MARKOV CHAIN in COVID-19

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Background

◈ What is Markov Chain*?*

A stochastic process containing random variables, transitioning from one state to another depending on certain assumptions and definite probabilistic rules.

------- Andrey Markov

In this process, the outcome of a given experiment can affect the outcome of the next experiment. This type of process is called a Markov chain.

Mathematical Technique

We have a set of states, S = {s1, s2,...,sr}. The process starts in one of these states and moves successively from one state to another. Each move is called a step. If the chain is currently in state si, then it moves to state sj at the next step with a probability denoted by pij , and this probability does not depend upon which states the chain was in before the current state.

Transition Probabilities:

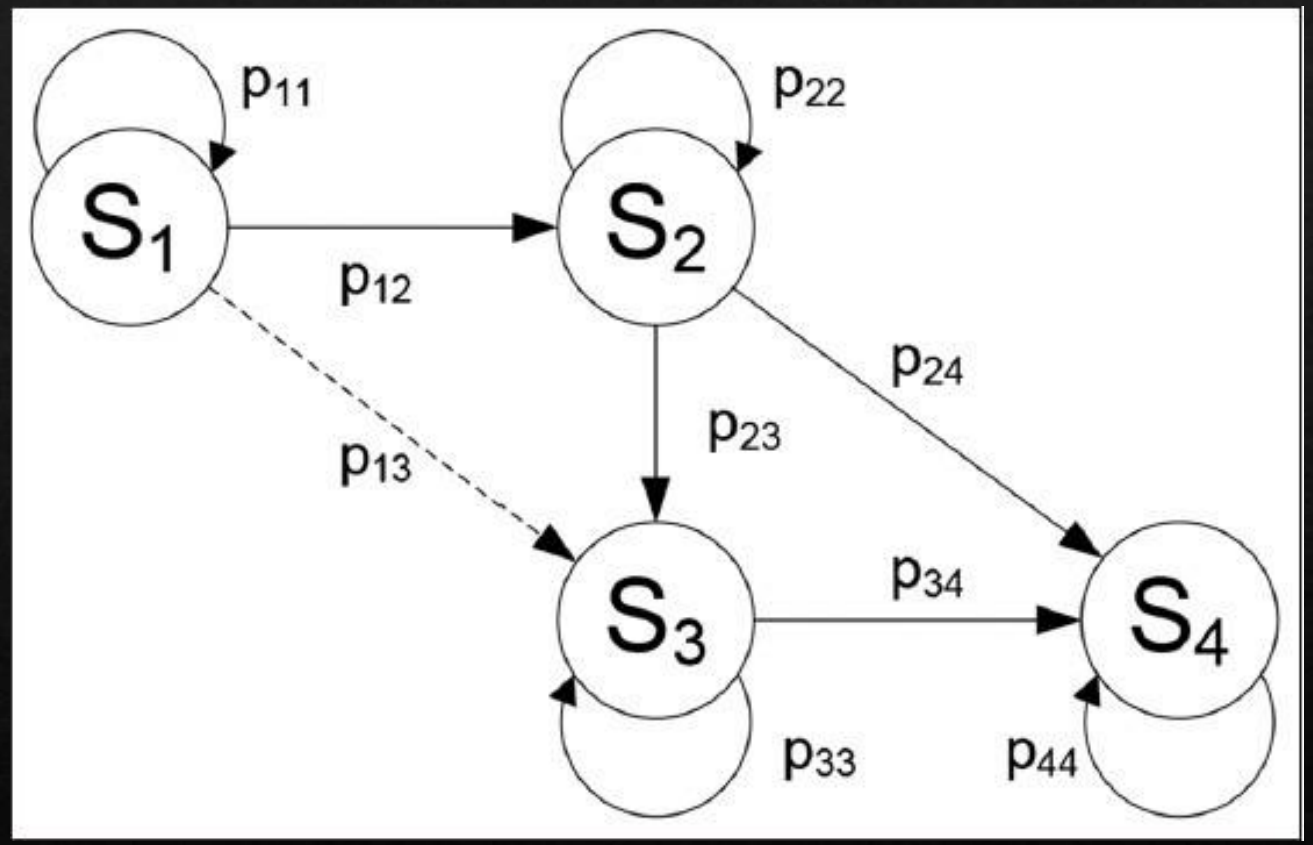
The probabilities pij are called transition probabilities. The process can remain in the state it is in, and this occurs with probability pii. An initial probability distribution, defined on S, specifies the starting state.

