Document 10: Multiplication of Block Matrices

Sometimes when matrices get large, we can use block matrices to reduce the complexity of operations we must perform. A matrix can be divided into submatrices, and a the original matrix can now be expressed as a matrix of matrices. Consider the following:

$$E = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} a & b \\ d & e \end{bmatrix} & \begin{bmatrix} c \\ f \end{bmatrix} \\ \begin{bmatrix} g & h \end{bmatrix} & \begin{bmatrix} i \end{bmatrix} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

When multiplying a block matrix, each submatrix is an element, and matrix multiplication can be performed as normal. Using the above example, we may demonstrate how EE is computed.

$$EE = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} AA + BC & AB + BD \\ AC + DC & CB + DD \end{bmatrix}$$

Since $A, B, C,$ and D are all matrices, their products and sums follow all of the rules of standard matrix multiplication and addition. Next, lets examine how the span function is used.

Let $L = \text{span}\left\{ \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} \right\}$, $\vec{x} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$. To find the reflection of $\vec{x}$ about L, we use: $\text{ref}_L(\vec{x}) = 2(\vec{x} \cdot \vec{u})\vec{u} - \vec{x}$. Define $\vec{v}$ such that $L = \text{span}\{\vec{v}\}$. Now, find $\vec{u}$; this vector is a unit vector pointing in the same direction as $\vec{v}$. Hence, $\vec{u} = \frac{\vec{v}}{||\vec{v}||}$. Recall $||\vec{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$, since $\vec{v}$ is in 3D space, and must have 3 elements.