Q4

(a)

Denote $A =< 1, \underbrace{0, 0, \ldots, 0}_{k}, 1 >$, the corresponding polynomial is $P_A(x) = 1 + x^{k+1}$. So the question can be thought as compute the convolution $P_A(x) * P_A(x)$. Thus, we simply multiply them because the form of two polynomials are easily to compute:

$P_A(x)P_A(x) = 1 + 2x^{k+1} + x^{2k+2}$

So the convolution of these two sequences is

$(1, \underbrace{0, 0, \ldots, 0}_{k}, 2, \underbrace{0, 0, \ldots, 0}_{k}, 1)$

(b)

As the above question mentioned, this sequence produces polynomial $P_A(x) = 1 + x^{k+1}$. Consequently, the DFT is equal to

$$\begin{aligned} DFT(A) &=< 1 + \omega_{k+2}^{0*(k+1)}, 1 + \omega_{k+2}^{k+1}, 1 + \omega_{k+2}^{2(k+1)}, \ldots, 1 + \omega_{k+2}^{(k+1)(k+1)} > \\ &=< 2, 1 + \omega_{k+2}^{k+1}, 1 + \omega_{k+2}^{2(k+1)}, \ldots, 1 + \omega_{k+2}^{(k+1)(k+1)} > \end{aligned}$$