

### Q3

According to this question, a train can arrive before the midnight and leave after midnight, so we first find how many trains followed this rule, those trains must be separate on different platforms.

Assume there are  $t$  trains that across midnight so the number of platforms must be greater or equal than  $t$ , which means  $total \geq t$ , now we sort these across midnight trains by departure time in increasing order. Denote list  $T$ .

For each trains in list  $T$ , we do a loop, we pick up in order and we can convert this problem to *Activity selection problem* talked on the lecture, we split the day as start at  $d_i$  and end at  $a_i$ . So the subquestion now is to find a maximum size subset of compatible trains. Note that trains here are non-across midnight trains. Like the answer of *Activity selection problem*, among the trains which do not conflict with the previously chosen trains always chose the one with the earliest end time. At the end of this loop, we delete those trains that picked in this loop and the rest trains as the next loop's non-across midnight trains.

After find all maximum size subset of trains for trains in list  $T$ , if there still have trains we continue do the above steps but the day has the normal start time and end time. Until there are no trains left. In this stage, when finishing one loop, means one more platform added. So finally we can find the number of platforms.

In this question, we followed the rules that find the maximum size subset of compatible trains and stay one platform, so when the total number of trains fixed, we can get the minimum number of platforms so that each train can stay without interfering with other trains.

**Optimality:** exactly like *Activity selection problem*, assume there is any different optimal solution, we find the first place where the chosen trains violates our algorithm choice. Show that replacing that trains with our method choice produces a non conflicting selection with the same number of trains. Continue in this manner till “morph” this optimal solution into our method solution, thus proving our method solution is also optimal.