Document 15: Bases

Given a base $B = \{\vec{v_1}, \vec{v_2}\}$, let $\vec{x} = c_1\vec{v_1} + c_2\vec{v_2}$. The span on the right are the coordinates of $\vec{x}$.

Definition: Set $\vec{x} = c_1[\vec{x_1}]_{\mathbb{B}} + c_2[\vec{x_2}]_{\mathbb{B}}$ where $\mathbb{B}$ is the basis for the vector space, and $\vec{x}$ is the vector whose coordinates we are interested in. The coordinates are c_1, c_2. The solution set is the set that contains both these values.

We also have a B-matrix $B = \begin{bmatrix}[T(\vec{v_1})]_{\mathbb{B}} & [T(\vec{v_n})]_{\mathbb{B}}\end{bmatrix}$. B is like our standard matrix, but it lets us perform a transformation from $[\vec{x}]_{\mathbb{B}}$ to $[T(\vec{x})]_{\mathbb{B}}$, as opposed to from $\vec{x}$ to $T(\vec{x})$. This lets us use different coordinate systems with a different basis, such as a rotated coordinate system, very easily. If our x-axis gets replaced with a line rotated 45 deg, and our y-axis remains orthogonal, we can transform x within this axis system by transforming it to $[\vec{x}]_{\mathbb{B}}$ and using B.

$$\begin{array}{ccc} \vec{x} & A :\rightarrow & T(\vec{x}) \\ S :\uparrow, S^{-1} :\downarrow & & S :\uparrow, S^{-1} :\downarrow \\ [\vec{x}]_{\mathbb{B}} & B :\rightarrow & [T(\vec{x})]_{\mathbb{B}} \end{array}$$

Further, $T(\vec{x}) = A\vec{x} = AS[\vec{x}]_{\mathbb{B}} = S([T(\vec{x})]_{\mathbb{B}}) = SB[\vec{x}]_{\mathbb{B}}$. Interesting to note, the simplest form of a matrix can be found using a different basis, and it is called the Jordan Canonical Form.

Definition Similarity: Two matrices A, B are similar if there exists a matrix S such that $AS = SB$. Two equations that allow us to find similar matrices are $A = SBS^{-1}$ and $B = S^{-1}AS$.

Considering harmonic functions, for 2nd Order Ordinary Differential Equations, $\frac{d^2x}{dt^2} + x = 0$. Claim, Any other solution of the ODE can be written as $x_g(t) = c_1 \sin(t) + c_2 \cos(t)$