Document 11: Summary Section

First, was linear equations. There are linear and nonlinear systems, and matrix representations. Additionally, elementary row operations. These are represented with E_i, E_j. One can compute a reduced row echelon form of a matrix, and classify solutions after reading the previous sections as well. Classification of solutions (Given $A_{m\times n}$):

1. $\text{rank}(A) \leq m, n$.
2. $\text{rank}(A) = m$, the system is consistent.
3. $\text{rank}(A) = n$, at most one solution.
4. $\text{rank}(A) < n$, Either infinite solutions or none.
5. $\text{rank}(A) \equiv$ Number of nonzero rows in rref.

Second, was Linear Transformations. One can prove if something is linear or nonlinear. Additionally, earlier sections covered the formulas for 2D transformations. Next, one can move to 3D, and learn the linear transformations in 3D from the previous sections as well.

SUMMARY TABLE: Transformations Table

Name	Formula	Matrix Representation
Line Projection	$\text{proj}_L(\vec{x}) = (\vec{x}\cdot\vec{u})\vec{u}$	$A = \begin{bmatrix} (u_1)^2 & u_1u_2 \\ u_1u_2 & (u_2)^2 \end{bmatrix}$
Line Reflection	$\text{ref}_L(\vec{x}) = 2\text{proj}_L(\vec{x}) - \vec{x}$	$A = \begin{bmatrix} 2(u_1)^2 - 1 & 2u_1u_2 \\ 2u_1u_2 & 2(u_2)^2 - 1 \end{bmatrix}$
Plane Projection	$\text{proj}_V(\vec{x}) = \vec{x} - (\vec{x}\cdot\vec{u})\vec{u}$	$A = \begin{bmatrix} 1-(u_1)^2 & u_1u_2 \\ u_1u_2 & 1-(u_2)^2 \end{bmatrix}$
Plane Reflection	$\text{ref}_V(\vec{x}) = \vec{x} - 2\text{proj}_L(\vec{x})$	$A = \begin{bmatrix} 1-2(u_1)^2 & -2u_1u_2 \\ -2u_1u_2 & 1-2(u_2)^2 \end{bmatrix}$
Rotation	$R_\theta \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = e^{i\theta}\vec{z}$	$R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

The inverse of a transformation is also something that previous sections cover. For a 2×2 matrix, $A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det(A) = ad - bc$. Finally, previous sections covered the algebra of matrices and vectors.