Q5

(a)

First, we can get $g(n) = 2\log_2^{(n^{\log_2^n})} = 2\log_2^n * \log_2^n = 2(\log_2^n)^2$
Second, taking $c = 1$: $0 \leq (\log_2^n)^2 \leq 2(\log_2^n)^2$ for all $n \geq 1$. Which is $f(n) = O(g(n))$
Also, taking $c = \frac{1}{2}$, $c = 1$: $0 \leq c2(\log_2^n)^2 \leq (\log_2^n)^2$ for all $n \geq 1$. Which is $f(n) = \Omega(g(n))$
Therefore, $f(n) = \theta(g(n))$

(b)

$f(n) = n^{10}$ $\qquad g(n) = 2^{\sqrt[10]{n}}$
We want to show that $f(n) = O(g(n))$, which means that we have to show that $f(n) \leq cg(n)$ for some positive c and all sufficiently large n. But, since the log function is monotonically increasing, this will hold just in case

$$10\log n \leq \log c + \sqrt[10]{n}$$

We now taking $c = 1$ then it is enough to show that

$$\frac{10\log n}{\sqrt[10]{n}} \leq 1$$

for sufficiently large n. To this end we use the L'Hoˆpital's to compute the limit

$$\lim_{n\to\infty} \frac{10\log n}{\sqrt[10]{n}} = \lim_{n\to\infty} \frac{(10\log n)^{'}}{(\sqrt[10]{n})^{'}} = \frac{100}{\ln 2} \lim_{n\to\infty} \frac{1}{\sqrt[10]{n}} = 0$$

Since $\lim\limits_{n\to\infty} \frac{10\log n}{\sqrt[10]{n}} = 0$ then, for sufficiently large n we will have $\frac{10\log n}{\sqrt[10]{n}} \leq 1$.
So, $f(n) = O(g(n))$.

(c)

Just note that $1 + (-1)^n$ cycles with one period equal to $\{2, 0\}$. Thus, for all even numbers for n we have $1+(-1)^n = 2$ and for all odd numbers for n we have $1+(-1)^n = 0$. Thus for any fixed constant $c > 0$ for all even numbers n eventually $n^{1+(-1)^n} = n^2 > n$ and for all odd numbers n eventually $n^{1+(-1)^n} = 1 < c * n$ when $n > 0$.
Thus, neither $f(n) = O(g(n))$ nor $f(n) = \Omega(g(n))$.