STMATH 381

HW7

1. $S_1(GG, GG)$

$\xrightarrow{1}$ $S_1(GG,GG)$ $\quad\xrightarrow{0}$ $S_2(GG,Gg)$ $\quad\xrightarrow{0}$ $S_3(GG,gg)$ $\quad\xrightarrow{0}$ $S_4(Gg,Gg)$ $\quad\xrightarrow{0}$ $S_5(Gg,gg)$ $\quad\xrightarrow{0}$ $S_6(gg,gg)$

$P(S_1 \to S_1) = 1$

$P(S_1 \to S_2) = 0$

$P(S_1 \to S_3) = 0$

$P(S_1 \to S_4) = 0$

$P(S_1 \to S_5) = 0$

$P(S_1 \to S_6) = 0$

$S_2(GG, Gg)$

$\xrightarrow{\frac{1}{4}}$ $S_1(GG,GG)$ $\quad\xrightarrow{\frac{1}{2}}$ $S_2(GG,Gg)$ $\quad\xrightarrow{0}$ $S_3(GG,gg)$ $\quad\xrightarrow{\frac{1}{4}}$ $S_4(Gg,Gg)$ $\quad\xrightarrow{0}$ $S_5(Gg,gg)$ $\quad\xrightarrow{0}$ $S_6(gg,gg)$

$P(S_2 \to S_1) = \frac{1}{4}$

$P(S_2 \to S_2) = \frac{1}{2}$

$P(S_2 \to S_3) = 0$

$P(S_2 \to S_4) = \frac{1}{4}$

$P(S_2 \to S_5) = 0$

$P(S_2 \to S_6) = 0$

$S_3(GG, gg)$

$\xrightarrow{0}$ $S_1(GG,GG)$ $\quad\xrightarrow{0}$ $S_2(GG,Gg)$ $\quad\xrightarrow{0}$ $S_3(GG,gg)$ $\quad\xrightarrow{1}$ $S_4(Gg,Gg)$ $\quad\xrightarrow{0}$ $S_5(Gg,gg)$ $\quad\xrightarrow{0}$ $S_6(gg,gg)$

$P(S_3 \to S_1) = 0$

$P(S_3 \to S_2) = 0$

$P(S_3 \to S_3) = 0$

$P(S_3 \to S_4) = 1$

$P(S_3 \to S_5) = 0$

$P(S_3 \to S_6) = 0$

[Photo]

$S_4(Gg,Gg)$

$S_4(Gg,Gg) \xrightarrow{\frac{1}{16}} S_1(GG,GG)$, $S_4(Gg,Gg) \xrightarrow{\frac{1}{4}} S_2(GG,Gg)$, $S_4(Gg,Gg) \xrightarrow{\frac{1}{8}} S_3(GG,gg)$, $S_4(Gg,Gg) \xrightarrow{\frac{1}{4}} S_4(Gg,Gg)$, $S_4(Gg,Gg) \xrightarrow{\frac{1}{4}} S_5(Gg,gg)$, $S_4(Gg,Gg) \xrightarrow{\frac{1}{16}} S_6(gg,gg)$

$P(S_4 \to S_1) = \frac{1}{16}$

$P(S_4 \to S_2) = \frac{1}{4}$

$P(S_4 \to S_3) = \frac{1}{8}$

$P(S_4 \to S_4) = \frac{1}{4}$

$P(S_4 \to S_5) = \frac{1}{4}$

$P(S_4 \to S_6) = \frac{1}{16}$

$S_5(Gg,gg)$

$S_5(Gg,gg) \xrightarrow{0} S_1(GG,GG)$, $S_5(Gg,gg) \xrightarrow{0} S_2(GG,Gg)$, $S_5(Gg,gg) \xrightarrow{0} S_3(GG,gg)$, $S_5(Gg,gg) \xrightarrow{\frac{1}{4}} S_4(Gg,Gg)$, $S_5(Gg,gg) \xrightarrow{\frac{1}{2}} S_5(Gg,gg)$, $S_5(Gg,gg) \xrightarrow{\frac{1}{4}} S_6(gg,gg)$

$P(S_5 \to S_1) = 0$

$P(S_5 \to S_2) = 0$

$P(S_5 \to S_3) = 0$

$P(S_5 \to S_4) = \frac{1}{4}$

$P(S_5 \to S_5) = \frac{1}{2}$

$P(S_5 \to S_6) = \frac{1}{4}$

$S_6(gg,gg)$

$S_6(gg,gg) \xrightarrow{0} S_1(GG,GG)$, $S_6(gg,gg) \xrightarrow{0} S_2(GG,Gg)$, $S_6(gg,gg) \xrightarrow{0} S_3(GG,gg)$, $S_6(gg,gg) \xrightarrow{0} S_4(Gg,Gg)$, $S_6(gg,gg) \xrightarrow{0} S_5(Gg,gg)$, $S_6(gg,gg) \xrightarrow{1} S_6(gg,gg)$

$P(S_6 \to S_1) = 0$

$P(S_6 \to S_2) = 0$

$P(S_6 \to S_3) = 0$

$P(S_6 \to S_4) = 0$

$P(S_6 \to S_5) = 0$

$P(S_6 \to S_6) = 1$

2. Probability transition Matrix

$$\begin{array}{c|cccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ \hline S_1 & 1 & \frac{1}{4} & 0 & \frac{1}{16} & 0 & 0 \\ S_2 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ S_3 & 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ S_4 & 0 & \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ S_5 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ S_6 & 0 & 0 & 0 & \frac{1}{16} & \frac{1}{4} & 1 \end{array}$$

