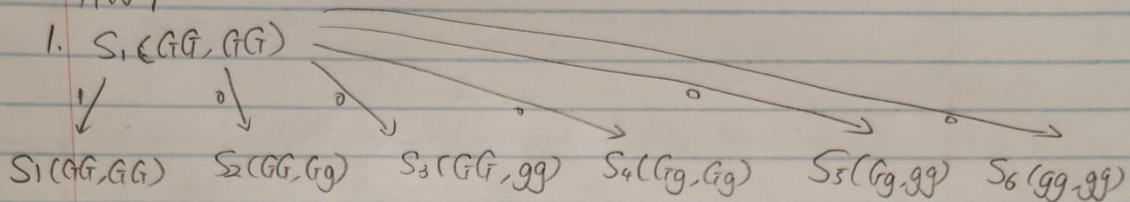


STMATH 381

HW7

1.  $S_1 \in (GG, GG)$



$$P(S_1 \rightarrow S_1) = 1$$

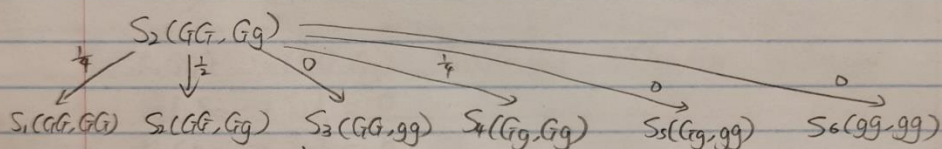
$$P(S_1 \rightarrow S_2) = 0$$

$$P(S_1 \rightarrow S_3) = 0$$

$$P(S_1 \rightarrow S_4) = 0$$

$$P(S_1 \rightarrow S_5) = 0$$

$$P(S_1 \rightarrow S_6) = 0$$



$$P(S_2 \rightarrow S_1) = \frac{1}{4}$$

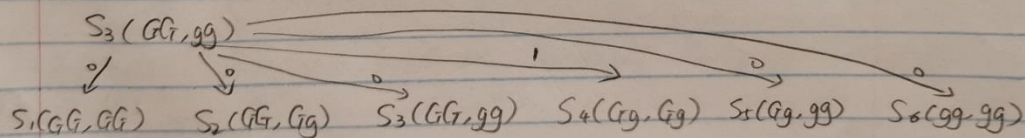
$$P(S_2 \rightarrow S_2) = \frac{1}{2}$$

$$P(S_2 \rightarrow S_3) = 0$$

$$P(S_2 \rightarrow S_4) = \frac{1}{4}$$

$$P(S_2 \rightarrow S_5) = 0$$

$$P(S_2 \rightarrow S_6) = 0$$



$$P(S_3 \rightarrow S_1) = 0$$

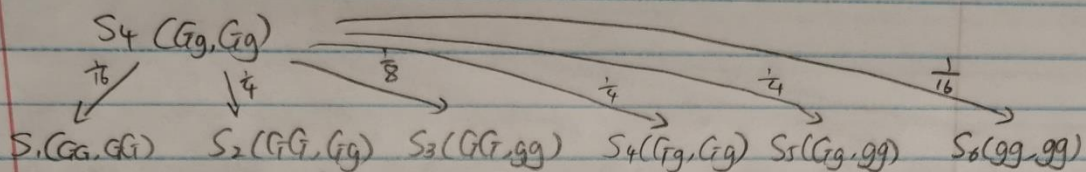
$$P(S_3 \rightarrow S_2) = 0$$

$$P(S_3 \rightarrow S_3) = 0$$

$$P(S_3 \rightarrow S_4) = 1$$

$$P(S_3 \rightarrow S_5) = 0$$

$$P(S_3 \rightarrow S_6) = 0$$



$$P(S_4 \rightarrow S_1) = \frac{1}{16}$$

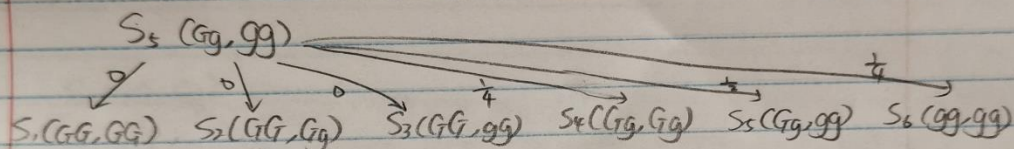
$$P(S_4 \rightarrow S_2) = \frac{1}{4}$$

