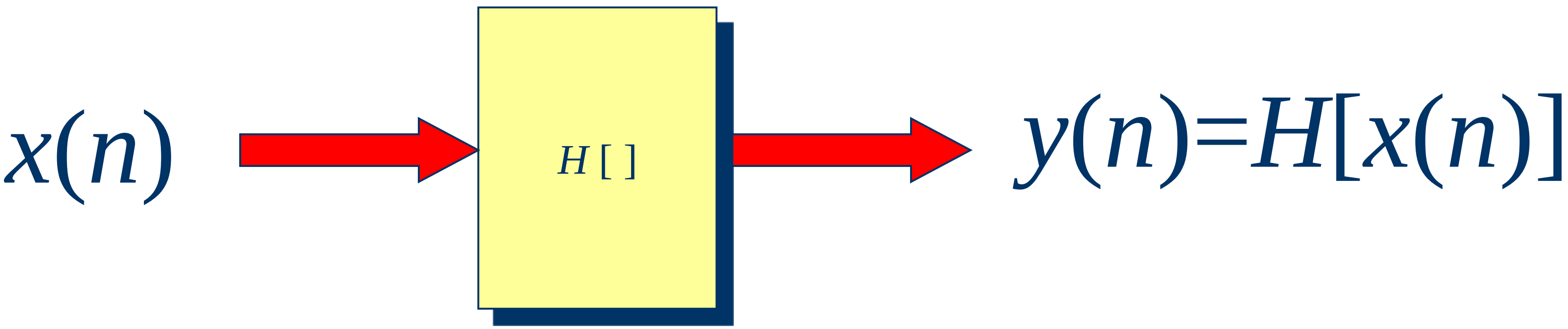
  - Linear Systems are analytically tractable providing basis for important tools and considerable insight

Linear Systems

$$x(n) \longrightarrow \boxed{H[\ ]} \longrightarrow y(n)=H[x(n)]$$

$$\boxed{H[ax_1(n)+bx_2(n)] = aH[x_1(n)]+bH[x_2(n)]}$$

Examples:



Ideal Delay System $$y(n) = x(n - n_d)$$

Moving Average $$y(n) = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{k=M} x(n-k)$$

Accumulator $$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Examples:

$x(n)$ Are these system linear? $H[x(n)]$

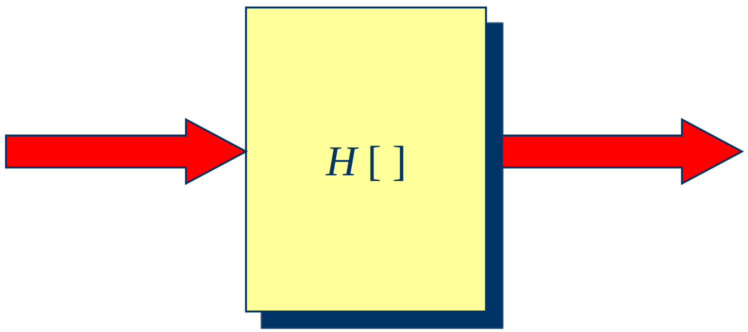
Ideal Delay System $\quad y(n) = x(n - n_d)$

Moving Average $\quad y(n) = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{k=M} x(n-k)$

Accumulator $\quad y(n) = \sum_{k=-\infty}^{n} x(k)$

Examples:

$x(n)$



$y(n) = H[x(n)]$

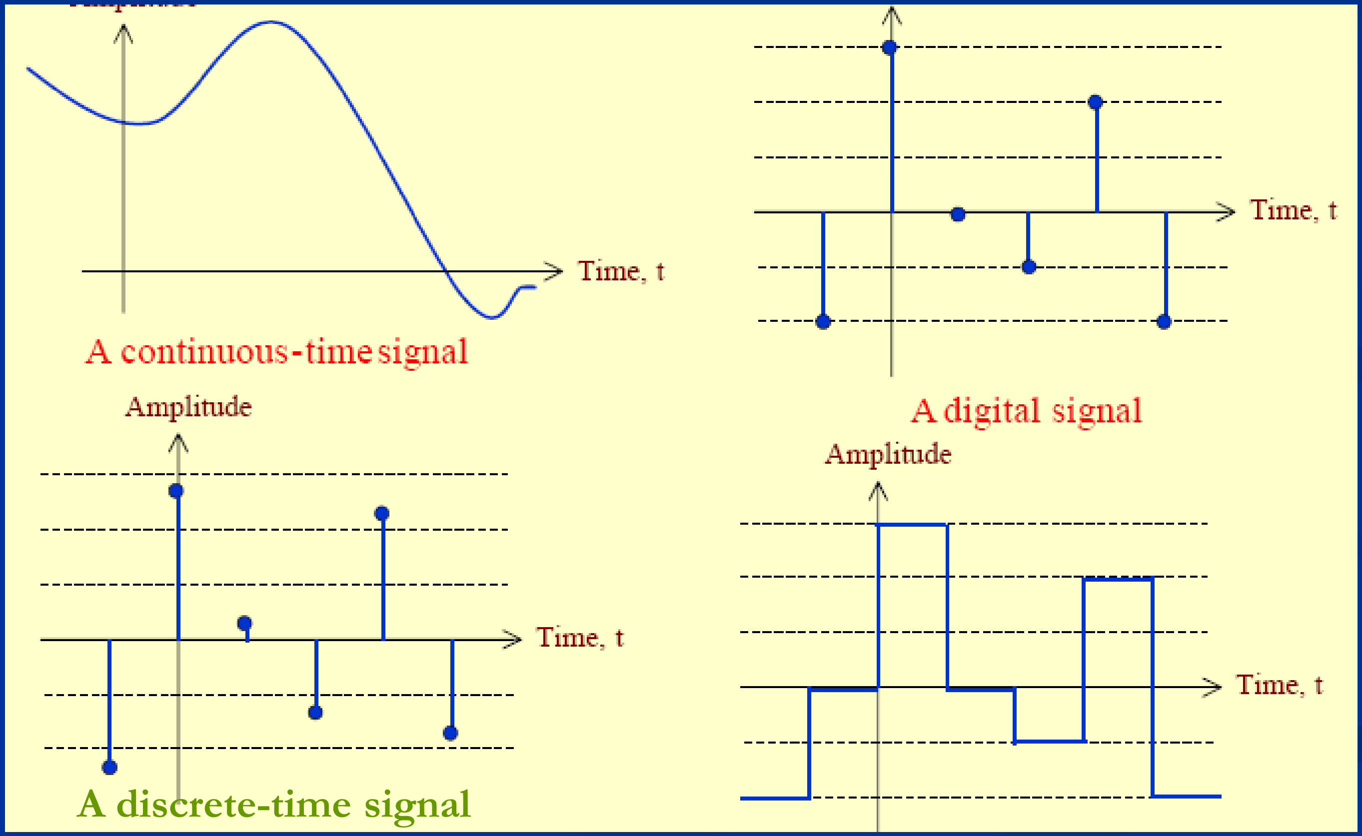
A Memoryless System $y(n) = [x(n)]^2$

Is this system linear?

Linear and Nonlinear System

- Sampling:

Nonlinear



Linearity

A system is linear if it has the superposition property:

If $x_1(t) \to y_1(t)$ and $x_2(t) \to y_2(t)$

then $ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t)$

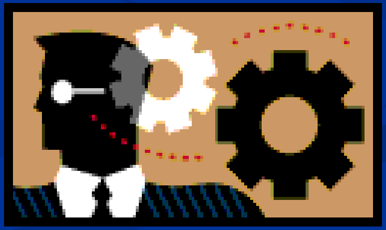
$y[n] = (x[n])^2$ Nonlinear, TI, Causal

$y(t) = x(2t)$ Linear, not TI, Noncausal

Find systems with other combinations ?

-e.g. Linear, TI, Noncausal

Linear, not TI, Causal



Properties of Linear Systems

- Superposition

$$\text{If} \qquad x_k[n] \;\longrightarrow\; y_k[n]$$

$$\text{Then} \qquad \sum_k a_k x_k[n] \;\longrightarrow\; \sum_k a_k y_k[n]$$

- For linear systems, zero input → zero output

$$0 = 0 \cdot x[n] \rightarrow 0 \cdot y[n] = 0$$

Properties of Linear Systems (Continued)

- A new system: linear time-invariant system (LTI, 线性时不变系统或线性移不变系统)

Linear Time-Invariant (LTI) Systems

- A particularly important class of systems: linear and time invariant (TI).
  - A linear system can be completely characterized by its impulse response.

Exploiting Superposition and TI

$$x[n] = \sum_k a_k x_k[n] \xrightarrow{Linear System} y[n] = \sum_k a_k y_k[n]$$

