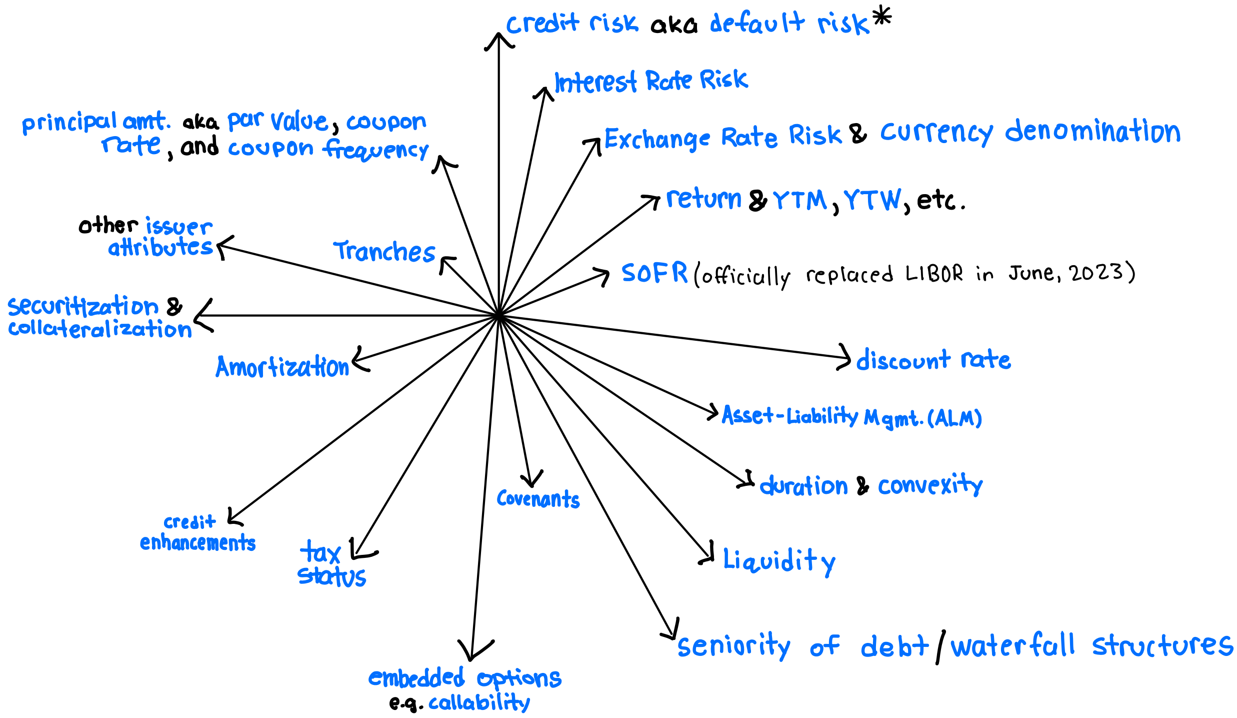
Pricing Fixed Income Securities is HIGH DIMENSIONAL,

less complex than equities, but a LOT more to keep track of...



some basics:

- Fundamentally, bonds are contractual agreements (debentures) between borrower & lender = main difference from stocks!
- "If you don't understand duration, then you don't understand Fixed Income"
- "A huge part of Fixed Income analysis is measuring & hedging risk"
- Flat/"clean" price = Full/"dirty" price +AI (pg.106-9) ≠ Present Value
- coupon types: fixed v.s. float v.s. zero-coupon discount bonds v.s. ...
- discount rate (YTM, r, i, y) and coupon rate get divided by n in PV formula
- U.S. Treasuries almost always pay semi-annual coupons | T-Bills (4w:1yr.); T-Notes (2yr.:10yr.); T-Bonds (≥30yr.)
- SOFR offically replaced LIBOR on June 30th, 2023 but many bond still "carry" LIBOR
- Present Value formulas assume all cash flows are reinvested at the bond's Internal Rate of Return (IRR) aka Yield to Maturity, which is unrealistic
- Good practice to quote bonds on a "per \$100 par" basis

Yields

$$BEY = \frac{par - pv}{pv}\left(\frac{365}{\tau}\right), \quad \tau = T - t \quad \text{or} \quad = YTM * n$$: annualized YTM

(& solves potential 360 days assumed in pricing formulas)

$$EAY = \left(\frac{par}{PV}\right)^{\left(\frac{365}{\tau}\right)} - 1$$: continuously compounded

$$YTM \underset{\text{coupon}}{\overset{\text{zero}}{=}} \sqrt{\frac{fv}{p}} - 1 \quad \sim= \left[1 + \left(\frac{BEY}{n}\right)\right]^n - 1$$