$$P(S_4 \rightarrow S_3) = \frac{1}{8}$$

$$P(S_4 \rightarrow S_4) = \frac{1}{4}$$

$$P(S_4 \rightarrow S_5) = \frac{1}{4}$$

$$P(S_4 \rightarrow S_6) = \frac{1}{16}$$



$$P(S_5 \rightarrow S_1) = 0$$

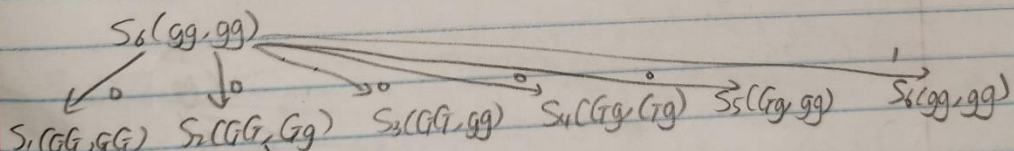
$$P(S_5 \rightarrow S_2) = 0$$

$$P(S_5 \rightarrow S_3) = 0$$

$$P(S_5 \rightarrow S_4) = \frac{1}{4}$$

$$P(S_5 \rightarrow S_5) = \frac{1}{2}$$

$$P(S_5 \rightarrow S_6) = \frac{1}{4}$$



$$P(S_6 \rightarrow S_1) = 0$$

$$P(S_6 \rightarrow S_2) = 0$$

$$P(S_6 \rightarrow S_3) = 0$$

$$P(S_6 \rightarrow S_4) = 0$$

$$P(S_6 \rightarrow S_5) = 0$$

$$P(S_6 \rightarrow S_6) = 1$$



## 2. Probability transition Matrix

	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
$S_1$	1	$\frac{1}{4}$	0	$\frac{1}{16}$	0	0
$S_2$	0	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0
$S_3$	0	0	0	$\frac{1}{8}$	0	0
$S_4$	0	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0
$S_5$	0	0	0	$\frac{1}{4}$	$\frac{1}{2}$	0
$S_6$	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	1

Absorbing State:  $S_1, S_6$

Transition State:  $S_2, S_3, S_4, S_5$

Canonical Form:  $P = \begin{matrix} Tr & Abs \\ Abs & R \end{matrix} \begin{pmatrix} Q & 0 \\ R & I \end{pmatrix}$

	$S_2$	$S_3$	$S_4$	$S_5$	$S_1$	$S_6$
$P = S_2$	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0
$S_3$	0	0	$\frac{1}{8}$	0	0	0
$S_4$	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0	0
$S_5$	0	0	$\frac{1}{4}$	$\frac{1}{2}$	0	0
$S_1$	$\frac{1}{4}$	0	$\frac{1}{16}$	0	1	0
$S_6$	0	0	$\frac{1}{16}$	$\frac{1}{4}$	0	1

$$3. (a) N = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & \frac{4}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{4}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} \end{pmatrix} \end{matrix}$$

$$(b) \text{ absorption probability } B = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 6 \end{matrix} & \begin{pmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} \end{pmatrix} \end{matrix}$$

$$(c) N_p = \left[ \frac{2}{2}, \frac{1}{2}, 2, \frac{3}{2} \right]$$

$$(d) B_p = \left[ \frac{1}{2}, \frac{1}{2} \right]$$

4. Regardless of starting state, the process will eventually be absorbed at probability of 1 as it is MC and  $P^\infty \cdot \lim_{n \rightarrow \infty} Q^n = 0$ , eventually 0 probability will be in transition state which probability of absorbed state will be 1.

5.  $N_{43} = \frac{8}{3}$ . in  $N$ , start from 3 end at 4 turns out at probability of  $\frac{18}{3}$  before absorbed.

6. 
$$\begin{array}{c|c|c|c} S_2 & S_3 & S_4 & S_5 \\ \hline \frac{2}{3} & \frac{1}{3} & 2 & \frac{3}{2} \end{array}$$

We assume  $\frac{1}{4}$  probability at each starting state,  $N_p$  and get  $[\frac{3}{2}, \frac{1}{2}, 2, \frac{3}{2}]$   
This are the probability of each transient states before absorbing.

