

## Q2

According to the question, we already know that we know the exactly  $R_a$ ,  $P_a$  and  $S_a$  number and  $R_a + P_a + S_a = N$ , same as  $R_b$ ,  $P_b$  and  $S_b$  also  $R_b + P_b + S_b = N$ , however, we don't need to throw in the consecutive order.

The question is how should we play to maximise the number of points, which can be convert to that to minimise the loss round, as the total round is  $N$ , so win round plus loss round plus tie round is equal to  $N$ . Minimise loss round is equal to maximise the win round plus tie round.

Assume we have two arrays, array A with  $[R_a, P_a, S_a]$  and array B with  $[R_b, P_b, S_b]$ , so the maximum win round is  $win\_round = \min(R_a, P_b) + \min(P_a, S_b) + \min(S_a, R_b)$ , Denote  $w_0 = \min(R_a, P_b)$ ,  $w_1 = \min(P_a, S_b)$  and  $w_2 = \min(S_a, R_b)$ , we throw Paper, Scissor and Rock in  $w_0$ ,  $w_1$  and  $w_2$  rounds respectively, after that we can derive new array  $A_1$  with  $[R_a - w_0, P_a - w_1, S_a - w_2] = [R'_a, P'_a, S'_a]$  and new array  $B_1$  with  $[R_b - w_2, P_b - w_0, S_b - w_1] = [R'_b, P'_b, S'_b]$ , then we can compute the maximum tie rounds is  $tie\_round = \min(R'_a, R'_b) + \min(P'_a, P'_b) + \min(S'_a, S'_b)$  and same as above, for number of array  $A_1$  and  $B_1$ , minus the minimum value at the same index, we finally get new array  $A_2$   $[R''_a, P''_a, S''_a]$  and new array  $B_2$   $[R''_b, P''_b, S''_b]$ , it is clearly that some of those values are 0. Finally the rest of number of array  $A_2$  and  $B_2$ , no matter how to throw are all loss so the loss round which is  $loss\_round = \sum(A_2) = \sum(B_2)$  so the total score that by doing this method is  $win\_round - loss\_round$  which is also the maximum score.

**Optimality:** assume we have another optimal solution with the same scores but not the same maximum rounds of  $win\_round$  plus  $tie\_round$  which is less than our algorithm. If so, this solution must have more loss rounds as the  $win\_round + tie\_round + loss\_round = N$ , in this situation, the scores must less than our's method, which is a contradictory. So my algorithm shows above can give the optimal solution.