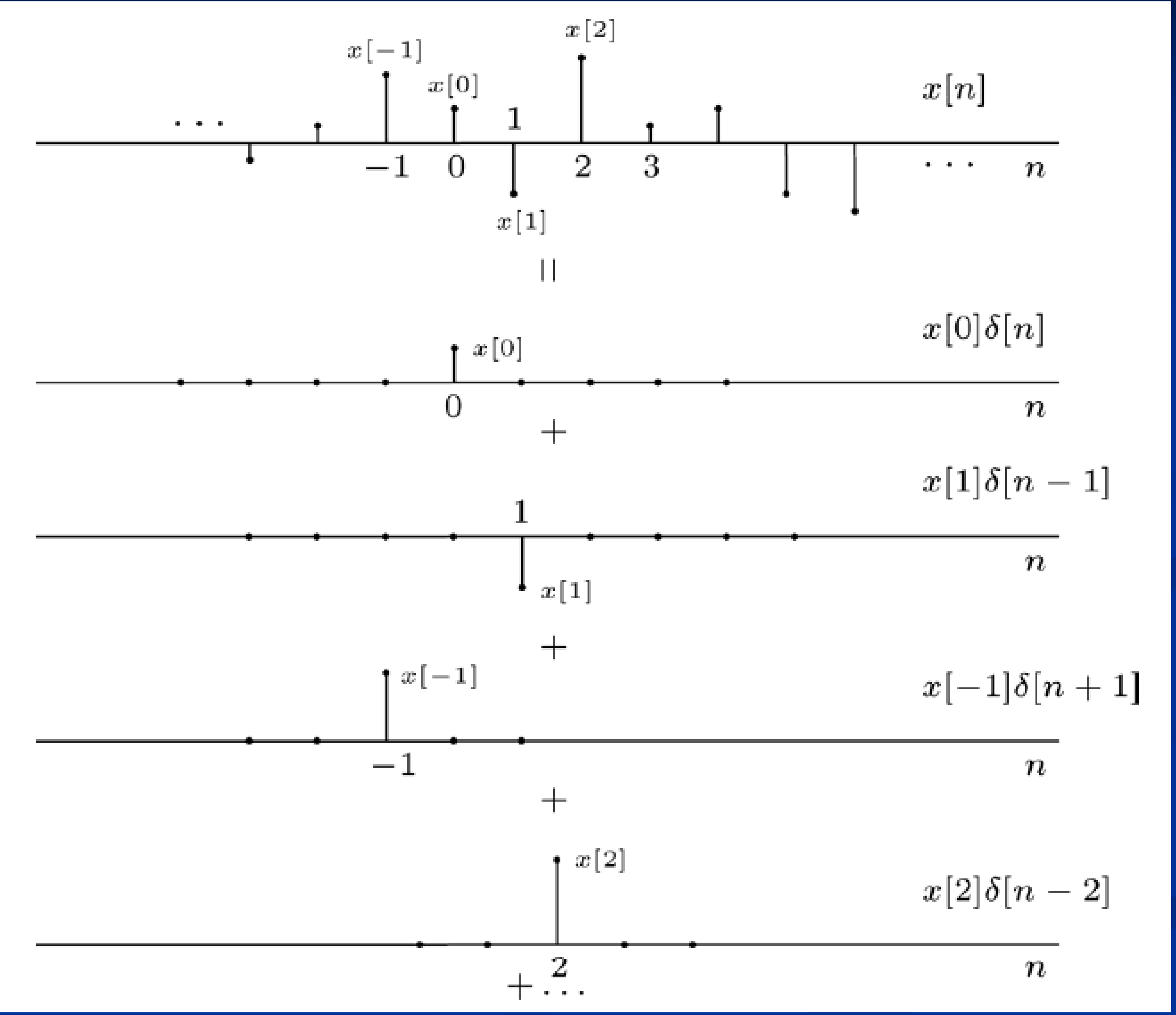
- Question: Are there sets of "basic" signals so that
  - We can represent rich classes of signals as linear combinations of these building block signals
  - The response of LTI systems to these basic signals are both simple and insightful.

?

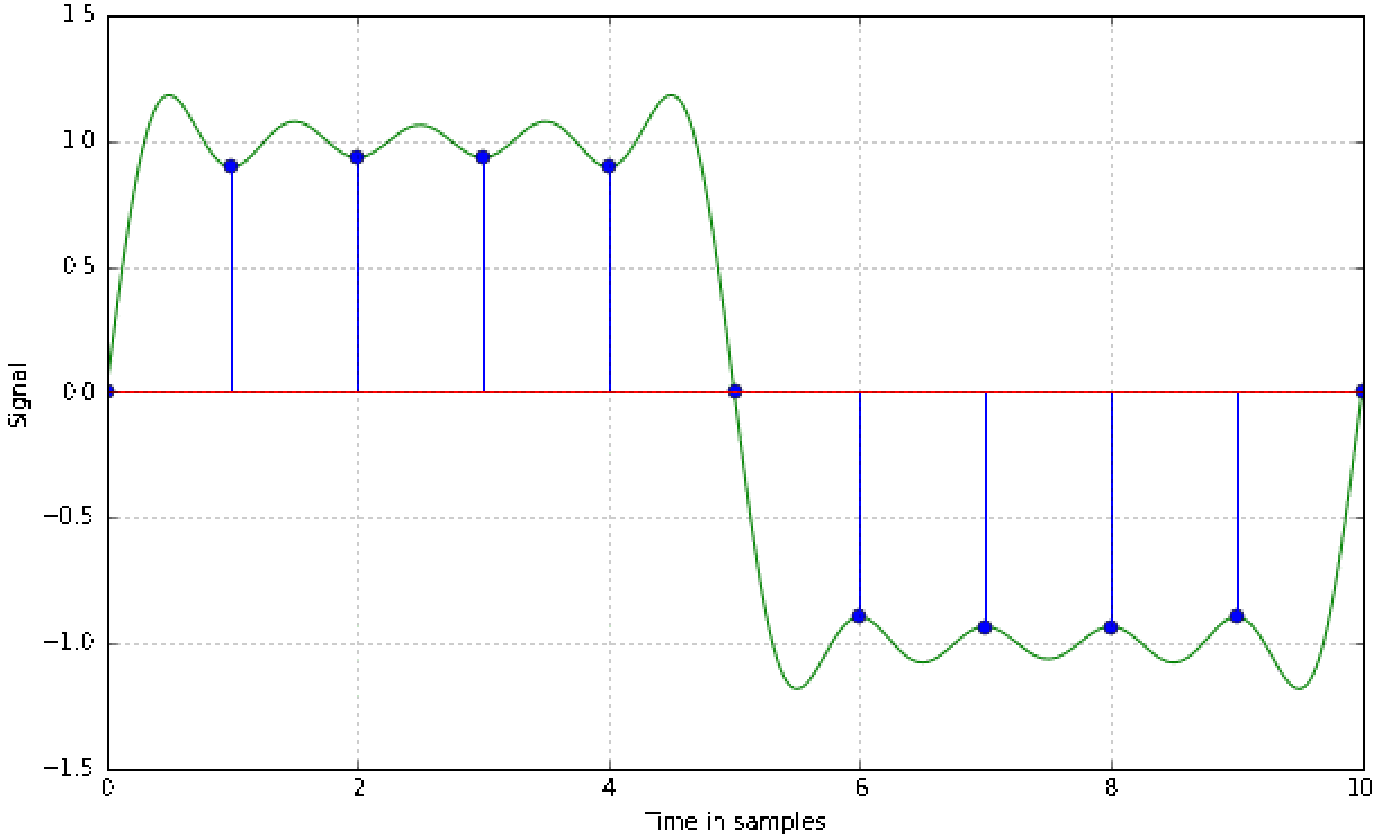
Fact: For LTI systems there are two natural choices for these building blocks

Focus for Now: DT Shifted unit samples

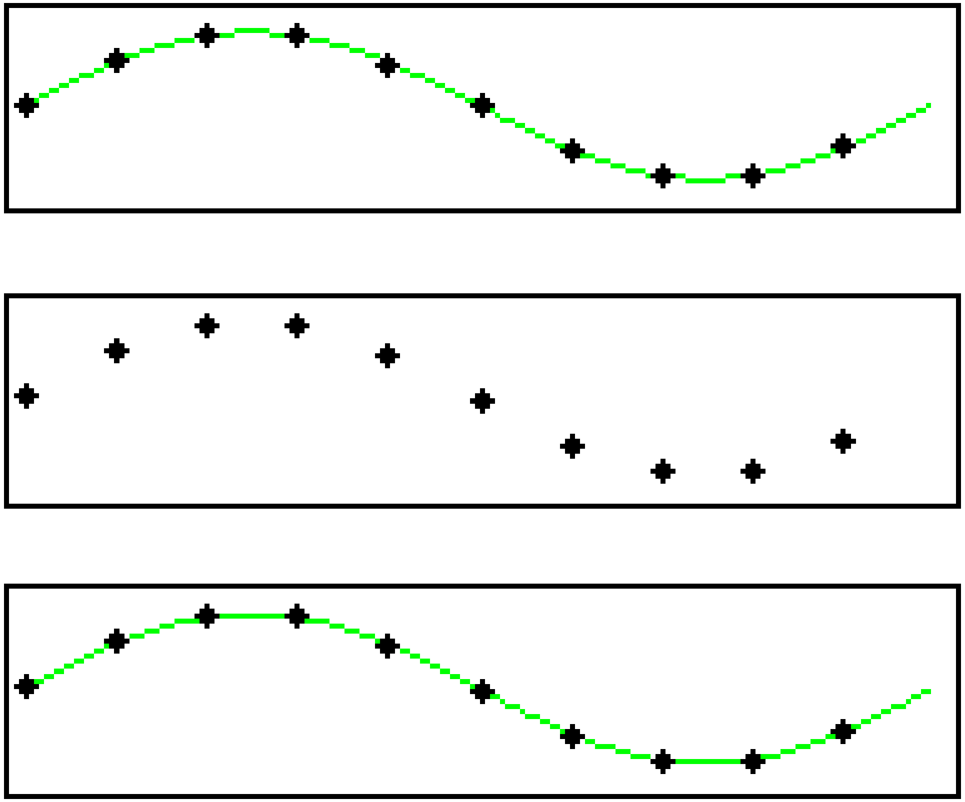
Representation of DT signals Using Unit Samples



Representation of DT signals Using Unit Samples



Representation of DT signals Using Unit Samples



- That is…

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \cdots$$

$$x[n] = \sum_{k=-\infty}^{\infty} \underbrace{\boxed{x[k]}}_{\text{Coefficients}} \underbrace{\boxed{\delta[n-k]}}_{\text{Basic Signals}}$$

The Sifting Property of the Unit Sample

- Suppose the system is *linear*, and define $h_k[n]$ as the response to δ

$$\delta[n-k] \rightarrow h_k[n]$$

- From superposition

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

- **Now suppose the system is LTI, and define the unit sample response $h_k[n]$**

$$\delta[n] \rightarrow h[n]$$

From TI:

$$\delta[n-k] \rightarrow h[n-k]$$

From LTI:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow \underbrace{\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}}_{\text{Convolution Sum}}$$

