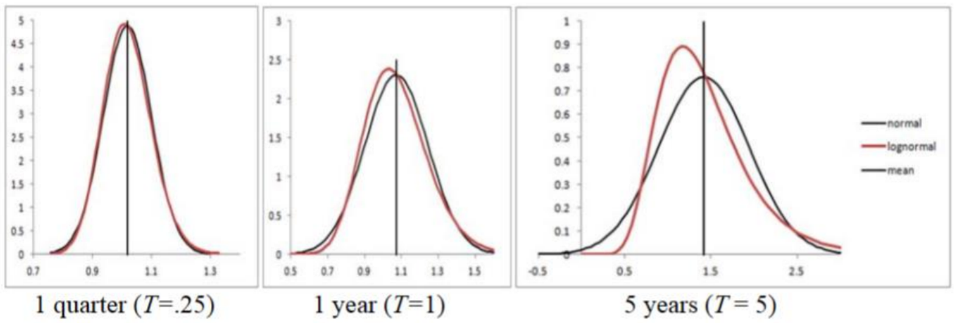
Portfolio Management with Time-Varying Parameters

- **Probem:** In any case, the covariance parameters that we estimate might be time-varying aka non-stationary (i.e., equity market volatility is <u>not</u> constant over time $\Rightarrow$ cannot reliably assume constant volatility of risky asset returns in our robust models for estimating the variance-covariance matrix w/ fewer parameters; yet these models are necessary to defeat the curse of dimensionality).

estimating volatility more accurately:

- first, take a moment to recognize that increasing frequency is better than increasing the sample period (history) in the case of estimating non-stationary return distributions and attacking the curse of dimensionality from the side of raising n... that's because increasing n by means of increasing frequency preserves improves our confidence in $\hat{\beta}$ (b/c $\hat{\beta}$'s confidence interval tightens as $n \to \infty$) while also preservin the normality assumption where as increasing n by means of increasing sample period improves our confidence in $\hat{\beta}$ **but at the expence of normality...**



At horizons of one quarter or shorter, the normal and lognormal distributions are almost indistinguishable. Significant divergences arise at the 1-year horizon and longer, with the nonnegativity of the lognormal distribution becoming important at 5 years.

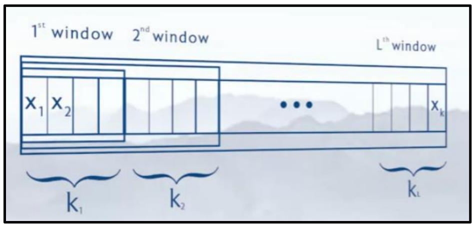
- now take another moment to realize that the situation depicted above suggests we are limited by how much we can attack the curse of dimensionality from the side of increasing n — to maintain normality, we're only allowed to increase frequency. We are <u>not</u> allowed to increase the sample period. So, reducing p just became a lot more important and we further awknowledge the need for robust covariance models in addressing the curse of dimensionality.

- statistical evidence that equity volatilities are time-varying can be plainly observed in the VIX:

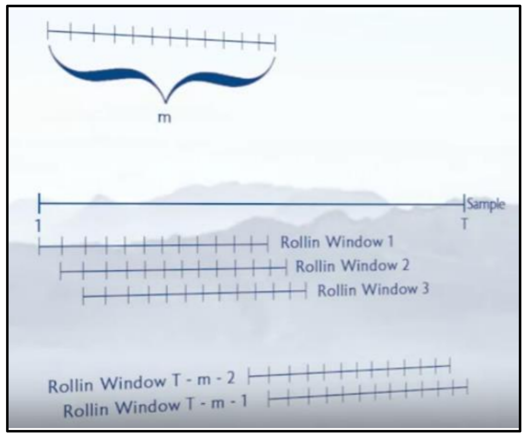
① let σ_T denote the volatility per day between day T and day T+1 s.t.

$$\sigma_T^2 = \frac{1}{T}\sum_{t=1}^{T}\left(R_t - \overline{R}\right)^2, \overline{R} = \frac{1}{T}\Sigma_{t=1}^{T} R_t$$

EXPANDING window analysis versus **ROLLING window analysis**



** better when input variable is stationary **



** better when input variable is time-varying/non-stationary **

IN PRACTICE, WE PREFER A ROLLING WINDOW TO AN EXPANDING WINDOW BECAUSE IT IS MORE PRESERVING OF THE NORMALITY ASSUMPTION AND INFORMS ANALYSIS WITH MORE RECENT DATA / LESS BACKWARD-LOOKING BIAS THAN EXPANDING WINDOW.

② But we can do even better than basic rolling window... exponentially-weighted autoregressive models such as **ARCH, GARCH, and AGARCH** grant all historical observations existing within the expanding or rolling frame **proportional influence** over our volatility predictions based on their relevance (in terms of how recent each obs. is to the estimation period).