7. 
$$\begin{aligned} S_2 &= \frac{8}{3} + \frac{1}{6} + \frac{4}{3} + \frac{2}{3} = \frac{29}{6} \\ S_3 &= \frac{4}{3} + \frac{4}{3} + \frac{8}{3} + \frac{4}{3} = \frac{20}{3} \\ S_4 &= \frac{4}{3} + \frac{1}{3} + \frac{8}{3} + \frac{4}{3} = \frac{17}{3} \\ S_5 &= \frac{2}{3} + \frac{1}{6} + \frac{4}{3} + \frac{8}{3} = \frac{29}{6} \end{aligned}$$

$$\begin{array}{c|c|c|c} S_2 & S_3 & S_4 & S_5 \\ \hline \frac{29}{6} & \frac{20}{3} & \frac{17}{3} & \frac{29}{6} \end{array}$$

From the fundamental matrix  $N$ , the steps expected for  $S_2$  is  $\frac{29}{6}$ ,  $S_3$  is  $\frac{20}{3}$ ,  $S_4$  is  $\frac{17}{3}$ ,  $S_5$  is  $\frac{29}{6}$ .

8.  $P^n, n \rightarrow \infty$

starting state	Result after a long time
$S_2$	0.75 $\rightarrow$ state 1, 0.25 $\rightarrow$ state 6
$S_3$	0.5 $\rightarrow$ state 1, 0.5 $\rightarrow$ state 6
$S_4$	0.5 $\rightarrow$ state 1, 0.5 $\rightarrow$ state 6
$S_5$	0.25 $\rightarrow$ state 1, 0.75 $\rightarrow$ state 6
trans states	very strong likelihood of absorbing

9.  $B_p = [\frac{1}{2}, \frac{1}{2}]$

From the absorption probability matrix  $B$  with equal probability in each starting state, we can get result that  $\frac{1}{2}$  to 1, and  $\frac{1}{2}$  to state 6.

```

1 #HW7
2 #3
3 Q = matrix(4,4,[1/2, 0, 1/4, 0, 0, 0, 1/8, 0, 1/4, 1, 1/4, 1/4, 0, 0, 1/4, 1/2])
4 I = matrix(4,4,[1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1])
5 R = matrix(2,4,[1/4, 0, 1/16, 0, 0, 0, 1/16, 1/4])
6 N = (I-Q)^-1
7 B = R*N
8 P = matrix(6,6,[1/2, 0, 1/4, 0, 0, 0, 0, 0, 1/8, 0, 0, 0, 1/4, 1, 1/4, 1/4, 0, 0, 0, 0, 1/4, 1/2, 0, 0, 1/4, 0, 1/16, 0, 1, 0, 0, 0, 1/16, 1/4, 0, 1])
9 p = matrix(4,1, [1/4, 1/4, 1/4, 1/4])
10 print
11 print 'I = ', I
12 print
13 print 'R = ', R
14 print
15 print 'N = ', N
16 print
17 print 'B = ', B
18 print
19 print 'Np = ', N*p
20 print
21 print 'Bp = ', B*p
22 print
23 print n(P^100)]

```

```

I = [1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]

R = [ 1/4 0 1/16 0]
[ 0 0 1/16 1/4]

N = [8/3 4/3 4/3 2/3]
[1/6 4/3 1/3 1/6]
[4/3 8/3 8/3 4/3]
[2/3 4/3 4/3 8/3]

B = [3/4 1/2 1/2 1/4]
[1/4 1/2 1/2 3/4]

Np = [3/2]
[1/2]
[ 2]
[3/2]

Bp = [1/2]
[1/2]

[1.63583591554910e-10 2.49933487888782e-10 2.02200439165429e-10 1.63583591554910e-10 0.000000000000000 0.000000000000000]
[3.12416859860977e-11 4.77330487233522e-11 3.86168476105193e-11 3.12416859860977e-11 0.000000000000000 0.000000000000000]
[2.02200439165429e-10 3.08934780884154e-10 2.49933487888782e-10 2.02200439165429e-10 0.000000000000000 0.000000000000000]
[1.63583591554910e-10 2.49933487888782e-10 2.02200439165429e-10 1.63583591554910e-10 0.000000000000000 0.000000000000000]
[ 0.749999999719695 0.499999999571733 0.499999999653524 0.249999999719695 1.000000000000000 0.000000000000000]
[ 0.249999999719695 0.499999999571733 0.499999999653524 0.749999999719695 0.000000000000000 1.000000000000000]

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