Problems for all classes (3121/3821/9101/9801)

1. A table composed of **N x M** cells, each cell having a certain quantity of apples, is given. You start from the upper-left corner. At each step you can go down or right one cell. Find the maximum number of apples you can collect.

2. You are traveling by a canoe down a river and there are n trading posts along the way. Before starting your journey, you are given for each $1 \leq i < j \leq n$ the fee $F(i, j)$ for renting a canoe from post i to post j. These fees are arbitrary. For example it is possible that $F(1, 3) = 10$ and $F(1, 4) = 5$. You begin at trading post 1 and must end at trading post n (using rented canoes). Your goal is to minimize the rental cost. Give the most efficient algorithm you can for this problem.

3. You have n_1 items of size s_1, n_2 items of size s_2 and n_3 items of size s_3 . You must pack all of these items into bins, each of capacity C, such that the total number of bins used is minimized. Design a polynomial time algorithm for such packaging.

4. Assume each student can borrow at most 10 books from the library, and the library has three copies of each title in its inventory. Each student submits a list of books he wishes to borrow. You have to assign books to students, so that a maximal number of volumes is checked out.

5. (a) Design an algorithm which finds a minimal cut in a flow network.
   (b) Can there be more than one minimal cut in a flow network? If so, give an example of a flow network with more than one min cut.

Additional problem for extended classes only (COMP3821 and COMP9801)

6. You are given an ordered sequence of n cities, and the distances between every pair of cities. You must partition the cities into two subsequences (not necessarily contiguous) such that person A visits all cities in the first subsequence (in order), person B visits all cities in the second subsequence (in order), and such that the sum of the total distances travelled by A and B is minimized. Assume that person A and person B start initially at the first city in their respective subsequences.