$YTM \underset{\text{Bond}}{\overset{\text{coupon}}{=}}$ use financial calculator (solve for I/Y)

expected rate of return for a bond only if held to maturity; no defaults; coupons reinvested at YTM

poor est. if

1. Interest Rates are Vol.
2. Steep yield curve
3. significant default risk
4. bond is callable

$$YTM < BEY < EAY$$

Spreads

$$G\text{-spread} = NYS = r^* - r_f$$

↳ aka Default Spread aka Credit Spread aka Yield Spread

$$I\text{-spread} = r^* - r_{swap}$$

Z-spread = the basis point spread that would need to be added to the default-free (risk-free) spot curve, usually a government or interest rate spot curve, for market price to equal PV

↳ aka Zero Volatility Spread (ZVS) aka Static Spread

$$OAS = Z\text{-spread} - \text{Option cost}$$

Duration Formulas

$$\text{macaulay} = \frac{\partial PV}{\partial YTM} \underset{\text{coupon}}{\overset{\text{zero}}{=}} N = nt \,,\quad n = \text{compounding frequency} \,,\quad t = \text{time to maturity (in years)}$$

$$\text{mod} = \text{macaulay} \div \left(1+\frac{r}{n}\right) \underset{\text{coupon}}{\overset{\text{zero}}{=}} \frac{N}{1+r} = \text{elasticity}$$

$$\cong \frac{(PV_-)-(PV_+)}{2 * \Delta\text{yield} * PV_0}$$

$$\text{PVBP/DV01} = [(PV_-)-(PV_+)]/2$$

$$\text{yield convexity} \cong \frac{[(PV_-)-(PV_+)]-[2*PV_0]}{(\Delta\text{yield})^2 * PV_0} \underset{\text{coupon}}{\overset{\text{zero}}{=}} N^2$$

$$\text{KRD} \cong T_i \frac{CF_i * Z(0,T_i)}{P_i} \longleftarrow \frac{P_i}{\text{par}}$$

$$\text{curve convexity} \cong \sum_{i=1} \frac{CF_i * Z(0,T_i)}{P_i} (T_i)^2$$

The 3 types of interest rate risk are:

1. Parallel Shifts
2. Curvature/Convexity (large changes)
3. Nonparallel Shifts

Other Formulas

PV based on spot: dynamic discount rate (price each individual coupon payment as a zero-coupon bond)

PV based on YTM: constant discount rate

$f(7,1)$ = future discount rate agreed on today on a one-year bond issued in seven years

$$f_n = \text{forward rate} = E_t(\text{spot}) = \frac{(1+S_n)^n}{(1+S_{n-1})^{n-1}} - 1$$

$$S = \text{spot rate} \underset{\text{coupon}}{\overset{\text{zero}}{=}} \left(\frac{\text{par}}{PV}\right)^{\frac{1}{T}} - 1$$

Valuation Formulas (plain vanilla bonds)

$$PV = \frac{\left(\frac{\text{coupon rate}}{n}\right) \times (\text{par})}{\left(1+\frac{YTM}{n}\right)^{1}} + \frac{\left(\frac{\text{coupon rate}}{n}\right) \times (\text{par})}{\left(1+\frac{YTM}{n}\right)^{2}} + \dots + \frac{\left[\left(\frac{\text{coupon rate}}{n}\right) \times (\text{par})\right] + \text{par}}{\left(1+\frac{YTM}{n}\right)^{(n)(T)}}$$

$n \times T$ number of cash flows, n = no. coupon payments per year

Valuation Formulas (ABS & callable bonds)

An example of an embedded call option is prepayment.

$$PV(MBS) = PV(\text{plain vanilla}) - PV(\text{call option})$$

$$\text{pool factor} = \frac{\text{outstanding principal balance}}{\text{par}}$$ = % of principal amt. still eligible for prepayment (exposure)

$$WAL = \frac{\sum_{t=1}^{T} t\,(SP + PR)}{\text{Principal Balance},\ t=0}$$

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 * \Delta\text{curve} * PV_0} \qquad \text{EffConv} = \frac{[(PV_-) - (PV_+)] - [2 * PV_0]}{(\Delta\text{curve})^2 * PV_0}$$

Valuation Formulas (REPO)