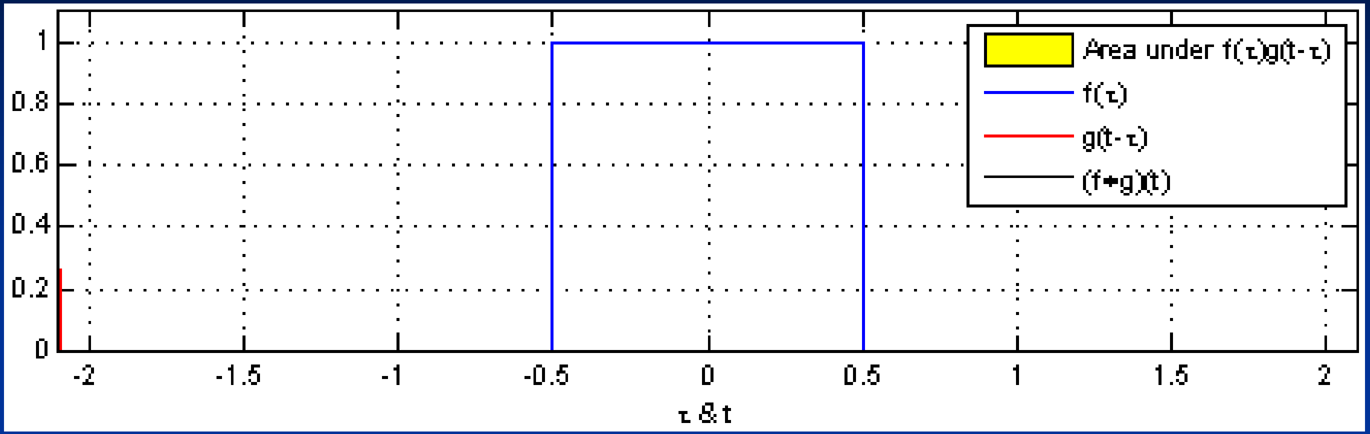
Convolution Sum Representation of Response of LTI Systems

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

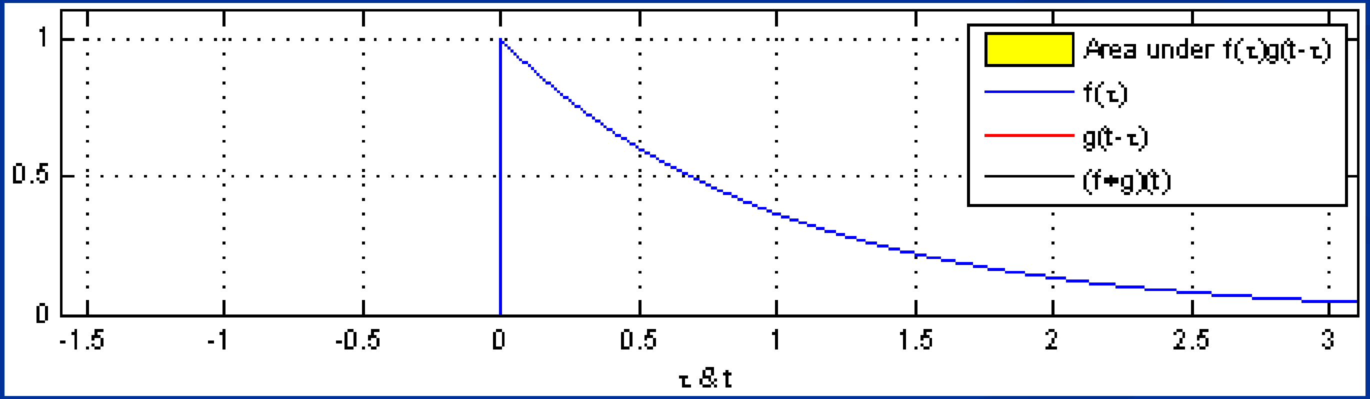
Convolution Sum

Loading...

Convolution Sum



Convolution of two square pulses: the resulting waveform is a triangular pulse



Convolution of a square pulse with an impulse response. The integral of their product is the area of the yellow region.

Properties of Convolution and DT LTI Systems

- A DT LTI system is ***completely characterized*** by its unit sample response

Ex. #1: $h[n] = \delta[n - n_0]$

There are *many* systems with this repsonse to $\delta[n]$

There is only *one* LTI System with this response to $\delta[n]$:

$$y[n] = x[n - n_0]$$

$$\Downarrow$$

$$\boxed{x[n] * \delta[n - n_0] = x[n - n_0]}$$

Ex. #2:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$ - An Accumulator

Unit Sample response

$$h[n] = \sum_{k=-\infty}^{n} \delta[k] = u[n]$$

$$\Downarrow$$

$$x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

The Commutative Property (可交换性)

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

- Ex: Step response $s[n]$ of an LTI system

$$s[n] = u[n] * h[n] = h[n] * u[n]$$

Step input

"input"

Unit Sample response of accumulator

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

Prove

Properties of Convolution Math

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$$

$$x(n) * h(n) = h(n) * x(n)$$

Properties of Convolution Math

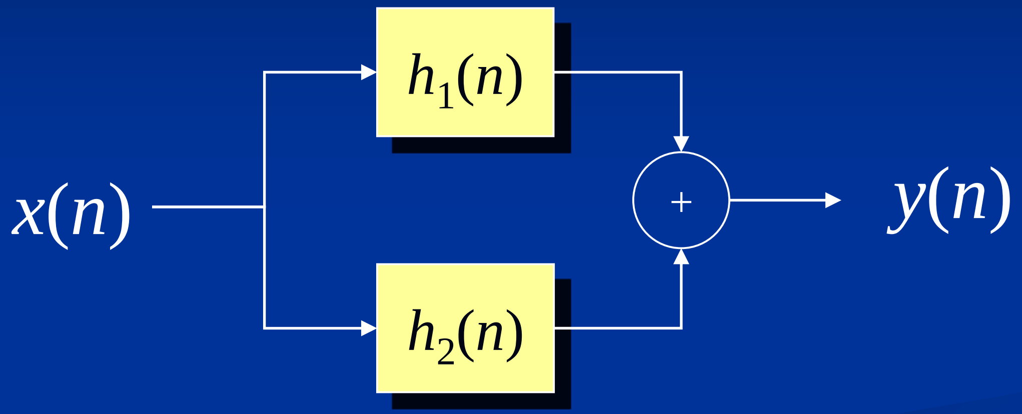
$$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y(n)$$

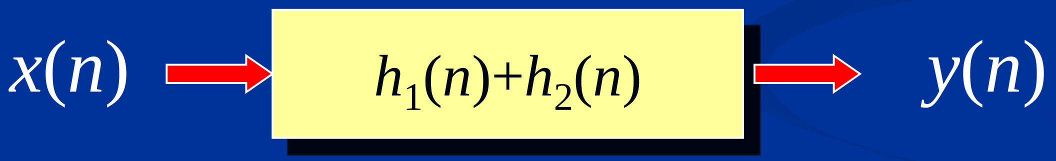
$$x(n) \rightarrow \boxed{h_2(n)} \rightarrow \boxed{h_1(n)} \rightarrow y(n)$$

$$x(n) \rightarrow \boxed{h_1(n)*h_2(n)} \rightarrow y(n)$$

These systems are identical.

Properties of Convolution Math





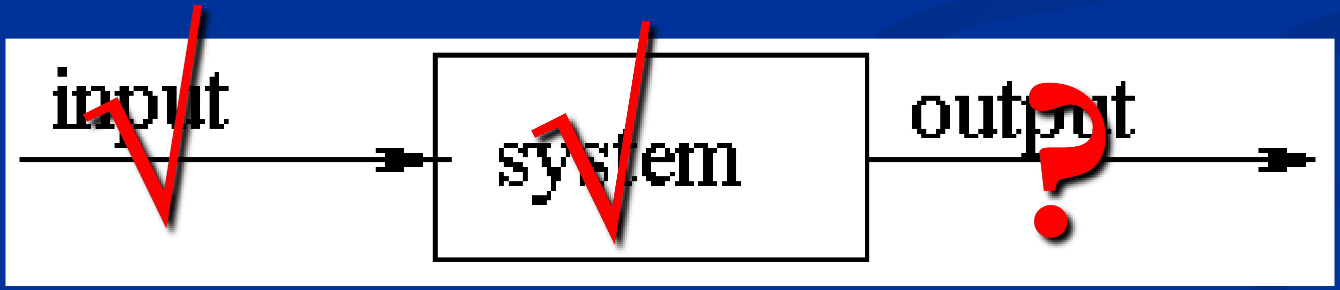
These two systems are identical.

System-Based Applications

- Response Analysis

Given the *system parameters* and *input signal*

Determine *output signal*



Example: signal filtering, denoising

System-Based Applications

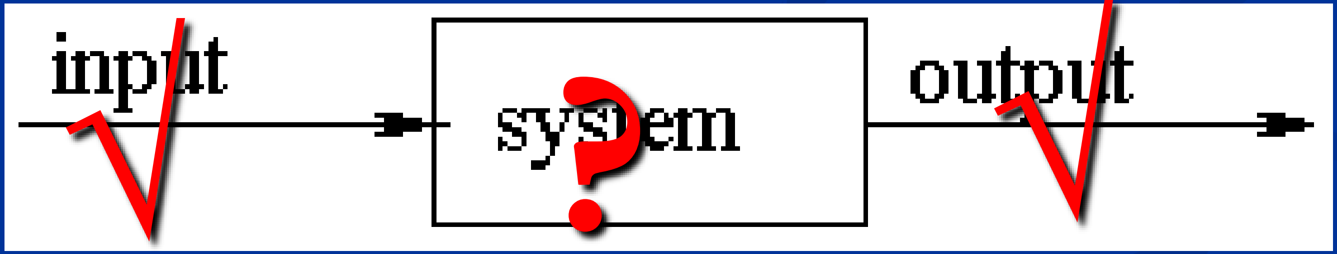
- System Design & System Identification

  System parameters design (no system)

