1. You have to cut a wood stick into several pieces. The most affordable company, Analog Cutting Machinery (ACM), charges money according to the length of the stick being cut. Their cutting saw allows them to make only one cut at a time.
   It is easy to see that different cutting orders can lead to different prices. For example, consider a stick of length 10 m that has to be cut at 2, 4, and 7 m from one end. There are several choices. One can cut first at 2, then at 4, then at 7. This leads to a price of 10 + 8 + 6 = 24 because the first stick was of 10 m, the resulting stick of 8 m, and the last one of 6 m. Another choice could cut at 4, then at 2, then at 7. This would lead to a price of 10 + 4 + 6 = 20, which is better for us.
   Your boss demands that you design an algorithm to find the minimum possible cutting cost for any given stick.

2. For bit strings $X = x_1 \ldots x_m$, $Y = y_1 \ldots y_n$ and $Z = z_1 \ldots z_{m+n}$ we say that Z is an interleaving of X and Y if it can be obtained by interleaving the bits in X and Y in a way that maintains the left-to-right order of the bits in X and Y. For example if X = 101 and Y = 01 then $x_1x_2y_1x_3y_2 = 10011$ is an interleaving of X and Y, whereas 11010 is not. Give the most efficient algorithm you can to determine if Z is an interleaving of X and Y.

3. You are given *n* points on the *x*-axis with coordinates $x_1, x_2, \ldots, x_n$. Describe an algorithm for covering them all with *k* intervals of arbitrary length, so that the total length of all intervals is minimal.

4. A lecturer has offered a list of topics $T_1, T_2, \ldots T_n$ to a group of students $S_1, S_2, \ldots S_m$. Each student submits a list of topics among those listed which he likes. Design an algorithm which determines if it is possible to assign a topic to each student so that each student gets a topic he likes and each topic is assigned to at most one student, and if such assignment exists it returns one such assignment.

5. Assume you have two algorithms A1 and A2 for the same task. For some inputs algorithm A1 runs 100 times faster than algorithm A2, and for some other inputs algorithm A2 runs 100 times faster than A1. Design an algorithm which runs for ALL inputs at most about twice slower than the faster of the two algorithms. Note that for each input you DO NOT know in advance how long it will take for either of the algorithms to execute on that input.

ADDITIONAL PROBLEM FOR THE EXTENDED CLASSES 3821 & 9801 ONLY:

6. Two children love cake and mathematics. For this reason, Jeremy convinces Marie to play the following game on two identical rectangular cakes. Jeremy will cut the first cake into two pieces, perhaps evenly, perhaps not. After seeing the cut, Marie will decide whether she will choose first or allow Jeremy to do so. If she goes first, she will take the larger piece. If she goes second, she can assume that Jeremy will take the larger piece. Next, Jeremy will cut the second cake into two pieces (remember that one of the pieces can be vanishingly small if he so chooses). If Marie had chosen first for the first cake, then Jeremy gets to take the larger piece of the second cake. If Marie had chosen second for the first cake, then she gets to take the larger piece of the second cake. Assuming each child will strive to get the most total cake possible, what is an optimal strategy for Jeremy?