Mathematical Technique

- Markov chain with stationary transition probabilities, which is represented as: $\{X(t), t \in T\}$ with time index set $T = \{0,1,\ldots\}$ and a finite state space $S = \{S_1, S_1, \ldots, S_N\}$. This model satisfies the following the Markovian property; $P\{X(t)=S_i | X(t-2)=S_k, \ldots\}$ $= P\{X(t)=S_i | X(t-)=S_j\}(1)$ for all t and all possible states.
- This Markovian property means that the probability of the random variable $X(t)$ being in state S_i at time t depends only on the variable's state S_i at time $t - 1$, but not on states at previous points in time. In other words, it does not depend on how the system has led to the current state.

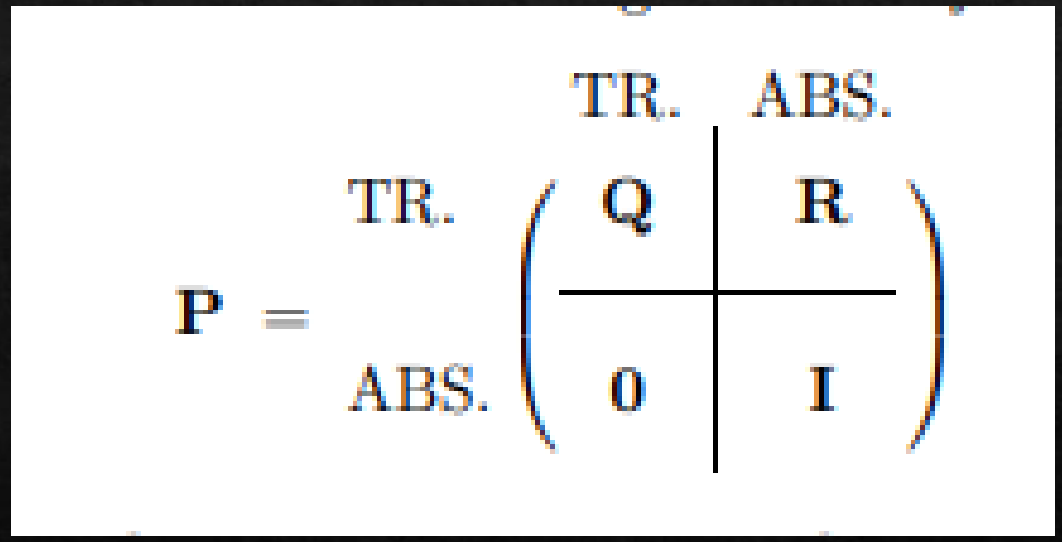
States

- Si will be each transition states, Pij will be probability from state i to state j.
- S4 we see no outlet arrows; we call it Absorbing State.



Canonical Form

- With Absorbing State, Canonical Form can help solve the problem.
- Renumber the states so that the transient states come first. If there are r absorbing states and t transient states, the transition matrix will have the following canonical form



I is an r-by-r identity matrix,
0 is an r-by-t zero matrix
R is a nonzero t-by-r matrix
Q is an t-by-t matrix

Example in Canonical Form

$$\mathbf{P} = \begin{array}{c} \\ \text{TR.} \\ \text{ABS.} \end{array} \begin{array}{c} \begin{array}{cc} \text{TR.} & \text{ABS.} \end{array} \\ \left(\begin{array}{c|c} \mathbf{Q} & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{I} \end{array}\right) \end{array}$$

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \hline 0 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc|cc} 1 & 2 & 3 & 0 & 4 \end{array} \\ \left(\begin{array}{ccc|cc} 0 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ \hline 1/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/2 & 0 & 1 \end{array}\right) \end{array}.$$

