H10

Q1 Maximize: $3X_1 + 4X_2 + X_3 + 7X_4$

Subject to: $8X_1 + 3X_2 + 4X_3 + X_4 \leq 7$

$2X_1 + 6X_2 + X_3 + 5X_4 \leq 3$

$X_1 + 4X_2 + 5X_3 + 2X_4 \leq 8$

$X_1, X_2, X_3, X_4 \geq 0$

Maximize Z

$Z = 3X_1 + 4X_2 + X_3 + 7X_4 + 0S_1 + 0S_2 + 0S_3$

Subject to

$8X_1 + 3X_2 + 4X_3 + X_4 + S_1 = 7$

$2X_1 + 6X_2 + X_3 + 5X_4 + S_2 = 3$

$X_1 + 4X_2 + 5X_3 + 2X_4 + S_3 = 8$

$X_1, X_2, X_3, X_4, S_1, S_2 \geq 0$

	Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	RHS	
	1	3	4	1	7	0	0	0	0	
S_1	0	8	3	4	1	1	0	0	7	$\frac{7}{1} = 7$
S_2	0	2	6	1	5	0	1	0	3	$\frac{3}{5} = 0.6$ →
S_3	0	1	4	5	2	0	0	1	8	$\frac{8}{2} = 4$
		-3	-4	-1	-7 ↑	0	0	0	$Z_j - C_j$	

Negative Minimum $Z_j - C_j$ is -7, column index is 4, entering variable X_4

Minimum Ratio is 0.6, row index 2, leaving basis variable is S_2.

Pivot element is 5.

B	Z	RHS	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum Ratio
S_1	0	$\frac{32}{5}$	$\frac{38}{5}$	$\frac{9}{5}$	$\frac{19}{5}$	0	1	$-\frac{1}{5}$	0	$\frac{32}{5} / \frac{38}{5} = 0.8421$ →
X_4	7	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{6}{5}$	$\frac{1}{5}$	1	0	$\frac{1}{5}$	0	$\frac{3}{5} / \frac{2}{5} = 1.5$
S_3	0	$\frac{34}{5}$	$\frac{1}{5}$	$\frac{8}{5}$	$\frac{23}{5}$	0	0	$-\frac{2}{5}$	1	$\frac{34}{5} / \frac{1}{5} = 34$
$Z = \frac{21}{5}$		Z_j	$\frac{14}{5}$	$\frac{42}{5}$	$\frac{7}{5}$	7	0	$\frac{7}{5}$	0	
		$Z_j - C_j$	$-\frac{1}{5}$ ↑	$\frac{22}{5}$	$\frac{2}{5}$	0	0	$\frac{7}{5}$	0	

Negative Minimum $Z_j - C_j$ is $-\frac{1}{5}$, column index 1, entering variable is X_1

Minimum Ratio is 0.8421, row index 1, entering variable is S_1,
Pivot element is $\frac{38}{5}$

B	Z	RHS	X_1	X_2	X_3	X_4	S_1	S_2	S_3
X_1	3	$\frac{16}{19}$	1	$\frac{9}{38}$	$\frac{1}{2}$	0	$\frac{5}{38}$	$-\frac{1}{38}$	0
X_4	1	$\frac{5}{19}$	0	$\frac{21}{9}$	0	1	$-\frac{1}{19}$	$\frac{4}{19}$	0
S_3	0	$\frac{126}{19}$	0	$\frac{59}{38}$	$\frac{9}{2}$	0	$-\frac{1}{38}$	$-\frac{15}{38}$	1
$Z=\frac{81}{19}$		Z_j	3	$\frac{321}{38}$	$\frac{3}{2}$	7	$\frac{1}{38}$	$\frac{53}{38}$	0
		Z_j-C_j	0	$\frac{169}{38}$	$\frac{1}{2}$	0	$\frac{1}{38}$	$\frac{53}{38}$	0

All $Z_j - C_j \geq 0$

$\therefore X_1 = \frac{16}{19}$

$X_2 = 0$

$X_3 = 0$

$X_4 = \frac{5}{19}$

Max $Z = \frac{83}{19}$

Q2

	Model A	Model B	Model C	Time
Assembly	2	2.5	3	6004
Painting	1.5	2	1.5	2695
Packaging	1	0.75	1.25	1500
Profit	45	60	55	

The total time taken for assembly model A, B and C must not exceed the total time duration of 6004 hours

$2A + 2.5B + 3C \leq 4006$ eq1

Painting.

$1.5A + 2B + 1.5C \leq 2695$ eq2

Packaging

$1A + 0.75B + 1.25C \leq 1500$ eq3

We get $A = 0$

$B = 813.6$

$C = 711.8$

Try Brute Force/ Trial and Error

Case 1: C = 710

B = 815

max = 87950 ✓ max

Case 2: C = 711

B = 814

max = 87945

Case 3: C = 712

B = 813

max = 87940

∴ Best solution is

A = 0
B = 815
C = 710

Iteration-1		C_j	45	60	55	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_2}$
S_1	0	6004	2	2.5	3	1	0	0	$\frac{6004}{2.5} = 2401.6$
S_2	0	2695	1.5	(2)	1.5	0	1	0	$\frac{2695}{2} = 1347.5 \rightarrow$
S_3	0	1500	1	0.75	1.25	0	0	1	$\frac{1500}{0.75} = 2000$
$Z = 0$		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-45	-60 ↑	-55	0	0	0	

Negative minimum = -60, entering variable x2

Minimum ratio = 1347.5, leaving basis is s2

Pivot element is 2

Iteration-2		C_j	45	60	55	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_3}$
S_1	0	2635.25	0.125	0	1.125	1	-1.25	0	$\frac{2635.25}{1.125} = 2342.4444$
x_2	60	1347.5	0.75	1	0.75	0	0.5	0	$\frac{1347.5}{0.75} = 1796.6667$
S_3	0	489.375	0.4375	0	(0.6875)	0	-0.375	1	$\frac{489.375}{0.6875} = 711.8182 \rightarrow$
$Z = 80850$		Z_j	45	60	45	0	30	0	
		$Z_j - C_j$	0	0	-10 ↑	0	30	0	

Negative minimum = -10, entering variable x3

Minimum ratio = 711.8182, leaving basis is s3

Pivot element is 0.6875

Iteration-3		C_j	45	60	55	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio
S_1	0	1834.4545	-0.5909	0	0	1	-0.6364	-1.6364	
x_2	60	813.6364	0.2727	1	0	0	0.9091	-1.0909	
x_3	55	711.8182	0.6364	0	1	0	-0.5455	1.4545	
Z = 87968.1818		**Z_j**	**51.3636**	**60**	**55**	**0**	**24.5455**	**14.5455**	
		$Z_j - C_j$	6.3636	0	0	0	24.5455	14.5455	

Since zj-cj >=0,

We get the solution

X1 = 0

X2 = 813.6

X3 = 711.8