Document 9: Computing the inverse of a matrix

To compute the inverse of a matrix, write the augmented matrix $[A_{n\times n}|I_n]$, compute the reduced row echelon form of the augmented matrix, yielding the identity matrix on the right. In other terms, $\text{rref}([A|I]) = [I|A^{-1}]$.

For the special case of a matrix $A_{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, A is invertible $\Leftrightarrow \det(A) = ad - bc \neq 0$. Let $A_{p\times n}, B_{m\times p}$. Then BA is defined and $T(\vec{x}) = B(A\vec{x}), T : \mathbb{R}^n \to \mathbb{R}^m$ and $(BA)_{m\times n}$. In general, $BA \neq AB$ unless $A = B^{-1}$ and $B = A^{-1}$

Definition: Matrix properties

1. Let $A_{q\times n}, B_{m\times p}$ be arbitrary matrices. Then BA is defined $\Leftrightarrow p = q$.
2. Let $A_{p\times n}, B_{m\times p}$ be arbitrary matrices. Then BA is defined, and $BA_{m\times n}$ is the standard matrix for a transformation $T : \mathbb{R}^n \to \mathbb{R}^m, T(\vec{x}) = B(A\vec{x})$
3. For arbitrary matrices A, B it is not guaranteed that $AB = BA$.

Properties of Invertable Matrices:

1. Let $A_{n\times n}$ be an invertable matrix. Then, $AA^{-1} = I_n, A^{-1}A = I_n$.
2. Let $A_{m\times n}$ be an invertable matrix. Then, $I_m A_{m\times n} = A_{m\times n} I_n = A$.
3. Matrix multiplication is associative, so $(AB)C = A(BC) = ABC$.
4. Let $A_{n\times n}, B_{n\times n}$ be invertable matrices. Then $(AB)^{-1} = A^{-1}B^{-1}$.
5. Let $A_{m\times p}, B_{m\times p}, C_{p\times n}, D_{p\times n}$ be matrices. Then $A(C+D) = AC+AD$ and $(A + B)C = AC + BC$

Criteria for Invertability: Let $A_{n\times n}, B_{n\times n}$. Set $BA = I_n$. Then,

1. A and B are both invertible.
2. $A^{-1} = B$, and $B^{-1} = A$
3. $AB = I_n$