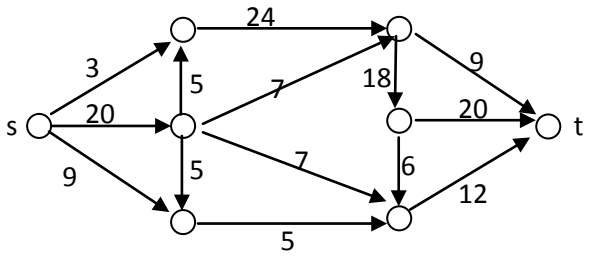
COMP 3121/3821/9101 FINAL

Semester 1 2008

1. You are given an ordered sequence of n cities, and the distances between every pair of cities. You must partition the cities into two subsequences (not necessarily contiguous) such that person A visits all cities in the first subsequence (in order), person B visits all cities in the second subsequence (in order), and such that the sum of the total distances travelled by A and B is minimized. Assume that person A and person B start initially at the first city in their respective subsequences. Design a polynomial time algorithm for producing such a partition.

2. You have n_1 items of size s_1 and n_2 items of size s_2. You must pack all of these items into bins, each of capacity C, such that the total number of bins used is minimized. Design a polynomial time algorithm for such packaging.

3. (a) Find a minimal cut in the network flow given below:



(b) Explain in detail why in a network the maximal flow from the source s to the sink t produced by adding augmenting paths using the Ford Fulkerson method is independent of the choice of augmenting paths.

4. The emergency services are responding to a major earthquake that has hit a wide region, and left *n* people injured who need to be sent to a hospital. Let *P* be the set of *n* people and *H* be the set of *k* hospitals. Several hospitals are available to treat these people, but there are some constraints:

   (a) Each injured person needs to be sent to a hospital no further than one hour drive away. Let H_p be the set of hospitals that are within range for person *p*.

   (b) Each hospital *h* has a capacity c_h, the maximum number of people that the hospital can receive.

   Develop a polynomial time algorithm that determines whether it is possible to assign each person to a hospital in a way that satisfies these constraints, and returns such an assignment if so.

- **Additional problem for extended classes 3821/9801 <u>ONLY.</u>**

5. Assume you are given a table *n* by *n* containing integers. At every step you can flip the sign of all numbers in a single column or a single row. Design an algorithm that transforms this table into a table that has the property that the sum of numbers in every row and every column is non-negative.

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