

Assignment 4 - Dynamic Programming
The number of points reflects the difficulty of problems

1. Some people think that the bigger an elephant is, the smarter it is. To disprove this you want to analyze a collection of elephants and place as large a subset of elephants as possible into a sequence whose weights are increasing but their IQs are decreasing. Design an algorithm which given the weights and IQs of n elephants, will find a longest sequence of elephants such that their weights are increasing but IQs are decreasing. (10pts)
2. You have to cut a wood stick into several pieces at the marks on the stick. The most affordable company, Analog Cutting Machinery (ACM), charges money according to the length of the stick being cut. Their cutting saw allows them to make only one cut at a time. It is easy to see that different cutting orders can lead to different prices. For example, consider a stick of length 10 m that has to be cut at 2, 4, and 7 m from one end. There are several choices. One can cut first at 2, then at 4, then at 7. This leads to a price of $10 + 8 + 6 = 24$ because the first stick was of 10 m, the resulting stick of 8 m, and the last one of 6 m. Another choice could cut at 4, then at 2, then at 7. This would lead to a price of $10 + 4 + 6 = 20$, which is better for us. Your boss demands that you design an algorithm to find the minimum possible cutting cost for any given stick. (10pts)
3. We say that a sequence of Roman letters A occurs as a subsequence of a sequence of Roman letters B if we can obtain A by deleting some of the symbols of B . Design an algorithm which for every two sequences A and B gives the number of different occurrences of A in B , i.e., the number of ways one can delete some of the symbols of B to get A . For example, the sequence `ba` has three occurrences in the sequence `baba`: **b**a**a**, **b**a**a**, **b**a**a**. (15pts)
4. A company is organising a party for its employees. The organisers of the party want it to be a fun party, and so have assigned a fun rating to every employee. The employees are organised into a strict hierarchy, i.e. a tree rooted at the president. There is one restriction, though, on the guest list to the party: an employee and their immediate supervisor (parent in the tree) cannot both attend the party (because that would be no fun at all). Give an algorithm that makes a guest list for the party that maximises the sum of the fun ratings of the guests. (20pts)
5. For bit strings $X = x_1 \dots x_m$, $Y = y_1 \dots y_n$ and $Z = z_1 \dots z_{m+n}$ we say that Z is an interleaving of X and Y if it can be obtained by interleaving the bits in X and Y in a way that maintains the left-to-right order of the bits in X and Y . For example if

$X = 101$ and $Y = 01$ then $x_1x_2y_1x_3y_2 = 10011$ is an interleaving of X and Y , whereas 11010 is not. Give an efficient algorithm to determine if Z is an interleaving of X and Y . (20pts)

6. You are given n turtles, and for each turtle you are given its weight and its strength. The strength of a turtle is the maximal weight you can put on it without cracking its shell. Find the largest possible number of turtles which you can stack one on top of the other without cracking any turtles. (25pts)

Hint: for each turtle consider the sum of its weight and its strength.