Q5

According to question, we know that each day chemicals will evaporate p percent of the amount you had at the end of previous day. So first we can determine how much of a chemical is left after k many days. We have N chemicals to produce so we need N days.

If we consider case that producing chemical in order. And according to my own understanding, I **assume the delivery is complete at the end of No.N day, and the last chemical produced doesn't evaporate but the previous produced chemicals will also evaporate**, so the total losing weight is

$$total = (W_1 - W_1(1-p\%)^n) + (W_2 - W_2(1-p\%)^{n-1}) + \cdots + (W_n - W_n(1-p\%)^0)$$

Consider $y = (1-p\%)^n$, when n decrease, the y will increase, and $1-(1-p\%)^n$ will decrease, so $lose = W(1-(1-p\%)^n) = W - W(1-p\%)^n$ will be decrease when n decrease which means one chemical will lose less if it be produced later. But according to question, chemicals have their own weight, what we should do is to arrange that those high-demand chemicals should be produced later and low-demand should be produced first.

Thus, sort chemicals in increasing-order of weight, and produce Ci in that order can minimise the total extra weight of all chemicals needed to produce.

Optimality: Assume we already sort the chemicals in increasing order of weight and denote the total loss is:

$total = (W_1 - W_1(1-p\%)^n) + (W_2 - W_2(1-p\%)^{n-1}) + \cdots + (W_k - W_k(1-p\%)^{n-k+1}) + (W_{k+1} - W_{k+1}(1-p\%)^{n-k}) + \cdots + (W_n - W_n(1-p\%)^0)$

(W_1, W_2, ... , W_n is already ordered). And let us see what happens if we swap to adjacent chemicals C_k and C_{k+1}, the total loss become:

$total^{'} = (W_1 - W_1(1-p\%)^n) + (W_2 - W_2(1-p\%)^{n-1}) + \cdots + (W_{k+1} - W_{k+1}(1-p\%)^{n-k+1}) + (W_k - W_k(1-p\%)^{n-k}) + \cdots + (W_n - W_n(1-p\%)^0)$

Thus,

$$total - total^{'} = W_k(1-p\%)^{n-k} - W_k(1-p\%)^{n-k+1} + W_{k+1}(1-p\%)^{n-k+1} - W_{k+1}(1-p\%)^{n-k}$$

Denote $x = (1-p\%)$

$$\begin{aligned} total - total^{'} &= W_k x^{n-k}(1-x) + W_{k+1}x^{n-k}(x-1) \\ &= (1-x)(W_k - W_{k+1})x^{n-k} \end{aligned}$$

As $W_k < W_{k+1}$ and $1-p\% > 0$, so $total - total^{'} < 0$, so as long as there are two chemicals in decreasing order, means there are two chemicals produced in decreasing order and will produce more extra weight when producing. Thus the optimal solutions is the one that in increasing order of weight to produce.