Document 1: Linear Equations and General Systems

The general linear system looks like:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n$$

These would have m equations with n unknowns each. Any system of linear equations can have only three types of solutions: 0 solutions, 1 solution, or ∞ solutions.

For cases in $2D$, we visualize the solutions as overlapping for infinite solutions, parallel for no solution, and intersecting for a unique solution. In $3D$, three intersecting planes represent infinite solutions, no universal intersection point represents no solution, and one solution would be represented by planes all at 90 degrees, intersecting at one place (i.e. a plane in xy-space, one in yz-space, and one in xz-space).

Elementary Operations Theorem: This theorem provides a system for solving a general system of linear equations.

1. Interchange two equations: $E_i \leftrightarrow E_j$
2. Multiplication by a scalar: $E_i \leftrightarrow sE_i$
3. Addition of a constant multiple of one equation to another: $E_i \leftrightarrow E_i + kE_j$

This theorem gives way to early concepts such as solving a linear system of equations by elimination. For more generalized algorithmic solutions, we use matrices. An example of a system of equations and its matrix representations is shown below.

$$\begin{array}{r|l} x_2 + x_3 = 4 & \\ x_1 - x_2 + 2x_3 = 1 & A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix} \\ 2x_1 + x_2 - x_3 = 6 & \end{array}$$