# Q2

## Setup

First, denote the 2D map called $Martix$, and $Martix[c][r](1 \leq c \leq C$ and $1 \leq r \leq R)$ representing the elevation of the terrain at that square.

## Subproblem

For every $r \in [1, C]$ and $c \in [1, C]$, we define the subproblem $P[c][r]$ to be "find the number of moves from lower elevation to higher elevation from $Martix[1][R]$ to $Martix[c][r]$ ".

Let $Opt[c][r]$ be the total moves from lower elevation to higher elevation along the path from $Martix[1][R]$ to $Martix[c][r]$, which is the optimal solution to the subproblem $P[c][r]$.

## Build-up order

As the question mentioned we can only move to the strictly right square or strictly down square, so for the first column and first row, we should solve these subproblems first and then do recursion.

For the first column $Martix[1][r]$ ($r$ from $R-1$ to 1), can be only derived by the previous square on the left. And same to the first row $Opt[c][R]$ ($c$ from 2 to $C$), can be only derived by the previous square on the up.

So solve the subproblems in the order $P[1][R], P[1][R-1], \ldots, P[1][1], P[2][R]P[3][R] \ldots P[C][R]$ and then $P[2][R-1], \ldots, P[2][1], P[3][R-1], \ldots, P[3][1], \ldots, P[C][R-1], \ldots, P[C][1]$. From left to right, up to bottom.

## Base case

Because the start square is $Martix[1][R]$, so we set $Opt[1][R] = 0$. So for the first column $Opt[1][r]$ ($r$ from $R-1$ to 1), can be only derived by the previous square on the left, so

$$Opt[1][r] = \begin{cases} Opt[1][r+1] + 0 & Martix[1][r+1] \geq Martix[1][r] \\ Opt[1][r+1] + 1 & Martix[1][r+1] < Martix[1][r] \end{cases}$$

The objective is to find the minimum moves from lower elevation to higher elevation, so if $Martix[1][r+1] > Martix[1][r]$, which means from high elevation to lower elevation, so $Opt[1][r]$ do not need to add one, otherwise $Opt[1][r]$ need to add one. And same to the first row $Opt[c][R]$ ($c$ from 2 to $C$),

$$Opt[c][R] = \begin{cases} Opt[c-1][R] + 0 & Martix[c-1][R] \geq Martix[c][R] \\ Opt[c-1][R] + 1 & Martix[c-1][R] < Martix[c][R] \end{cases}$$

## Recursion

Now we do recursion, in order to make it clear, we denote

$$E(p1, p2) = \begin{cases} 0 & Martix(p1) \geq Martix(p2) \\ 1 & Martix(p1) < Martix(p2) \end{cases}$$

$p1$ and $p2$ are two points, and format is $p = [c, r]$.

For $r$ from $R-1$ to 1, and $c$ from 2 to $C$, we have

$$Opt[c][r] = min\{Opt[c-1][r] + E([c-1,r],[c,r]), Opt[c][r+1] + E([c,r+1],[c,r])\}$$

## Final solution

The final solution is given by $Opt[c][1]$.

## Time complexity

The complexity is $O(C * R)$ because we only traverse the entire matrix elements once.