

**Q4****(a)**

Denote  $A = \langle 1, \underbrace{0, 0, \dots, 0}_k, 1 \rangle$ , the corresponding polynomial is  $P_A(x) = 1 + x^{k+1}$ . So the question can be thought as compute the convolution  $P_A(x) * P_A(x)$ . Thus, we simply multiply them because the form of two polynomials are easily to compute:

$$P_A(x)P_A(x) = 1 + 2x^{k+1} + x^{2k+2}$$

So the convolution of these two sequences is

$$(1, \underbrace{0, 0, \dots, 0}_k, 2, \underbrace{0, 0, \dots, 0}_k, 1)$$

**(b)**

As the above question mentioned, this sequence produces polynomial  $P_A(x) = 1 + x^{k+1}$ . Consequently, the DFT is equal to

$$\begin{aligned} DFT(A) &= \langle 1 + \omega_{k+2}^{0 \cdot (k+1)}, 1 + \omega_{k+2}^{1 \cdot (k+1)}, 1 + \omega_{k+2}^{2 \cdot (k+1)}, \dots, 1 + \omega_{k+2}^{(k+1) \cdot (k+1)} \rangle \\ &= \langle 2, 1 + \omega_{k+2}^{k+1}, 1 + \omega_{k+2}^{2(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)(k+1)} \rangle \end{aligned}$$