Document 23: Eigenvalues / Eigenvectors Continued

Reexamining the methods for computing Eigenvalues and Eigenvectors, we have a new formula for the characterisitc polynomial of the vector.

$$\text{Given } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ we have } A - \lambda I = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$$P_A(\lambda) = (a-\lambda)(d-\lambda) - bc$$
$$\text{Or, } P_A(\lambda) = \lambda^2 + \text{trace}(A)\lambda + \det(A)$$

These formulas only work for 2×2 matrices. Another theorem that exists for 2×2 matrices is that for any given matrix A, defined above, the following holds true.

$$\text{Define } B_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, B_2 = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}, B_3 = \begin{bmatrix} \sigma & -\tau \\ \tau & \sigma \end{bmatrix}, \lambda = \sigma \pm \tau i$$

$$A \sim B_1 \vee A \sim B_2 \vee A \sim B_3$$

It is worthy of noting that B_1, B_2, B_3 are each referred to as the Jordan Cannonical Forms of the matrix A. Shifting to diagonalization, lets define Algebraic Multiplicity as the number of times an eigenvalue apears in the characteristic polynomial.

The eigenvalues of triangular matricies are the diagonal entries.

Given a characteristic polynomial $P_A(\lambda) = (1-\lambda)^3(2-\lambda)^2 = 0$, we have $\lambda_1 = 1$, with an Algebraic Multiplicity of 3, and $\lambda_2 = 2$, with an Algebraic Multiplicity of 2.

Definition Geometric Multiplicity: Let $A\vec{v} - \lambda\vec{v} = 0 \Rightarrow (A - \lambda I)\vec{v} = 0 \Rightarrow \vec{v} \in \ker(A - \lambda I)$. Let $E_\lambda = \ker(A - \lambda I)$. Then, $\dim(E_\lambda)$ is the Geometric Multiplicity of λ.

Definition diagonalizable: Let the function yielding the Algebraic Multiplicity be a.m.(λ_i). Let the function yielding the Geometric Multiplicity be g.m.(λ_i). $A_{m\times n}$ is diagonalizable $\Leftrightarrow$ a.m.(λ_i) = g.m.(λ_i). Then, $\exists B$ matrix such that $A \sim B$ with $S = [\text{eigenvector}(A)]$.