ZID: z5230310 Name: Tian Liu Date: 01/08/2020

## $\mathbf{Q}\mathbf{1}$

As the question mentioned, we first construct a bipartite graph with all the cities as vertices on the left side and all cities as vertices on the right side. Cities on the left side represent populations of the corresponding cities, cities on the right side represent the set of pods in the corresponding cities.

If there exists a way from city  $C_i$  on the left side to city  $C_j$  in cities on the right side that cost less than X days (i.e.  $t(i,j) \leq X$ ), whatever a direct road from  $C_i$  to  $C_j$  or a sequence of intermediate cities connected by direct roads from  $C_i$  to  $C_j$ , connect  $C_i$  to  $C_j$  as one direction edge with infinite capacity. Thus, we add a super source S and a super sink T and connect S with all the cities  $C_i$  in cities on the left side with edges of inhabitant and with capacity of the population. Also, connect all cities on the right side with T as edges with capacity of the number of pods.

We now run the Edmons-Karp algorithm to find the maximal flow through such a network, which is the largest number of invaders the Earth will have to deal with.