  System parameters identification

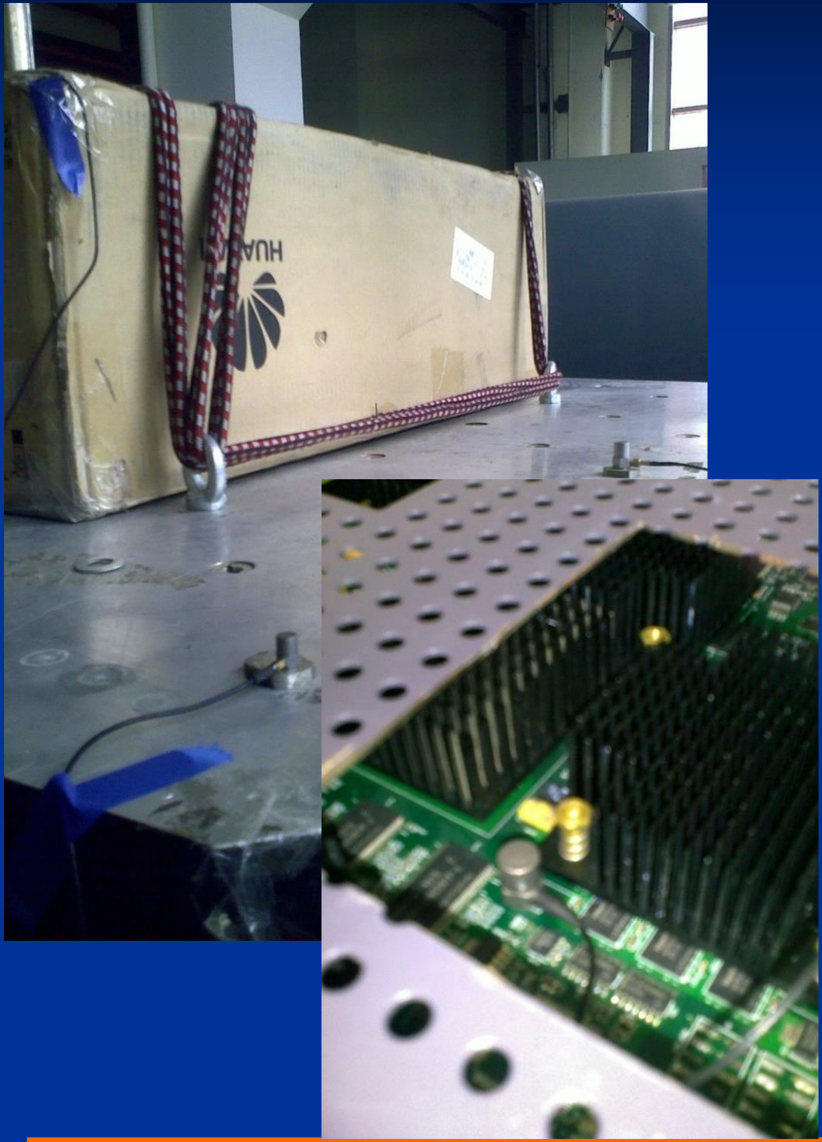
  Given the *input signal* and *response (output signal)*

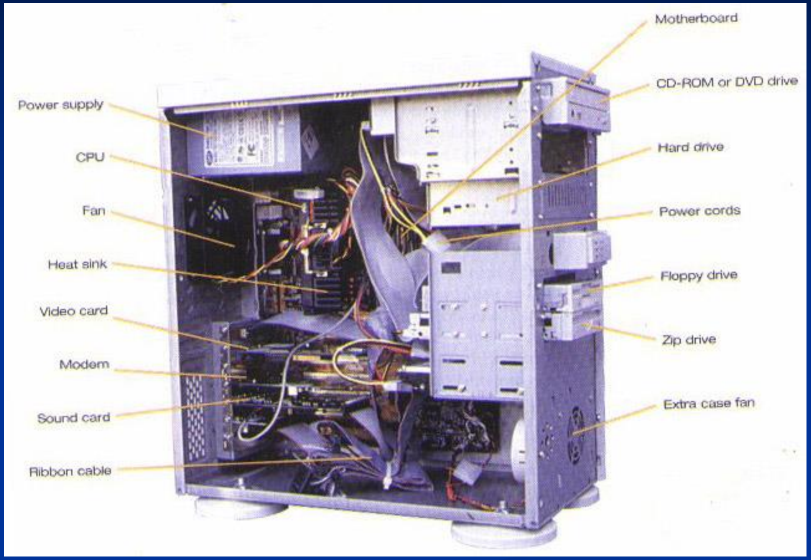
  Determine *system parameters*

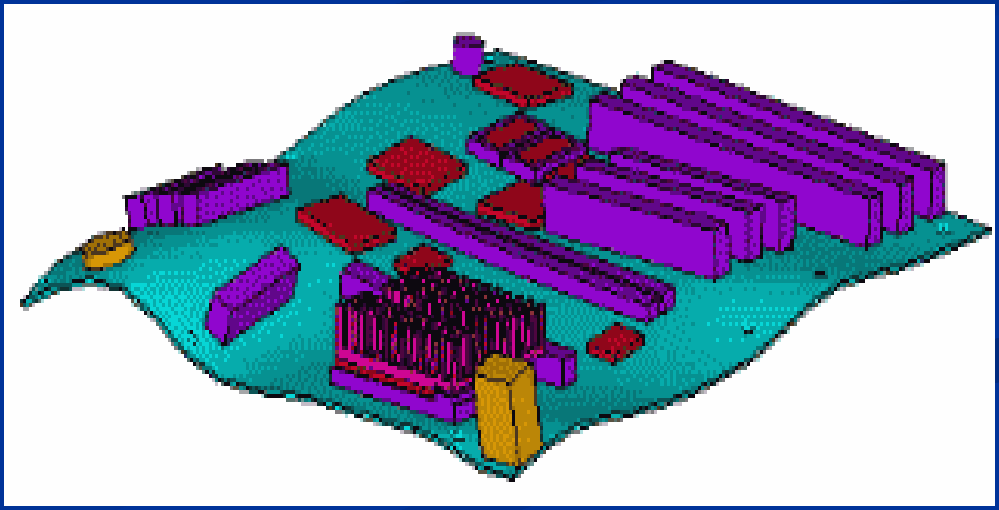


Example: filter design

Example




Motherboard
CD-ROM or DVD drive
Power supply
CPU
Hard drive
Fan
Power cords
Heat sink
Floppy drive
Video card
Zip drive
Modem
Extra case fan
Sound card
Ribbon cable

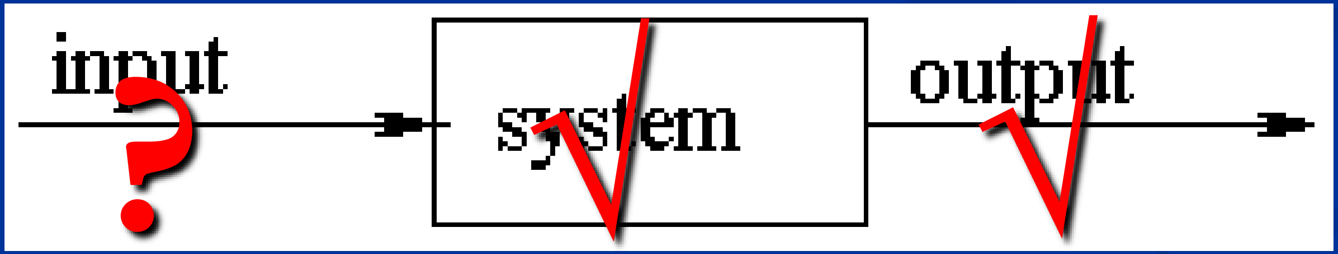


System-Based Applications

- Prediction

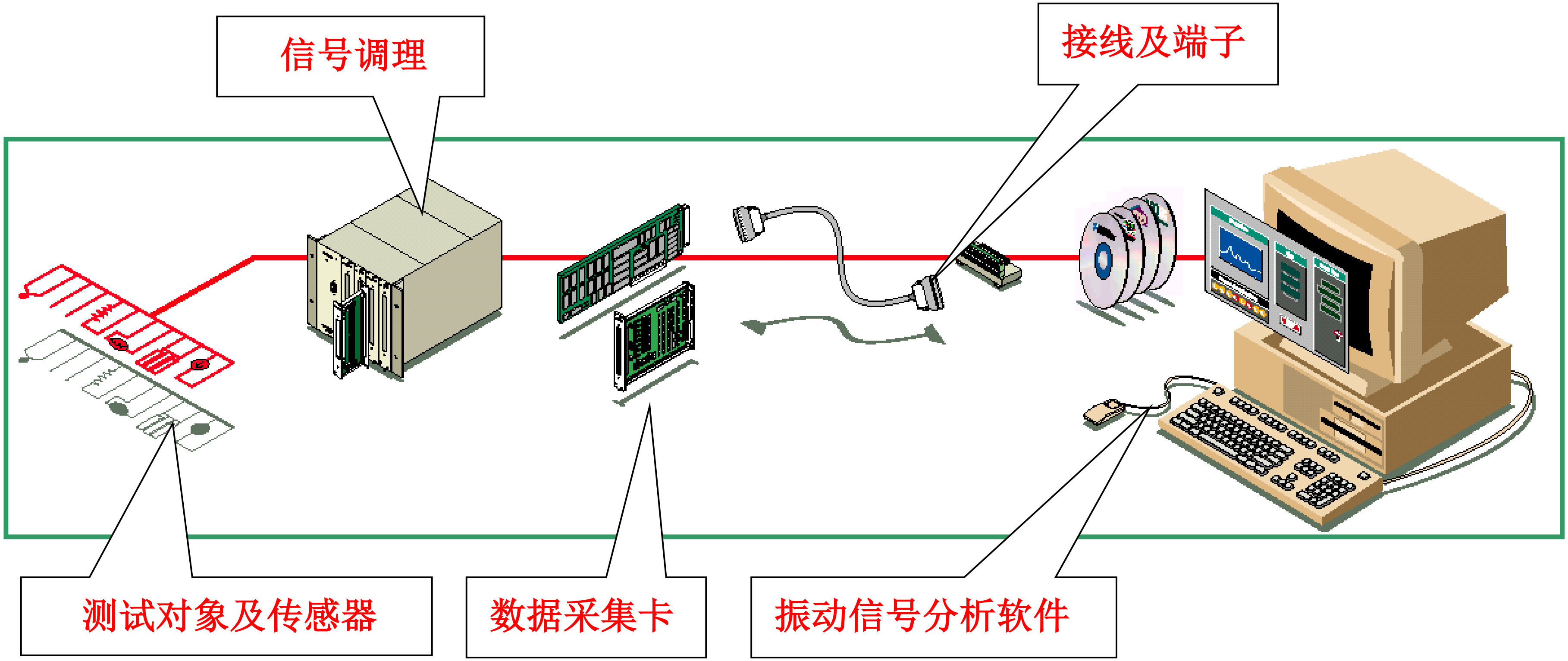
Given the *output signal* and *system parameters*