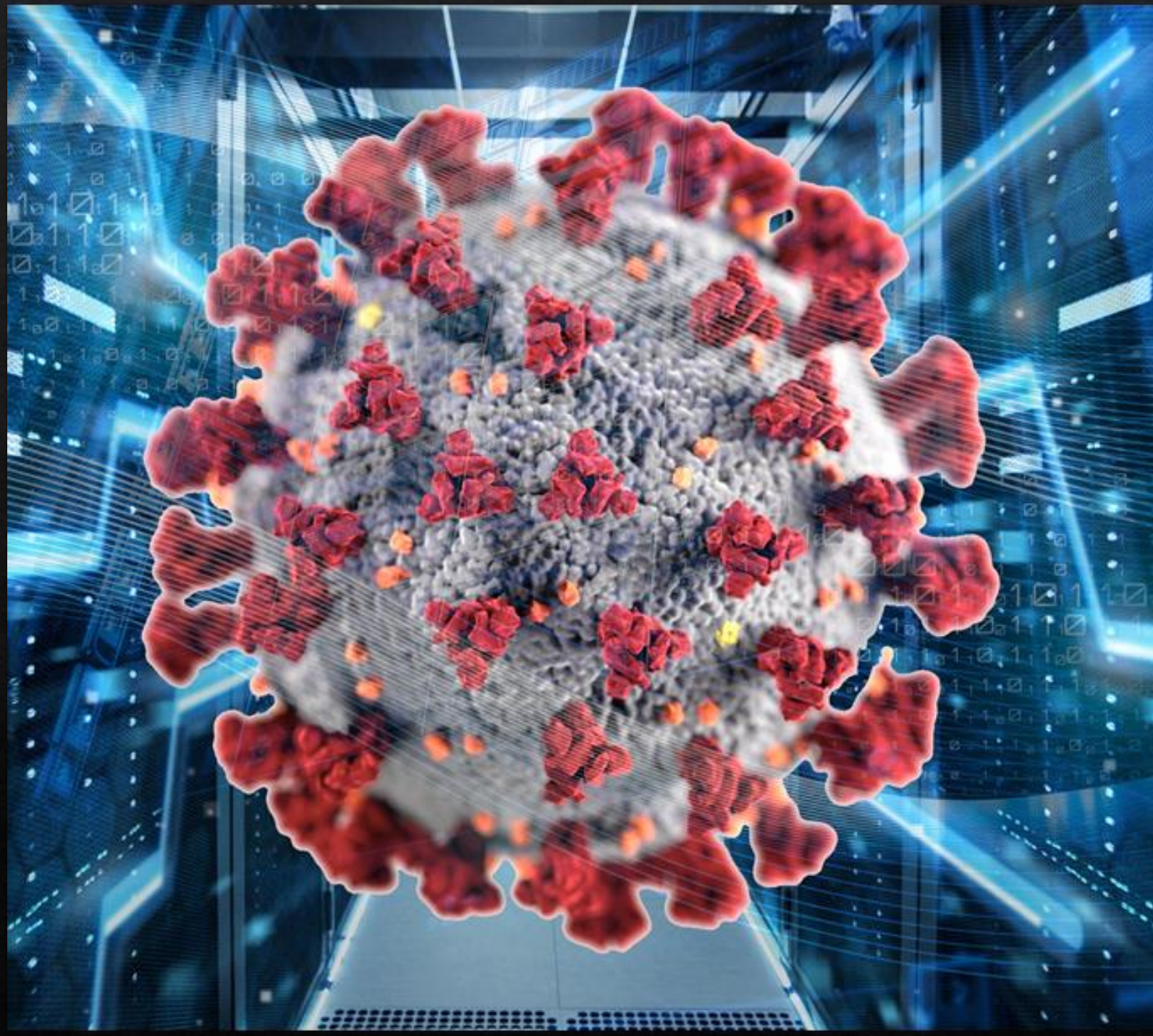
$$\mathbf{Q} = \begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix}$$

Transition State Probability

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 3/2 & 1 & 1/2 \\ 1 & 2 & 1 \\ 1/2 & 1 & 3/2 \end{pmatrix} \end{array}.$$

