

# COMP3121/3821/9101/9801 Final 2018

COMP3121/9101: Q1-4; COMP3821/9801: Q1-5

June 23, 2018

These are from memory, but should be right.

1. (a) {10 Marks} You are the owner of a shipping company and have just received  $N$  orders for products you have in stock. The  $i^{\text{th}}$  order requires  $t_i$  trucks, and gives you a profit of  $d_i$ . You have  $T$  trucks at your disposal. Design an efficient algorithm to choose a (not necessarily proper) subset of the orders, such that all chosen orders combined use no more than  $T$  trucks, and the total profit is maximised.
- (b) {15 Marks} You have just recently employed some people, and have once again received  $N$  orders. The  $i^{\text{th}}$  order requires  $t_i$  many trucks and  $e_i$  many employees, and gives you a profit of  $d_i$ . You have  $T$  trucks and  $E$  employees at your disposal. Design an efficient algorithm to choose a (not necessarily proper) subset of the orders, such that all chosen orders combined use no more than  $T$  trucks and  $E$  employees, and the total profit is maximised.
2. {25 Marks} You are given an array of  $n$  many integers,  $A[1 \dots n]$ , along with infinitely many tiles of each of the following types:



You need to cover the array  $A$  with these tiles. Every element in the array must be covered by exactly one tile. No tiles can “hang” off the end of the array. A tile may only cover consecutive elements in the array (two for  $W$  and  $X$ , and three for  $Y$  and  $Z$ ). Each covering of the array is assigned a score. This score is the sum of the elements in the array covered by white tiles. Design an efficient algorithm that finds the maximum score possible.

3. {20 Marks} You are the king of a country that has  $n$  cities. You are in city 1. You know that there are  $k < n$  many spies, and that the spies reside in cities  $c_1, c_2, \dots, c_k$ , but none are in the same city as you. There are one-way roads connecting cities (not necessarily between every city). Design an efficient algorithm that finds the minimum number of roads you must close to ensure that the enemy spies cannot reach you.
4. {30 Marks} You are given an  $n \times n$  array of pixels. Each pixel is either black or white. You are given a list of  $m \leq n^2$  pixels, telling you the colour of those specific pixels, but the rest of the image seems to have been corrupted. You also know the total number of black pixels in every row, and in every column. Design an efficient algorithm that creates an image (that is, fills in the  $n \times n$  array), such that the number of black pixels in each row and column are the same as those given, and the  $m$  pixels you know are also the same.
5. {20 Marks} You have been asked to create an algorithm called `rcat` that prints a single line from a file at random, with the probability that each line is printed being the same. You are given a function `rand`, where `rand(x)` returns a random number between 1 and  $x$  inclusive, with equal probability. Each line is at most 100 characters long. You do not know how many lines are in the file. You can only read each line once, and in the order they are in the file. You can only read the file once. You can only use  $O(1)$  memory (so you can’t store all the lines, nor any non-constant number of them). Design such an algorithm, and argue that your algorithm prints any line in the file with equal probability.