Determine *input signal*



Example: fault diagnosis for machine

Measurement System

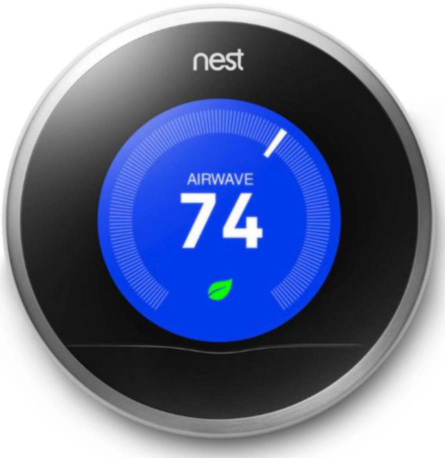


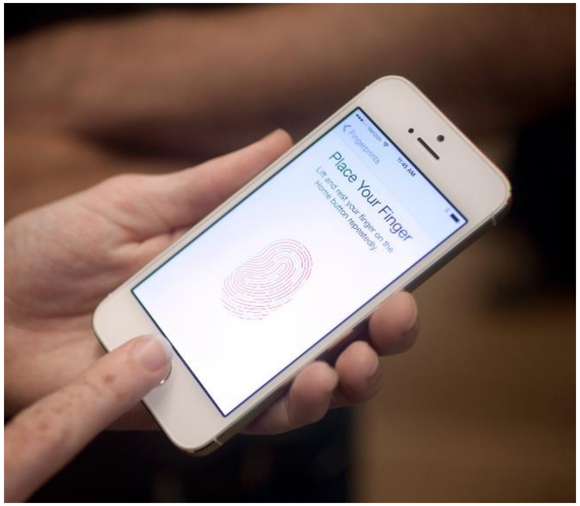
Measurement System

Measurement System

Measurement System

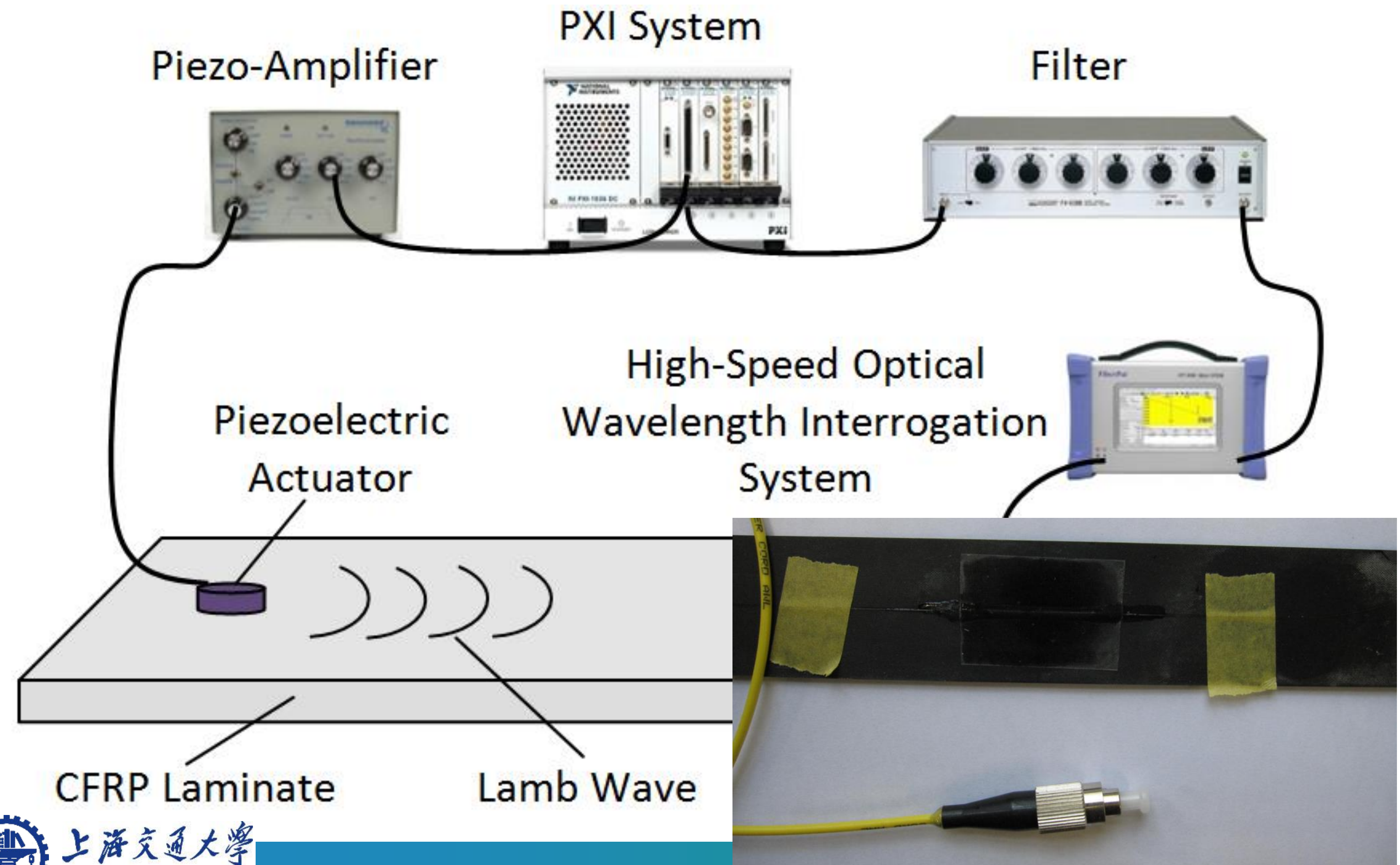
Measurement System



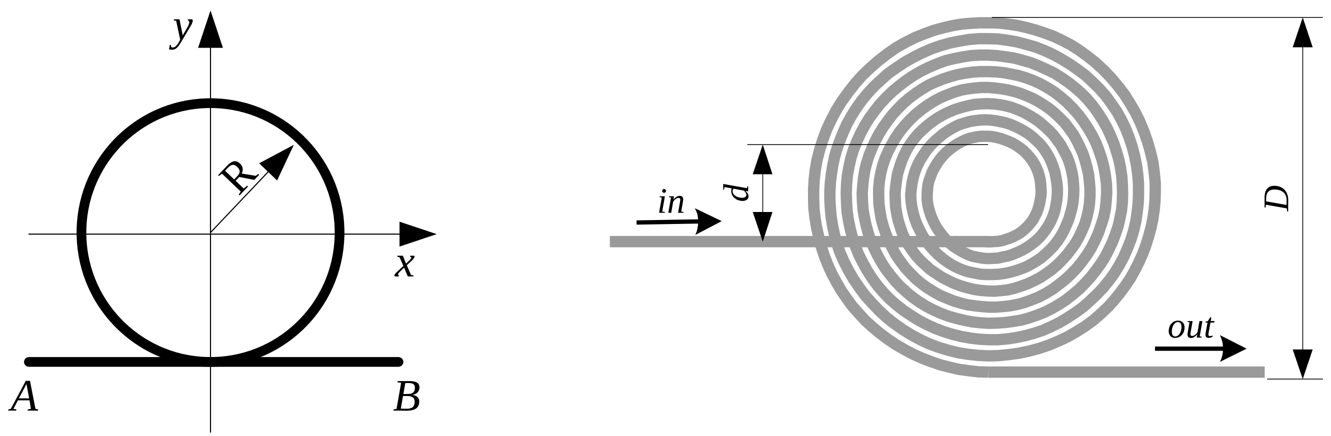


Measurement System

Sensor: Fiber Bragg grating (FBG) sensor



Sensor: Doppler effect-based fiber optic (FOD) sensor

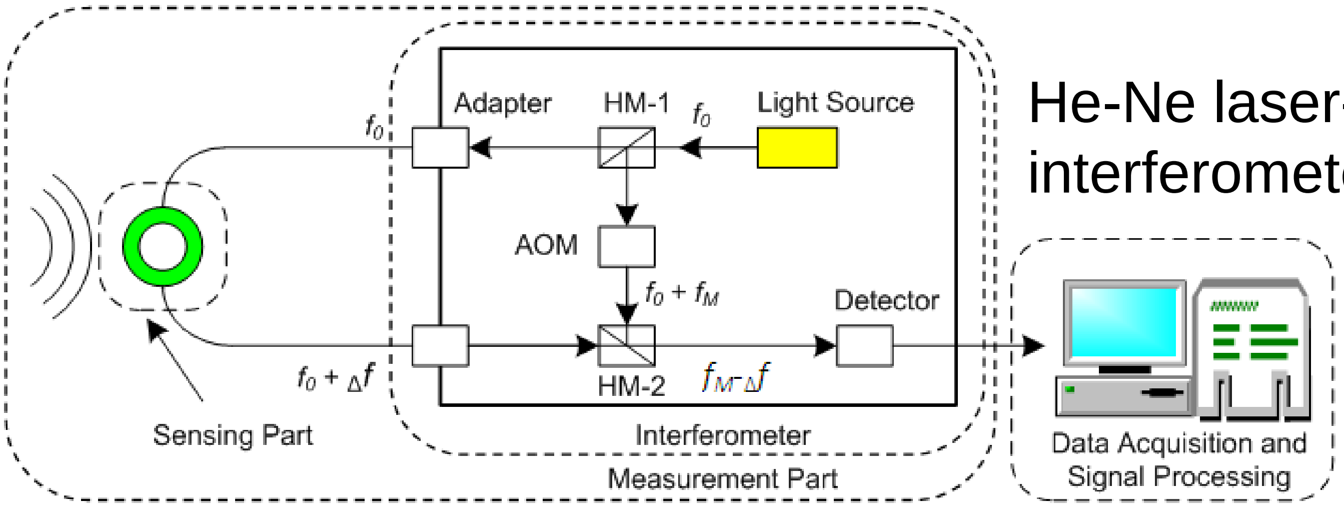


- If an accident causes length of the optical fiber to change from L to $L+dL$ in infinitesimal time dt, Doppler frequency shift f_D can be obtained by

