

Min cost s.t. Production Constraint (Cobb-Douglas)

Cobb-Douglas production fnc: $Q = 72x^{3/4}y^{1/4}$, $\underbrace{x = \text{Labor} \quad y = \text{Capital}}_{\text{decision variables}}$

cost fnc (objective): $C = 108x + 576y$

constraint: $Q = 9216 \text{ units} \Rightarrow 72x^{3/4}y^{1/4} = 9216$

step 1 $\mathcal{L} = [\text{objective function}] + \lambda [\text{constraint set} = 0]$

$$= 108x + 576y + \lambda (72x^{3/4}y^{1/4} - 9216)$$

step 2 Take the first-order partial derivatives

step 3 set equal to zero

$$L_x = 108 + \lambda (72(\frac{3}{4})x^{-1/4}y^{1/4} - 0) = 108 + 54\lambda x^{-1/4}y^{1/4} = 0$$

$$L_y = 576 + \lambda (72(\frac{1}{4})x^{3/4}y^{-3/4} - 0) = 576 + 18\lambda x^{3/4}y^{-3/4} = 0$$

$$L_\lambda = 72x^{3/4}y^{1/4} - 9216 = 0$$

step 4 Solve L_x and L_y for λ

$$L_x: 108 + 54\lambda x^{-1/4}y^{1/4} = 0 \Rightarrow \lambda = \frac{-108}{54} x^{1/4} y^{-1/4}$$

$$L_y: 576 + 18\lambda x^{3/4}y^{-3/4} = 0 \Rightarrow \lambda = \frac{-576}{18} x^{-3/4} y^{3/4}$$

step 5 Set the λ 's equal to each other. Solve for x in terms of y , y in terms of x

$$\frac{-108}{54} x^{1/4} y^{-1/4} = \frac{-576}{18} x^{-3/4} y^{3/4} \Rightarrow 2 x^{1/4+3/4} = 32 y^{3/4+1/4} \Rightarrow$$

$$\Rightarrow x = 16y, \quad y = \left(\frac{1}{16}\right)x$$

step 6 Plug x or y into the constraint L_λ - suppose x

$$L_\lambda: 72x^{3/4}y^{1/4} - 9216 = 0 \Rightarrow 72(16y)^{3/4}y^{1/4} - 9216 = 0 \Rightarrow$$

$$\Rightarrow 72(16^{3/4})y^{3/4}y^{1/4} - 9216 = 0 \Rightarrow 576y^{3/4+1/4} = 9216 \Rightarrow$$

$$\Rightarrow \boxed{y^* = \frac{9216}{576} = 16 \Rightarrow x^* = 16 \underbrace{(16)}_y = 256}$$