Document 5: Linear Transformations

We define a test used to see if a transformation is linear. For the definition of linearity and test, we use two vectors that have the following definitions.

$$\text{Let } k \text{ be an arbitrary scalar. Set } \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Definition: A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear if and only if.

1. $T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$.

2. $T(k\vec{v}) = kT(\vec{v})$.

Let $m, n \in \mathbb{Z}$ be arbitrary. Define $T : \mathbb{R}^m \to \mathbb{R}^n$. If the transformation is linear, then there must be a matrix, A, called the standard matrix, such that $T(\vec{x}) = A\vec{x}$.