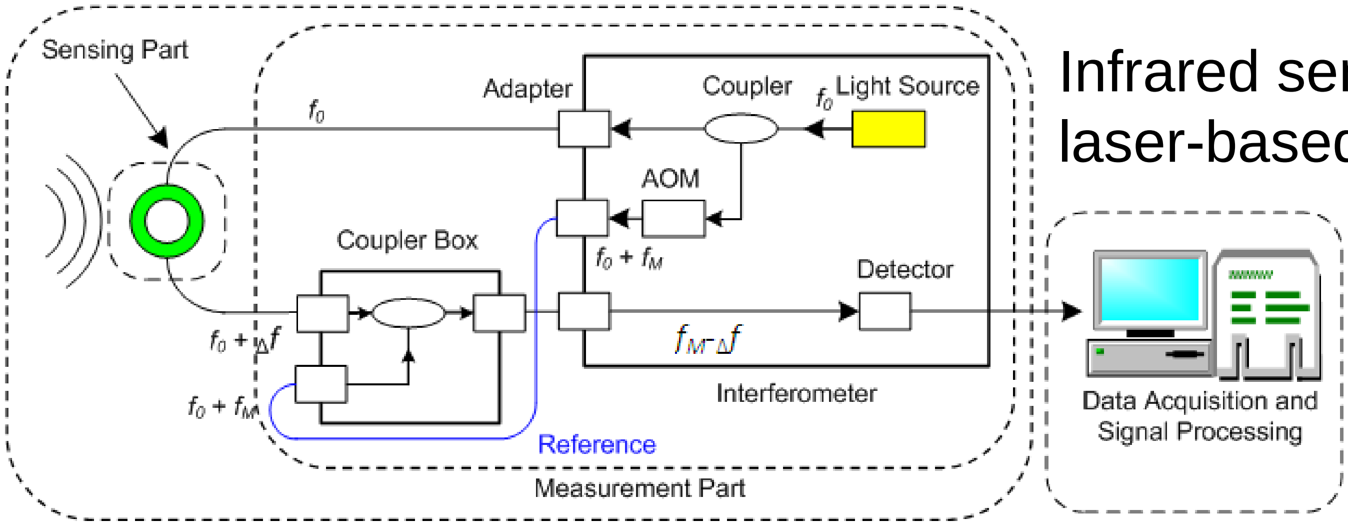
$$f_D = -\frac{n}{\lambda_0} \cdot \frac{dL}{dt} \quad \Rightarrow \quad f_D = -\frac{\pi D n_{eq}}{2\lambda_0}\left(\dot{\varepsilon}_x + \dot{\varepsilon}_y\right)$$

Measurement System

Interferometers for FOD sensor

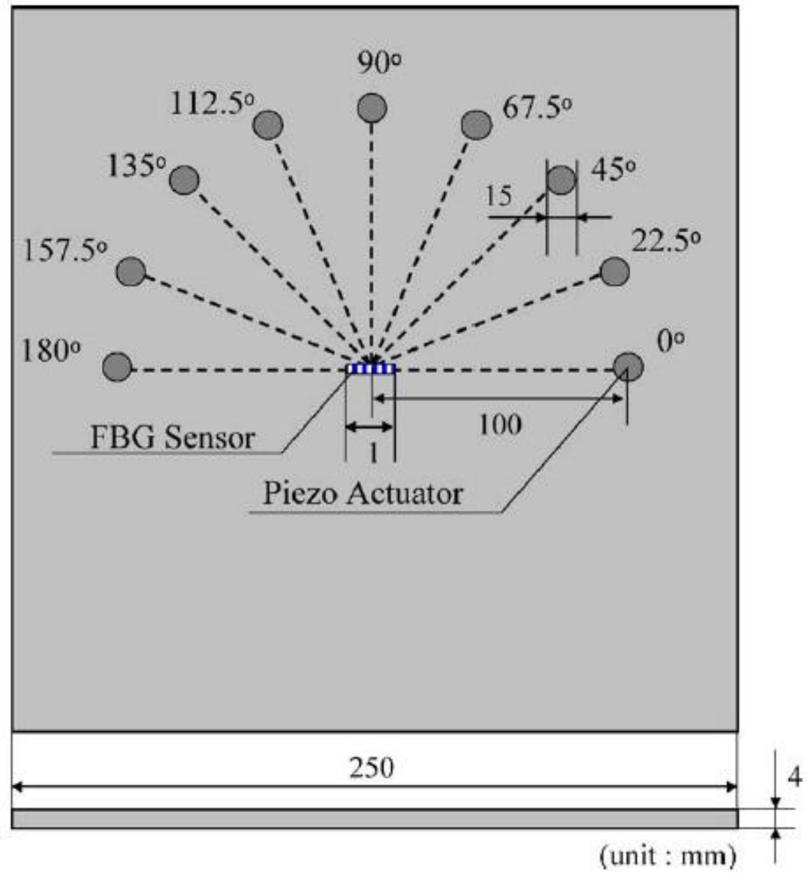


He-Ne laser-based interferometer



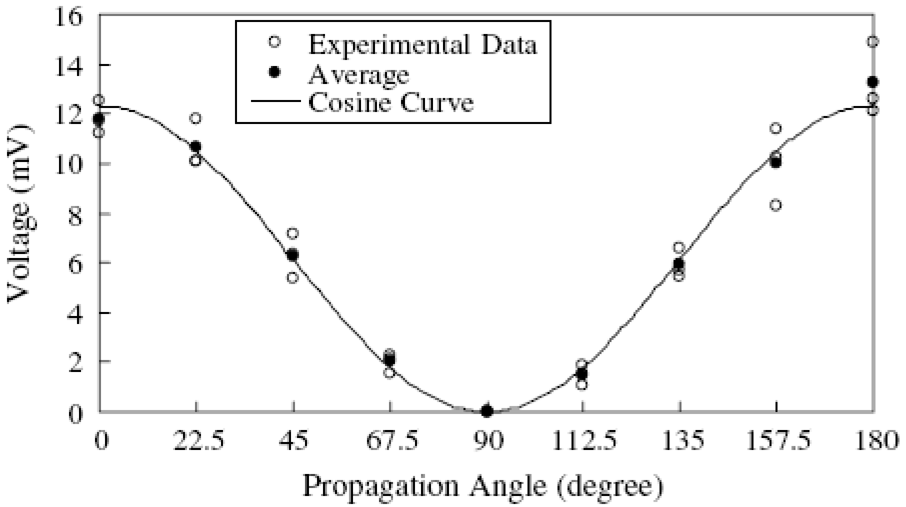
Infrared semiconductor laser-based interferometer

