COMP3121/3821/9101/9801 final practice problems

1. You have n_1 items of size s_1 and n_2 items of size s_2. You must pack all of these items into bins, each of capacity C, such that the total number of bins used is minimised. Design a polynomial time algorithm for such packaging.

2. Given a sequence of n real numbers $A_1, \ldots, A_n$, determine a contiguous subsequence $A_i, A_{i+1}, \ldots, A_j$ for which the sum of elements in the subsequence is maximised.

3. You are given a set of n types of rectangular 3-D boxes, where the ith box has height h_i, width w_i and depth d_i (all real numbers). You want to create a stack of boxes which is as tall as possible, but you can only stack a box on top of another box if the dimensions of the 2-D base of the lower box are each strictly larger than those of the 2-D base of the higher box. Of course, you can rotate a box so that any side functions as its base. It is also allowable to use multiple instances of the same type of box.

4. Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with $x-$coordinates $a_1, \ldots, a_n$ and n cities on the northern bank with $x-$coordinates $b_1, \ldots, b_n$. You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you are only allowed to connect the the ith city on the northern bank to the ith city on the southern bank.

5. You are given a Boolean expression consisting of a string of the symbols true, false, and, or, and xor. Count the number of ways to place parentheses in the expression such that it will evaluate to true.

6. Consider a row of n coins of values $v(1), \ldots, v(n)$, where n is even. We play a game against an opponent by alternating turns. In each turn, a player selects either the first or last coin from the row, removes it from the row permanently, and receives the value of the coin. Determine the maximum possible amount of money we can definitely win if we move first.

7. You are given an ordered sequence of n cities, and the distances between every pair of cities. You must partition the cities into two subsequences (not necessarily contiguous) such that person A visits all cities in the first subsequence (in order), person B visits all cities in the second subsequence (in order), and such that the sum of the total distances travelled by A and B is minimised. Assume that person A and person B start initially at the first city in their respective subsequences. Design a polynomial time algorithm for producing such a partition.

8. You are given a tree which is a source, capacities of all edges of the tree and all leaves acting as sinks. Find a fast algorithm for finding the max flow in such a flow network.

9. Use the max flow algorithm to solve the following problem. You are given n families; family i has $n(i)$ many members, $1 \leq i \leq n$. You are also given k tables, such that table j can seat $m(j)$ people. You have to assign to everyone their seats so that no two members of the same family sit at the same table, or return "impossible" if this cannot be done.

10. You are given a flow network where not only edges have capacities, but also each node has a capacity of flow that can go through the node. Design an algorithm which finds a maximal flow through such a network.

11. Assume that you are the administrator of a network of computers; each computer is connected by unidirectional fiberoptic cables of the same capacity to a few other computers on the same network (so the network can be modelled by a directed graph). You noticed that computers $P_1, P_2, ...P_n$ are mounting an attack on computers $Q_1, Q_2, ..., Q_m$. The total number of computers on the network is $N > m + n$. Since it is a real emergency, you must disconnect some of the optical cables of the network or entirely remove some of the computers so that none of computers $P_1, P_2, ...P_n$ can send packets to any of $Q_1, Q_2, ..., Q_m$. Since you must send crews to disconnect some of the fiberoptic cables or to take away some of the computers, for each cable c_{ij} for traffic from a computer X_i to a computer X_j there is an associated cost c_{ij} for disconnecting it and for each computer X_i there is a cost C_i of removing it. Your task is to design an algorithm for determining which cables to disconnect and which computers to remove to isolate computers $Q_1, Q_2, ..., Q_m$ from all of the computers $P_1, ..., P_n$ so that the total cost incurred is minimal.

12. When solving Max Flow problems, it is convenient to assume that the flow graph does not contain anti-parallel edges, i.e., if a directed edge $(i, j) \in E$ then $(j, i) \notin E$. However, many practical problems involve undirected graphs, which can be seen as directed graphs with pairs of antiparallel edges. Show how a flow network with antiparallel edges can be transformed into a network without antiparallel edges.

13. (Escape Problem) This is a problem which is faced by people who design printed circuit boards. An $n \times n$ grid is an undirected graph consisting of n rows, each row containing n vertices, with vertices connected to all of their immediate neighbours (2 at all of the 4 corners, 3 at all of the 4 sides and 4 in the interior of the grid. The escape problem is, given $m \leq n^2$ many vertices in the gid, connect them with m many distinct vertices located on the 4 sides by non intersecting paths or return "impossible" when there is no such a solution.

14. (very tricky - for extended classes only) You are given a directed acyclic graph. A path cover of the graph is a collection of paths so that every vertex belongs to exactly one path; thus the paths do not intersect. The paths can be of any length including 0 (a single vertex). Find a path cover which contains a minimal number of paths.