Absorbing State : S_1 , S_6

Transition State : S_2, S_3, S_4, S_5

Canonical Form : $P = \begin{array}{c} \text{Tr} \\ \text{Abs} \end{array} \begin{array}{c} \begin{array}{cc} \text{Tr} & \text{Abs} \end{array} \\ \left(\begin{array}{c|c} Q & O \\ \hline R & I \end{array}\right) \end{array}$

$$P = \begin{array}{c|cccc|cc} & S_2 & S_3 & S_4 & S_5 & S_1 & S_6 \\ \hline S_2 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ S_3 & 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ S_4 & \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ S_5 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ \hline S_1 & \frac{1}{4} & 0 & \frac{1}{16} & 0 & 1 & 0 \\ S_6 & 0 & 0 & \frac{1}{16} & \frac{1}{4} & 0 & 1 \end{array}$$

3. (a) $N = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & \frac{4}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{4}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} \end{pmatrix} \end{array}$

(b) absorption probability $B = \begin{array}{c} \\ 1 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} \end{pmatrix} \end{array}$

(c) $N_p = [\frac{3}{2}, \frac{1}{2}, 2, \frac{3}{2}]$

(d) $B_p = [\frac{1}{2}, \frac{1}{2}]$

4. Regardless of starting state, the process will eventually be absorbed at probability of 1 as it is MC and P^{∞}. $\lim_{n\to\infty} Q^n = 0$, eventually 0 probability will be in transition state which probability of absorbed state will be 1.

5. $N_{43} = \frac{8}{3}$. in N, start from 3 end at 4 turns out at probability of $\frac{8}{3}$ before absorbed.

6.

S_2	S_3	S_4	S_5
$\frac{3}{2}$	$\frac{1}{2}$	2	$\frac{3}{2}$

We assume $\frac{1}{4}$ probability at each starting state, N_p and get $[\frac{3}{2}, \frac{1}{2}, 2, \frac{3}{2}]$
This are the probability of each transient states before absorbing.

7. $S_2 : \frac{8}{3} + \frac{1}{6} + \frac{4}{3} + \frac{2}{3} = \frac{29}{6}$
$S_3 : \frac{4}{3} + \frac{4}{3} + \frac{8}{3} + \frac{4}{3} = \frac{20}{3}$
$S_4 : \frac{4}{3} + \frac{1}{3} + \frac{8}{3} + \frac{4}{3} = \frac{17}{3}$
$S_5 : \frac{2}{3} + \frac{1}{6} + \frac{4}{3} + \frac{8}{3} = \frac{29}{6}$

S_2	S_3	S_4	S_5
$\frac{29}{6}$	$\frac{20}{3}$	$\frac{17}{3}$	$\frac{29}{6}$

From the fundamental matrix N, the steps expected for S_2 is $\frac{29}{6}$, S_3 is $\frac{20}{3}$, S_4 is $\frac{17}{3}$, S_5 is $\frac{29}{6}$.

8. P^n, $n \to \infty$

Starting state	Result after a long time
S_2	0.75 → state 1, 0.25 → state 6
S_3	0.5 → state 1, 0.5 → state 6
S_4	0.5 → state 1, 0.5 → state 6
S_5	0.25 → state 1, 0.75 → state 6
trans states	very strong likelyhood of absorbing

9. $B_p = [\frac{1}{2}, \frac{1}{2}]$
From the absorption probability matrix B with equal probability in each starting state, we can get result that $\frac{1}{2}$ to 1, and $\frac{1}{2}$ to state 6.

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#HW7
#3
Q = matrix(4,4,[1/2, 0, 1/4, 0, 0, 0, 1/8, 0, 1/4, 1, 1/4, 1/4, 0, 0, 1/4, 1/2])
I = matrix(4,4,[1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1])
R = matrix(2,4,[1/4, 0, 1/16, 0, 0, 0, 1/16, 1/4])
N = (I-Q)^-1
B = R*N
P = matrix(6,6,[1/2, 0, 1/4, 0, 0, 0, 0, 0, 1/8, 0, 0, 0, 1/4, 1, 1/4, 1/4, 0, 0, 0, 0, 1/4, 1/2, 0, 0, 1/4, 0, 1/16, 0, 1,0, 0, 0, 1/16, 1/4, 0, 1])
p = matrix(4,1, [1/4, 1/4, 1/4, 1/4])
print
print 'I = ', I
print
print 'R = ', R
print
print 'N = ', N
print
print 'B = ', B
print
print 'Np = ', N*p
print
print 'Bp = ', B*p
print
print n(P^100)
```

```
I =  [1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]

R =  [ 1/4    0 1/16    0]
[   0    0 1/16  1/4]

N =  [8/3 4/3 4/3 2/3]
[1/6 4/3 1/3 1/6]
[4/3 8/3 8/3 4/3]
[2/3 4/3 4/3 8/3]

B =  [3/4 1/2 1/2 1/4]
[1/4 1/2 1/2 3/4]

Np =  [3/2]
[1/2]
[  2]
[3/2]

Bp =  [1/2]
[1/2]

[1.63583591554910e-10 2.49933487888782e-10 2.02200439165429e-10 1.63583591554910e-10    0.000000000000000    0.000000000000000]
[3.12416859860977e-11 4.77330487233522e-11 3.86168476105193e-11 3.12416859860977e-11    0.000000000000000    0.000000000000000]
[2.02200439165429e-10 3.08934780884154e-10 2.49933487888782e-10 2.02200439165429e-10    0.000000000000000    0.000000000000000]
[1.63583591554910e-10 2.49933487888782e-10 2.02200439165429e-10 1.63583591554910e-10    0.000000000000000    0.000000000000000]
[   0.749999999719695    0.499999999571733    0.499999999653524    0.249999999719695     1.00000000000000    0.000000000000000]
[   0.249999999719695    0.499999999571733    0.499999999653524    0.749999999719695    0.000000000000000     1.00000000000000]
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