Optical fiber sensors in Lamb wave detection

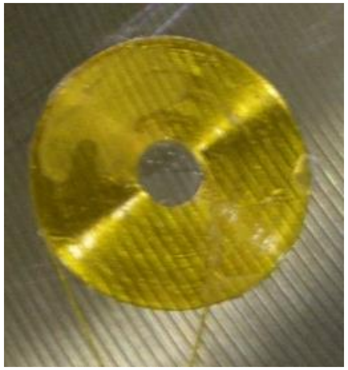


FBG: directionality

N. Takeda et al. Comp. Sci. & Tech. 65(2005) 2575-2587

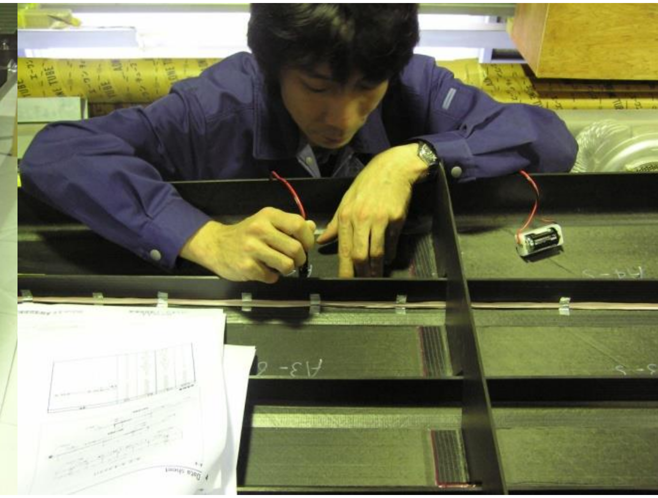


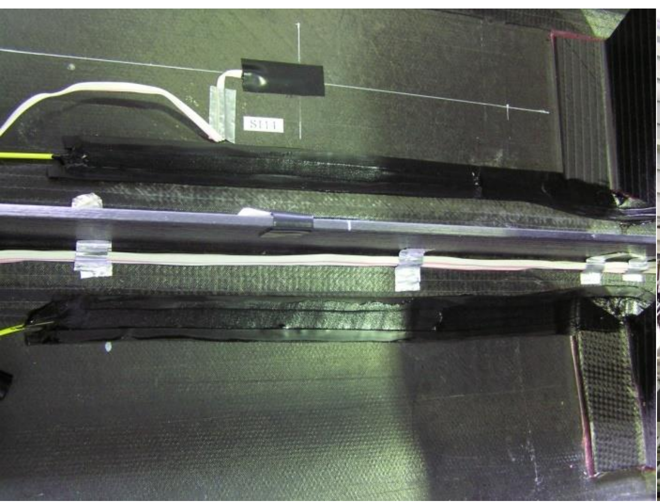
FBG: directionality

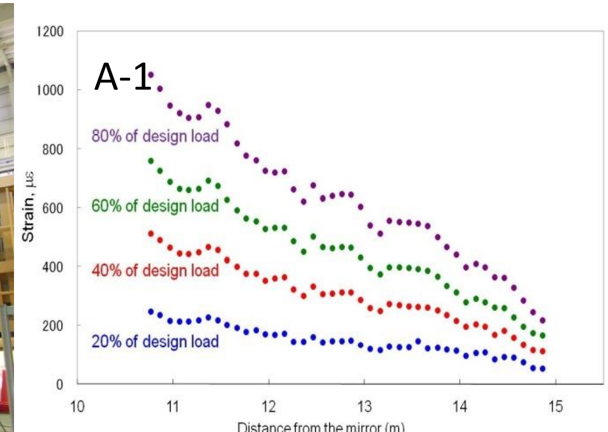


FOD: **Omnidirectional!!!**

Measurement System



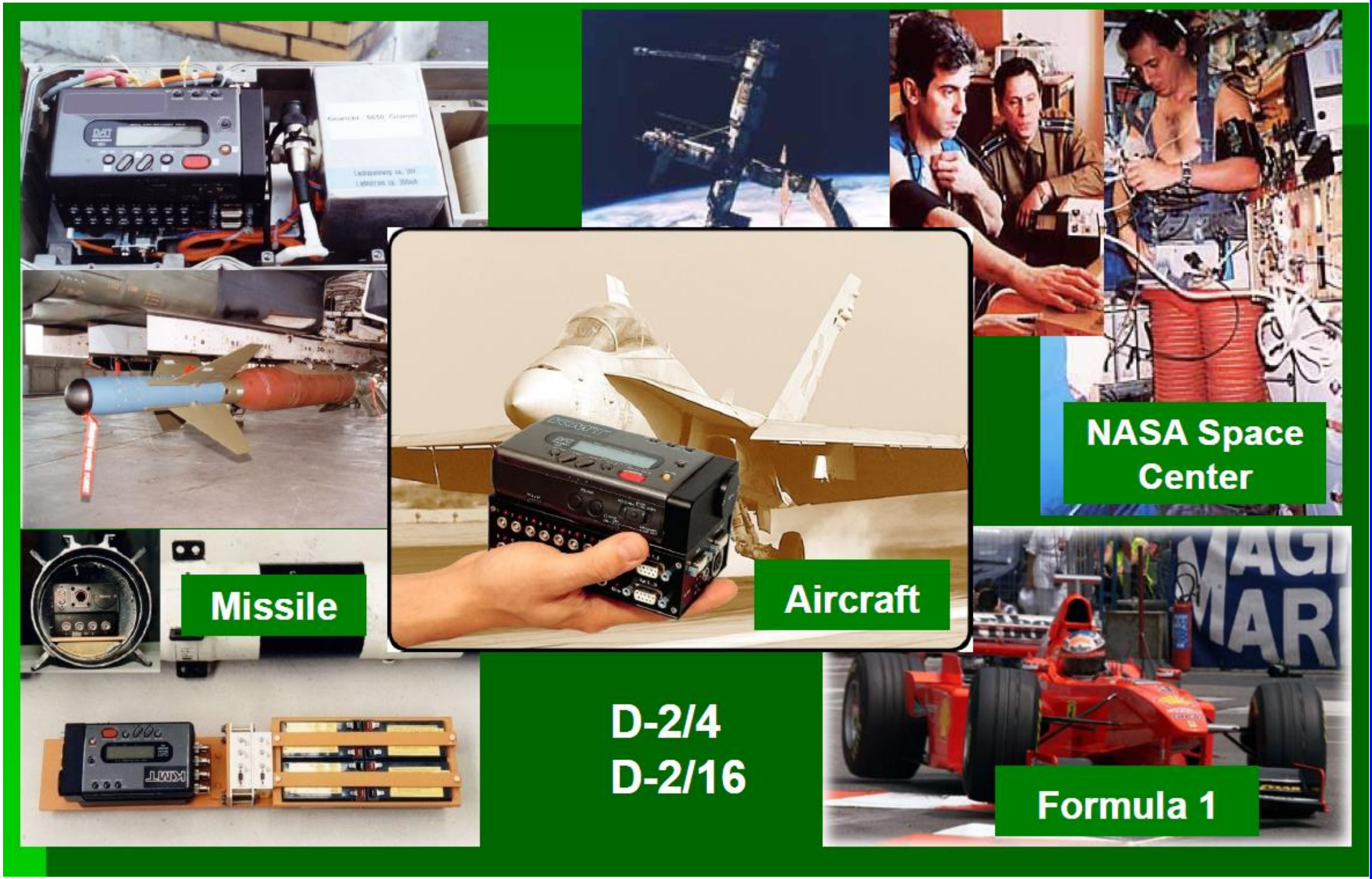




各荷重におけるひずみ分布

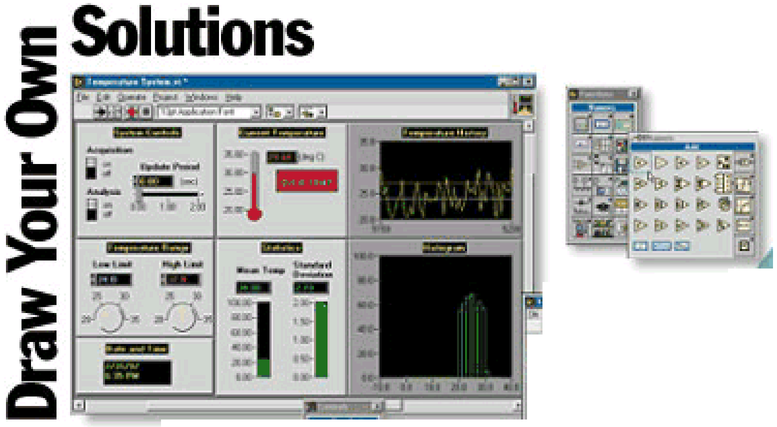
Measurement System

Measurement System



3、测量分析仪器

美国国家仪器公司(全球最大的计算机虚拟仪器生产商)



http://www.ni.com/

美国Agilent公司(原惠普公司仪器部，著名的测试仪器商)

http://www.agilent.com.cn/

1、工业自动化类传感器

美国**霍尼威尔**公司（有全球最大**传感器**技术研究中心）

http://www.honeywell.com/china

2、振动/噪声传感器

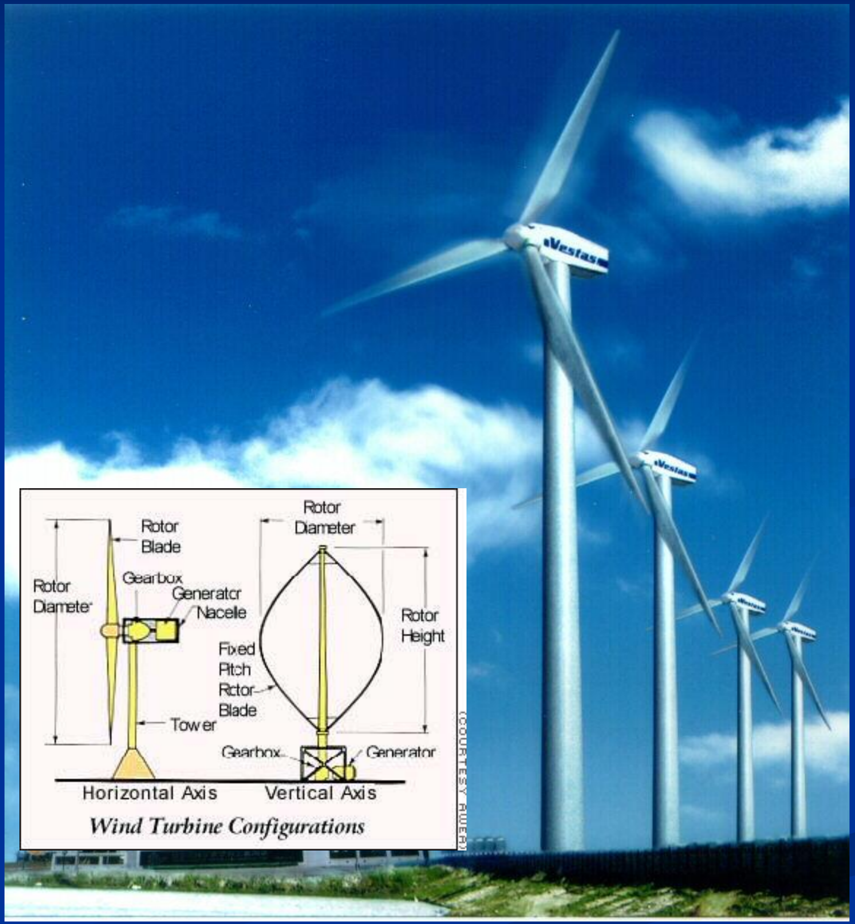
丹麦 **B&K**（振动测量、声学测量领域最富盛名）

Brüel & Kjær

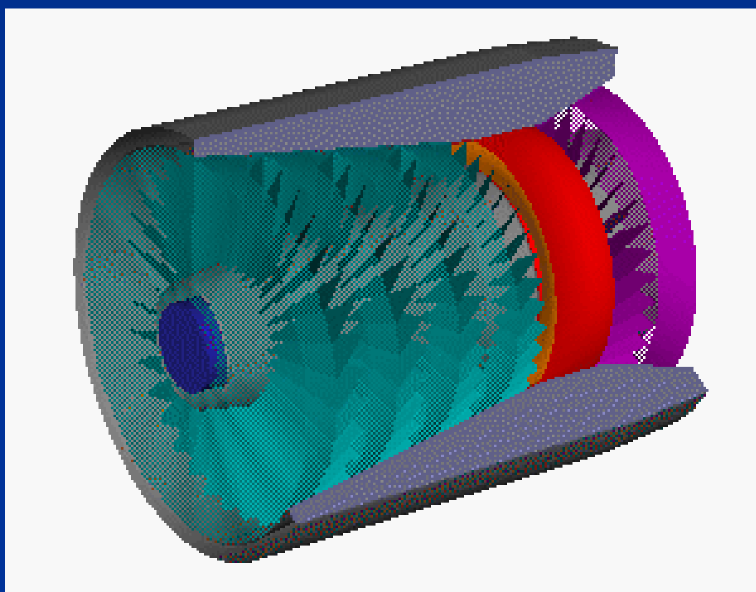
creating sustainable value

http://www.bksv.com/

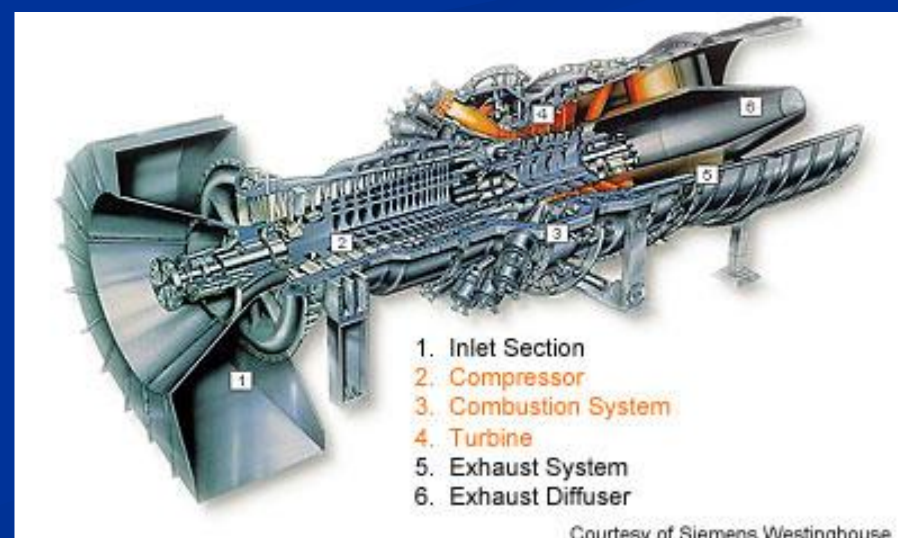
Example: Rotor Unbalance



Example: Rotor Unbalance



Every gas turbine engine has a combustion section (red), a compressor (cyan) and a turbine (magenta)