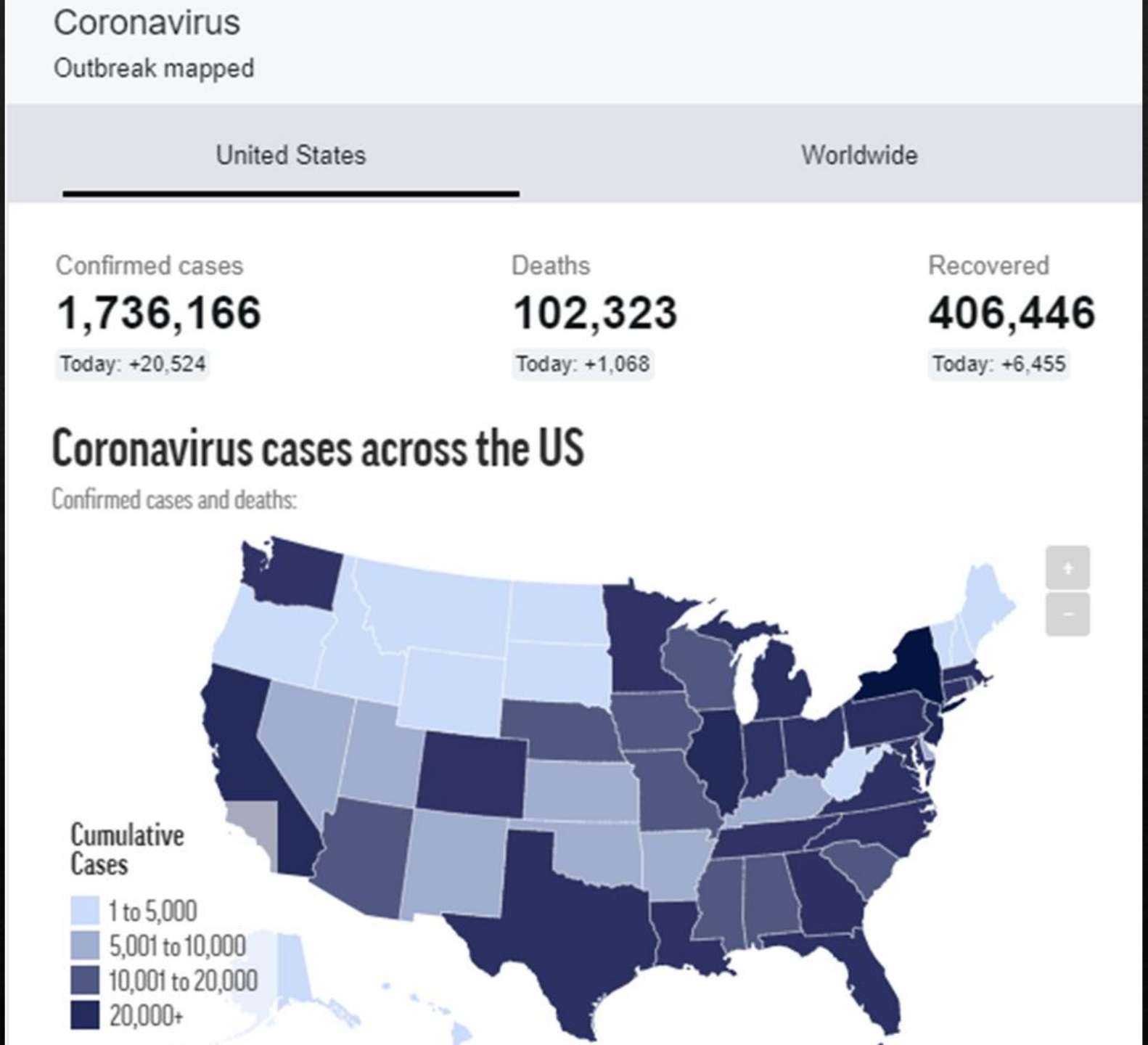
Application

COVID-19 in the US
Recorded until June 1st, 2020



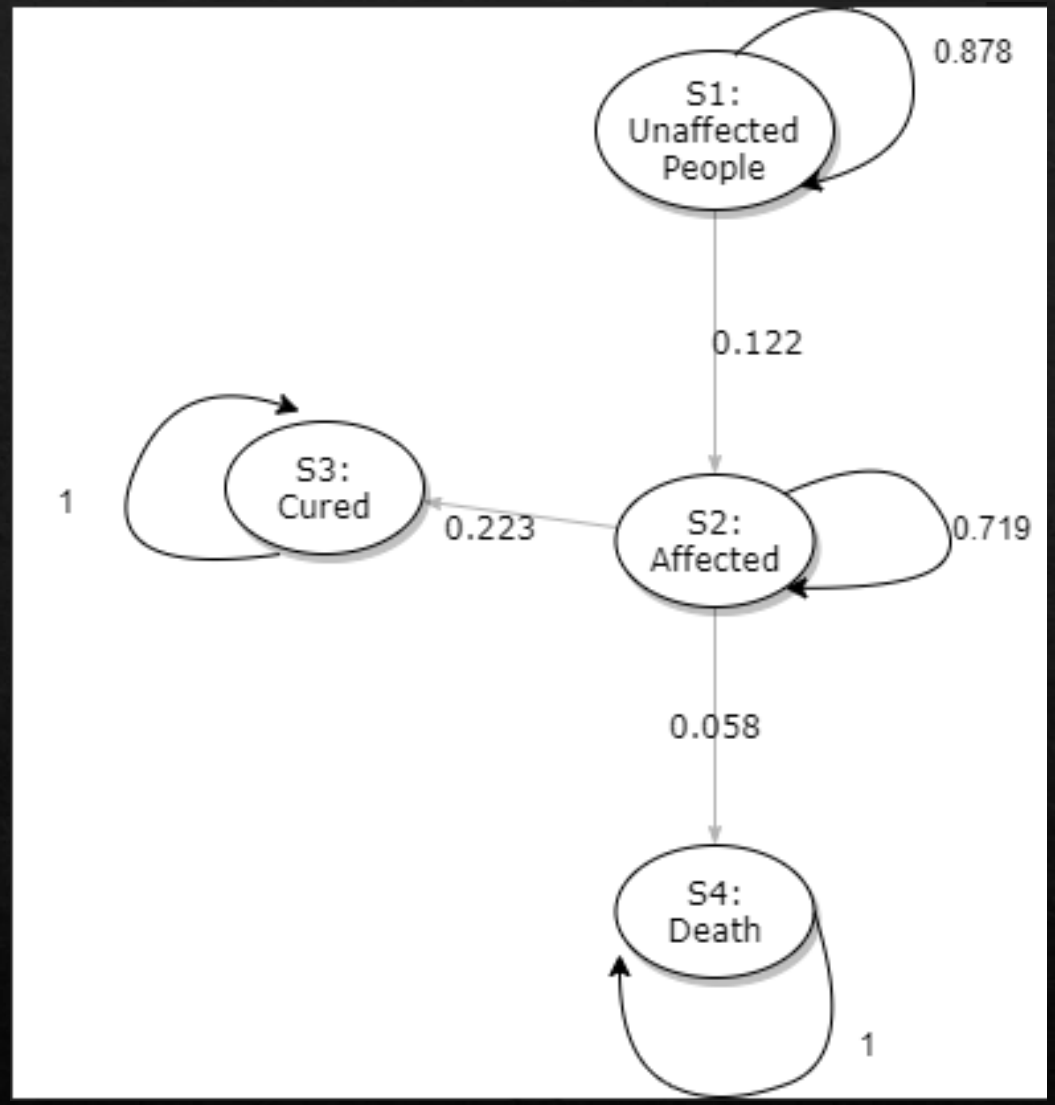
Application Background

- Coronaviruses are a large family of viruses that are known to cause illness ranging from the common cold to more severe diseases such as Severe Acute Respiratory syndrome (SARS) and Middle East Respiratory Syndrome (MERS).
- COVID-19 has the largest effect in 2020 and freeze the human activity and causing dramatic death.



