

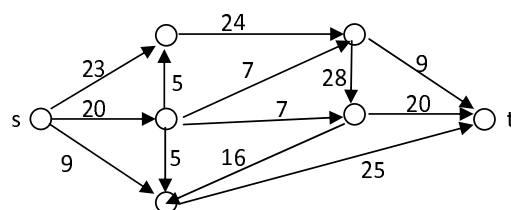
COMP 3121/3821/9101 PRACTICE FINAL
SEMESTER 1 2008

1. Total variation of a finite sequence $\vec{x} = \{x_1, x_2, \dots, x_n\}$ is defined as

$$V(\vec{x}) = \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2.$$

Given a sequence \vec{x} , partition it into two disjoint subsequences such that the sum of total variations of the two subsequences is minimal.

2. (a) Find Maximal Flow of the following network flow:



- (b) Can there be a network flow that has at least two distinct minimal cuts? If so, give an example; if not prove that there is no such network.

3. Every student of the University of Elbonia can check out at most 5 books from the University's library. No one can check out more than one copy of the same book. Given the list of books each student wants to check out and the number of copies of each book, assign books to students so that the number of checked out books is maximal.

4. In Elbonia denomination of coins are 1c, 7c, 13c, 19c, 25c, 31c, 52c, and 91c. Design an algorithm that gives any sum of money in coins, so that the total number of coins for this sum is minimal.

5. Consider a 2-D map with a river flowing straight horizontally from the West to the East. There are n cities C_1, \dots, C_n on the southern bank with x-coordinates c_1, \dots, c_n (note: the coordinates are not necessarily in an increasing sequence). Each city C_i , $1 \leq i \leq n$, has its corresponding airport P_i , $1 \leq i \leq n$, on the northern bank with x-coordinate p_i (again, the coordinates are not necessarily increasing). Connect with bridges as many cities as possible with their corresponding airports. No two bridges can cross.

6. Optimal Strategy for a Game. Consider a row of n coins of values $v(1) \dots v(n)$, where n is even. We play a game against an opponent by alternating turns. In each turn, a player selects either the first or last coin from the row, removes it from the row permanently, and receives the value of the coin. Determine the maximum possible amount of money we can definitely win if we move first.