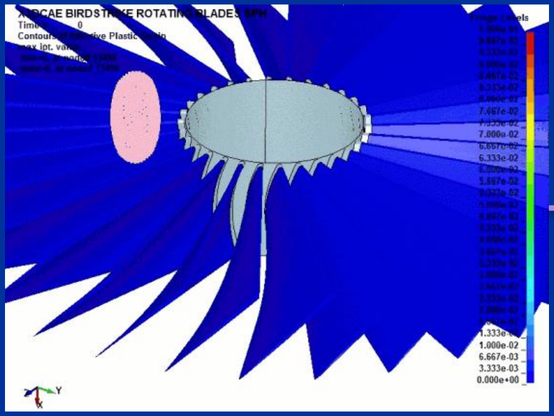


Gas turbine

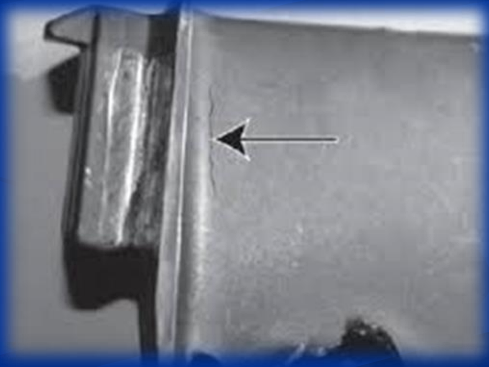
Example: Rotor Unbalance

风扇叶片脱落（Fan Blade Out, FBO）

航空发动机事故



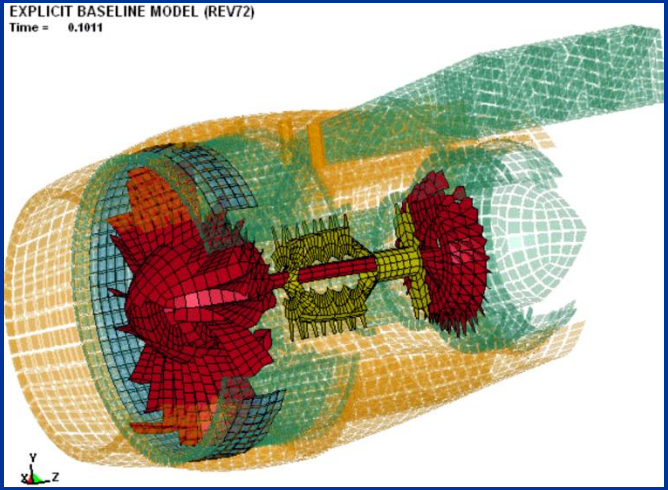
外物撞击（鸟撞）

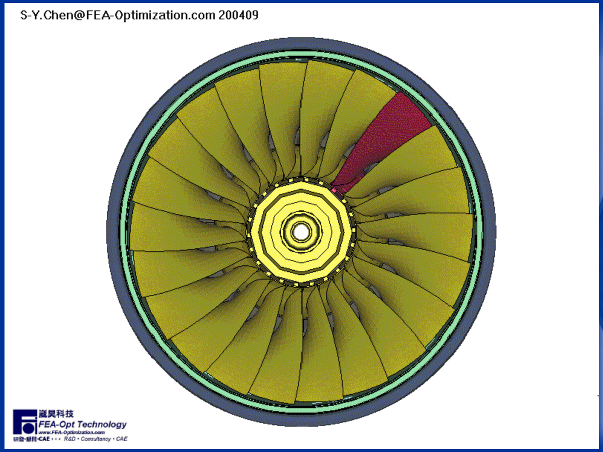


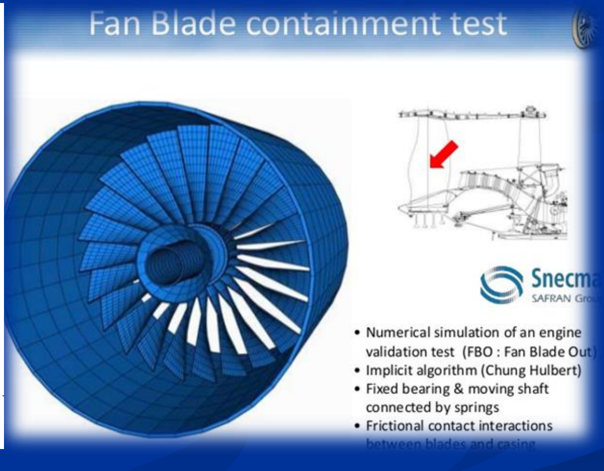
疲劳裂纹等会发生断裂

Example: Rotor Unbalance

….The NASA Glenn Research Center has teamed with GE Aircraft Engines, Pratt & Whitney, Boeing Commercial Aircraft, Rolls-Royce, and MSC. Software. Progress to date on this project includes expanding NASTRAN to perform rotordynamic analysis of complete engine-airframe systems.

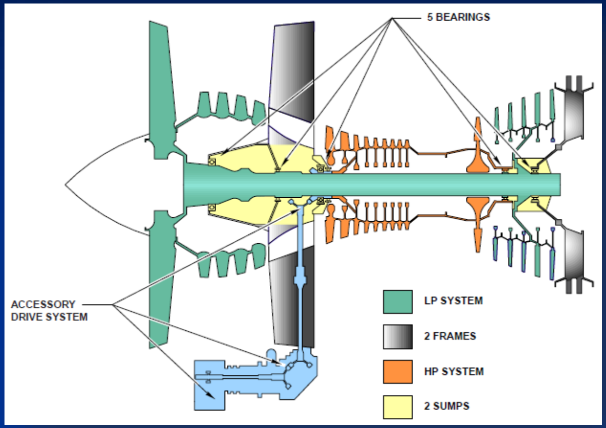


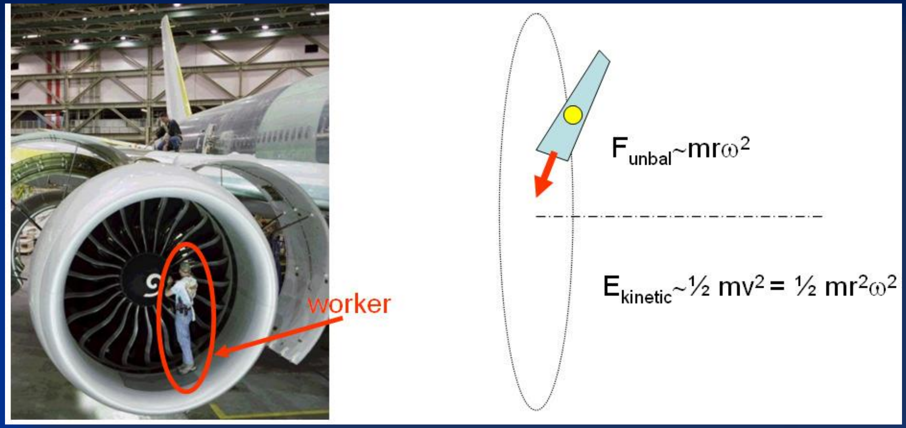


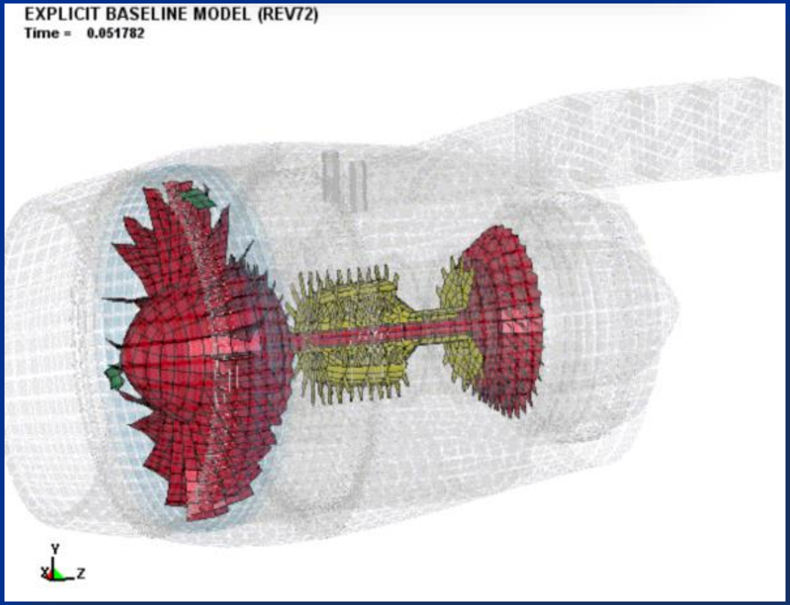


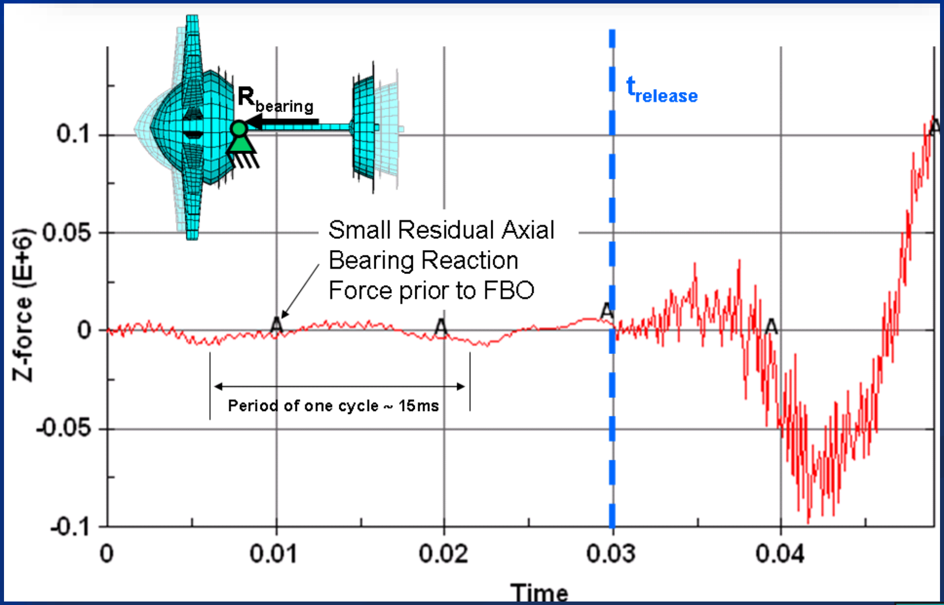
LS-Dyna modeling of fan blade impact and NASTRAN modeling of engine rotor imbalance

Example: Rotor Unbalance









Thank You