Document 17: Isomorphisms

In this section, we are investigating Isomorphisms. Previously, we had linearity, where we could perform transformations back and forth between two sets, isomorphisms are similar.

Definition: Isomorphisms are invertible linear transofmrations.

We say that $V \overset{\text{iso}}{\sim} W \Leftrightarrow \exists$ an isomorphism $T : V \rightarrow W$. We also know that any n-dimentional linear space V is isomorphic to $\mathbb{R}^n$.

Definition: Similarity between A, B exists if $\exists S, B = S^{-1} A S$.

Properties of Isomorphism:

1. $T : V \rightarrow W$ is an isomorphism $\Leftrightarrow \ker(T) = \{0\} \wedge \text{im}(T) = W$

2. Let V, W be finite dimensional. $V \sim W \rightarrow \dim(V) = \dim(W)$.

3. Let $T : V \rightarrow W$ be a linear transformation where $\text{Kernel}(T) = \{0\}$. $\dim(V) = \dim(W) \rightarrow T$ is an isomorphism.

Now, lets look at a matrix representation.