Document 22: Eigenvalues and Eigenvectors

This section covers eigenvalues and eigenvectors, which will be important in further mathematics and useful for applications of linear algebra.

Definition Eigenvectors/Eigenvalues: A nonzero vector $\vec{v} \in \mathbb{R}^n$ is an eigenvector of A if $\exists \lambda \in \mathbb{R}, A\vec{v} = \lambda\vec{v}$. The Eigenvalue is λ.

When finding Eigenvalues/Eigenvectors, we must find values such that $(A - \lambda I)\vec{v} = \vec{0}$. This is a homogeneous linear system. Recall $A\vec{x} = \vec{0}$ has either 1 or infinite solutions. The eigenvalues are produced with $\det(A - \lambda I) = 0$, and the eigenvectors are produced by substituting into $(A - \lambda I)\vec{v} = 0$.

For finding eigenvalues, we need to find the characteristic polynomial of the matrix A. We begin with $P_\lambda(A) = \det(A - \lambda I)$. We will solve and get some values, lets say λ_1, λ_2. Then we will solve $(A - \lambda_1 I)\vec{v} = \vec{0}$. We can use an augmented matrix and compute the $\mathrm{rref}(A)$ to solve this. This may have infinitely many solutions, for instance:

$$\begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \end{bmatrix} = s \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

This was a basic introduction to eigenvalues and eigenvectors. This should be informative, yet basic. Further exploration is necessary.