Price	Loan
− haircut	− $ interest
Loan	repayment = RI
* ann. repo rate	
$ interest	

borrower's profit = P(T) − P(t) − RI

lender's profit = P(t) − P(T) + RI

$$ROIC = \frac{\text{profit}}{\text{haircut}}$$

30/360
- corporate
- MBSS
- Agency
- muni

Actual/360
- T-Bills (4w - 1y)*
- money market

Actual/365L
- T-Notes (2y - 10y)*
- T-Bonds (>10y)*

* semiannual coupons

Calculating Accrued Interest and the Full Price of a plain vanilla Bond

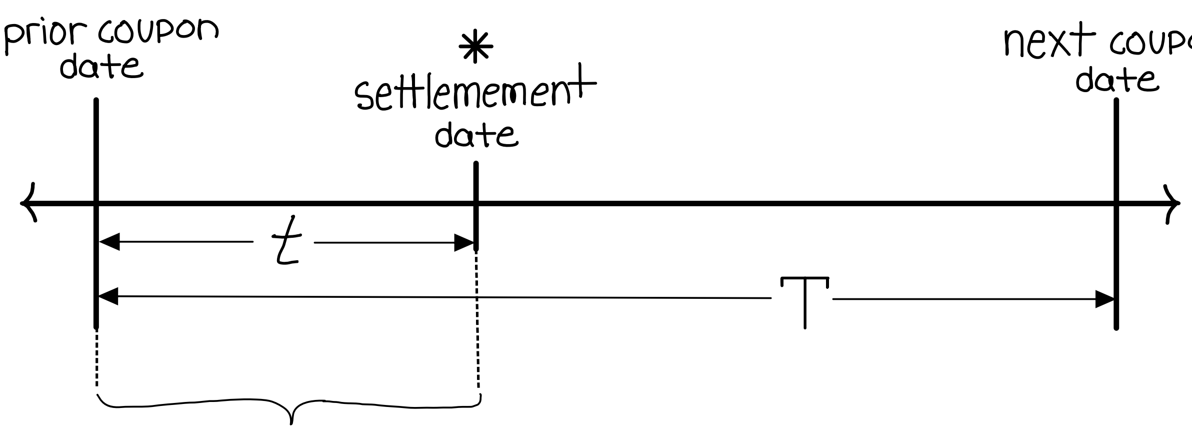
hw1q9 (Excel)

days since prior coupon date

$$\underbrace{P_{FULL}}_{\text{Dirty}} = \underbrace{P_{FLAT}}_{\text{clean}} + AI, \; AI = \frac{\boxed{t}}{\boxed{T}}(PMT) \text{ or } P_{full} = PV * \left[\left(1+ \text{discount rate}\right)^{\left(\frac{t}{T}\right)}\right]$$

Total # days between coupon dates

- 30/360 for corporate bonds
- Actual/360 for short-term debt including commercial paper & T-Bill
- Actual/365 for longer term govt. debt such as T-Bonds



accrural period $\Rightarrow$ $\frac{t}{T}$ = fraction of time between coupon dates accruing interest

- The full price of a fixed-rate bond between coupon payments given the market discount rate per period (*r*) can be calculated as:

$$P_f = \frac{\text{PMT}}{(1+r)^{1-t/T}} + \frac{\text{PMT}}{(1+r)^{2-t/T}} + \cdots + \frac{\text{PMT}+\text{FV}}{(1+r)^{N-t/T}}$$

where ***N – t/T*** represents the time before the appropriate payment is made and **FV** is the face value of the bond.

- The above formula can be simplified to:

$$P_f = \text{PV} \times (1+r)^{t/T}$$

AN IMPORTANT DETAIL ABOUT FIXED INCOME IS THE COMPOUNDING INTEREST RATE FREQUENCY. UNLESS OTHERWISE STATED, IT IS ALWAYS CORRET TO ASSUME...

In USA: government bonds pay <u>semi-annual</u> coupons

In asia: government bonds pay <u>quarterly</u> coupons

In Germany: government bonds (the "bund") pay <u>annual</u> coupons

The assumed compounding frequencies hold even for zero-coupon governtment bonds:

In USA: $PV_{\text{zero coupon}} = \dfrac{par}{(1 + r/2)^{t*2}} \implies ModDur = \dfrac{t*2}{(1 + r/2)}$

In asia: $PV_{\text{zero coupon}} = \dfrac{par}{(1 + r/4)^{t*4}} \implies ModDur = \dfrac{t*4}{(1 + r/4)}$

In Germany: $PV_{\text{zero coupon}} = \dfrac{par}{(1 + r/1)^{t*1}} \implies ModDur = \dfrac{t*1}{(1 + r/1)}$

In general: $PV_{\text{zero coupon}} = \dfrac{par}{(1 + r/n)^{nt}} \implies ModDur = \dfrac{nt}{(1 + r/n)}$

- "par" = Face Value
- n = compounding frequency
- t = time to maturity in years