The Model

According to the report from CDC on June 1st 2020



USA
17,612,125
TESTS REPORTED
CDC | Updated: Jun 1 2020 5:45PM

USA
2,140,439
POSITIVE TESTS
CDC | Updated: Jun 1 2020 5:45PM

	Cases	Deaths	Recovered	Fatality
United States	+4,612 1,856,584	+185 106,883	+503 413,581	5.8%

Analysis

- From the state diagram created, we only estimate 4 states which S1, S2 are transition state and S3, S4 are absorbing state.
- Using the data CDC updated on June 1st, 2020, estimate the percentage for general condition in the US.
- S1 is state where everyone hasn't been affected and test for COVID-19, from the report, 12.2% of tested results are positive, which means 0.122 probability of unaffected tested people is US are affected by COVID-19 and goes to S2 state.
- From the report in 1acre3points.com, 22.3% of affected cases are recovered, goes to S3 cured state. Cured patient will have COVID updated immune system, it would not affect again shortly in period, it will have probability of 1 staying in S3 state.
- From the report in 1acre3points.com, 5.8% of death are recorded, goes to S4 death state.

	S1	S2	S3	S4
S1	0.878000000000000	0.000000000000000	0.000000000000000	0.000000000000000
S2	0.122000000000000	0.719000000000000	0.000000000000000	0.000000000000000
S3	0.000000000000000	0.223000000000000	1.000000000000000	0.000000000000000
S4	0.000000000000000	0.058000000000000	0.000000000000000	1.000000000000000

Q=

	S1	S2
S1	0.878000000000000	0.000000000000000
S2	0.122000000000000	0.719000000000000

R=

	S1	S2
S3	0.000000000000000	0.223000000000000
S4	0.000000000000000	0.058000000000000

I=

$$\begin{bmatrix} 1.000000000000000 & 0.000000000000000 \\ 0.000000000000000 & 1.000000000000000 \end{bmatrix}$$

Analysis

- Using the data and canonical form described, write the function and run 1000 times so that we can estimate the long-term results of COVID-19.
- For long-term keeping the general activity, without vaccine, the results showing that about 79.3% of people in the US will be cured and 20.6% of people in the US will die because of COVID-19.
- For currently 330856331 population in US, we can estimate that for a long-term effect, 68290630 people will die in this COVID-19 disaster.

```
#COVID-19 ESTIMATE FUNCTION
P = matrix(4,4,[878/1000, 0, 0, 0, 122/1000, 719/1000, 0, 0, 0, 223/1000, 1, 0, 0, 58/1000, 0, 1])
Q = matrix(2,2,[878/1000, 0,122/1000, 719/1000])
R = matrix(2,2,[0,223/1000,0,58/1000])
I = matrix(2,2,[1, 0, 0, 1])
N = (I-Q)^-1
B = R * N
p = matrix(2,1, [1, 0])
print P
print
print 'After many time, the probability p will be ', n(P^1000)
print
print 'Q  = ', Q
print
print 'I  = ', I
print
print 'R  = ', R
print
print 'N  = ', N
print
print 'B  = ', B
print
print 'Np = ', N*p
print
print 'Bp = ', B*p
print
```

```
[ 439/500        0        0        0]
[  61/500 719/1000        0        0]
[       0 223/1000        1        0]
[       0   29/500        0        1]

After many time, the probability p will be  [ 3.12259676971859e-57     0.000000000000000     0.000000000000000     0.000000000000000]
[ 2.39595475412370e-57 5.35661438034266e-144     0.000000000000000     0.000000000000000]
[    0.793594306049822     0.793594306049822      1.00000000000000     0.000000000000000]
[    0.206405693950178     0.206405693950178     0.000000000000000      1.00000000000000]

Q  =  [ 439/500        0]
[  61/500 719/1000]

I  =  [1 0]
[0 1]

R  =  [       0 223/1000]
[       0   29/500]

N  =  [  500/61        0]
[1000/281 1000/281]

B  =  [223/281 223/281]
[ 58/281  58/281]

Np =  [  500/61]
[1000/281]

Bp =  [223/281]
[ 58/281]
```

Recommendation

- As the results calculated using Markov Chain, the US is urgently needing the vaccine of COVID-19 and effective treatment for COVID-19.
- Keeping the social distance, limit human activity and interaction can accumulate lower the probability of affected COVID-19.
- Since the cured patients will not be able to get affected in period, we can encourage the cured patients out for working to keep our economy running and keeping the older ages staying home as they have larger probability being affected and less probability getting cured.

Reference

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