

Q5

The result sequence of $x * \langle 1, 1, -1 \rangle$ (denote P_B) is $\langle 1, 0, -1, 2, -1 \rangle$, and the corresponding polynomial is $P_C(x) = 1 - x^2 + 2x^3 - x^4$.

First, the resulting polynomial $P_C(x)$ length $(n-1) + (m-1) + 1 = 5$ as $m = 3$, so n should be 3.

Now, assume x has the sequence like $\langle x_1, x_2, x_3 \rangle$ and the first coefficient of $P_C(x)$ is 1 which equals to $x_1 * 1$ (first coefficient of P_B), so $x_1 = 1$.

And, the first coefficient of $P_C(x)$ is 0 which equals to x_1 multiply the second coefficient of P_B plus x_2 multiply the first coefficient of P_B which is $1 * 1 + x_2 * 1 = 0$ so, $x_2 = -1$.

Last, the fifth coefficient of $P_C(x)$ equals to x_3 multiply the third coefficient of P_B is -1 which is $x_3 * -1 = -1$, so $x_3 = 1$.

Actually, I use part of convolution coefficients formula to solve this problem, also we can use convolution to prove that sequence s is correct.

Thus, the sequence $\langle 1, -1, 1 \rangle$ can be satisfied.